A high-statistics study of the nucleon axial charge and quark momentum fraction

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Motivation

**Nucleon axial charge $g_A$**

- Experimental value is well determined: \[ g_A = 1.2701(25) \] \[ \text{[PDG, 2013]} \]
- Ideal benchmark quantity for Lattice QCD
  - Simple matrix element → local operator with quark bilinears
  - No momentum involved at initial and final state
  - Isovecor quantity → No disconnected diagrams
- So far Lattice results are typically $\sim 10\%$ below experimental value

**Quark momentum fraction $\langle x \rangle_{u-d}$**

- Benchmark quantity for Lattice QCD calculations
- Lattice computations tend to overestimate $\langle x \rangle_{u-d}$
- Important quantity to understand hadron structure
Nucleon axial charge $g_A$

- Improved local axial current:
  $$O_3(x) = \overline{\psi}(x)\gamma_3\gamma_5\psi(x) + a\overline{c}_a\partial_3 P + \mathcal{O}(a^2)$$

- Build ratio of 3-pt and 2-pt: $R(t, t_s) : = \frac{C_A^3(t, t_s)}{C_2(t_s)}$

- Extract $g_A^{\text{bare}}$ from ratio $R(t, t_s)$
  $$R(t, t_s)^{t, (t_s-t)\gg0} \rightarrow g_A^{\text{bare}} + \mathcal{O}(e^{-\Delta t}) + \mathcal{O}(e^{-\Delta(t_s-t)})$$

- Ratio should be independent of $t$ and $t_s$

- Renormalize $g_A = Z_A(1 + b_a m_q) g_A^{\text{bare}}$  

[Della Morte et al., 2008]
**Simulation details**

- $\mathcal{O}(a)$ improved Wilson fermions (Wilson clover) with $N_f = 2$
- CLS ensembles:

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<th>$L$ [fm]</th>
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<th>$m_\pi L$</th>
<th>Label</th>
<th># meas.</th>
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Nucleon 2-pt function

\[ a m_{\text{eff}} \]

\[ N6 \, \beta = 5.5, \, m_\pi = 340 \, \text{MeV}, \, L = 2.4 \, \text{fm} \]

2-pt function: excited states have died out \( t \sim 12 \)
Nucleon axial charge $g_A$

Excited states still present from source and sink

Simple plateau fits depend on source-sink separation $t_s$
Nucleon axial charge $g_A$

\[ f(t, t_s) = g_A + c_1 e^{-\Delta t} + c_2 e^{-\Delta (t_s-t)} + c_3 e^{-\Delta t_s} \]

- Included excitates states to fit ansatz

$N6 \beta = 5.5, m_\pi = 340 \text{ MeV}, L = 2.4 \text{ fm}$
Nucleon axial charge $g_A$

\[ S(t_s) := \sum_{t=1}^{t_s-1} R(t, t_s) \xrightarrow{t_s \gg 0} c + t_s \left( g_{A}^{\text{bare}} + \mathcal{O}(e^{-\Delta t_s}) \right) \]

- Summed operator insertion method [L.Maiani et al., 1987]
- Extract $g_A$ from the slope of a linear fit
Nucleon axial charge $g_A$

\[ S(t_s) := \sum_{t=1}^{t_s-1} R(t, t_s) \xrightarrow{t_s \gg 0} c + t_s \left( g^\text{bare}_A + O(e^{-\Delta t_s}) \right) \]
Including excited states lead to higher value for $g_A$

Summation and excited state fit agree
Nucleon axial charge $g_A$

Check summation and excited state fit by larger $t_s$ (up to $t_s \sim 1.4$ fm)

Signal-to-Noise ratio deteriorates quickly for large $t_s$
Results for $g_A$ a year ago arXiv:1205.0180
Update: increased statistics and more chiral ensemble.
Chiral behaviour of nucleon axial charge $g_A$

No strong dependence on $m_\pi^2 \to$ linear fit
Chiral behaviour of nucleon axial charge $g_A$

Heavy Baryon ChPT inspired fit [T.R. Hemmert et al., 2003]
Chiral behaviour of nucleon axial charge $g_A$

Restrict fit to chiral ensembles ($m_\pi \leq 365$ MeV)

Plateau $t_s \sim 1.1$ fm

Summation
Restrict fit to chiral ensembles ($m_\pi \leq 365$ MeV)
Outline

1. Nucleon axial charge

2. Quark momentum fraction of the nucleon $\langle x \rangle_{u-d}$
Quark momentum fraction of the nucleon $\langle x \rangle_{u-d}$

1. Insert operator with derivatives (with zero momentum transfer):

$$O(x) = \overline{\psi}(x) \left( \gamma_0 \vec{D}_0 - \frac{1}{3} \gamma_k \vec{D}_k \right) \psi(x)$$

2. Build ratio of 3-pt and 2-pt: $R(t, t_s) := \frac{C^O_{3}(t,t_s)}{C^O_{2}(t_s)}$

3. Extract $\langle x \rangle_{u-d}^{\text{bare}}$ from ratio $R(t, t_s)$:

$$R(t, t_s) \xrightarrow{t,(t_s-t) \gg 0} m_N \langle x \rangle_{u-d}^{\text{bare}} + \mathcal{O}(e^{-\Delta t}) + \mathcal{O}(e^{-\Delta (t_s-t)})$$

4. Ratio should be independent of $t$ and $t_s$

5. Renormalize $\langle x \rangle_{u-d}^{\text{bare}} \rightarrow \langle x \rangle_{u-d}$ using RI-MOM (not yet included)
Quark momentum fraction of the nucleon $\langle x \rangle_{u-d}$

- Plateaus depend on source-sink separation $t_s$
  - Very clear sign for excited states
- Summed operator insertion method works as for $g_A$
Conclusion

- Summed operator insertion method allows a systematic control of excited states
- Including excited states leads to agreement for the nucleon axial charge
- Chiral extrapolation improved by additional ensembles ($m^2_\pi < 200$ MeV)
- The quark momentum fraction suffers even more from excited states

Outlook

- Further improvements
  - Include renormalization for $\langle x\rangle_{u-d}$ using RI-MOM
  - Study finite size and volume effects (so far mild effect)
  - Simulations at the physical pion mass
  - Include a dynamical strange (and charm) quark
- Electromagnetic form factors $\rightarrow$ T. Rae’s talk
Thank you for your attention!
Checking summation method on N6 for larger $t_s$

- $g_A [\text{all } t_s] = 1.201(0.037) \Leftrightarrow g_A [t_s \leq 22] = 1.211(0.034)$
$g_A$ $B6$ $\beta = 5.2$, $m_\pi = 262$ MeV, $L = 3.8$ fm

$g_A$ $F7$ $\beta = 5.3$, $m_\pi = 277$ MeV, $L = 3.0$ fm

$O7$ $\beta = 5.5$, $m_\pi = 270$ MeV, $L = 3.2$ fm

$t_s = 10$ $t_s = 16$ $t_s = 12$ $t_s = 17$ $t_s = 13$ $t_s = 22$ $t_s = 16$ $t_s = 22$ $t_s = 19$
Fit individual $\beta$ to a constant

No clear sign of lattice artefacts ($\beta = 5.5$ tends to smaller $g_A$)
Heavy Baryon ChPT formula

- 6 free parameters: 3 are fixed to (physical) values

\[ c_A = 1.5 \text{ GeV}, \Delta_0 = 0.2711 \text{ GeV} \text{ and } \lambda = 1 \text{ GeV} \]

\[ g_A(m_{\pi}^2) = g_A^0 - \frac{(g_A^0)^3 m_{\pi}^2}{16 \pi^2 f_{\pi}^2} + 4 \left( C_{SSE}(\lambda) + \frac{c_A^2}{4 \pi^2 f_{\pi}^2} \left[ \frac{155}{972} g_1 - \frac{17}{36} g_A^0 \right] \right) \]

\[ + \gamma \ln \left( \frac{m_\pi}{\lambda} \right) m_{\pi}^2 + \frac{4 c_A^2 g_A^0}{27 \pi^2 f_{\pi}^2 \Delta_0} m_{\pi}^2 + \frac{8}{27 \pi^2 f_{\pi}^2} c_A^2 g_A^0 m_{\pi}^2 R(m_{\pi}) \]

\[ + \frac{c_A^2 \Delta_0^2}{81 \pi^2 f_{\pi}^2} (25 g_1 - 57 g_A^0) \left( \ln \frac{2 \Delta_0}{m_{\pi}} - R(m_{\pi}) \right) \]

with

\[ \gamma = \frac{1}{16 \pi^2 f_{\pi}^2} \left( \frac{50}{81} c_A g_1 - \frac{1}{2} g_A^0 - \frac{2}{9} c_A g_A^0 - (g_A^0)^3 \right) \]

\[ R(m_{\pi}) = \sqrt{1 - \frac{m_{\pi}}{\Delta_0}} \left( \frac{\Delta_0}{m_{\pi}} + \sqrt{\frac{\Delta_0}{m_{\pi}} - 1} \right) \]