Onset Transition to Cold Nuclear Matter from Lattice QCD with Heavy Quarks

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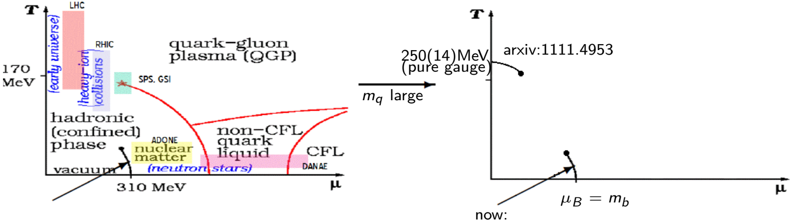
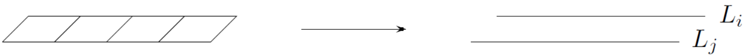


- Motivation: simulate cold dense nuclear matter
- Langevin Dynamics is applied to our effective theory derived by a strong coupling expansion

The Model



- We start with the partition function

$$Z = \int [dU_0][dU_i] \det[D] \, e^{\frac{\beta}{6}\sum_P (trU_P + trU_P^\dagger)}$$

- Effective fermionic action is introduced by a strong coupling expansion in the hopping parameter $\kappa = \frac{1}{2aM+8}$
- Effective gauge action is derived by an expansion in $\beta = \frac{2N}{g}$
- Spatial links are integrated over
- The resulting model describes QCD with large quark masses

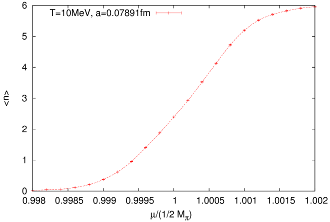
Leading Order

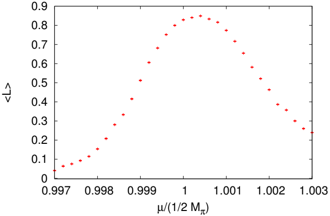
- Leading Order :
  System of static quarks, described by Polyakov Loops W winding through the temporal dimension

Static quarks

$$\det[Q]_{static} = \prod_i \det[1 + h_1 W_i]^2$$

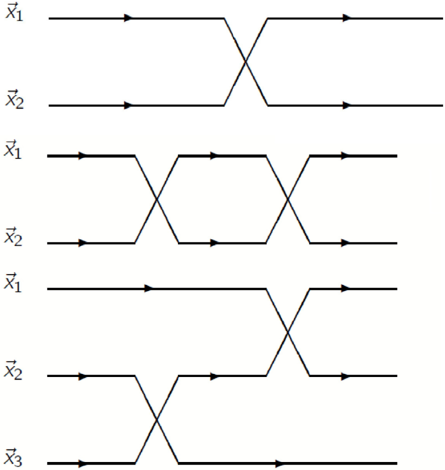
$$W_i = \prod_{\tau=1}^{N_\tau} U_0(\vec{x}_i, \tau)$$

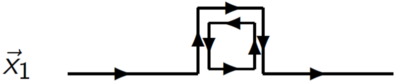




- Already simulated in the 80s using the Langevin Method [Karsch and Wyld, 1985]

Higher Orders





Higher orders: spatial hops between nearest and next to nearest neighbors ($O(\kappa^2)$) and $O(\kappa^4)$) introduce interaction, gauge corrections modify quark propagators

Langevin equation

$$\frac{\partial\phi(x,\theta)}{\partial\theta} = -\frac{\delta S[\phi]}{\delta\phi(x,\theta)} + \eta(x,\theta)$$

- System evolves as a stochastical process [Parisi and Wu, 1983]
- If S is complex all degrees of freedom will become complex too (doubling the degrees of freedom $\phi \to \phi^R + i\phi^I$)
- Stochastic process generates a well defined propability distribution $P(x,y)$ in the complex plane
- Observables are calculated over the resulting configurations

$$< O >_P = \frac{\int DxDyP(x,y)O(x,y)}{\int DxDyP(x,y)}$$

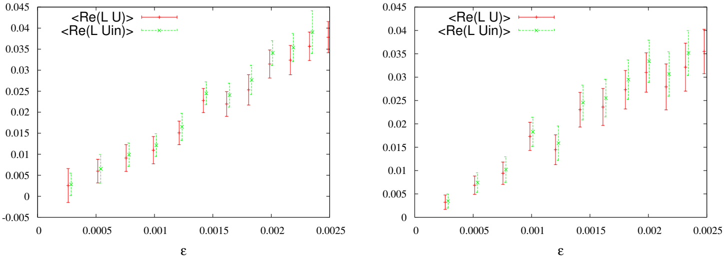
Convergence to the wrong limit

- It is long known that the Complex Langevin process converges to the wrong limit in some cases [Ambjorn, 1986]
- Have to ensure $< O >_P = < O >_\rho$ with $\rho \sim e^{-S}$
- To show correct convergence it has to be checked if $P(x, y)$ is sufficiently localised in the complex plane
- This can be checked via: [Aarts et al 2011]

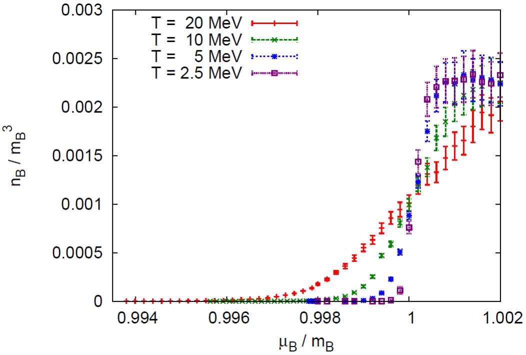
Criteria for correctnes

$$< LO > = 0, \quad L = \sum_{a,x} \left(\frac{\partial}{\partial \phi_{x,a}} - \frac{\partial S}{\partial \phi_{x,a}}\right) \frac{\partial}{\partial \phi_{x,a}}$$

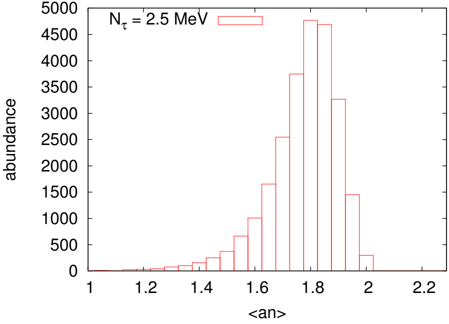
- We tested our model including κ^2(left) and κ^4(right) corrections



- The criterion is fullfilled in the limit of vanishing stepsize, indicating convergence to the right limit
- Additionally, results reproduce Monte Carlo simulations where these are possible

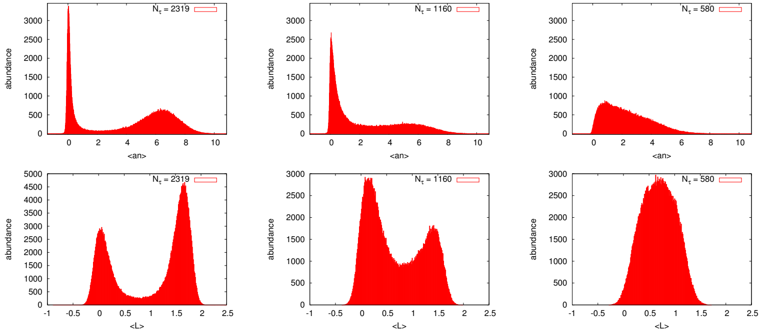


- $O(\kappa^2)$ action: simulate the transition for heavy quarks from the vacuum to finite density [Fromm, Langelage, Lottini, Neuman and Philipsen, 2012]
- The saturation density in units if m_B is of the same order of magnitude as the physical nuclear density
- Silver blaze property: μ-independece of observables for $\mu_b < m_b$



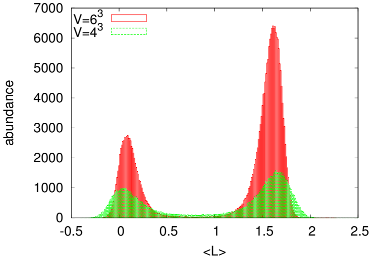
- Transition stays crossover for all investigated temperature
- Reason: nuclear binding energy $E_b \sim \frac{e^{-m_\pi r}}{r}$
- Heavy quarks $\rightarrow E_b \approx 0$

New Results (preliminary)

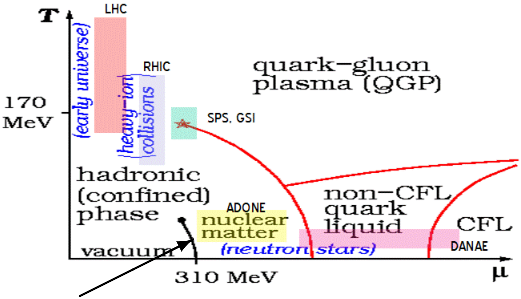


- $O(k^4)$:Stretching the hopping series, $\kappa = 0.12, \beta = 5.6$
- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

New Results (preliminary)



- Comparison between two different volumes (4^3 and 6^3) confirm the first order nature of the transition



- Effective theory makes correct qualitative predictions for nuclear liquid gas transition!

- Langevin Dynamics is capable to simulate an effective theory of lattice QCD in the limit of heavy quarks
- It seems possible to simulate the first order transition to cold nuclear matter for heavy quarks
- More work is needed to investigate the applicability to light quarks $\rightarrow$ calculate more orders in κ