K-π scattering from Lattice QCD

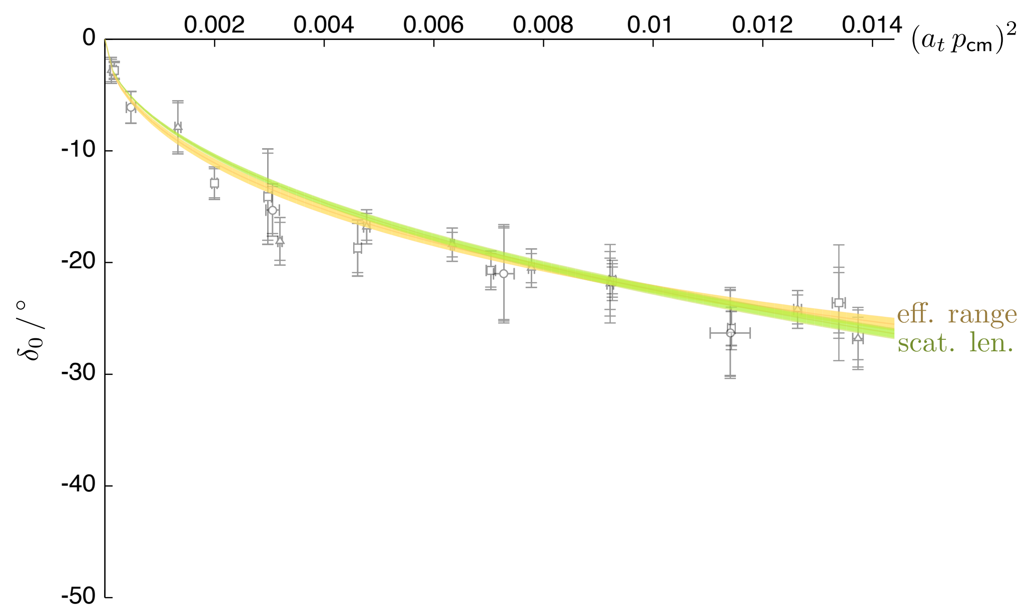
David Wilson
for the Hadron Spectrum Collaboration

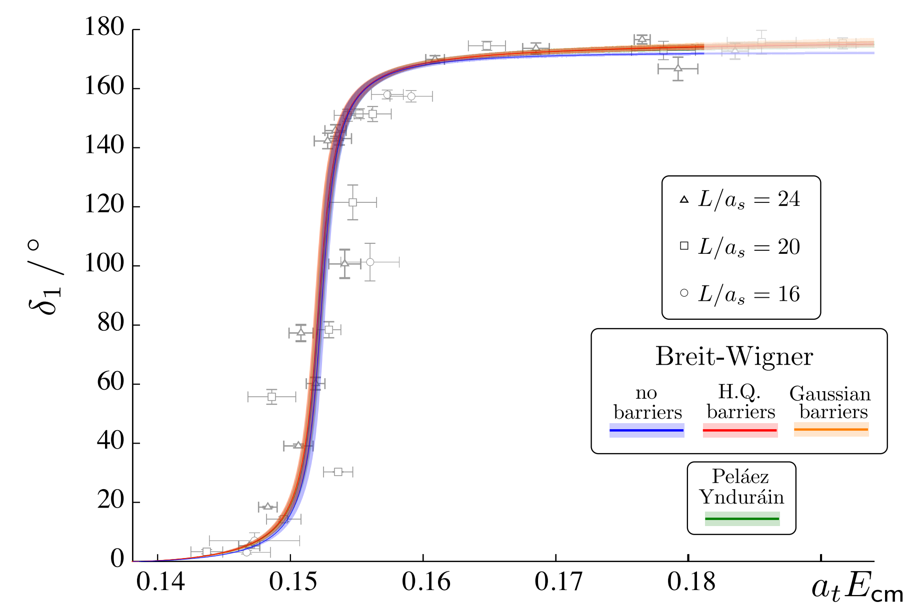
Old Dominion University

Motivation

Recent results from the Hadron Spectrum Collaboration:

I=2 and *I*=1 ππ scattering:





J. J. Dudek, R. G. Edwards and C. E. Thomas, arxiv:1203.6041, Phys. Rev. D 86, 034031 (2012)
arxiv:1212.0830 Phys. Rev. D 87, 034505 (2013)

Next: Kπ scattering.

Similar problem.
I=3/2 ~ non-resonant, weakly repulsive scattering for low energies.
I=1/2 ~ resonances are expected K*($J^P=1^-$), (κ ($J^P=0^+$)?)

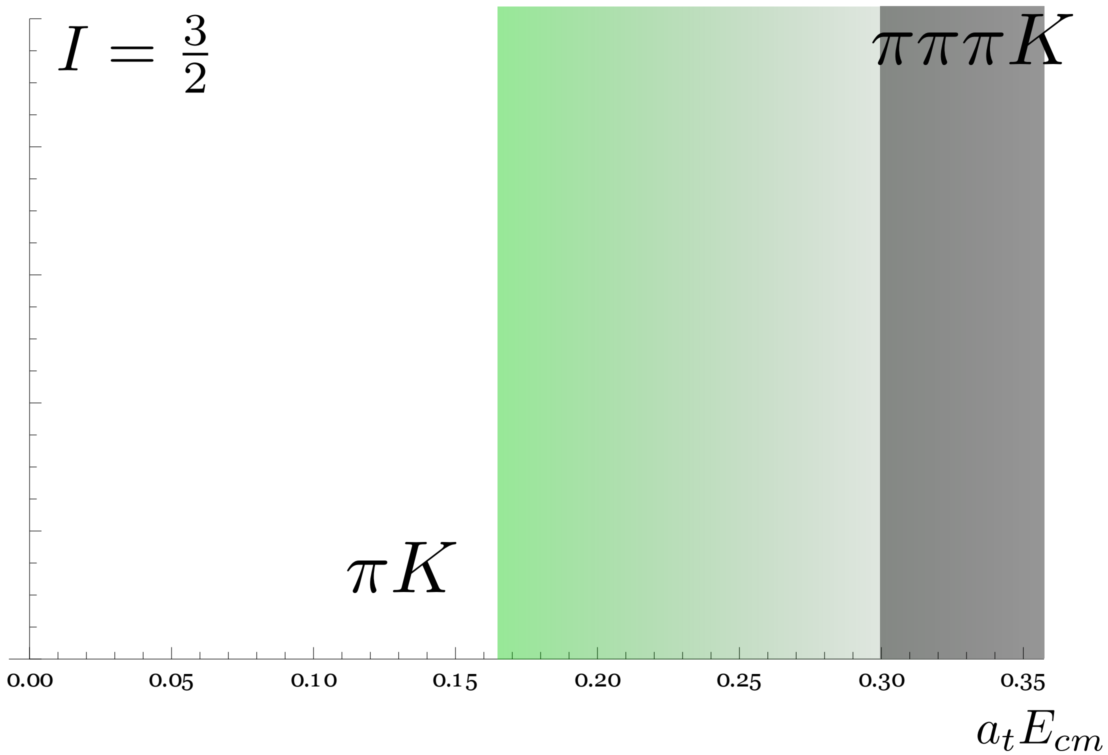
This talk: *I*=3/2 only.

Calculation Details

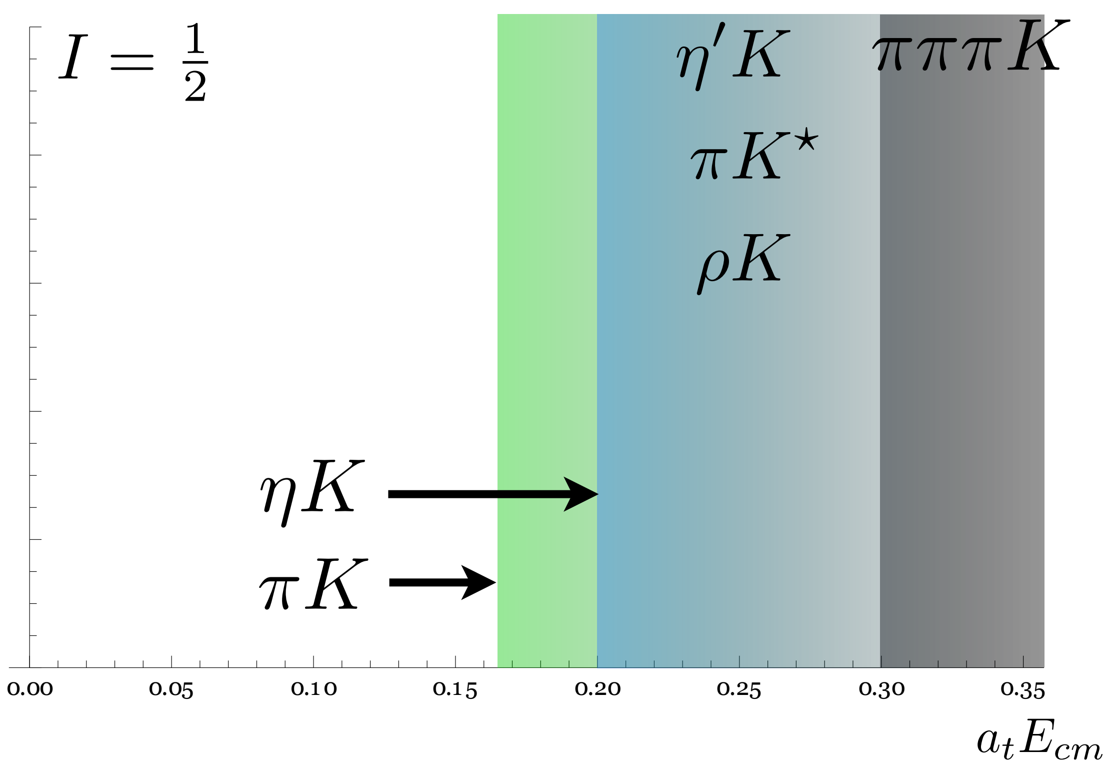
$(L/a_s)^3 \times T/a_t$	L	$m_\pi L$
$16^3 \times 128$	~2 fm	~3.8
$20^3 \times 128$	~2.5 fm	~4.8
$24^3 \times 128$	~3 fm	~5.7

- Anisotropic 2+1 Clover Lattices, with anisotropy $\xi = a_s/a_t = 3.444(6)$.
- ~500 configurations per volume.
- $m_K \approx 550$ MeV, $m_\pi \approx 392$ MeV.
- Use a large basis of operators: fermion bilinears plus derivative operators.
- Distillation method of smearing - numerically efficient for a large basis of operators.
- Use Lüscher method and extensions to relate the finite volume interacting energies to phase shifts.

Orientation: Masses and Energies



- No low lying resonances
- Few thresholds
- Large region of elastic two-particle scattering.



- $K^\star(J^P=1^-)$ expected around πK threshold
- Many thresholds around $a_t E_{cm}$=0.25.

Multi-meson operators

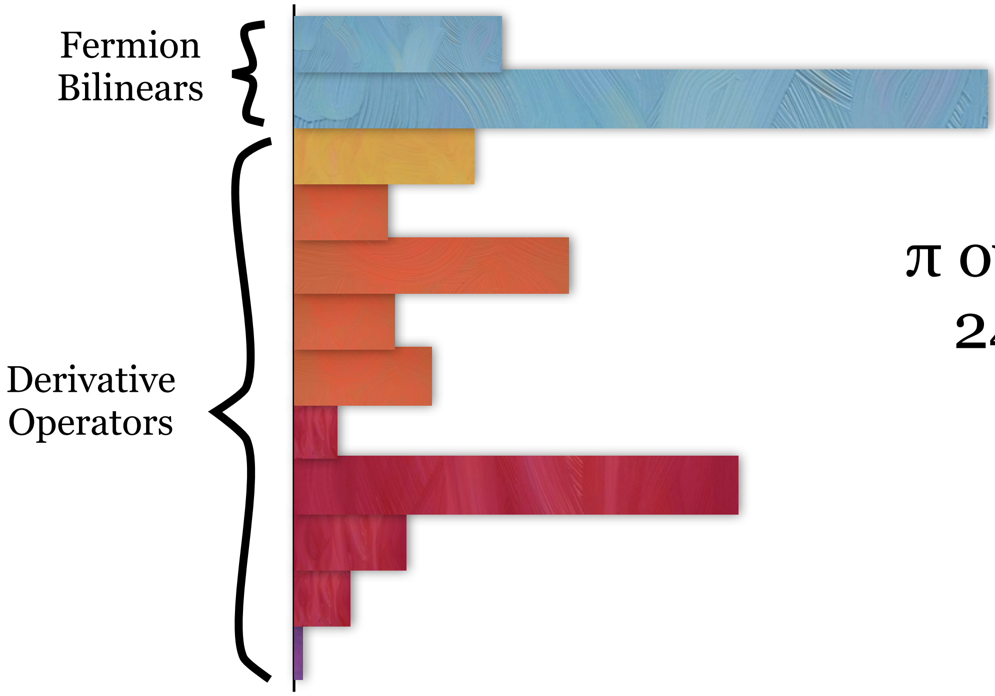
Variational Method:

Use a large basis of operators:
Fermion Bilinears + Derivatives.

Make use of orthogonality of state vectors:
Straightforward extraction of nearly-degenerate states.

$$C(t)v_\alpha = \lambda_\alpha(t)C(t_0)v_\alpha$$

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j(0)|0\rangle = \sum_\alpha \frac{Z_i^\star Z_j}{2m_\alpha} e^{-m_\alpha t}$$



π overlaps on 24^3 at rest

Projected Meson:

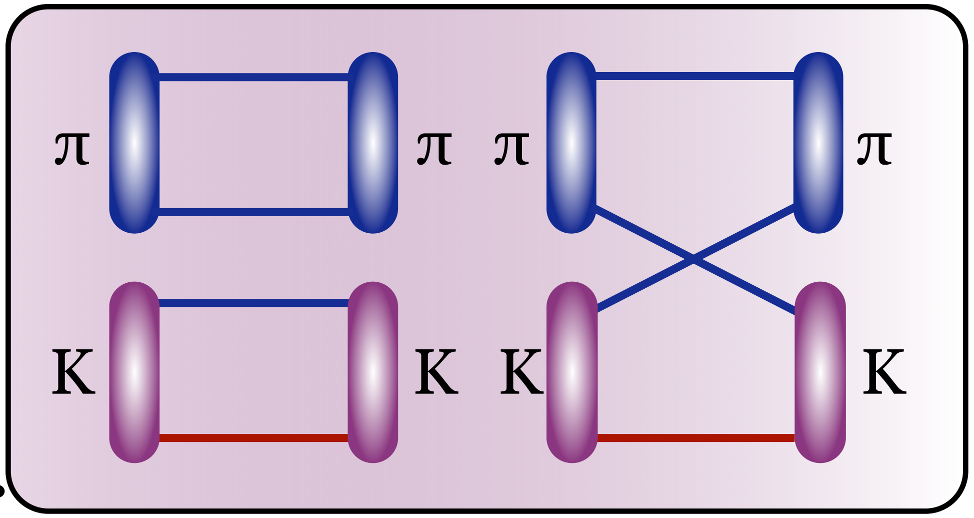
Mimic a single meson by creating a new operator as a sum of these components

$$\pi^\dagger(\vec{k}) = \sum_i v_i \mathcal{O}_i^{(\pi)\,\dagger}$$

Multi-Meson operator: (subduced) Product of single meson operators.

Wick contractions and operator basis

$$\pi(\vec{k}_1)K(\vec{k}_2)$$



Key feature of I=3/2 scattering:
There are no quark-line annihilation graphs.

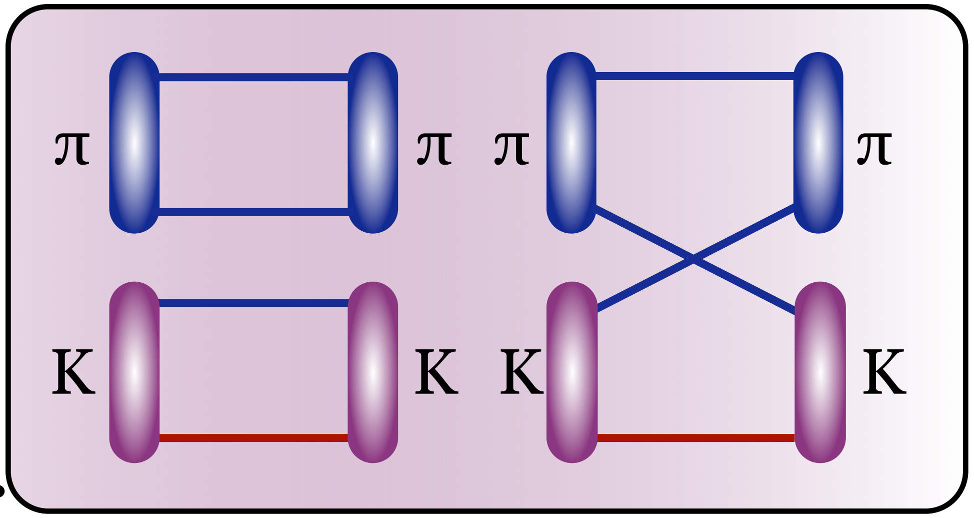
All operators must project onto meson-meson states.
Use projected meson operators in the different momentum combinations allowed by an irrep.

e.g. $\mathrm{Dic}_2 A_1 \quad \vec{p} = [110]$

$$\pi[000]K[110]$$
$$\pi[110]K[000]$$
$$\pi[100]K[100]$$
$$\pi[100]K[111]$$
$$\pi[111]K[100]$$
$$\pi[110]K[110]$$

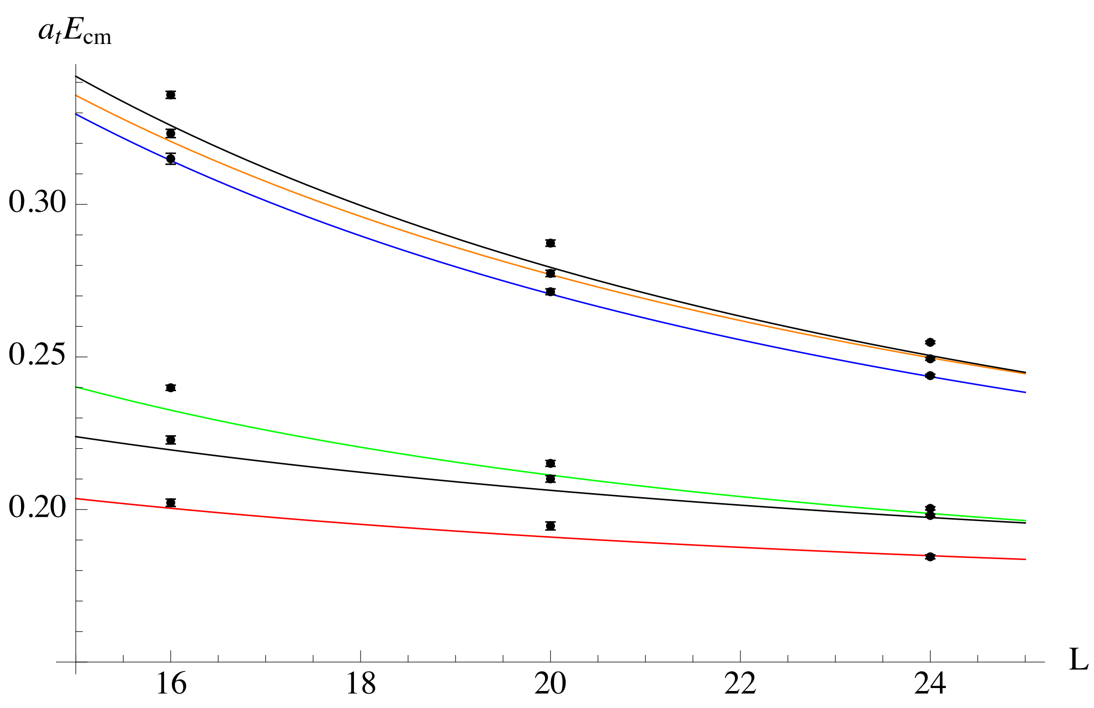
Wick contractions and operator basis

$$\pi(\vec{k}_1)K(\vec{k}_2)$$



Key feature of I=3/2 scattering:
There are no quark-line annihilation graphs.

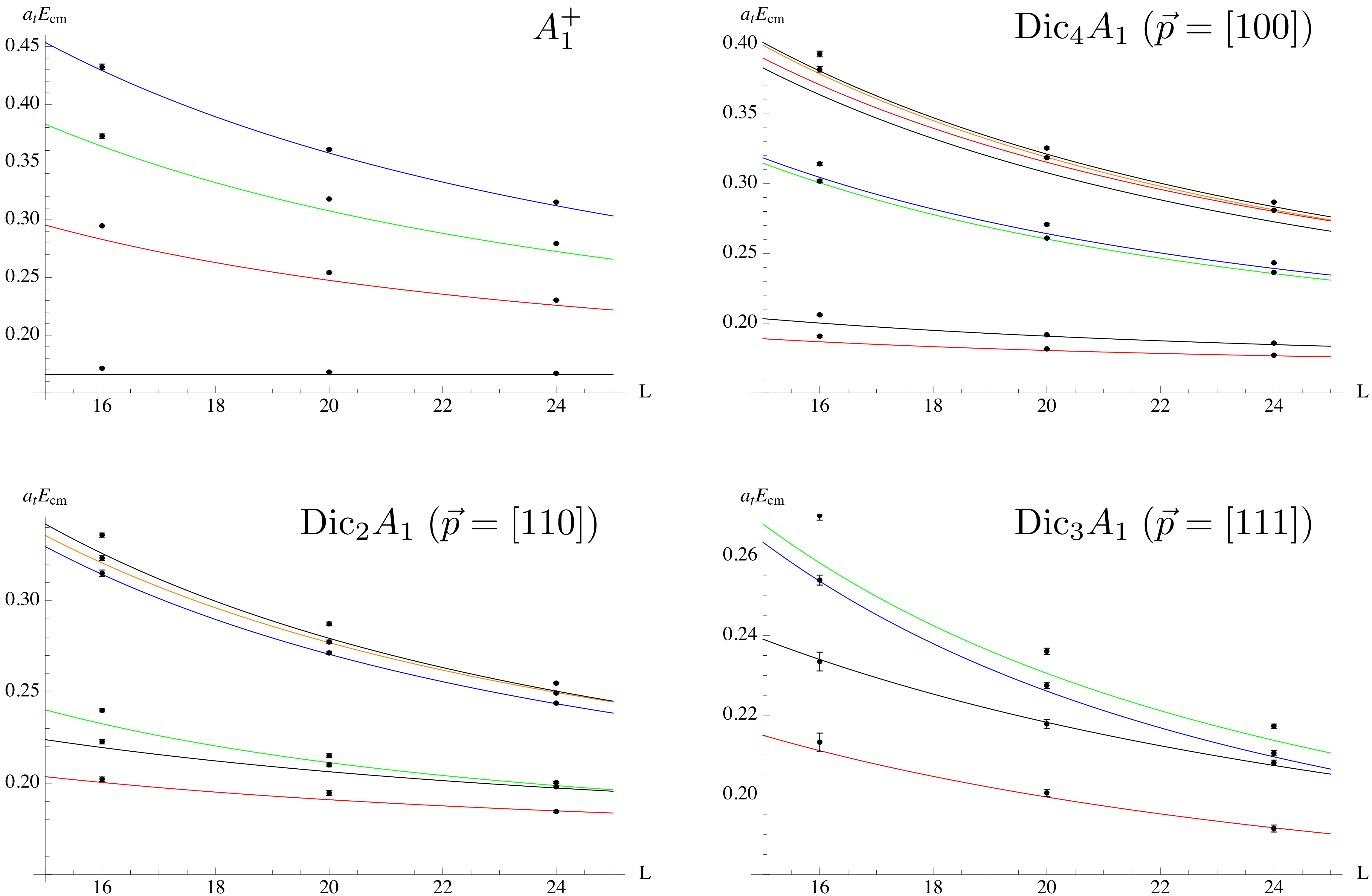
All operators must project onto meson-meson states.
Use projected meson operators in the different momentum combinations allowed by an irrep.



e.g. $\mathrm{Dic}_2 A_1 \quad \vec{p} = [110]$

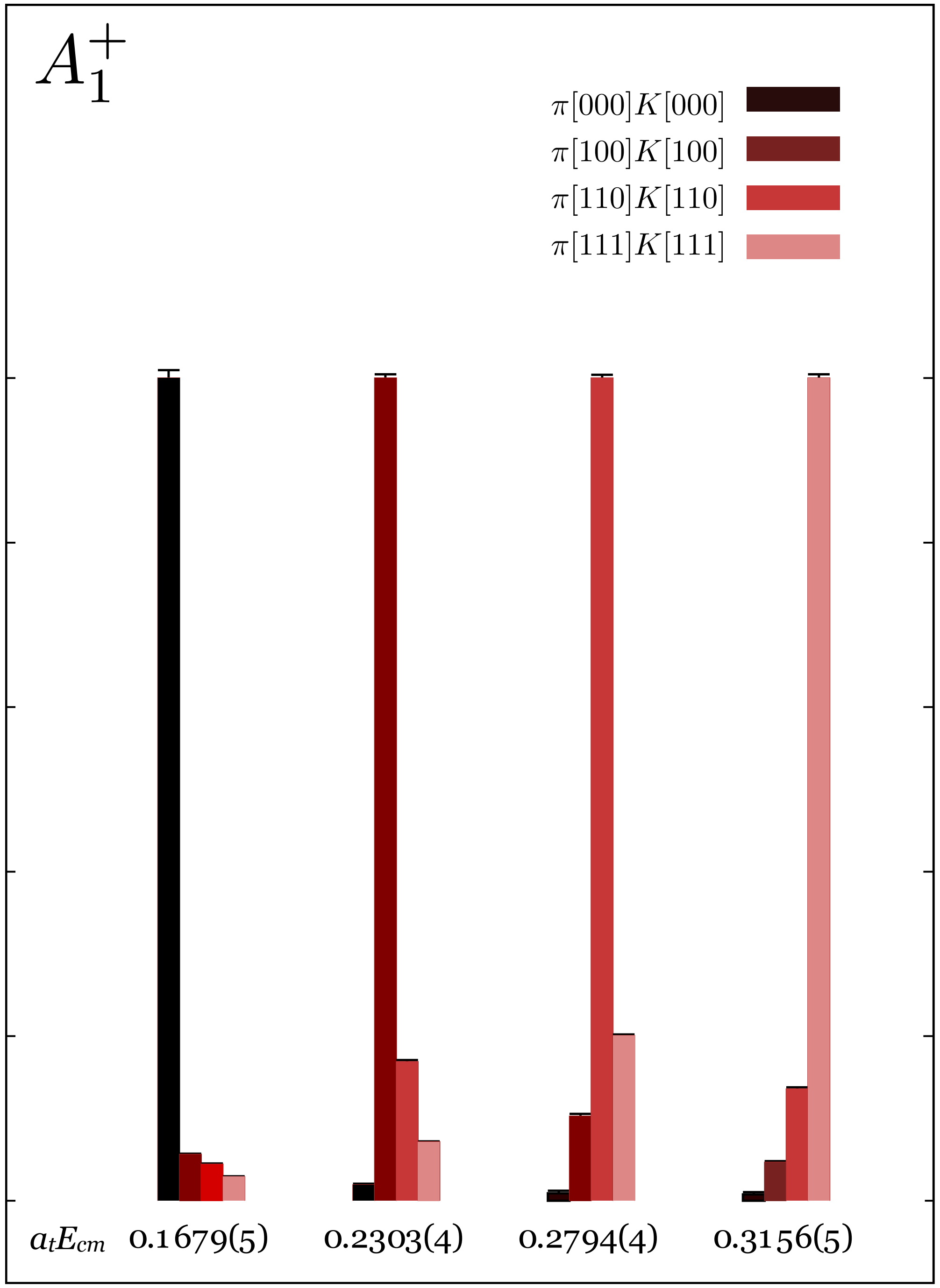
$$\pi[000]K[110]$$
$$\pi[110]K[000]$$
$$\pi[100]K[100]$$
$$\pi[100]K[111]$$
$$\pi[111]K[100]$$
$$\pi[110]K[110]$$

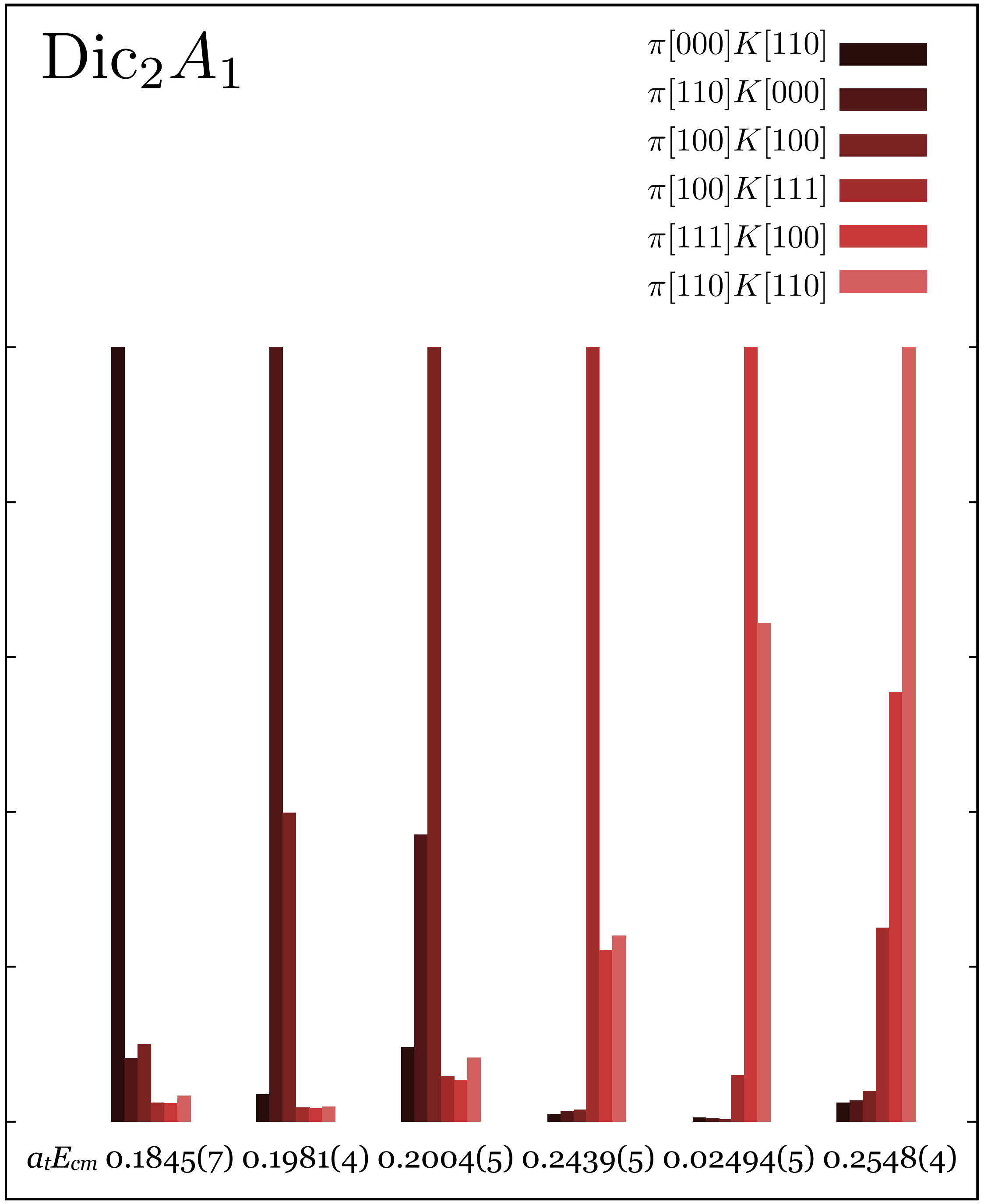
Volume dependence in different frames



Preliminary!

I=3/2 finite volume spectrum

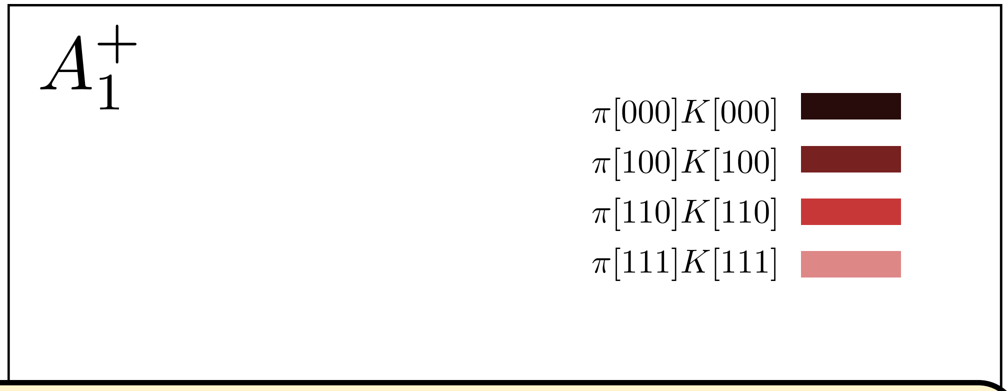


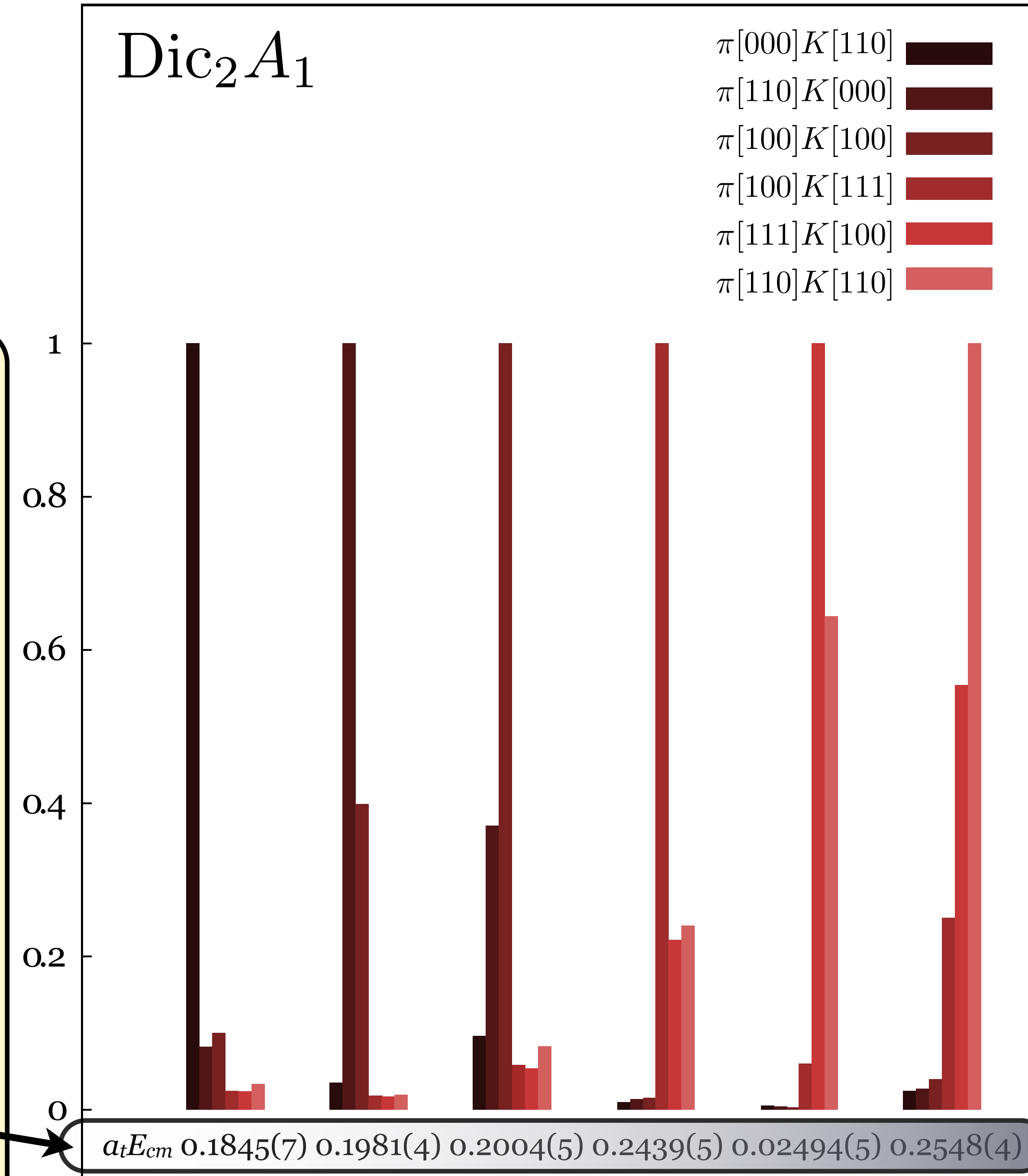


- Multi-meson operators overlap strongly onto their intended states.
- Variational method easily extracts nearly degenerate states.
- Small splitting due to having a relatively small m_K-m_π.

Preliminary!

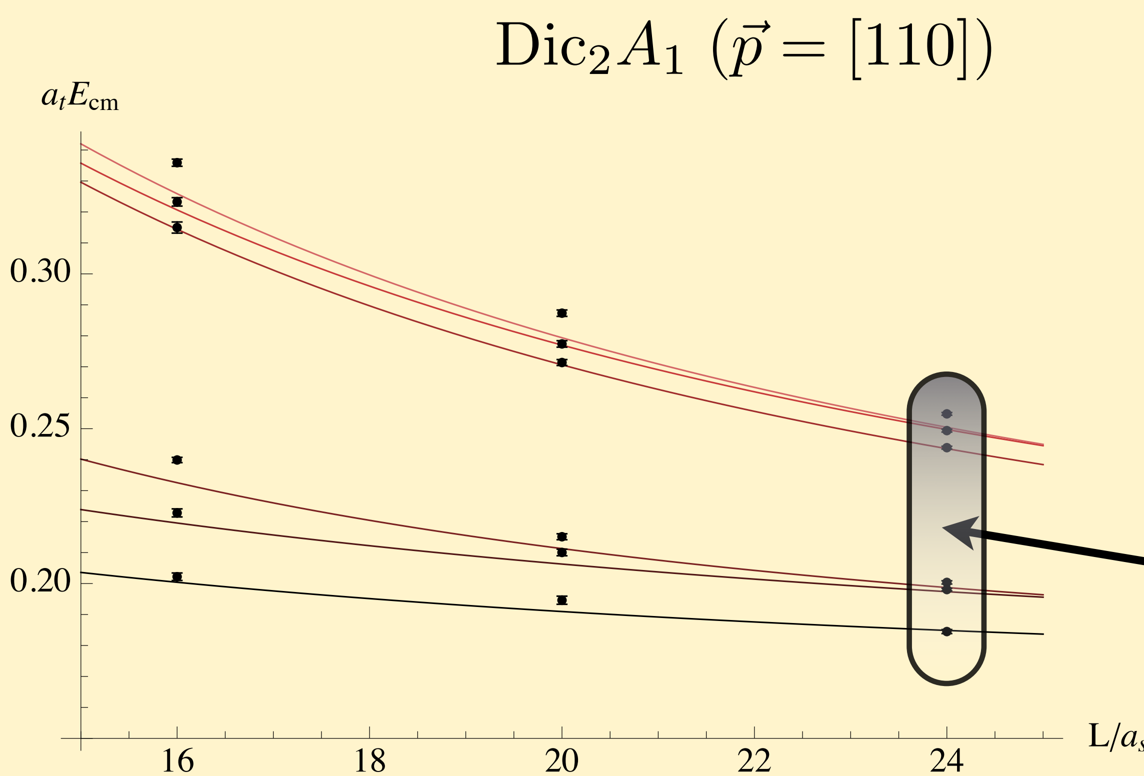
I=3/2 finite volume spectrum





Operator basis appears to work well:

- Roughly diagonal.
- Some mixing due to interactions (Good!)
- Variational method resolves finely spaced energies.



…gly onto their intended states.

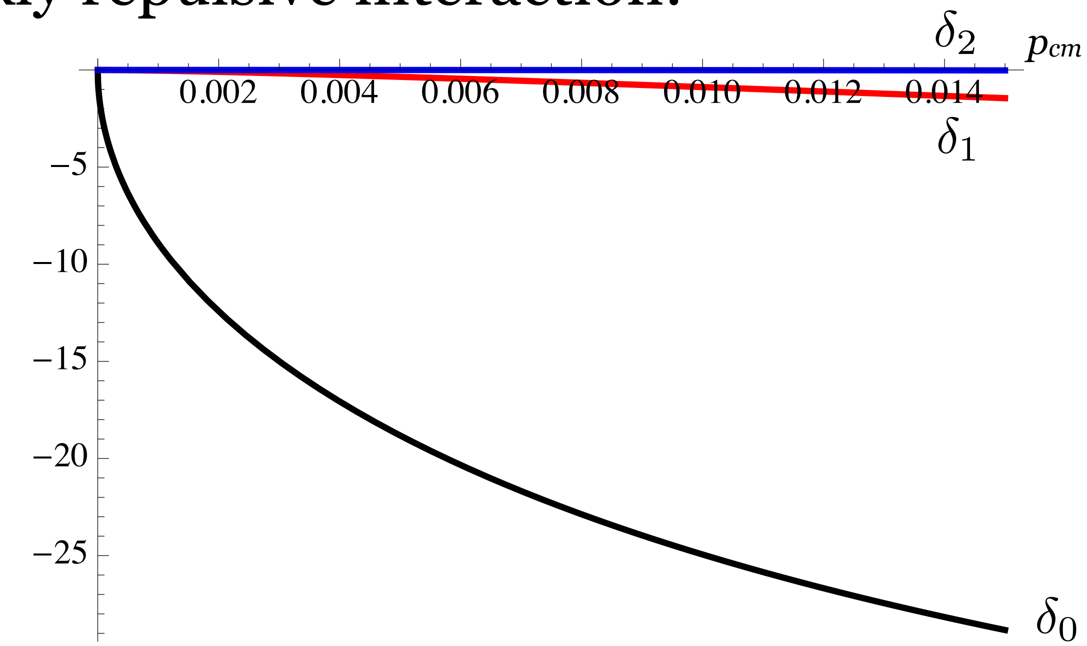
…arly degenerate states.

- Small splitting due to having a relatively small m_K-m_π.

Preliminary!

I=3/2 Phase shift analysis

- No resonances: S-wave expected to be largest.
- Effective Range approximation should be a good guide: $p_{\rm cm}^{2\ell+1}\cot\delta_\ell = \frac{1}{a_\ell} + \frac{1}{2} r_\ell\, p_{\rm cm}^2$
- Expect a hierarchy of partial waves: $\delta_{\ell=0} > \delta_{\ell=1} > \delta_{\ell=2} \quad \ldots$
- Typical effective range phase shifts for a weakly repulsive interaction:



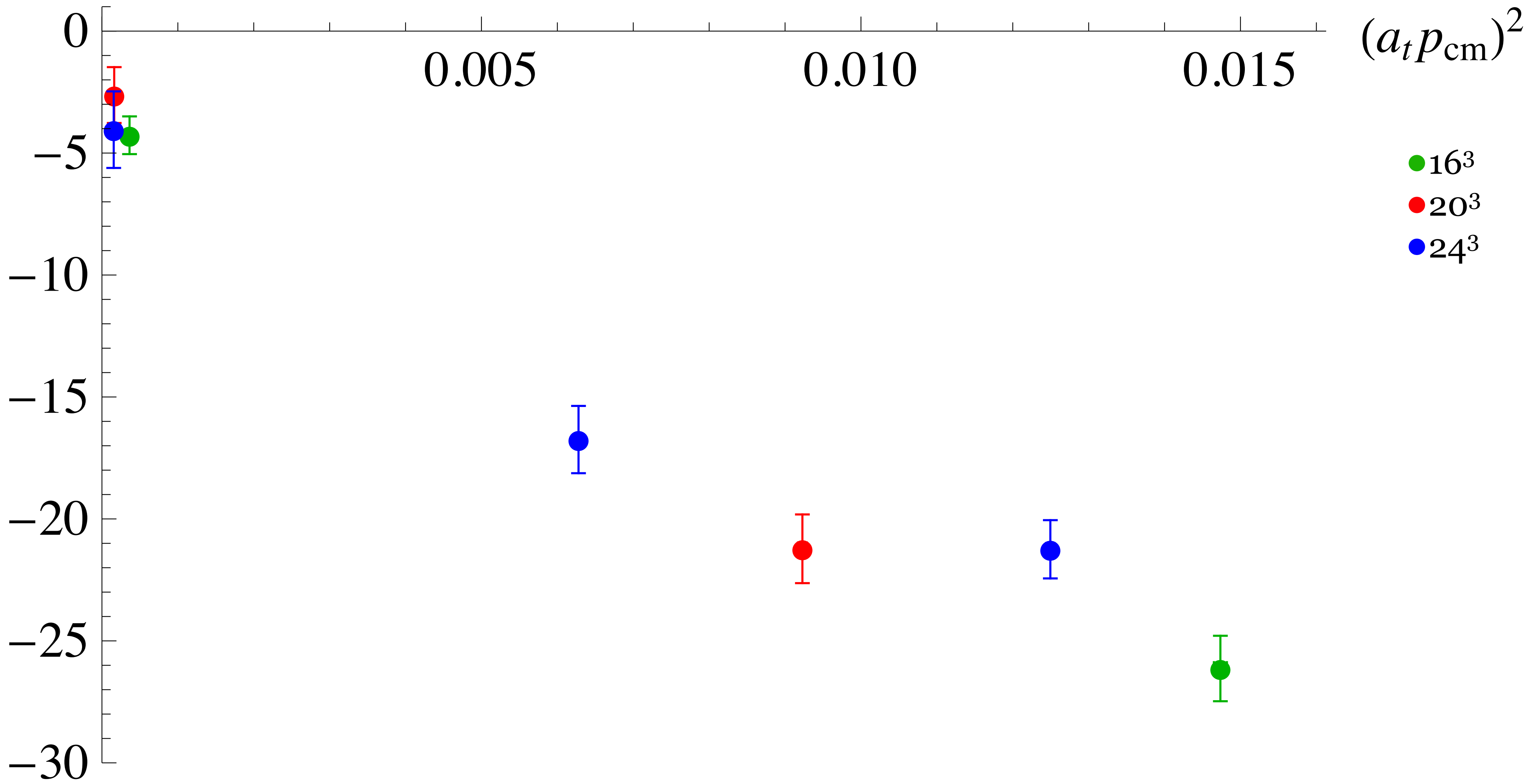
$\Longrightarrow$ Begin by neglecting higher waves, start with *S*-wave.

Use Lüscher method and its extensions to moving frames and unequal mass systems to relate the shifts in the Lattice energies due to interactions to the phases.

I=3/2 S-wave phase shift

Preliminary!

At rest points only, A_1^+

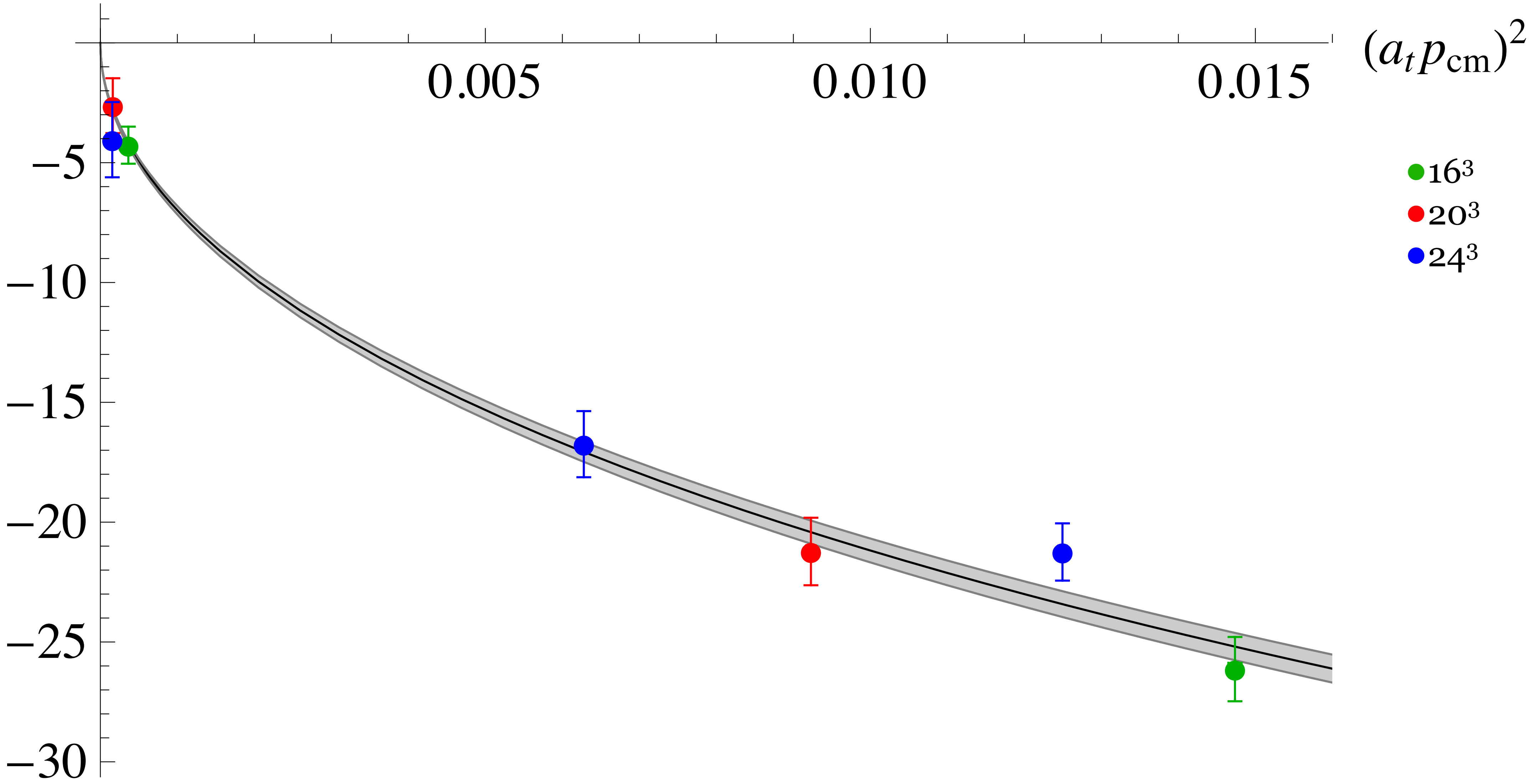


Phases from A_1^+ irreps from the 3 volumes.
Assuming one channel and only one (l=0) partial wave.
At rest there is the least contamination from higher partial waves: Only l=0, 4, ... in A_1^+

I=3/2 S-wave phase shift

Preliminary!

At rest points only, A_1^+



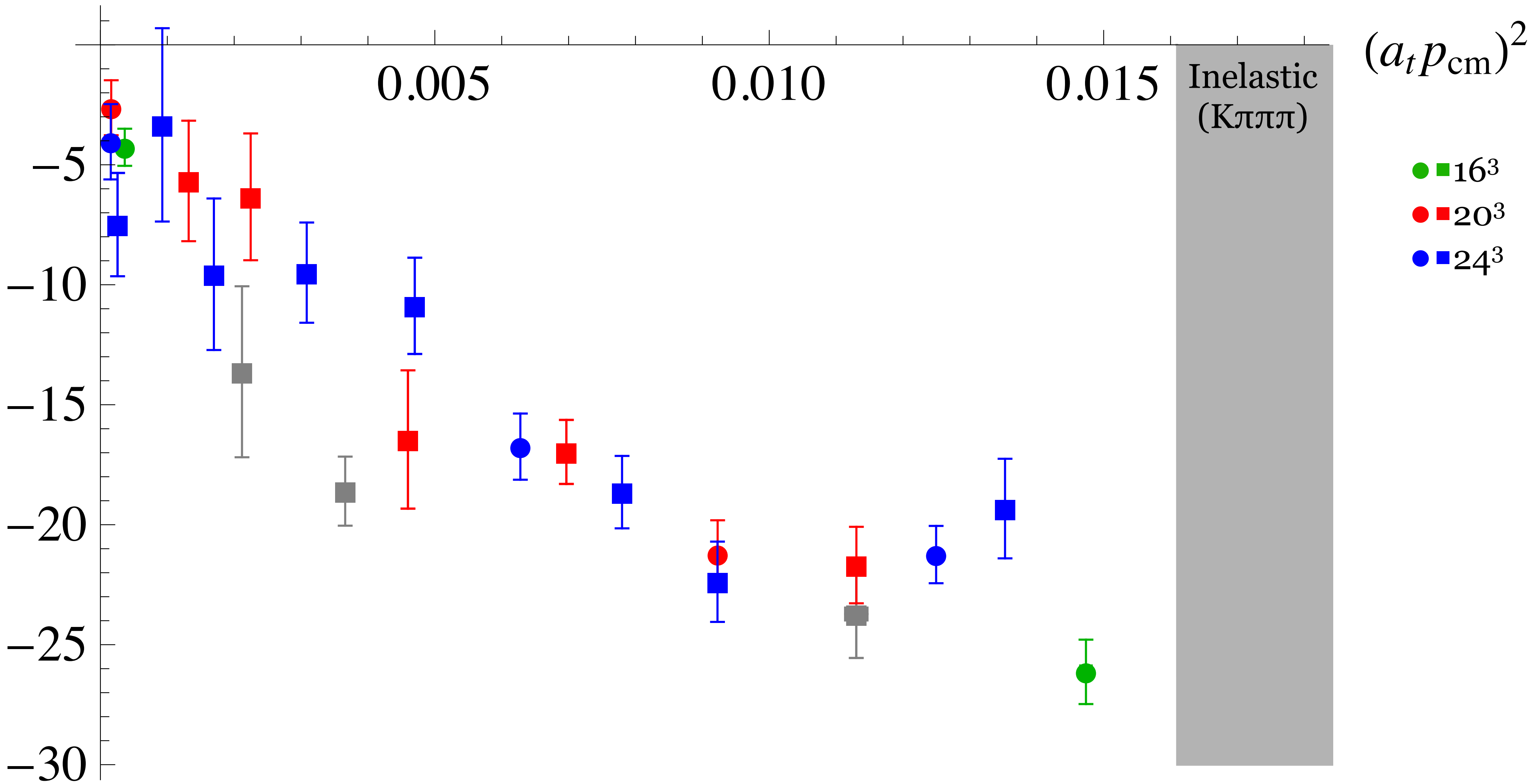
One parameter fit: $a_{l=0} = -3.88 \pm 0.10\ a_t$

$\chi^2/N_{\mathrm{dof}} = 12.1/6$

I=3/2 *S*-wave phase shift

Preliminary!

Add moving frame states:

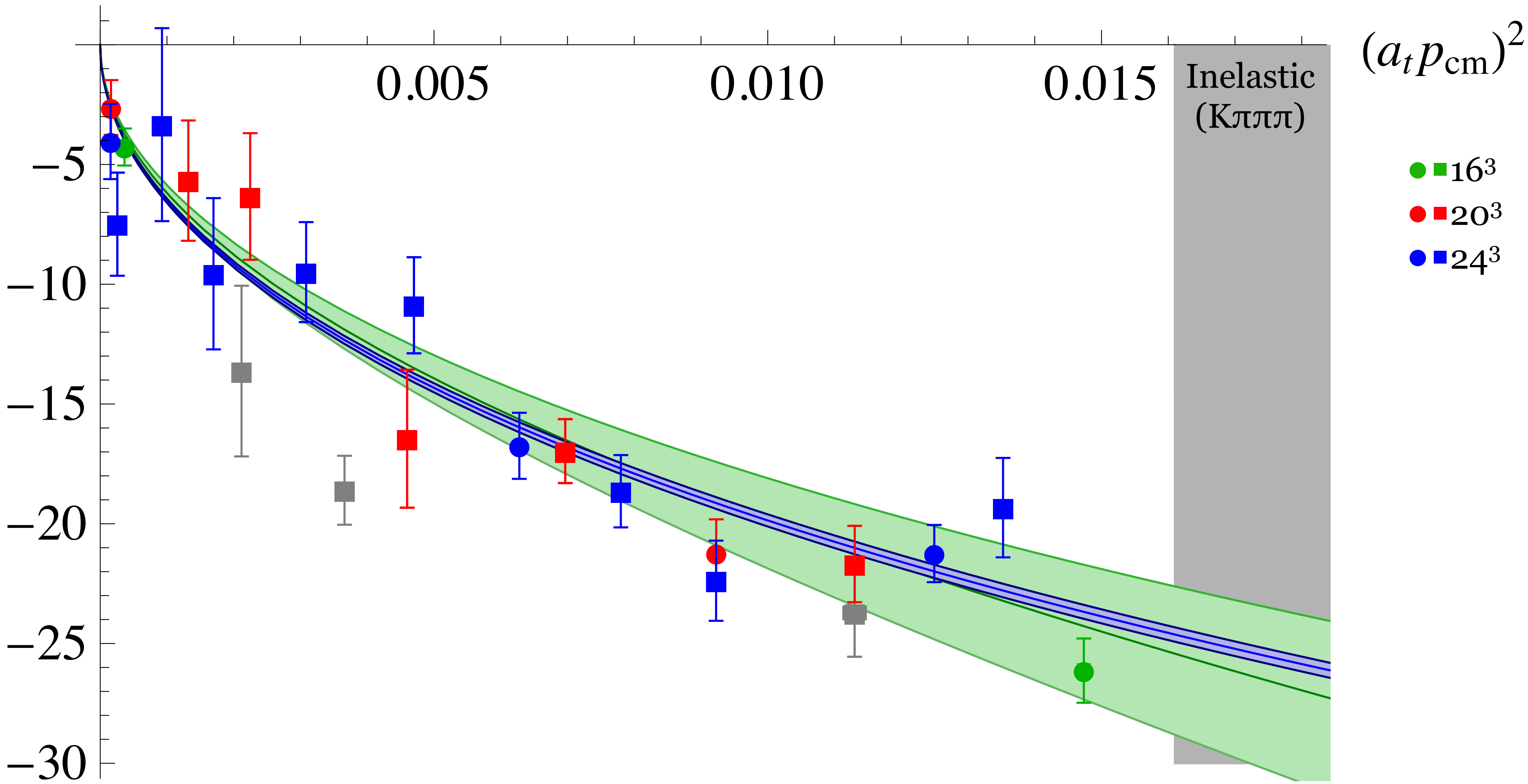


All A_1 irreps: lowest partial wave is l=0, but all others are present in flight.

I=3/2 *S*-wave phase shift

Preliminary!

Effective range and scattering length fits:



Scattering length fit (blue): $a_{l=0} = (-3.62 \pm 0.05)\, a_t$ with $\chi^2/N_{\rm dof} = 33.2/20$

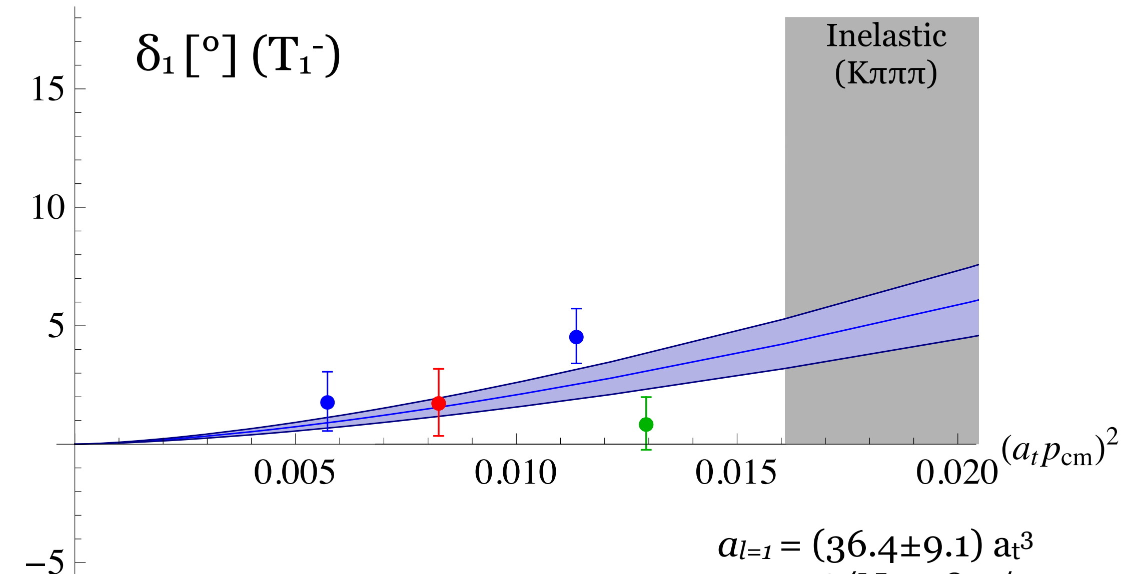
Effective range fit: $a_{l=0} = (-3.41 \pm 0.17)\, a_t$
(green) $r_{l=0} = (-3.26 \pm 2.71)\, a_t$ with $\chi^2/N_{\rm dof} = 31.8/19$ and correlation matrix: $\begin{pmatrix} 1 & 0.97 \\ 0.97 & 1 \end{pmatrix}$

I=3/2 Higher waves

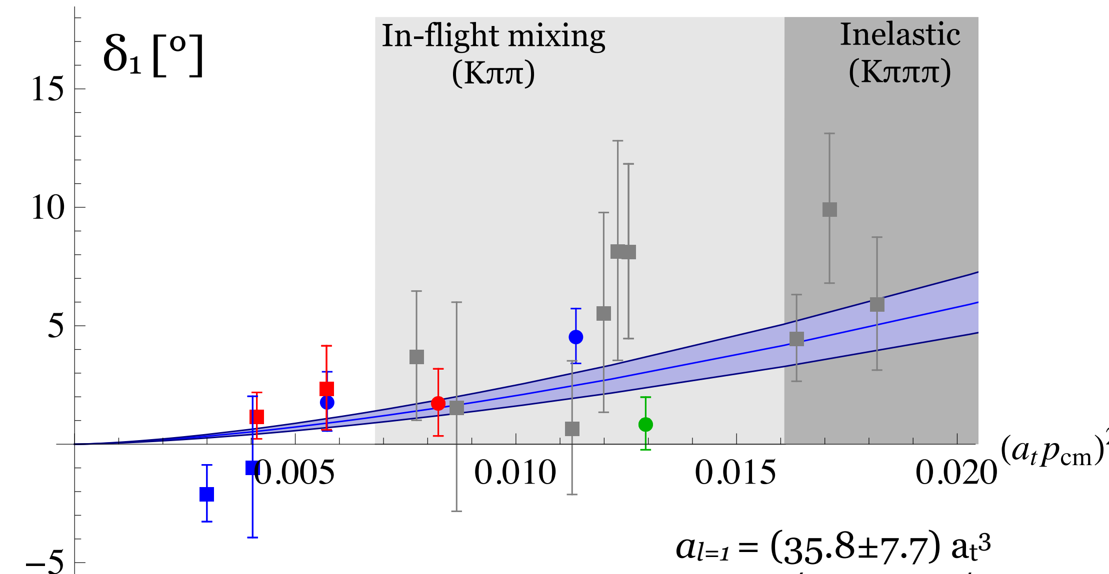
Preliminary!

Use the same method:
Neglect higher waves.
Use only irreps where the partial wave of interest is the lowest.

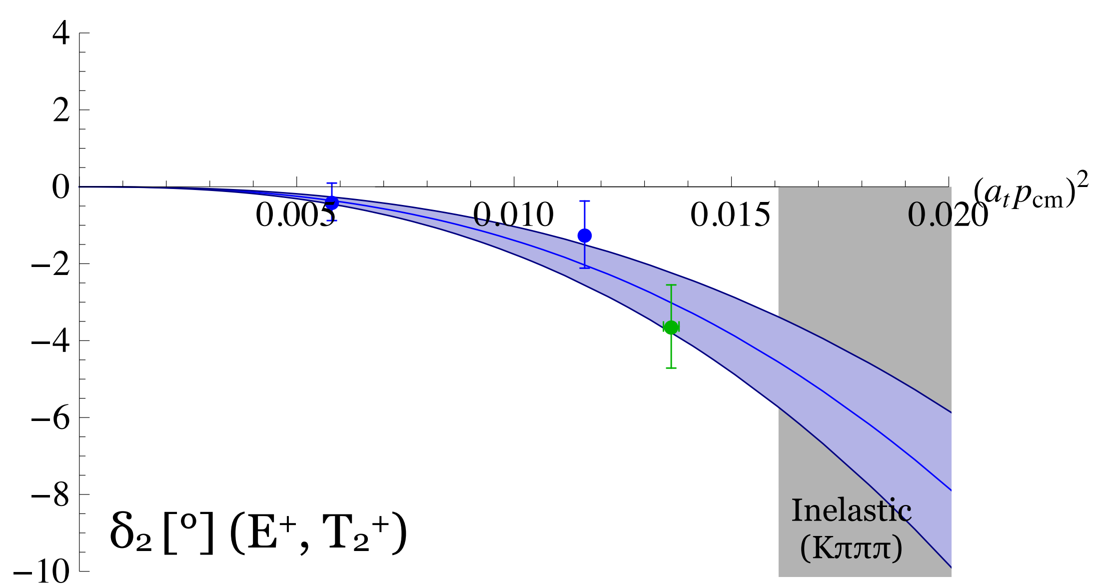
16^3 (green)
20^3 (red)
24^3 (blue)



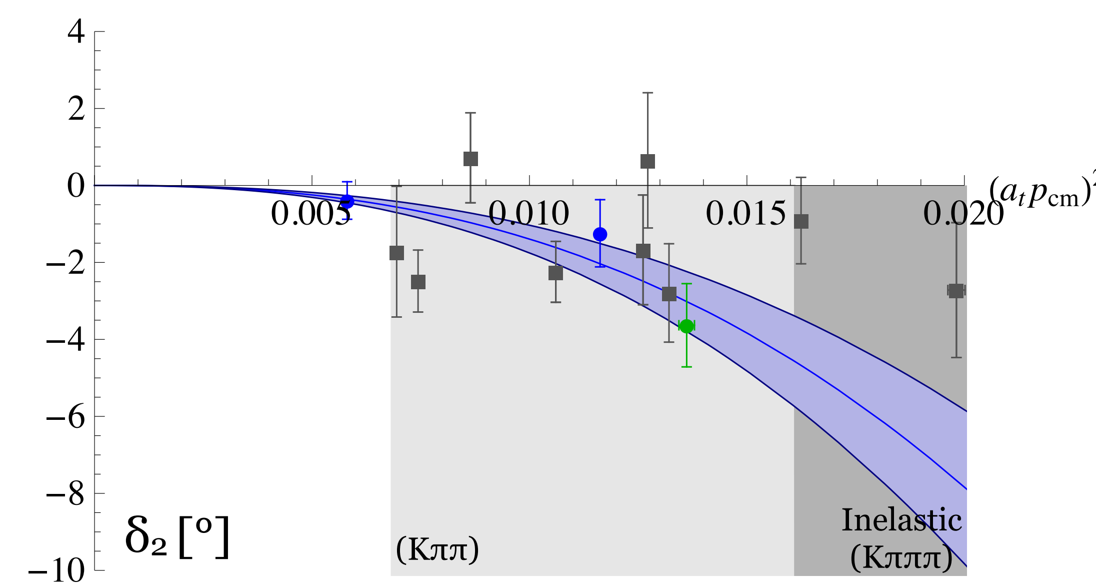
$a_{l=1}$ = (36.4±9.1) a_t^3
χ^2/N_{dof}=8.3/3



$a_{l=1}$ = (35.8±7.7) a_t^3
χ^2/N_{dof}=12.7/7



$a_{l=2}$ = (-2430±630) a_t^5
χ^2/N_{dof}=1.6/2



(No additional elastic points)

(Circles from irreps at rest; Squares in flight)

I=3/2 Global fit

Preliminary!

Fit all states simultaneously

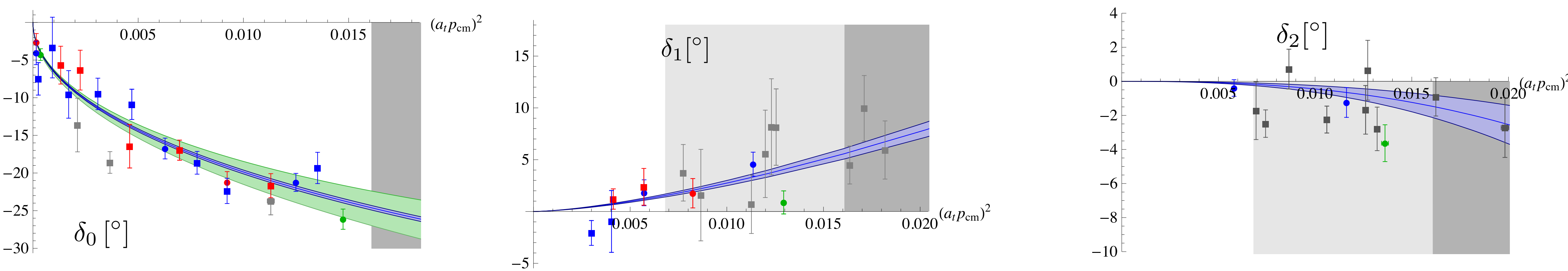
In $\text{Dic}_4 A_1$ p=[100] the helicity 0 parts of the l=1, 2 waves have a component.

In $\text{Dic}_4 E_2$ p=[100] helicities 1, 3 appear and the helicity 1 part of the l=2 wave may appear.

In $\text{Dic}_2 A_1$ p=[110] there are helicities 0, 2, 4 and the helicity 0 parts of the l=1, 2 waves appear.

In this system, these extra components appear to be small: very little correlation between fit parameters.

(only lattice states corresponding to coloured points are included in the fit)



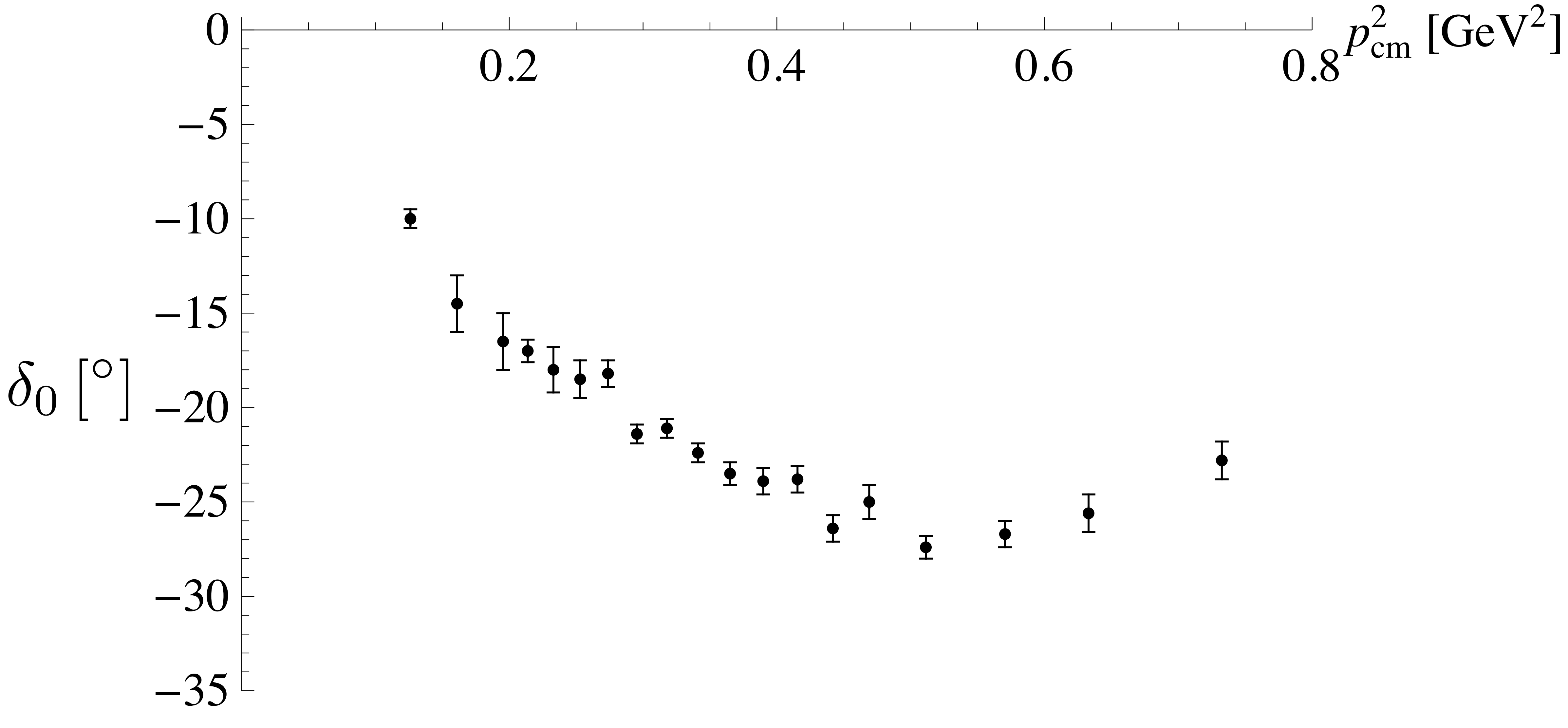
$a_{l=0} = (-3.62 \pm 0.04)\, a_t$	1	0.06	0.05
$a_{l=1} = (47.9 \pm 3.2)\, a_t^3$	0.06	1	-0.05
$a_{l=2} = (-783 \pm 386)\, a_t^5$	0.05	-0.05	1

$\chi^2/N_{\text{dof}}=41.6/28$

(Expected near-threshold behaviour: $\delta_3<\delta_2<4°$ in region of interest.)

Experimental Comparison

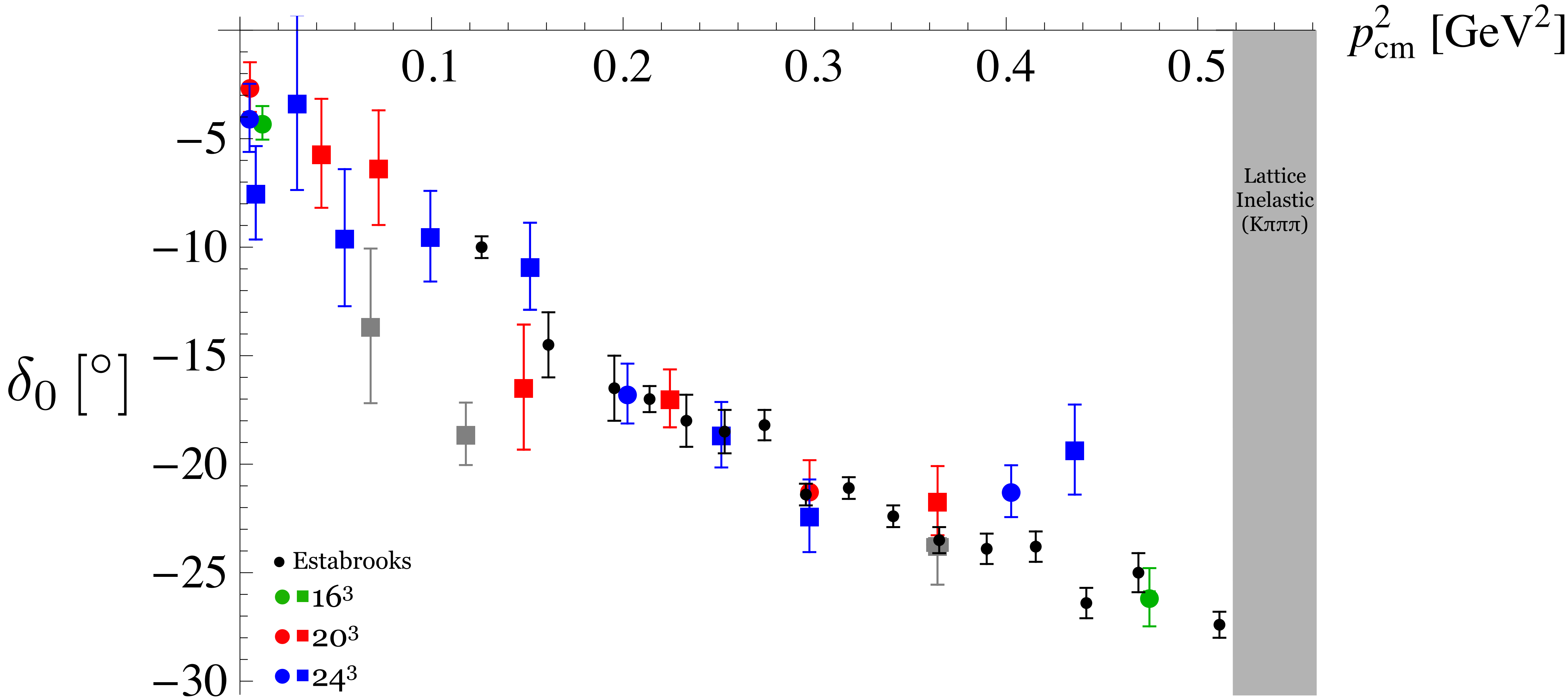
- Experimental data exist for this process [Estabrooks 1978].



Experimental Comparison

Preliminary!

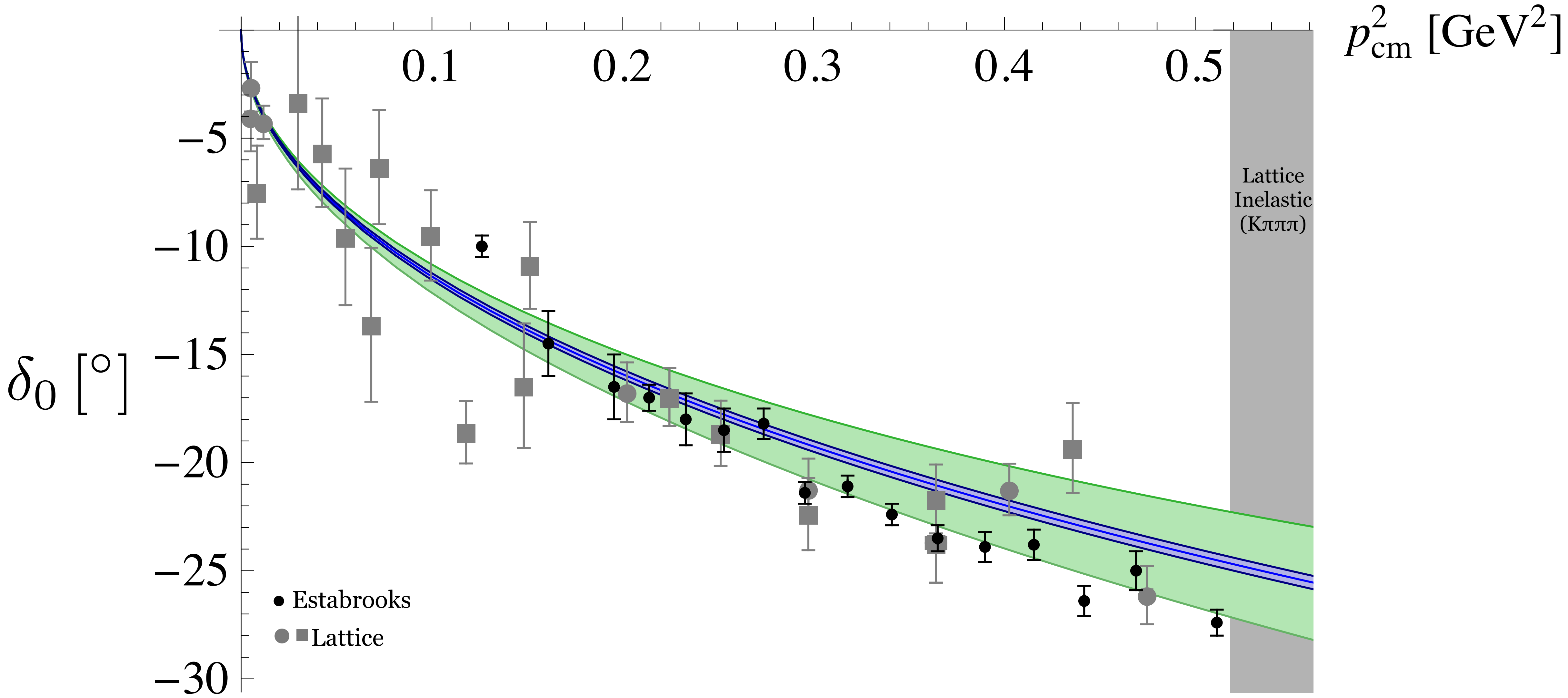
- Experimental data exist for this process [Estabrooks *et al* 1978].



Experimental Comparison

Preliminary!

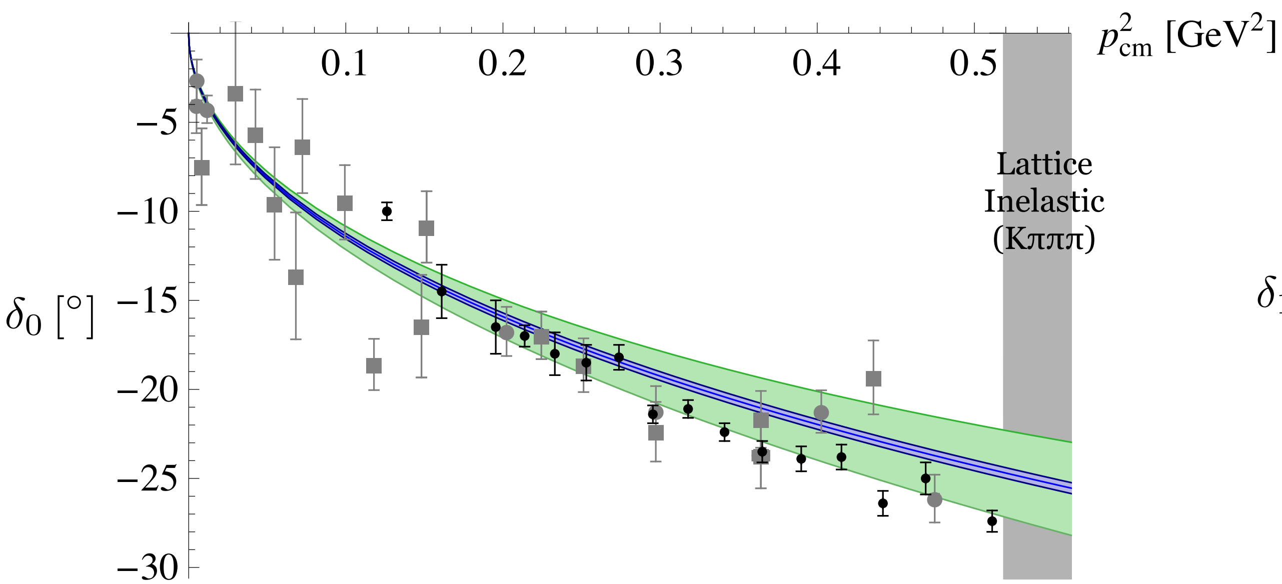
- Experimental data exist for this process [Estabrooks 1978].

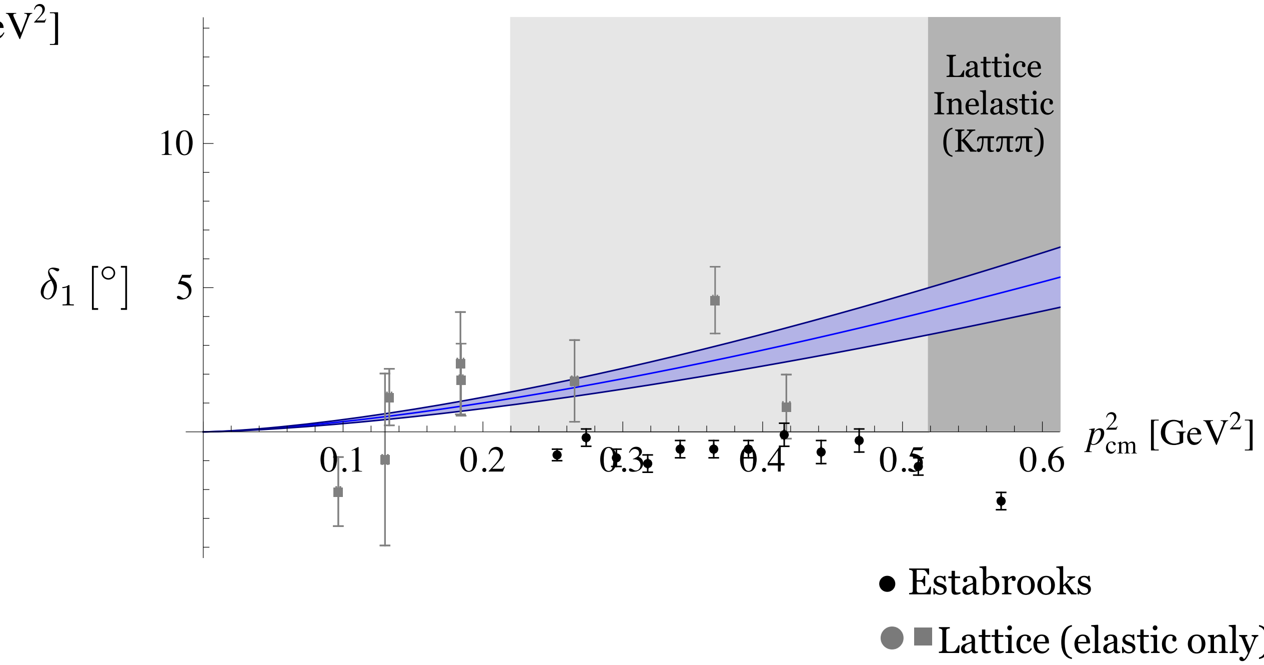


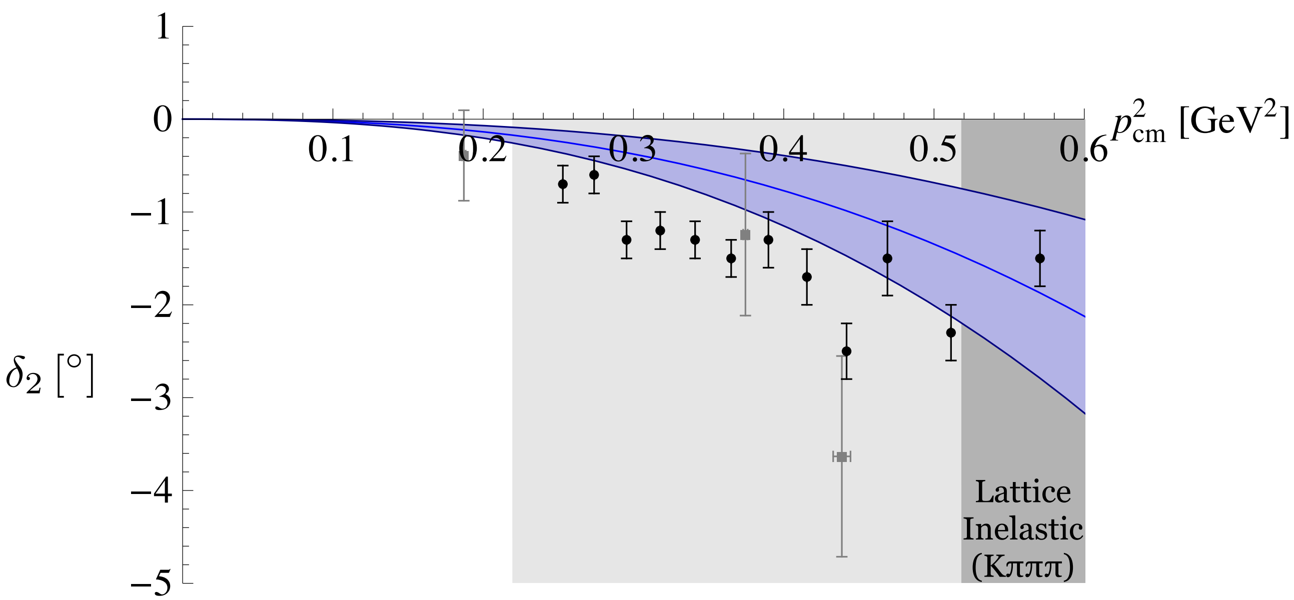
Experimental Comparison

Preliminary!

- Experimental from Estabrooks *et al* 1978.
- Fits performed to extracted Lattice energies.
- 3 partial wave effective range parameterisation with finite volume energies calculated from the Lüscher method.







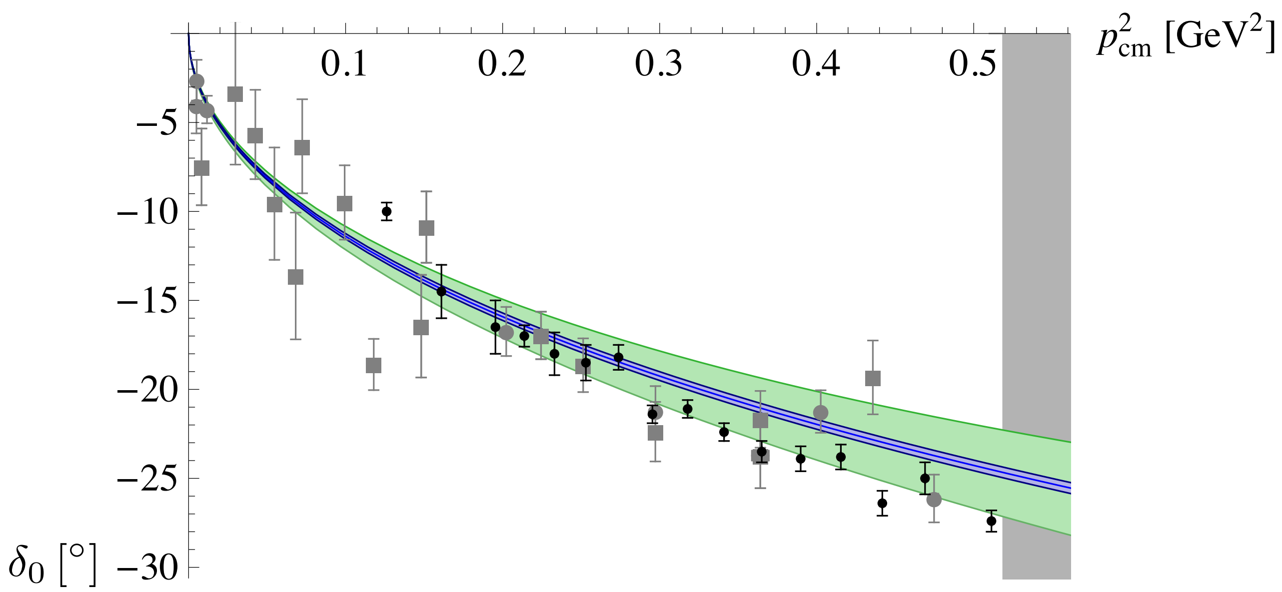
Important differences:
Scattering particles have different masses:
~392 vs. 139 MeV

Inelastic thresholds are very different:

$3m_\pi + m_K$ threshold:
Physical: $p_{cm}^2 = 0.09$ GeV².
Lattice: $p_{cm}^2 = 0.5$ GeV².

Summary and next steps…

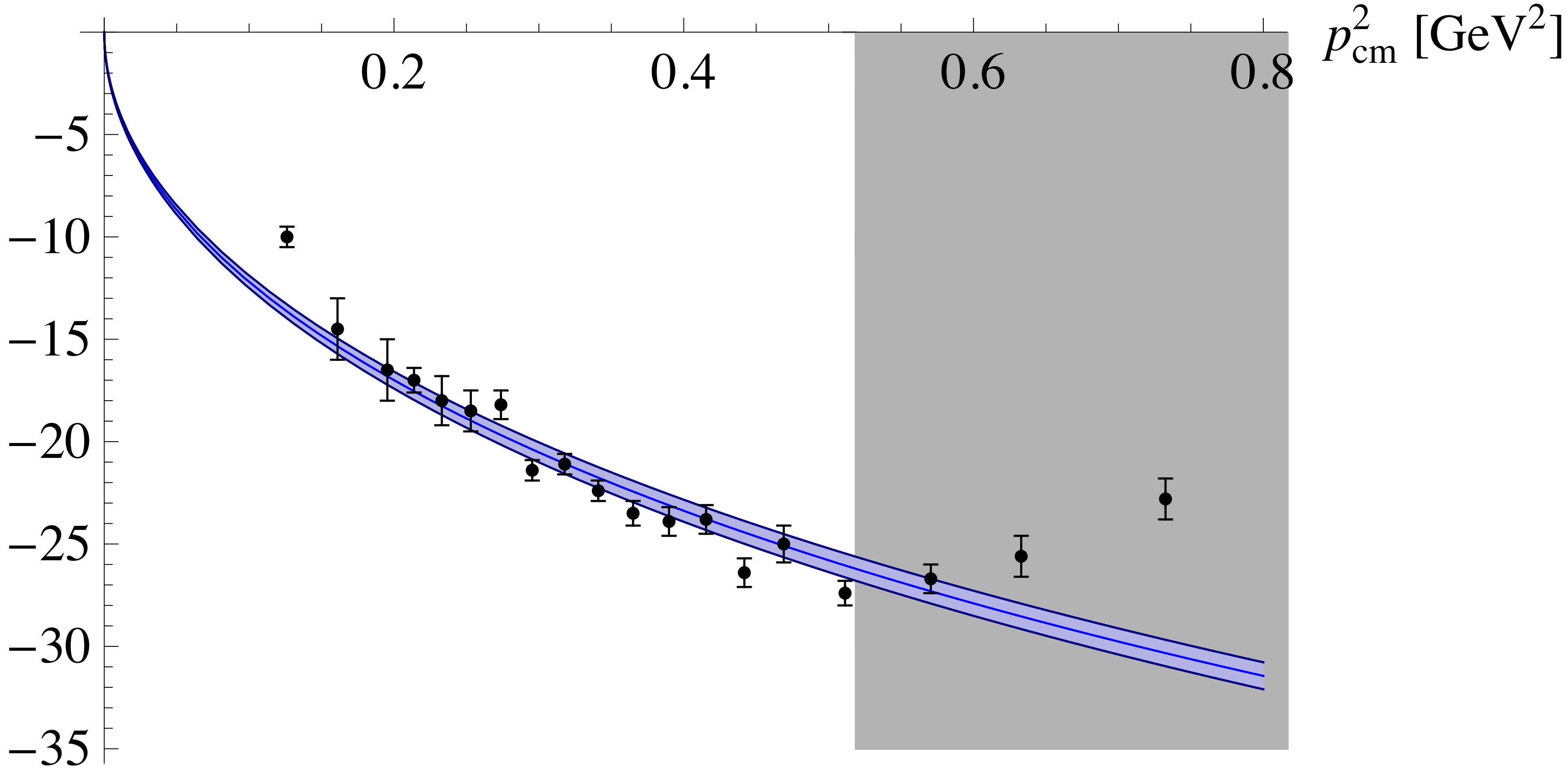
Preliminary!



- We extracted lattice energies and corresponding phase shifts for I=3/2 Kπ scattering at $m_K \approx 550$ MeV and $m_\pi \approx 392$ MeV in three different volumes around 2.5fm.
- We used a variational basis of products of projected meson operators which allows fine energy resolution.
- The S-wave phase is close to the experimental data, despite quark mass differences.
- Analysis of the associated and potentially more interesting I=1/2 data is underway.

Extra slides

At rest effective range and Estabrooks data

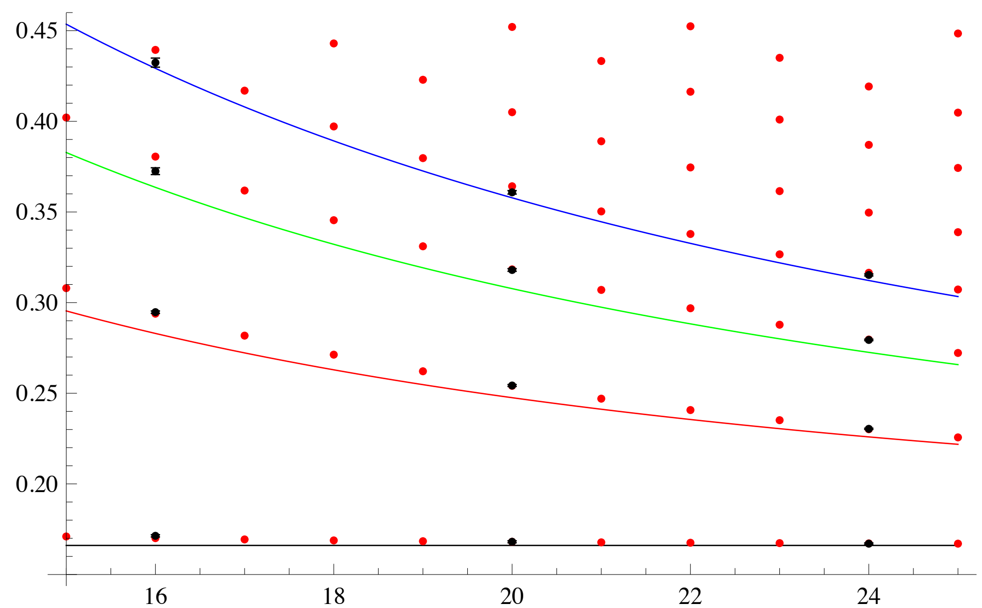


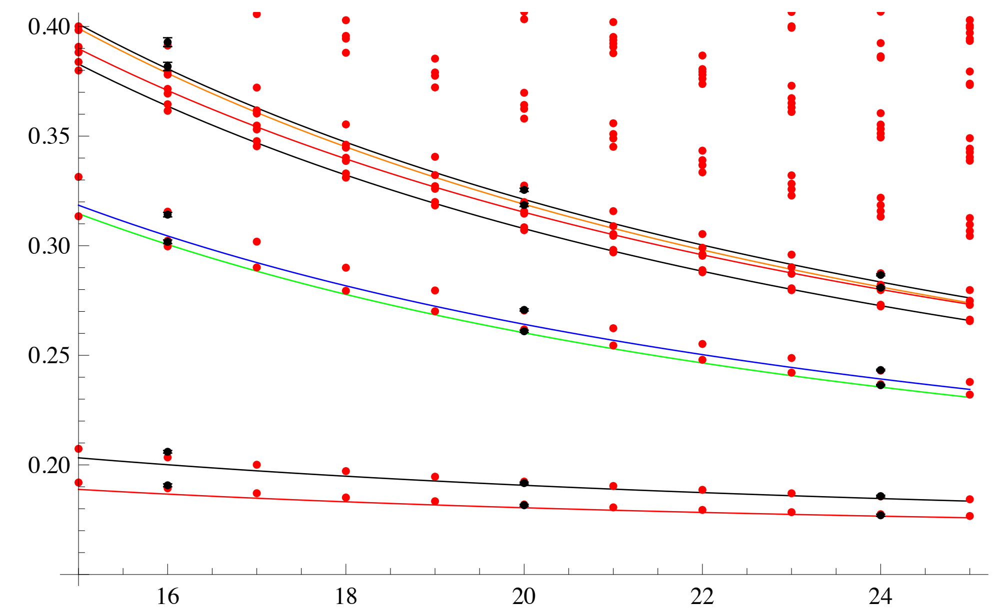
Working backwards

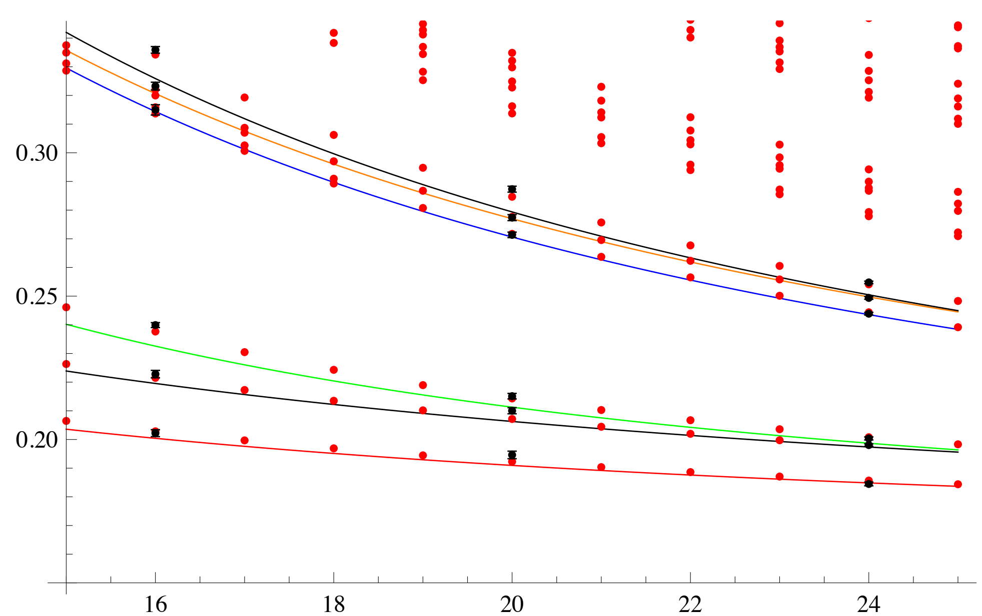
Multiple partial wave fit: Define interaction model of individual waves.

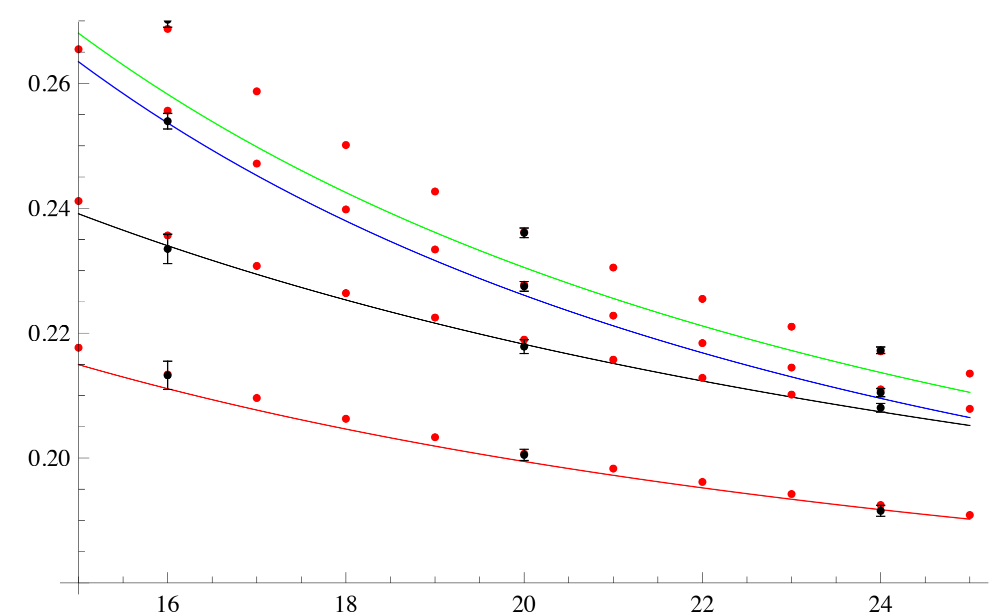
Interaction Model ⇒ Lüscher's Method ⇒ Model Finite Volume Energy Spectrum

Fit model parameters by minimising difference between extracted energies and model energies.









Partial wave contributions in each irrep

$\vec{P}$	LG($\vec{P}$)	Λ	$J^P(\vec{P}=0)$ $\lvert\lambda\rvert^{(\tilde{\eta})}$ $(\vec{P}\neq 0)$	$(K\pi)$ ℓ^N
		A_1^+	0^+, 4^+	0^1, 4^1
		T_1^-	1^-, 3^-, (4^-)	1^1, 3^1
		T_1^+	(1^+), (3^+), 4^+	4^1
$[0,0,0]$	O_h^D	T_2^-	(2^-), 3^-, (4^-)	3^1
		T_2^+	2^+, (3^+), 4^+	2^1, 4^1
		E^+	2^+, 4^+	2^1, 4^1
		A_2^-	3^-	3^1
		A_1	0^+, 4	0^1, 1^1, 2^1, 3^1, 4^2
		A_2	(0^-), 4	4^1
$[0,0,n]$	Dic_4	E_2	1, 3	1^1, 2^1, 3^2, 4^2
		B_1	2	2^1, 3^1, 4^1
		B_2	2	2^1, 3^1, 4^1
		A_1	0^+, 2, 4	0^1, 1^1, 2^2, 3^2, 4^3
$[0,n,n]$	Dic_2	A_2	(0^-), 2, 4	2^1, 3^1, 4^2
		B_1	1, 3	1^1, 2^1, 3^2, 4^2
		B_2	1, 3	1^1, 2^1, 3^2, 4^2
		A_1	0^+, 3	0^1, 1^1, 2^1, 3^2, 4^2
$[n,n,n]$	Dic_3	A_2	(0^-), 3	3^1, 4^1
		E_2	1, 2, 4	1^1, 2^2, 3^2, 4^3

Table 2: The separation of $K\pi$ spins, helicities and partial waves in each lattice irrep, with $J \leq 4$. The first three columns are taken from Refs. [13, 8, 7]. The final column is derived by considering how the helicity components are projected on each ℓ. The brackets around (J^P) donote a J^P at rest that does not contribute to elastic $K\pi$ scattering (e.g. 0^-, 1^+, 2^-, ...) and also $|\lambda|^{\tilde{\eta}} = 0^-$.