Structure of the sigma meson from lattice QCD

SCALAR Collaboration

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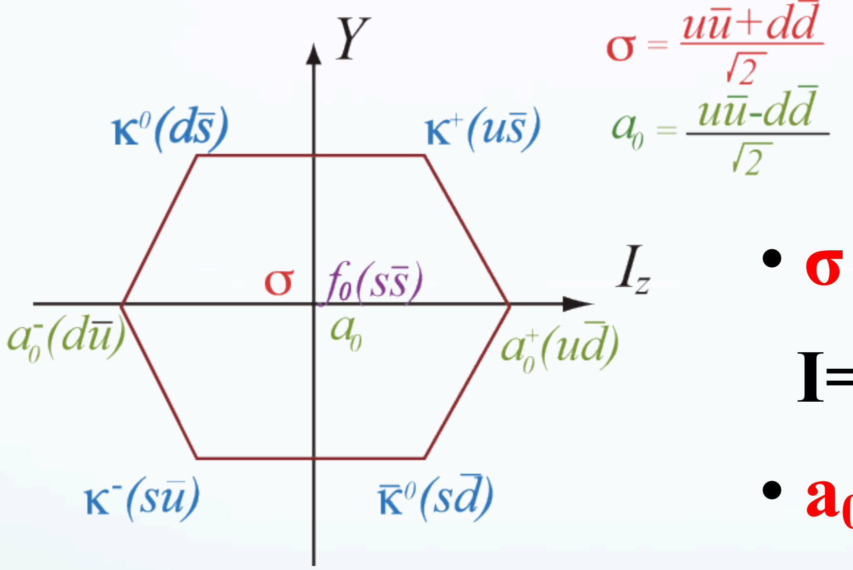
Structure of light scalar mesons?

- **Particle Data Group (2012)**

Light scalar mesons ($J^P = 0^+$) :



- **σ or f_0(500) :**

  I=0, mass = 400 - 550 MeV

- **a_0(980) :**

  I=1, mass = 980 ± 20 MeV

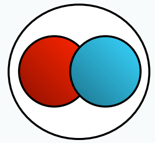
What is the structure of the light scalar meson?

Motivation

Using Lattice QCD,

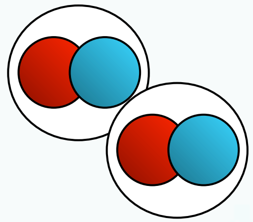
two-quark state ?

$$1_{\bar{q}\otimes q} = 1$$



molecular state ?

$$1_{\bar{q}\otimes q} \otimes 1_{\bar{q}\otimes q} = 1$$



tetra-quark state ?

$$\bar{3}_{q\otimes q} \otimes 3_{\bar{q}\otimes \bar{q}} = 1 \oplus 8$$

Structure of the sigma meson ?

Previous works for the light scalar meson from lattice QCD

two-quark state for σ meson with full QCD
SCALAR Collaboration, Phys. Rev. D70 (2004) 034504

two-quark state for κ meson
SCALAR Collaboration, Phys. Let. B652 (2007) 250

two-quark state for σ meson with full QCD
UKQCD Collaboration, Phys. Rev. D74 (2006) 114504

two-quark state for κ and a_0 mesons
BGR Collaboration, Phys. Rev. D85 (2012) 034508

tetra-quark state for σ, κ and a_0 mesons
S. Prelovsek *et al*, Phys. Rev. D79 (2009) 014503

molecular and tetra-quark state for κ and a_0 mesons
ETM Collaboration, JHEP 1304 (2013) 137

two-quark, molecular, tetra-quark, mixing state ?

Analysis method

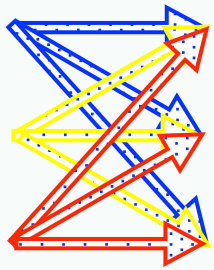
For sigma meson,
we consider all combinations of them.

Source | Variational method | Sink

two : $\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = \mathbf{1}$

molec : $\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} \otimes \mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = \mathbf{1}$

tetra : $\bar{\mathbf{3}}_{\mathbf{q}\otimes\mathbf{q}} \otimes \mathbf{3}_{\bar{\mathbf{q}}\otimes\bar{\mathbf{q}}} = \mathbf{1} \oplus \mathbf{8}$



$\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = \mathbf{1}$: two

$\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} \otimes \mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = \mathbf{1}$: molec

$\bar{\mathbf{3}}_{\mathbf{q}\otimes\mathbf{q}} \otimes \mathbf{3}_{\bar{\mathbf{q}}\otimes\bar{\mathbf{q}}} = \mathbf{1} \oplus \mathbf{8}$: tetra

<u>Prepared operators</u>

- two-quark state : two-quark operator
- molecular state : two pion operators
- tetra-quark state : (anti-) diquark operator

two-quark operator

$$\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = \mathbf{1}$$

- two-quark operator for σ meson

$$\mathcal{O}_S^{\text{two}}(t) = \sum_{\mathbf{x},\mathbf{y}}\sum_{a,b}\sum_{\alpha} \bar{q}_\alpha^a(t,\mathbf{x}) S_t^{ab}(\mathbf{x},\mathbf{y}) q_\alpha^b(t,\mathbf{y})$$

$S_t^{ab}(\mathbf{x},\mathbf{y})$: Smearing function at timeslice t

T.Burch *et al*, Phys. Rev. D73 (2006) 094505

$S = 1 \Rightarrow$ Point Source

$S = 2 \Rightarrow$ Narrow Source

$S = 3 \Rightarrow$ Wide Source

(Gaussian shaped source)

molecular operator $\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} \otimes \mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = \mathbf{1}$

- pion operator

$$\mathcal{O}_S^{\pi^+}(t) = -\sum_{\mathbf{x},\mathbf{y}}\sum_{a,b} \bar{d}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})u^b(t,\mathbf{y})$$

$$\mathcal{O}_S^{\pi^-}(t) = \sum_{\mathbf{x},\mathbf{y}}\sum_{a,b} \bar{u}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})d^b(t,\mathbf{y})$$

$$\mathcal{O}_S^{\pi^0}(t) = \frac{1}{\sqrt{2}}\sum_{\mathbf{x},\mathbf{y}}\sum_{a,b} \Big[\bar{u}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})u^b(t,\mathbf{y}) - \bar{d}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})d^b(t,\mathbf{y})\Big]$$

- molecular operator

$$\mathcal{O}_S^{\mathrm{molec}}(t) = \frac{1}{\sqrt{3}}\Big[\mathcal{O}_S^{\pi^+}(t)\mathcal{O}_S^{\pi^-}(t) - \mathcal{O}_S^{\pi^0}(t)\mathcal{O}_S^{\pi^0}(t) + \mathcal{O}_S^{\pi^-}(t)\mathcal{O}_S^{\pi^+}(t)\Big]$$

tetra-quark operator $\bar{\mathbf{3}}_{\mathbf{q}\otimes\mathbf{q}} \otimes \mathbf{3}_{\bar{\mathbf{q}}\otimes\bar{\mathbf{q}}} = \mathbf{1} \oplus \mathbf{8}$

◆ diquark operator for σ meson

$$[ud]^a_S(t) = \frac{1}{2}\sum_{\mathbf{x},\mathbf{y}}\sum_{b,c,d}\epsilon^{abc}\Big[u^{Tb}(t,\mathbf{x})\; C\gamma_5 S^{cd}_t(\mathbf{x},\mathbf{y})\; d^d(t,\mathbf{y}) - d^{Tb}(t,\mathbf{x})\; C\gamma_5 S^{cd}_t(\mathbf{x},\mathbf{y})\; u^d(t,\mathbf{y})\Big]$$

C : Charge conjugate matrix

◆ anti-diquark operator for σ meson

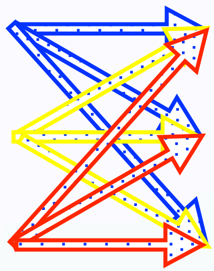
$$[\bar{u}\bar{d}]^a_S(t) = \frac{1}{2}\sum_{\mathbf{x},\mathbf{y}}\sum_{b,c,d}\epsilon^{abc}\Big[\bar{u}^b(t,\mathbf{x})\; C\gamma_5 S^{cd}_t(\mathbf{x},\mathbf{y})\; \bar{d}^{Td}(t,\mathbf{y}) - \bar{d}^b(t,\mathbf{x})\; C\gamma_5 S^{cd}_t(\mathbf{x},\mathbf{y})\; \bar{u}^{Td}(t,\mathbf{y})\Big]$$

◆ tetra-quark operator

$$\mathcal{O}^{\mathrm{tetra}}_S(t) = \sum_a [ud]^a_S(t)[\bar{u}\bar{d}]^a_S(t)$$

Propagators for sigma meson

Variational method

Source			Sink	
two :	$\mathbf{1}_{\bar{q}\otimes q} = \mathbf{1}$		$\mathbf{1}_{\bar{q}\otimes q} = \mathbf{1}$	: two
molec :	$\mathbf{1}_{\bar{q}\otimes q} \otimes \mathbf{1}_{\bar{q}\otimes q} = \mathbf{1}$		$\mathbf{1}_{\bar{q}\otimes q} \otimes \mathbf{1}_{\bar{q}\otimes q} = \mathbf{1}$	: molec
tetra :	$\bar{\mathbf{3}}_{q\otimes q} \otimes \mathbf{3}_{\bar{q}\otimes \bar{q}} = \mathbf{1} \oplus \mathbf{8}$		$\bar{\mathbf{3}}_{q\otimes q} \otimes \mathbf{3}_{\bar{q}\otimes \bar{q}} = \mathbf{1} \oplus \mathbf{8}$	: tetra

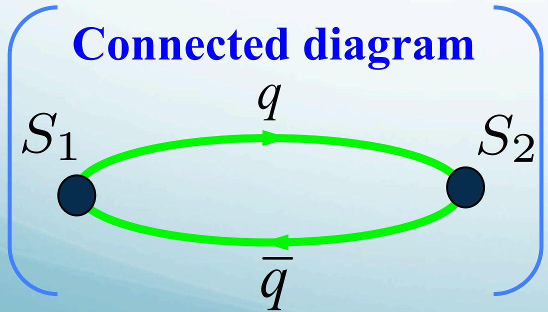
$$G^{\rm two-two}_{S_1S_2}(t), G^{\rm molec-molec}_{S_1S_2}(t), G^{\rm tetra-tetra}_{S_1S_2}(t), G^{\rm molec-tetra}_{S_1S_2}(t), G^{\rm molec-two}_{S_1S_2}(t), G^{\rm tetra-two}_{S_1S_2}(t)$$

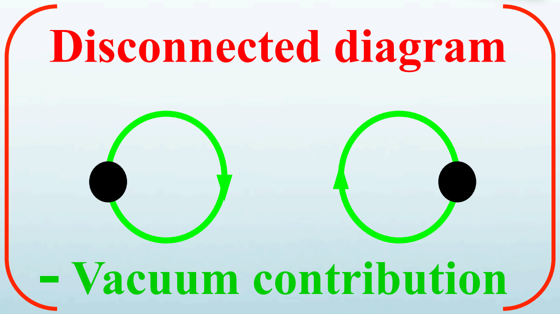
E.g.

$$G^{\rm two-two}_{S_1S_2}(t) = \left\langle \mathcal{O}^{\rm two}_{S_2}(t)\mathcal{O}^{\rm two\,\dagger}_{S_1}(0)\right\rangle$$

$$= -G^{\rm conn}_{S_1S_2}(t) + 2G^{\rm disc}_{S_1S_2}(t)$$

Difficult to evaluate!
Z_2 noise method with truncated eigenmode approach (TEA)
Optimal # of eigenmode?





Optimal cost calculation

$$\begin{aligned} \text{Variance (Error estimate)} &= \frac{f_1}{N_{ev}} + \frac{f_2(N_{ev})}{N} \\ \text{Total Cost} &= C_0 + C_1 N_{ev} + C_2(N_{ev}) N \end{aligned}$$

N : # of Noise

N_{ev} : # of Eigen vector

Constant : $f_1,\ C_0,\ C_1$

Function of N_{ev} : $f_2,\ C_2$

Contribution of TEA (first terms) | Contribution of Noise method (second terms)

Optimal N_{ev} under fixed Total Cost ? (Input : N)

$$\begin{cases} \text{Total Cost} = \text{fixed} \\ \dfrac{\partial}{\partial N_{ev}} [\text{Variance}] = 0 \end{cases}$$

Total Cost = fixed **: constraint condition**

⇓

$N = 1440 \Rightarrow N_{ev} = 12$

Simulation parameters

Gauge configuration

Two-flavor full QCD configurations by CP-PACS

(Phys. Rev. D65 (2002) 054505)

- Renormalization-group improved gauge action
- Mean field improved clover quark action

$$\text{Lattice size} = 12^3 \times 24 \qquad a = 0.2150(22)\ [\text{fm}]$$

$$\beta = 1.8 \quad , \quad \kappa = 0.143 \qquad m_\pi/m_\rho = 0.753(1)$$

$$C_{sw} = 1.6 \qquad m_\pi = 578.6(8)\ [\text{MeV}]$$

Quark Propagator

- Clover fermion action
- Z_2 noise method with TEA

(J.Foley *et al*, Comp. Phys. Comm.172 (2005) 145)

$$\#\ \text{of Noise} = 5 \times 4 \times 3 \times 24$$ ← Time dilution

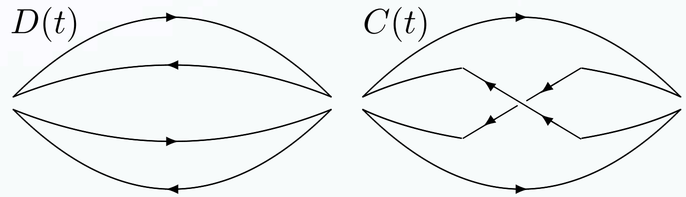
$$\#\ \text{of Eigenvector} = 12$$ ← The optimal cost calculation

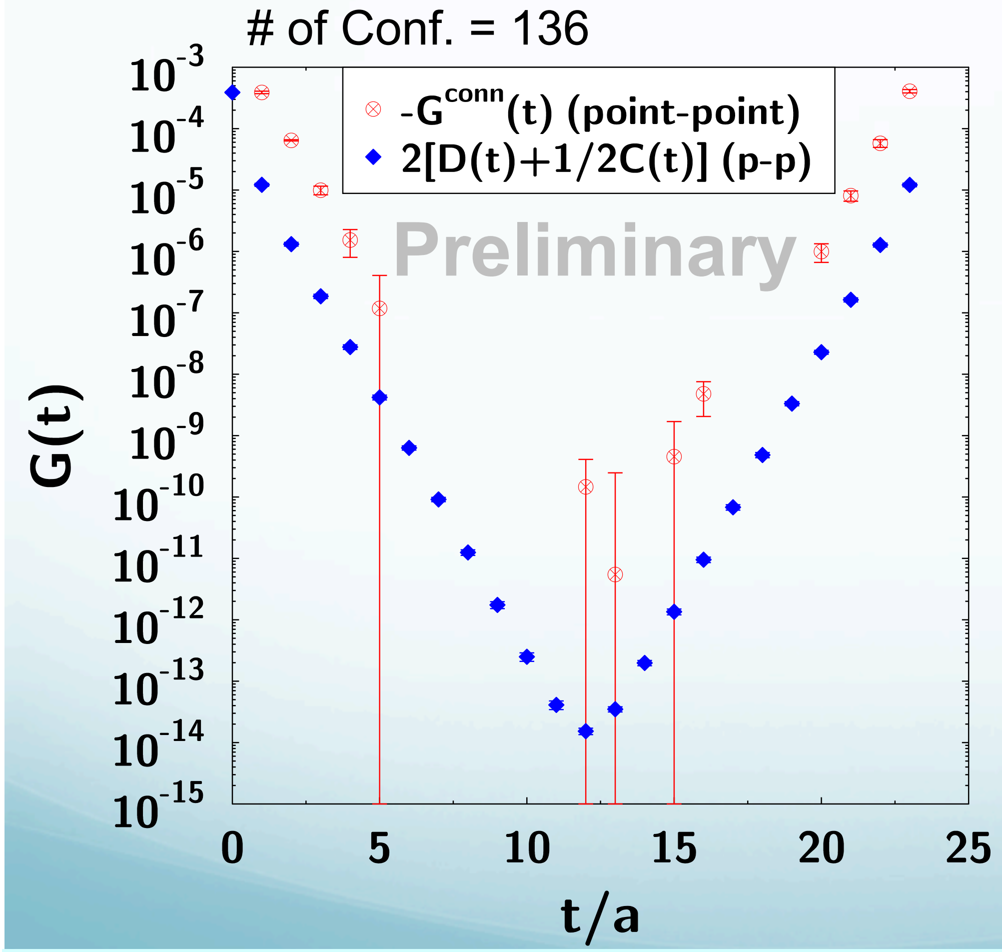
σ's connected part result

For connected part,

$$G^{\text{two-two}}_{S_1S_2}(t) = -G^{\text{conn}}_{S_1S_2}(t)$$

$$G^{\text{molec-molec}}_{S_1S_2}(t) = 2\left[D_{S_1S_2}(t) + \frac{1}{2}C_{S_1S_2}(t)\right]$$





- Two-quark state has much larger error than molecular state.

Two-quark : Excited state ?
$P=(+1)(-1)(-1)^L$

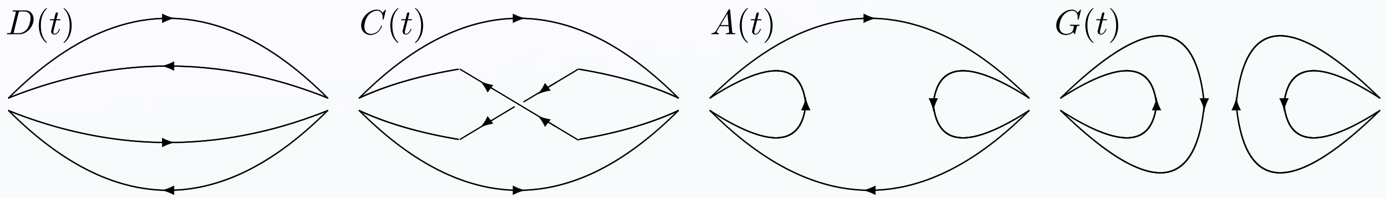
Molecular : Ground state ?
$P=(+1)^2(-1)^2(-1)^L$

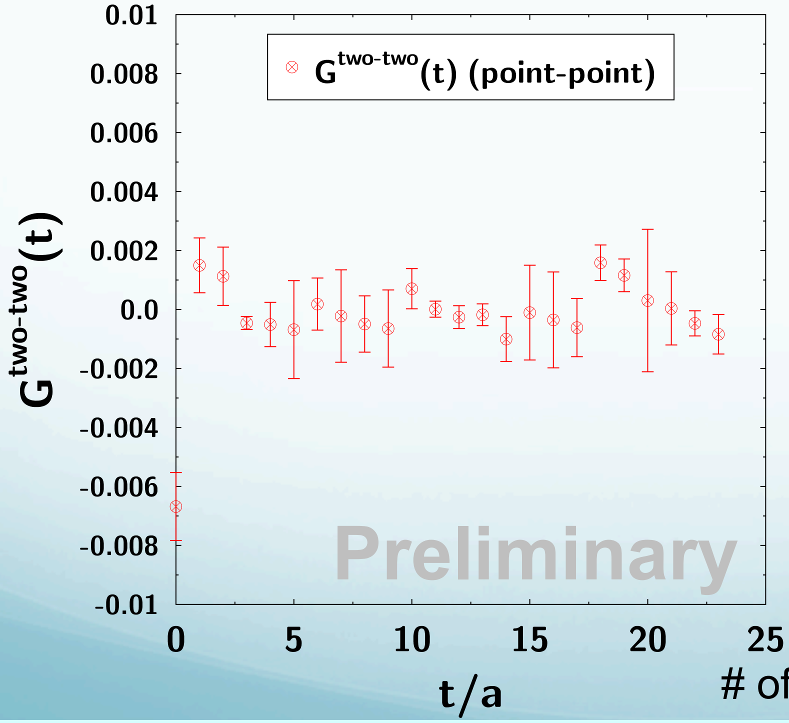
➢ Pion mass dependence

Sigma meson result (1)

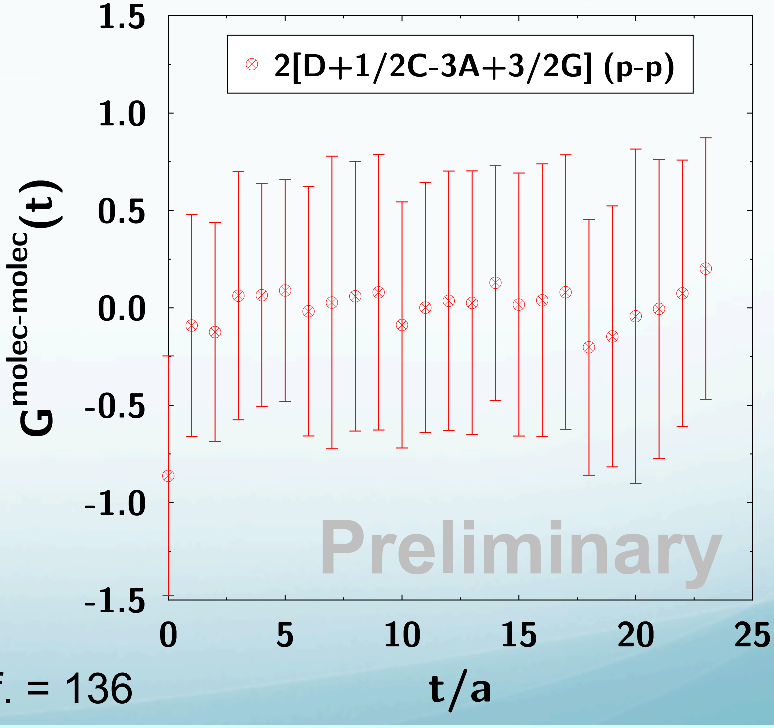
$$G^{\text{two-two}}_{S_1S_2}(t) = -G^{\text{conn}}_{S_1S_2}(t) + 2G^{\text{disc}}_{S_1S_2}(t)$$

$$G^{\text{molec-molec}}_{S_1S_2}(t) = 2\left[D_{S_1S_2}(t) + \frac{1}{2}C_{S_1S_2}(t) - 3A_{S_1S_2}(t) + \frac{3}{2}G_{S_1S_2}(t)\right]$$





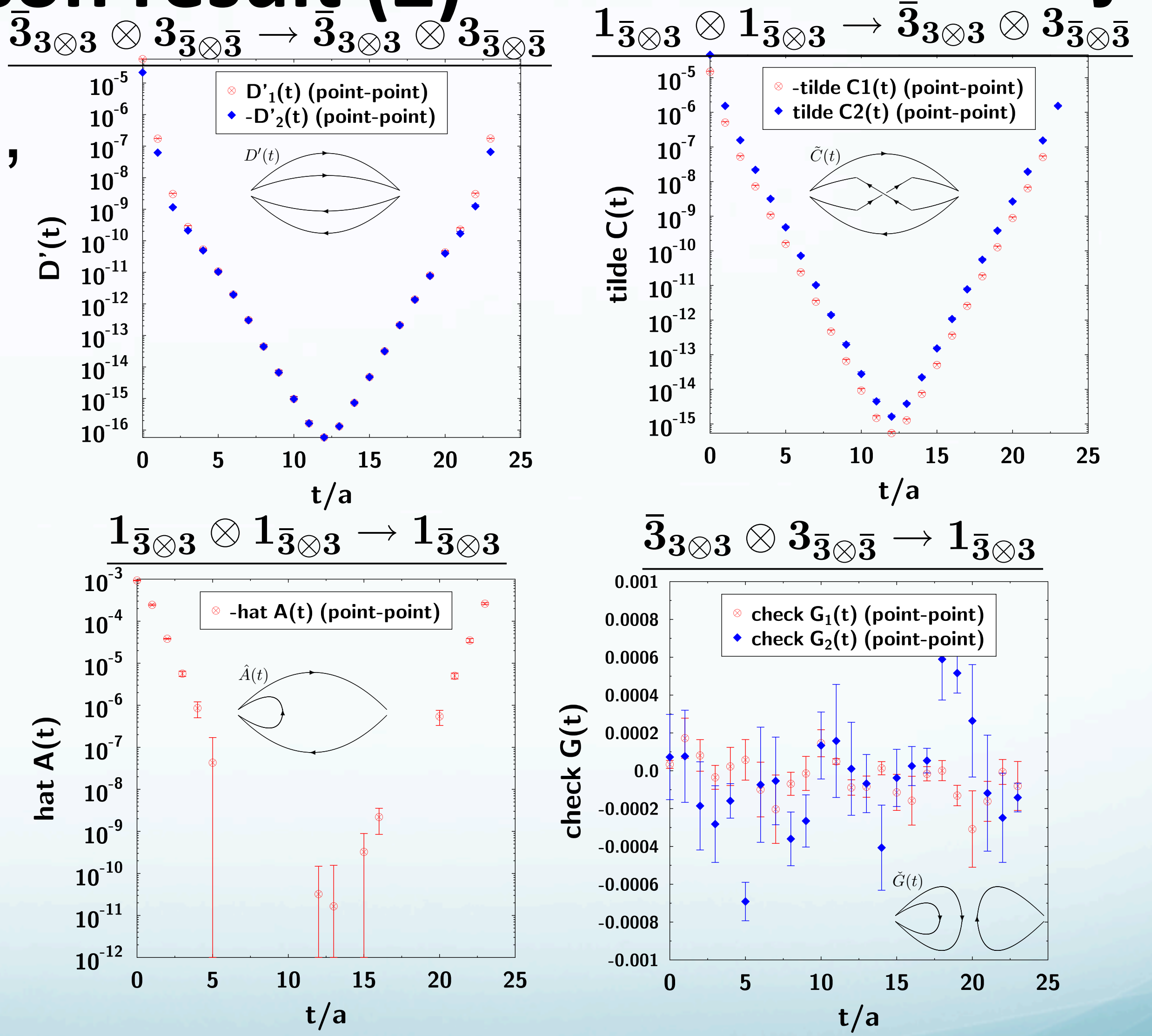
# of Conf. = 136



Sigma meson result (2)

Preliminary!

For example,



# of Conf. = 136

Summary

- Structure of the light scalar meson;
  two-quark, molecular, tetra-quark state ?
- σ's connected part :
  - two-quark's error > molecular's error
  - two-quark state : Excited state ?
  - molecular state : Ground state ?
  - (more statistics, m_π dependence, variational method)
- σ : more statistics !

Future work

- Mixing angle from variational method
  (m_π dependence)
- κ meson

Optimal cost calculation

$$\begin{aligned}\text{Variance (Error estimate)} &= \frac{f_1}{N_{ev}} + \frac{f_2(N_{ev})}{N} \\ \text{Total Cost} &= C_0 + C_1 N_{ev} + C_2(N_{ev})N\end{aligned}$$

N : # of Noise

N_{ev} : # of Eigen vector

Constant : $f_1,\ C_0,\ C_1,\ f_3,\ f_4,\ a,\ b$

Contribution of TEA (first terms) | Contribution of Noise method (second terms)

Optimal N_{ev} under fixed Total Cost ? (Fixed : N)

$$\begin{cases} \text{Total Cost} = \text{fixed} \\ \frac{\partial}{\partial N_{ev}}[\text{Variance}] = 0 \end{cases}$$

:contraint condition

Approximately,

$$f_2(N_{ev}) = f_3 - f_4 N_{ev}$$

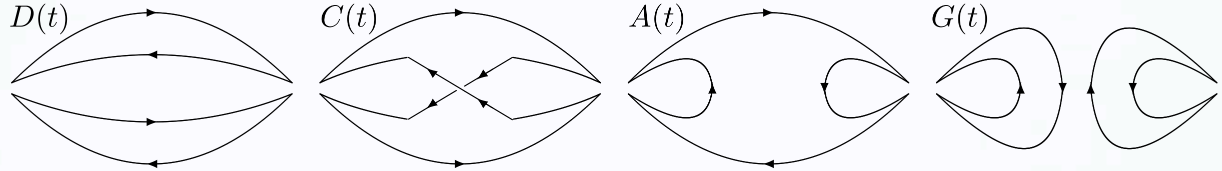
$$C_2(N_{ev}) = \frac{a}{N_{ev}} + b$$

$$-f_1 a N^2 - (f_1 b N + f_3 a) N N_{ev} + (-f_4 b N + f_3 C_1) N_{ev}^3 - f_4 C_1 N_{ev}^4 = 0$$

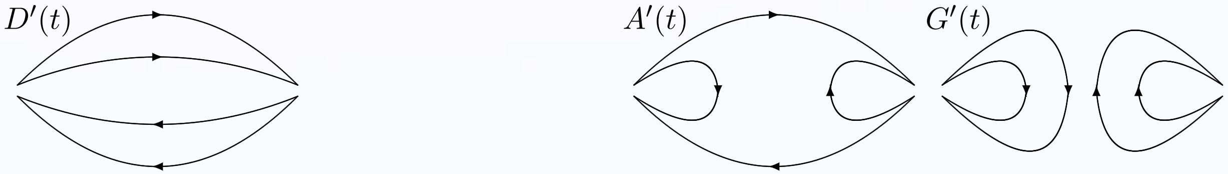
$$N = 1440 \Rightarrow N_{ev} = 12$$

Sigma meson result (2)

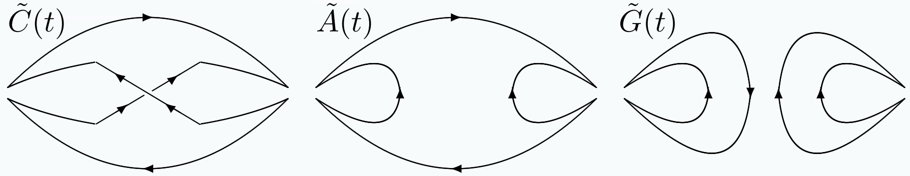
$\mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \otimes \mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \to \mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \otimes \mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$



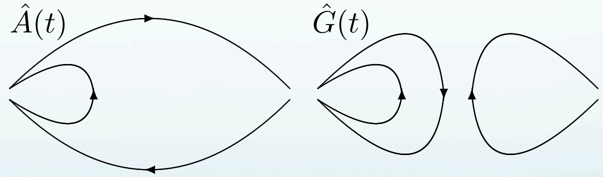
$\bar{\mathbf{3}}_{\mathbf{3}\otimes\mathbf{3}} \otimes \mathbf{3}_{\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}} \to \bar{\mathbf{3}}_{\mathbf{3}\otimes\mathbf{3}} \otimes \mathbf{3}_{\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}}$



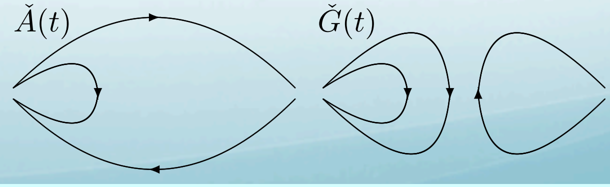
$\mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \otimes \mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \to \bar{\mathbf{3}}_{\mathbf{3}\otimes\mathbf{3}} \otimes \mathbf{3}_{\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}}$



$\mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \otimes \mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \to \mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$



$\bar{\mathbf{3}}_{\mathbf{3}\otimes\mathbf{3}} \otimes \mathbf{3}_{\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}} \to \mathbf{1}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$



SCALAR Collaboration (Lattice 2013)