Nucleon structure from stochastic estimates

J. Najjar

in collaboration with

G. Bali, S. Collins, B. Gläßle, M. Göckeler, R. Rödl, A. Schäfer, A. Sternbeck, W. Söldner

University of Regensburg- Institute for Theoretical Physics

August 1, 2013
1 Theoretical background
   • Motivation
   • Definitions
   • Related work
   • Optimization of signal to noise
   • Extracting form factors

2 Simulation details
   • Lattice Setup
   • Set up of the comparison

3 Results
   • Electromagnetic form factors
   • Axial form factors
   • Scalar form factor

4 Conclusion
Hadrons are composite objects with a non trivial internal structure. We call the building blocks partons. To describe them we can use GPDs, which are functions of the Bjorken $x_B$ and the momentum transfer $Q^2$. 
Our theoretical tool to compute (moments of) GPDs are matrix elements of operators of the type

\[ \mathcal{O}_\Gamma = \bar{\Psi} \Gamma \Psi , \]

which can be extracted on the lattice via three-point functions:

In this talk we will restrict ourselves to form factors \( FF(Q^2) \), which are the first Mellin Moment of the \( GPD(x_B, Q^2) \):

\[ FF(Q^2) = \int dx_B GPD(x_B, Q^2) \]
Inserting a current in a proton

\[ \vec{q} \]

\[ \vec{p}_i \quad \vec{p}_f \]

baryon source \hspace{1cm} baryon sink

To construct \( G(x, y) \) we have two possibilities:

1. fix all indices in the sink by fixing the polarization \( (T_{\alpha\bar{\alpha}}) \), \( \vec{p}_f \)
   \( (\sum \vec{x} e^{-i\vec{p}_f \vec{x}}) \), and the insertion flavor. (Sequential Propagator)

2. use a stochastic time-slice to all propagator: statistical noise
Related work

- The Regensburg group computed meson three-point functions with All-to-all propagators
  
  Evans, et. al. arXiv:1008.3293

- The ETM collaboration tried this for baryon three-point functions and computed $g_A$
  
  Alexandrou, et. al. arXiv:1302.2608

This talk

- We have explored other quantities with this method

- and have computed form factors
All-to-all propagators

- Random $\mathbb{C}_2 \equiv \mathbb{Z}_2 \times i\mathbb{Z}_2$ source vectors $\eta^i_{\alpha,a,x} = \frac{1}{\sqrt{2}}(v + iw)$ with $v, w \in \{\pm 1\}$ fulfill

\[
\frac{1}{N} \sum_{n=1}^{N} \eta^{n}_{i} \eta_{j}^{n\dagger} = \delta_{ij} + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right) \tag{1}
\]

- Solve $\chi^i = M^{-1}_{ij} \eta^j$ and recover $M^{-1}$ by

\[
\frac{1}{N} \sum_{n=1}^{N} \chi^{n}_{\beta,b,y} \eta^{n\dagger}_{\alpha,a,x} = M^{-1}_{\beta,b,y;\alpha,a,x} \left(1 + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)\right) \tag{2}
\]

S.-J. Dong, K.-F. Liu 94, G.S. Bali, H. Neff, T. Düssel, T. Lippert, K. Schilling (SESAM) 05
Note: $t \rightarrow N_t - t$, $\gamma_4 \rightarrow -\gamma_4$ is a symmetry and thus yields the same expectation values, but we can measure the transformed nucleon independently on each gauge configuration.
Note: \( t \rightarrow N_t - t, \gamma_4 \rightarrow -\gamma_4 \) is a symmetry and thus yields the same expectation values, but we can measure the transformed nucleon independently on each gauge configuration.
Note: $t \rightarrow N_t - t, \gamma_4 \rightarrow -\gamma_4$ is a symmetry and thus yields the same expectation values, but we can measure the transformed nucleon independently on each gauge configuration.
We extract a matrix element via a ratio of the three-point function and two-point functions:

\[
R_{t, \tau; \vec{p}_i, \vec{p}_f; O, \Gamma} = \frac{C_{3pt}^{t, \tau; \vec{p}_f, \vec{p}_i, O}}{C_{2pt}^{t; \vec{p}_f}} \times \left[ \frac{C_{2pt}^{\Gamma_u; \vec{p}_f} C_{2pt}^{t; \vec{p}_f} C_{2pt}^{t - \tau; \vec{p}_i}}{C_{2pt}^{\Gamma_u; \vec{p}_i} C_{2pt}^{t; \vec{p}_i} C_{2pt}^{t - \tau; \vec{p}_f}} \right]^{1/2}.
\]

Lorentz invariance permits to decompose the matrix elements into several structures:

\[
\langle P' | \bar{\Psi}_q \gamma^\mu_M \Psi_q | P \rangle = \langle \langle \gamma^\mu_M \rangle \rangle F_1(Q^2) + \frac{i}{2m_N} \langle \langle \sigma^{\mu\nu}_M \rangle \rangle \Delta^M_{\nu} F_2(Q^2),
\]

where

\[
\langle \langle O \rangle \rangle = \bar{U}(P') OU(P).
\]

This defines an overdetermined system of equations which we can solve using SVD. More combinations of momenta/polarizations for a given virtuality yield more equations and thus smaller errors.
Simulation details

- Configurations of the QCDSF collaboration
- Wilson Clover action
- Lattice Volume $32^3 \times 64$
- $\beta = 5.5$, $a = 0.074\text{fm}$, set with $w_0$ scale

- $N_f = 2 + 1$, $\kappa_{u,d} = \kappa_s = 0.1209$, $m_\pi = 440\text{ MeV}$
- $m_\pi L = 5.28$
- We use 400 steps of Wuppertal quark smearing

- on APE smeared gauge links

Borsanyi, et. al. arXiv:1203.4469


Set up of the comparison

- Unrenorm. data, insertion flavor u, no disconnected contrb.
- measured on a set of 103 cfgs using the same source position for the point-to-all propagator.
- Counting inversions: 12 (point-to-all), sequential: 12 per flavor and polarization (12x4), stochastic: 60
- To guide the eye: results for higher statistics (827 cfgs.) employing the stochastic method.
- We went up to $\vec{q}^2 |_{\text{max}} = 6$
- The sequential data was measured with $\vec{p}_f = \vec{0}$, unpolarized or pol. in $z$.
- For the stochastic 3pt-fns we use $\vec{p}_f^2 |_{\text{max}} = \vec{p}_i^2 |_{\text{max}} = 2$ and all polarizations
- Slightly different virtualities as we can split the transferred momentum between source and sink.
Electromagnetic form factor $F_1$
Electromagnetic form factor $F_2$

$F_2(t)$ vs $Q^2 [GeV]^2$

- Stoch. 103 configs
- Seq. 103 configs
- Stoch. 827 configs
Axial form factor $G_A$

\begin{align*}
\text{unrenormalized } G_A(t) \quad Q^2[\text{GeV}]^2
\end{align*}

stoch. 103 cfgs
seq. 103 cfgs
stoch. 827 cfgs
Pseudoscalar form factor $G_P$

Diagram showing the unrenormalized $G_P(t)$ as a function of $Q^2 [\text{GeV}]^2$. The graph includes data from stochastic 103 configs and sequential 103 configs, along with stochastic 827 configs.
Scalar form factor $G_S$
Conclusion

Stochastic three-point functions

- Stochastic baryon three-point functions work
- Cost efficiency is reached when many quantities are looked at
- Having many sink momenta one can access high $Q^2$ without using two-point functions with high momenta.
- Flexibility can be used to compute different $t_{\text{sink}}$, sink smearings, baryons at little additional cost

Thank you for the attention!