Nucleon structure from stochastic estimates

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in collaboration with

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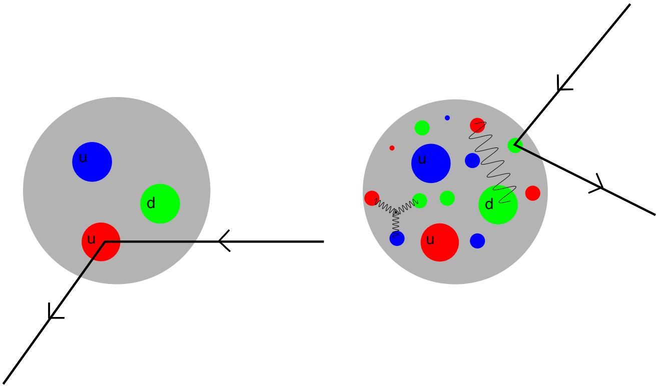
Partons

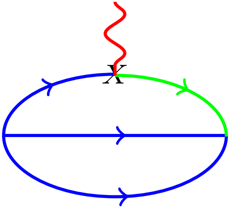
Hadrons are composite objects with a non trivial internal structure. We call the building blocks partons. To describe them we can use GPDs, which are functions of the Bjorken x_B and the momentum transfer Q^2.



Our theoretical tool to compute (moments of) GPDs are matrix elements of operators of the type

$$\mathcal{O}_\Gamma = \overline{\Psi}\Gamma\Psi\,,$$

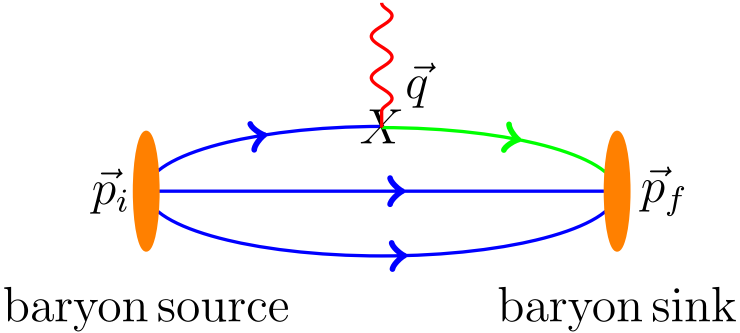
which can be extracted on the lattice via three-point funtions:



In this talk we will restrict ourselves to form factors $FF(Q^2)$, which are the first Mellin Moment of the $GPD(x_B, Q^2)$:

$$FF(Q^2) = \int \mathrm{d}x_B\, GPD(x_B, Q^2)$$

Inserting a current in a proton



To construct $G(x, y)$ we have two possibilities:
1. fix all indices in the sink by fixing the polarization $(T_{\alpha\overline{\alpha}})$, $\vec{p}_f$ $(\sum_{\vec{x}} e^{-\mathrm{i}\vec{p}_f\vec{x}})$, and the insertion flavor. (Sequential Propagator)
2. use a stochastic time-slice to all propagator: statistical noise

State of the art

Related work

- The Regensburg group computed meson three-point functions with All-to-all propagators Evans, et. al. arXiv:1008.3293
- The ETM collaboration tried this for baryon three-point functions and computed g_A Alexandrou, et. al. arXiv:1302.2608

This talk

- We have explored other quantities with this method
- and have computed form factors

All-to-all propagators

- Random $\mathbb{C}_2 \equiv \mathbb{Z}_2 \times \mathrm{i}\mathbb{Z}_2$ source vectors $\eta^i_{\alpha,a,x} = \frac{1}{\sqrt{2}}(v + \mathrm{i}w)$ with $v, w \in \{\pm 1\}$ fulfill

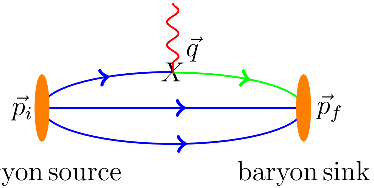
$$\frac{1}{N}\sum_{n=1}^{N} \eta_i^n \eta_j^{n\dagger} = \delta_{ij} + \mathcal{O}(\frac{1}{\sqrt{N}}) \tag{1}$$

- Solve $\chi^i = M^{-1}_{ij}\eta^j$ and recover M^{-1} by

$$\frac{1}{N}\sum_{n=1}^{N} \chi^n_{\beta,b,y}\eta^{n\dagger}_{\alpha,a,x} = M^{-1}_{\beta,b,y;\alpha,a,x}\left(\mathbb{1} + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)\right) \tag{2}$$

S.-J.Dong, K.-F.Liu 94 , G.S. Bali, H. Neff, T. Düssel, T. Lippert, K. Schilling (SESAM) 05

Note: $t \to N_t - t\,, \gamma_4 \to -\gamma_4$ is a symmetry and thus yields the same expectation values, but we can measure the transformed nucleon independently on each gauge configuration.



T-symmetry partner averaging

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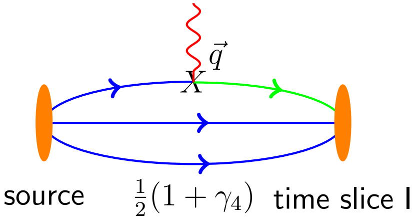

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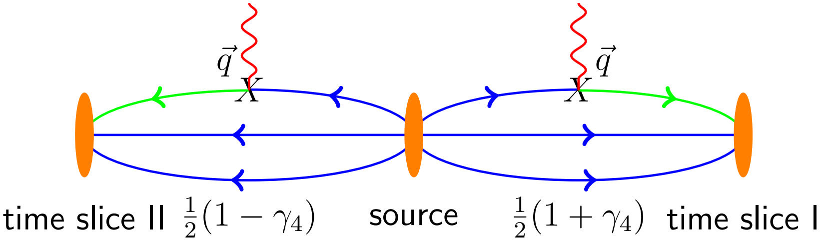


Extracting form factors

We extract a matrix element via a ratio of the three-point function and two-point functions:

$$R_{t,\tau;\vec{p}_i,\vec{p}_f;O,\Gamma} = \frac{C^{3pt}_{\Gamma}(t,\tau;\vec{p}_f,\vec{p}_i,O)}{C^{2pt}_{\Gamma_u}(t;\vec{p}_f)} \times \left[\frac{C^{2pt}_{\Gamma_u}(\tau;\vec{p}_f)C^{2pt}_{\Gamma_u}(t;\vec{p}_f)C^{2pt}_{\Gamma_u}(t-\tau;\vec{p}_i)}{C^{2pt}_{\Gamma_u}(\tau;\vec{p}_i)C^{2pt}_{\Gamma_u}(t;\vec{p}_i)C^{2pt}_{\Gamma_u}(t-\tau;\vec{p}_f)}\right]^{\frac{1}{2}} .$$

Lorentz invariance permits to decompose the matrix elements into several structures:

$$\langle P' \mid \overline{\Psi}_q \gamma^\mu_M \Psi_q \mid P\rangle = \langle\langle \gamma^\mu_M \rangle\rangle\, F_1(Q^2) + \frac{\mathrm{i}}{2m_N} \langle\langle \sigma^{\mu\nu}_M \rangle\rangle\, \Delta^M_\nu F_2(Q^2)\,,$$

where

$$\langle\langle O \rangle\rangle = \overline{U}(P')OU(P)\,.$$

This defines an overdetermined system of equations which we can solve using SVD. More combinations of momenta/polarizations for a given virtuality yield more equations and thus smaller errors.

Simulation details

- Configurations of the QCDSF collaboration
- Wilson Clover action
- Lattice Volume $32^3 \times 64$
- $\beta = 5.5$, $a = 0.074$fm, set with w_0 scale

Borsanyi, et. al. arXiv:1203.4469

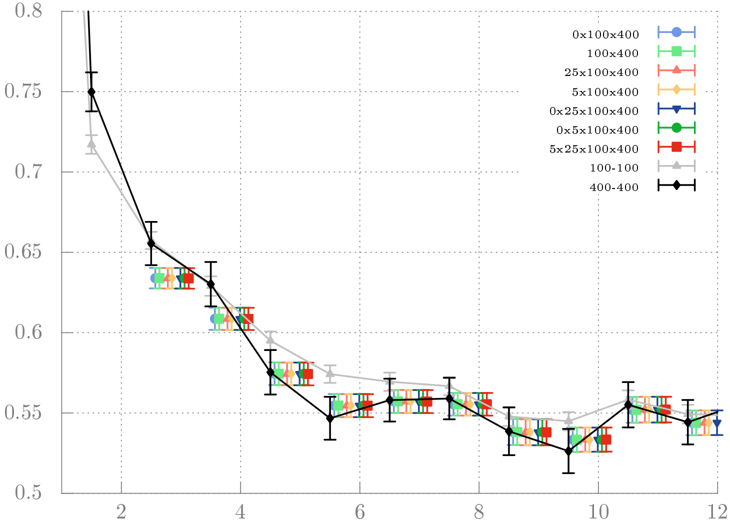
- $N_f = 2 + 1$, $\kappa_{u,d} = \kappa_s = 0.1209$, $m_\pi = 440$ MeV
- $m_\pi L = 5.28$
- We use 400 steps of Wuppertal quark smearing

Gusken, Nucl.Phys.Proc.Suppl. 17 (1990)

- on APE smeared gauge links

Albanese, M. Phys. Lett. B192 (1987)

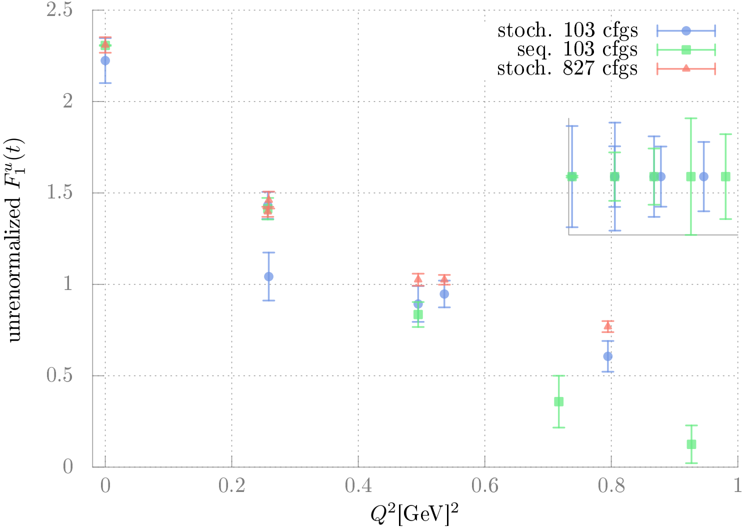
Smearing


0.8
0.75
0.7
0.65
0.6
0.55
0.5
2
4
6
8
10
12
0x100x400
100x400
25x100x400
5x100x400
0x25x100x400
0x5x100x400
5x25x100x400
100-100
400-400

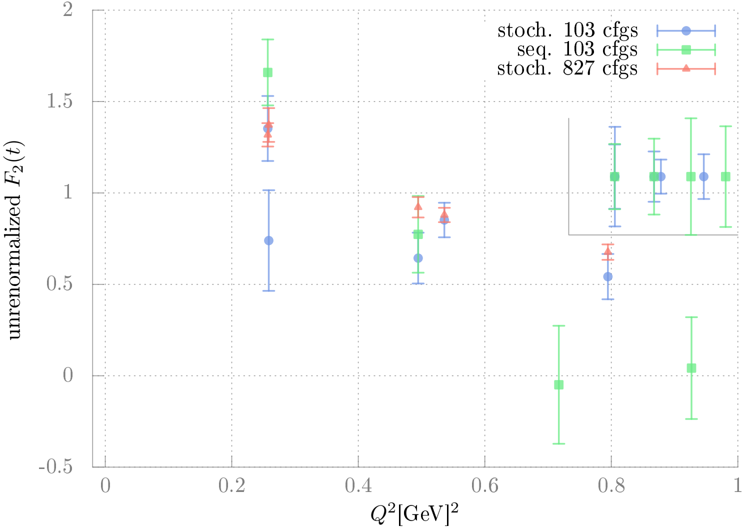
Set up of the comparison

- Unrenorm. data, insertion flavor u, no disconnected contrb.
- measured on a set of 103 cfgs using the same source position for the point-to-all propagator.
- Counting inversions: 12 (point-to-all), sequential: 12 per flavor and polarization (12x4), stochastic: 60
- To guide the eye: results for higher statistics (827 cfgs.) employing the stochastic method.
- We went up to $\vec{q}^{\,2}\mid_{\max} = 6$
- The sequential data was measured with $\vec{p}_f = \vec{0}$, unpolarized or pol. in z.
- For the stochastic 3pt-fns we use $\vec{p}_f^{\,2}\mid_{\max} = \vec{p}_i^{\,2}\mid_{\max} = 2$ and all polarizations
- Slightly different virtualities as we can split the transfered momentum between source and sink.

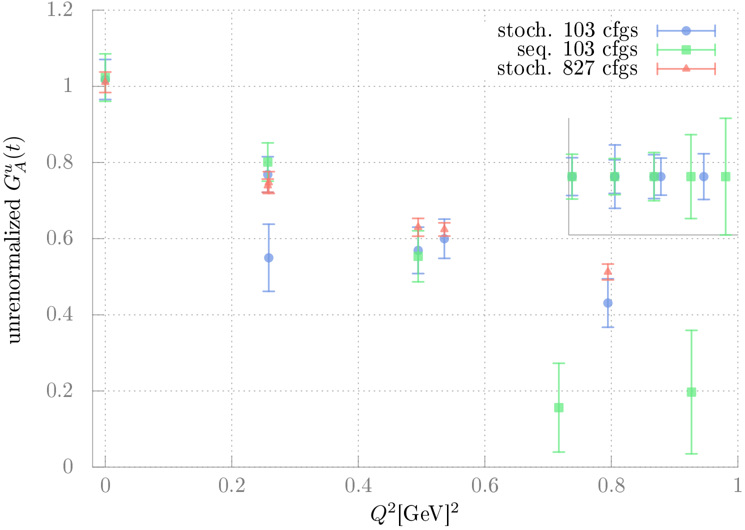
Electromagnetic form factor F_1



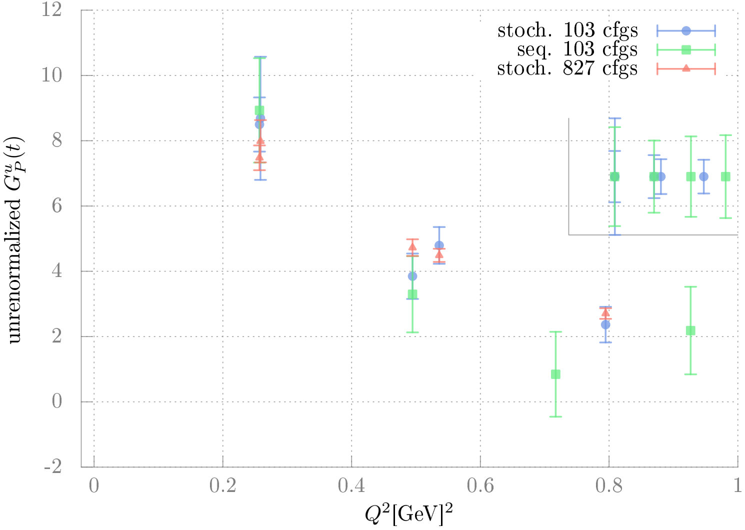
Electromagnetic form factor F_2



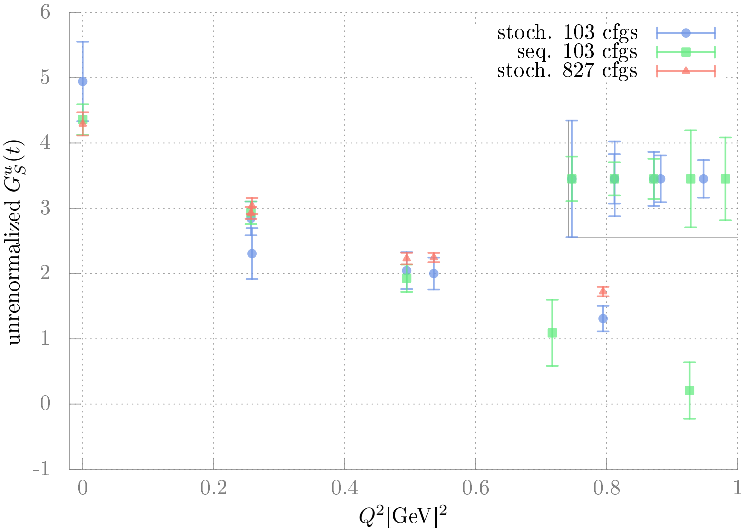
Axial form factor G_A



Pseudoscalar form factor G_P



Scalar form factor G_S



Conclusion

Stochastic three-point functions

- Stochastic baryon three-point functions work
- cost efficiency is reached when many quantities are looked at
- Having many sink momenta one can access high Q^2 without using two-point functions with high momenta.
- Flexibility can be used to compute different t_{sink}, sink smearings, baryons at little additional cost

Thank you for the attention!