

The critical endpoint of the finite temperature phase transition for three flavor QCD with clover type fermions

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in collaboration with

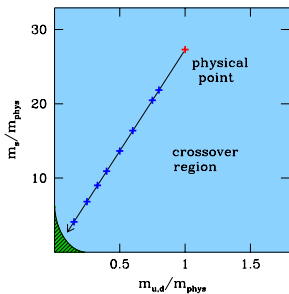
X.-Y. Jin, Y. Kuramashi, S. Takeda & A. Ukawa

Lattice 2013, 1 Aug. 2013, Mainz

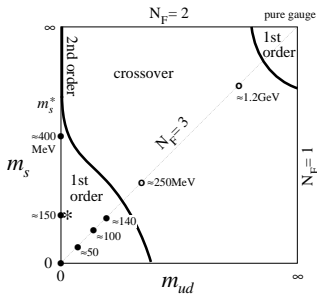


Critical endpoint(line) of $N_f = 3$ QCD at $\mu = 0$

- staggered type: (no continuum limit yet) [de Forcrand, Philipsen '07, Karsch, et. al. '03, Endrődi, et. al. '07]
 - m_π^E decreases with decreasing lattice spacing
- Wilson type: (no continuum limit yet)
 - heavy m_q region: boundary determined, [Saito, et. al. '11]
 - light m_q region: 1st order at rather heavy m_q (standard Wilson glue + Wilson fermion), [Iwasaki, et. al. '96]



[Endrődi, et. al. '07]



[Iwasaki, et. al. '96]



Motivation

- Critical endpoint obtained with staggered and Wilson type fermions is inconsistent
- Results in the continuum limit is necessary and $N_f = 3$ study is a stepping stone
 - the order of phase transition around the physical point
 - curvature of critical surface at $\mu = 0$

We determine the critical endpoint on **SU(3)** flavor symmetric line with clover type fermions

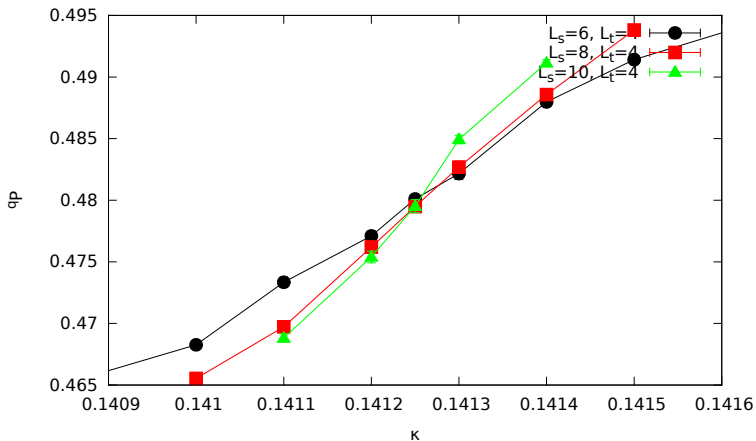


Simulations

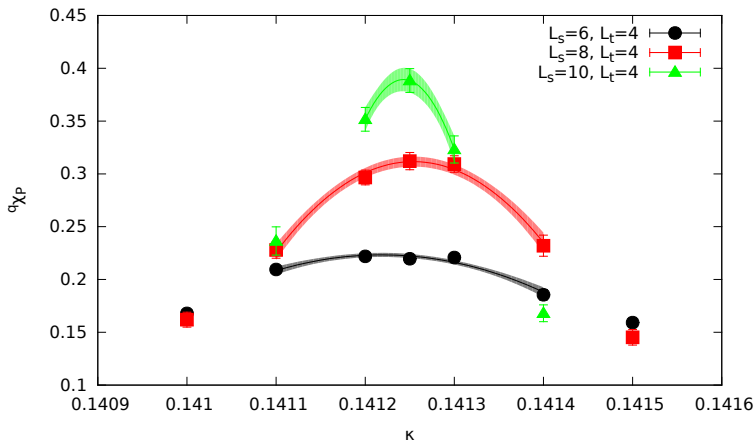
- action: Iwasaki gluon + $N_f = 3$ clover
(non perturbative c_{SW} , degenerate)
- temporal lattice size $L_t = 4, 6, 8$ for continuum extrapolation
- 3 spatial lattice sizes and a couple of β for each L_t to determine the critical endpoint by using intersection points of the Binder cumulants (kurtosis)
 - at $L_t = 4, L_s = 6, 8, 10, \beta = 1.60 - 1.73$
 - at $L_t = 6, L_s = 10, 12, 16, \beta = 1.73 - 1.77$
 - at $L_t = 8, L_s = 12, 16, 20, \beta = 1.73 - 1.78$
- statistics: O(10,000) - O(100,000) traj.
- machines:
 - K computer and Xeon cluster at AICS
 - FX10 at Uni. Tokyo
 - FX10 at Kyushu Uni.



plaquette at $\beta = 1.65$, $L_t = 4$

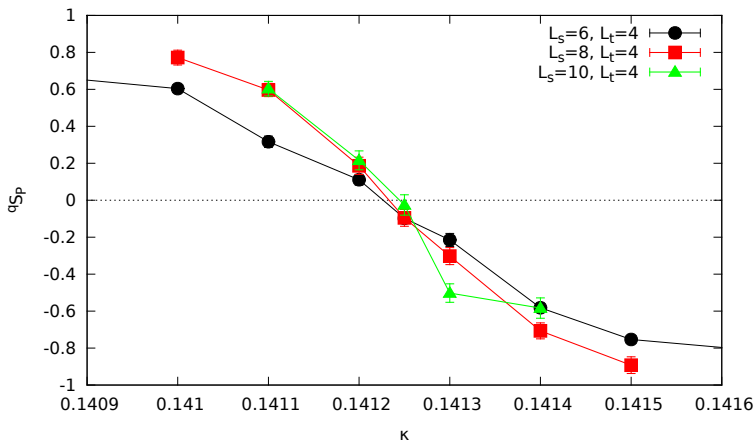


plaquette susceptibility at $\beta = 1.65$, $L_t = 4$

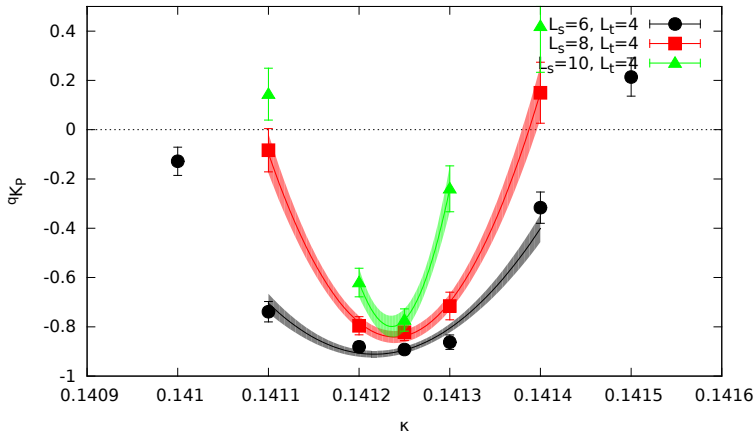


quadratic fit

plaquette skewness at $\beta = 1.65$, $L_t = 4$



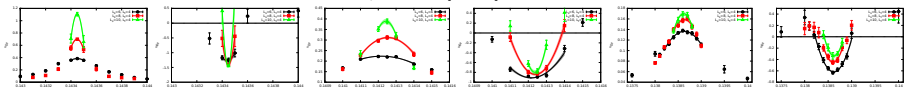
plaquette kurtosis ($= B_4 - 3$) at $\beta = 1.65$, $L_t = 4$



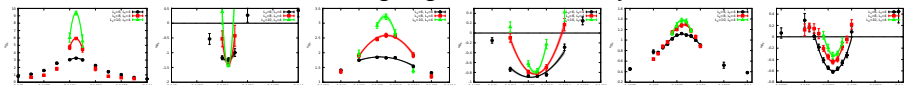
quadratic fit

P, S_g, L

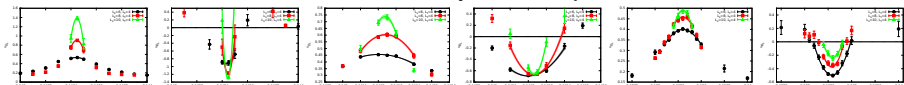
$L_t = 4$, plaquette



$L_t = 4$, gauge action density



$L_t = 4$, Polyakov loop



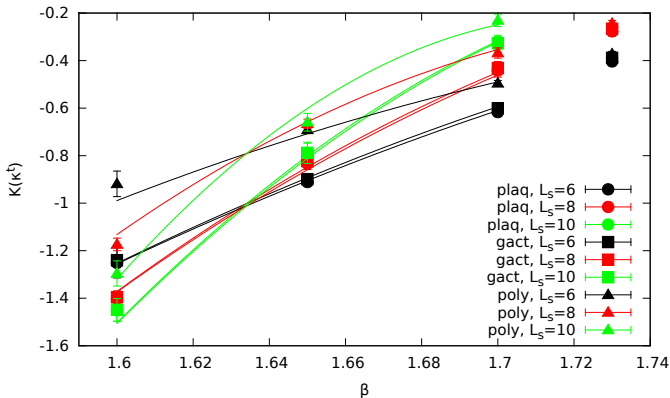
χ K
 $\beta = 1.60$

χ K
 $\beta = 1.65$

χ K
 $\beta = 1.70$

Critical endpoint at $L_t = 4$

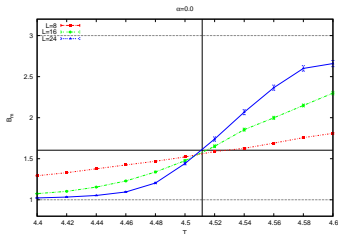
- $K(\kappa^t)$ is kurtosis value at transition point κ^t which is determined from the peak position of susceptibility
- fit(FSS inspired ansatz) : $K^E + a_0 L_s^{a_1} (\beta - \beta^E) + a_2 L_s^{2a_1} (\beta - \beta^E)^2$
- we have tried other fitting ansatz, linear, quadratic....



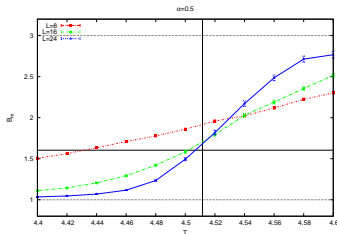
Intersection point

3D Ising

$$O = M$$



$$O = M + 0.5E$$



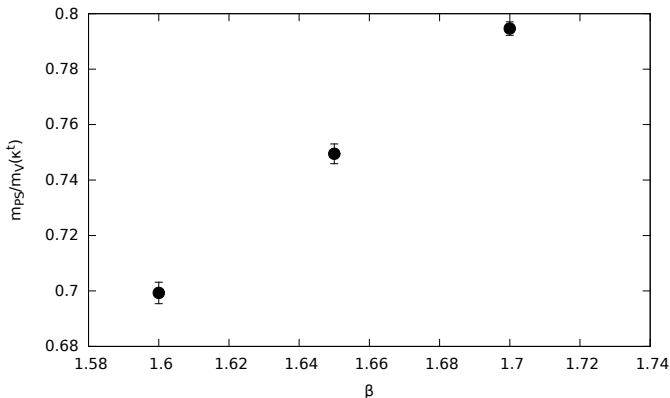
- B_4 intersection point for non-order parameter ($O = M + \alpha E$) is shifted due to finite volume effects

QCD with finite quark masses

- no order parameter
- larger lattice size, multiple observables analysis are necessary
 - heavy m_q region : Polyakov loop
 - light m_q region : chiral condensate

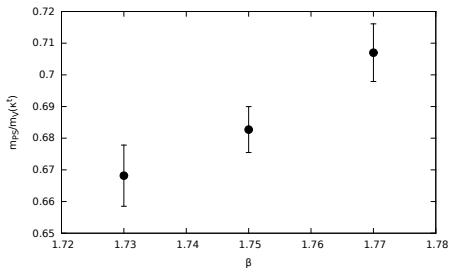
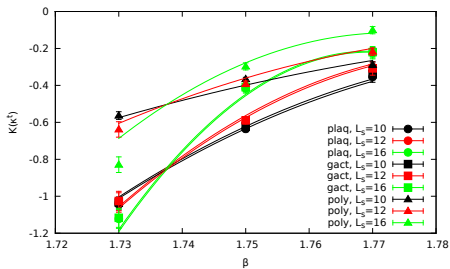
physical scale at $L_t = 4$ critical endpoint

- we measure hadron masses at transition points
- linear interpolation/extrapolation gives physical scale at the endpoint



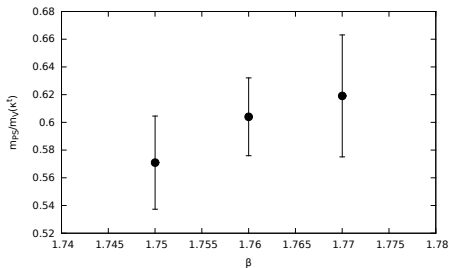
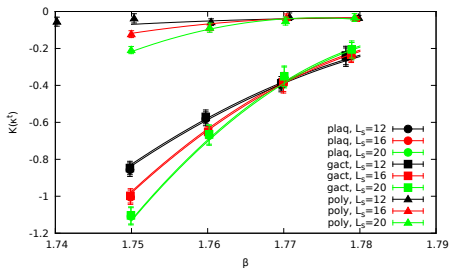
$$m_{PS}^E/m_V = 0.7326(36) \text{ at } L_t = 4$$

physical scale at $L_t = 6$ critical endpoint



$$m_{PS}^E / m_V = 0.6732(66) \text{ at } L_t = 6$$

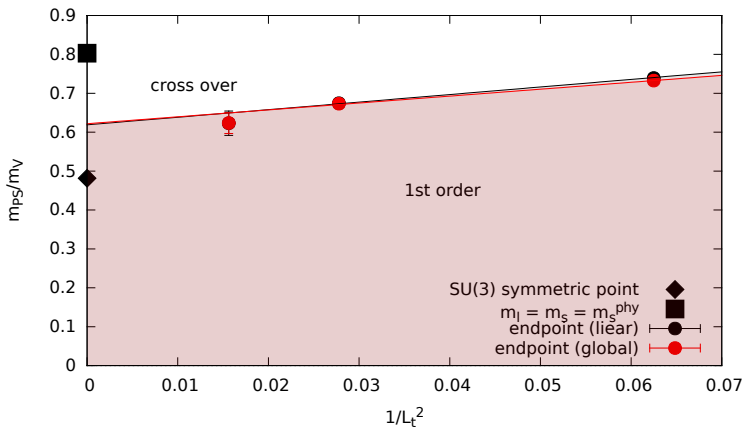
physical scale at $L_t = 8$ critical endpoint



$$m_{PS}^E/m_V = 0.624(27) \text{ at } L_t = 8$$

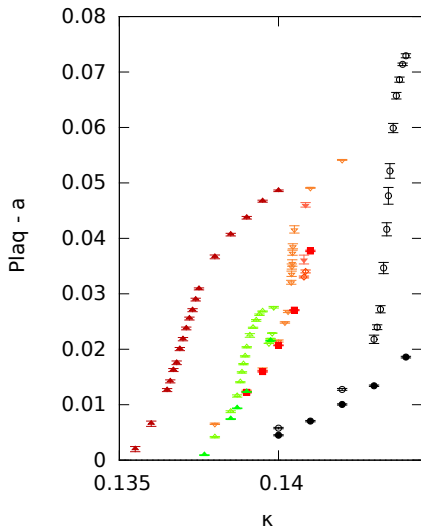
continuum extrapolation for m_{PS}^E / m_V

$$m_{PS}^{E;sym} / m_{PS}^{phy;sym} \sim 1.29(2)$$



$$\text{cf. } \frac{m_{PS}^{phy;sym}}{m_V^{phy;sym}} = \frac{\sqrt{(M_\pi^2 + 2M_K^2)}/3}{(M_\rho + 2M_{K^*})/3} \sim 0.4817, \quad \blacksquare : \frac{M_{\eta_{ss}}}{(M_\rho + 2M_{K^*})/3} \sim 0.8030$$

Bulk or NF



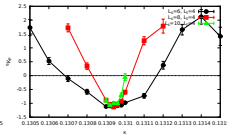
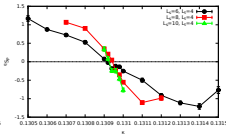
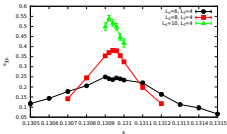
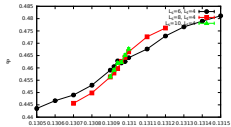
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b=1.60, a=0.42, Ls= 6, Lt=12
b=1.73, a=0.48, Ls= 6, Lt= 4
b=1.73, a=0.48, Ls=12, Lt= 6
b=1.73, a=0.48, Ls=12, Lt= 8
b=1.73, a=0.48, Ls=12, Lt=10
b=1.73, a=0.48, Ls=12, Lt=24
b=1.73, a=0.48, Ls=16, Lt=32
b=1.77, a=0.52, Ls=12, Lt= 6
b=1.77, a=0.52, Ls=24, Lt=10
b=1.77, a=0.52, Ls=12, Lt=24
b=1.77, a=0.52, Ls=16, Lt=32



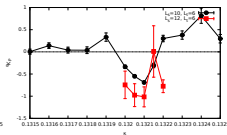
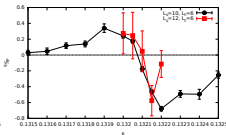
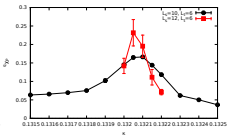
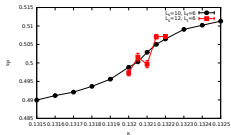
Smeared clover

- action: Iwasaki gluon + $N_f = 3$ smeared clover
 - stout link smearing both hopping and clover term
 - smearing parameter $\alpha = 0.1$, $n = 1$
 - non perturbative c_{SW}
- temporal lattice size $L_t = 4, 6$ and 2 spatial lattice sizes and a few β

$L_t = 4, \beta = 1.60$



$L_t = 6, \beta = 1.70$



Summary

- We have investigated the critical endpoint of QCD at $\mu = 0$ with clover type fermions
- We have determined the critical endpoint by using the intersection points of the Binder cumulants at $L_t = 4, 6, 8$ and extrapolated to the continuum limit
- We have found the critical endpoint at $1.66(5) \times (m_{uds}^{phy}, m_{uds}^{phy})$

where $m_{uds}^{phy} \equiv (m_u^{phy} + m_d^{phy} + m_s^{phy})/3$

- plan
 - smeared clover action to investigate discretization error (ongoing)
 - $\mu > 0$ for curvature of critical surface at $\mu = 0$ (ongoing, [Takeda, Wed])
 - $m_l \neq m_s$

sketch for endpoint

