Computation of disconnected contributions to nucleon observables

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August 1st, 2013

The Cyprus Institute

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Outline

- Stochastic procedures
- The Truncated Solver Method (TSM)
- The one-end trick
- Time-dilution
- The Hopping Parameter Expansion (HPE)
- Results
- Conclusions

Stochastic procedures

- **Exact computation of the all-to-all unfeasible nowadays**
- **We can use stochastic techniques**
  - Invert a random set of sources $|\eta_j\rangle$ that form a basis up to stochastic errors
  - Properties $\begin{cases} \frac{1}{N}\sum_{j=1}^{N} |\eta_j\rangle = O\left(\frac{1}{\sqrt{N}}\right) \\ \frac{1}{N}\sum_{j=1}^{N} |\eta_j\rangle\langle\eta_j| = I + O\left(\frac{1}{\sqrt{N}}\right) \end{cases}$
  - In this work we use $\mathbf{Z}_2$ and $\mathbf{Z}_4$ noise sources
- **So we get an unbiased estimation of the all-to-all propagator**

$$M|s_j\rangle = |\eta_j\rangle \longrightarrow M_E^{-1} := \frac{1}{N}\sum_{j=1}^{N} |s_j\rangle\langle\eta_j| \approx M^{-1}$$

- **Error decresases as** $1/\sqrt{N}$

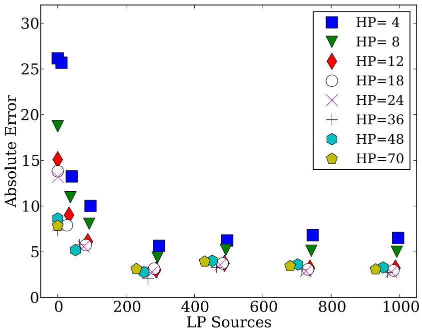
The Truncated Solver Method

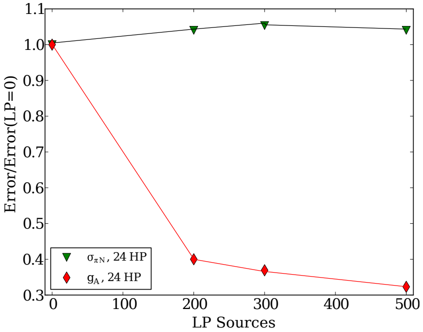
- **Instead of solving $M\,|s_j\rangle = |\eta_j\rangle$ exactly, we aim at a low precision estimation** Bali, Collins, Schäffer 2007
  - Cut the inverter (CG) at a certain number of iterations OR at a given precision $\rho^2 \sim 10^{-4}$
- **Cheap but inaccurate $\longrightarrow$ We introduce a bias we correct stochastically**

$$M_E^{-1} := \frac{1}{N_{HP}} \sum_{j=1}^{N_{HP}} \left(|s_j\rangle \langle \eta_j|_{HP} - |s_j\rangle \langle \eta_j|_{LP} \right) + \frac{1}{N_{LP}} \sum_{j=N_{HP}+1}^{N_{HP}+N_{LP}} |s_j\rangle \langle \eta_j|_{LP}$$

- **If the convergence in the inversions is fast, we can get away with a low N_{HP}**
- **Error should decrease essentially as $1/\sqrt{N_{LP}}$**
- **Requires loop-dependent fine-tuning**

Determination of the TSM parameters





- Data for $\bar{\psi}\gamma_3 D_3 \psi$
- ~12HP/300LP seem enough for this loop

- Data for $\sigma_{\pi N}$ and g_A
- g_A has not converged, extended to 24HP/500LP

The one-end trick

- **General trick that reduces variance, generally applied to 2pt** Foster, Michael 1998; McNeile, Michael 2006
- **Propagators in tmQCD can be arranged in a way that allows the application of the one-end trick**
- **The difference of propagators in the twisted basis is**

$$M_u - M_d = 2i\mu\gamma_5 \qquad M_u^{-1} - M_d^{-1} = -2i\mu M_d^{-1}\gamma_5 M_u^{-1}$$

$$\sum X \left(M_u^{-1} - M_d^{-1}\right) = -2i\mu \sum_r \left\langle s^\dagger X \gamma_5 s \right\rangle_r$$

- **Errors are considerably reduced**
  - The μ factor suppresses the noise
  - The volume sum enhances statistics
  - Improves signal-to-noise ratio from $\left(\frac{1}{\sqrt{V}}\right)$ to $O(1)$

The one-end trick

- **In principle, the trick only works for the difference, but an alternative version can be developed for the sum**

$$\bar{\psi}\gamma_5\gamma_\mu\psi \to \bar{\psi}\gamma_5\gamma_\mu\psi$$

$$\sum X\left(M_u^{-1} + M_d^{-1}\right) = 2\sum_r \left\langle s^\dagger \gamma_5 X \gamma_5 D_W s\right\rangle_r$$

- **Unfortunately, the results are not so good**
  - We lack the μ suppressing factor here
  - The Dirac operator in the loop can increase the noise
  - We still have the volume sum
  - Similar results to time-dilution + HPE for physical strange mass

Time-dilution

- **Dilution splits the source in m pieces, eliminating interactions among neighbors** Bernardson et al. 1993
- **We will use here dilution in time**
- **The Coherent Source Method allows us to calculate several time-slices in a single inversion**
  - Put noise in distant time-slices
  - Noise decreases exponentially
  - If distant enough, the time-slices won't interact

The hopping parameter expansion (HPE)

- Expansion of the inverse fermionic matrix in the hopping parameter
- For twisted mass fermions, Foster, McNeile, Michael 1999

$$M_u^{-1} = B - BHB + (BH)^2 B - (BH)^3 B + (BH)^4 M_u^{-1}$$

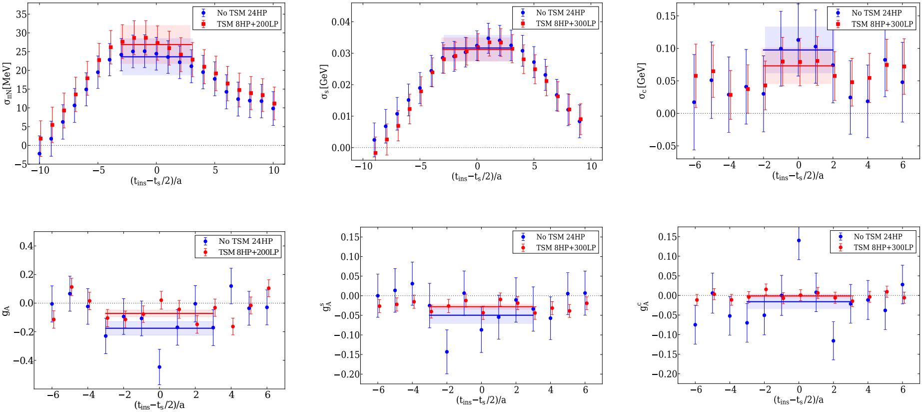
$$B = (1 + i2\kappa\mu a\gamma_5)^{-1} \qquad H = 2\kappa D$$

- The first four terms are computed exactly. The last is

$$\frac{1}{N}\sum_r^N \left[X (BH)^4 s_r \eta_r^\dagger\right] = \mathbf{Tr}\left[X (BH)^4 M_u^{-1}\right] + O\left(\frac{1}{\sqrt{N}}\right)$$

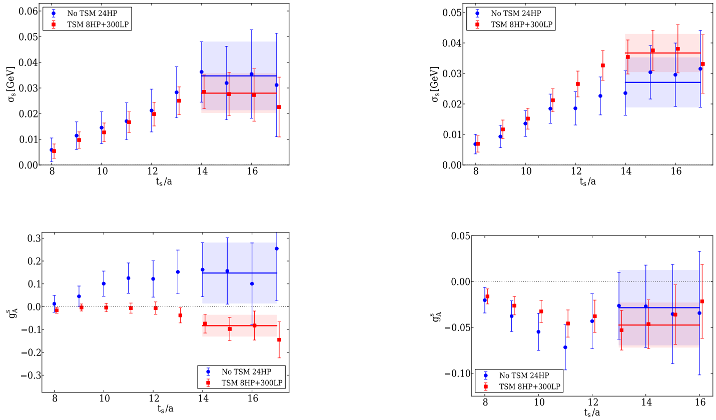
- First terms in the expansion are expected to be the noisiest

Results: TSM performance, the one-end trick



- N_F=2+1+1, stats 56400 light, 58560 strange and charm
- $m_\pi \approx 380$MeV, m_s and m_c physical
- $\sigma_q \to m_q \langle N| \bar{q}q |N\rangle$, good for $g_A^q = \to \langle N| i\bar{q}\gamma_\mu\gamma_5 q |N\rangle$

Results: TSM performance, time-dilution, HPE



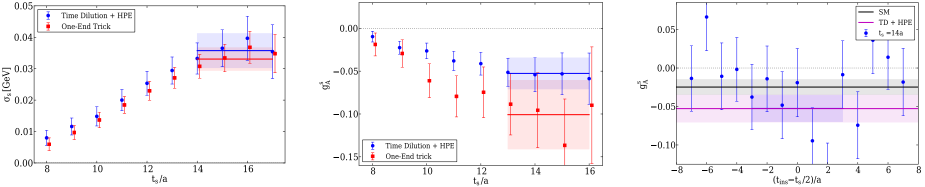
- N_F=2+1+1, results for strange quark, stats 18628
- $m_\pi \approx 380$MeV, m_s and m_c physical
- TSM always improves

Results: TSM performance, a word on overhead

Method	Quark sector	HP Inv	LP Inv	Overhead	$R_{HP/LP}$
One-end tck Local	Light	$\approx$ 90s	$\approx$ 2.2s	$\approx$ 2.5s	$\approx$ 26.7
One-end tck Dev	Light	$\approx$ 90s	$\approx$ 2.2s	$\approx$ 8.0s	$\approx$ 10.0
One-end tck Local	Strange	$\approx$ 29s	$\approx$ 0.6s	$\approx$ 2.5s	$\approx$ 16.9
One-end tck Dev	Strange	$\approx$ 29s	$\approx$ 0.6s	$\approx$ 8.0s	$\approx$ 5.8
Time-dil.	Strange	$\approx$ 29s	$\approx$ 0.6s	$\approx$ 1.0s	$\approx$ 20.7
Time-dil. + HPE	Strange	$\approx$ 29s	$\approx$ 0.6s	$\approx$ 1.2s	$\approx$ 19.1
One-end tck Local	Charm	$\approx$ 3.3s	$\approx$ 0.3s	$\approx$ 2.5s	$\approx$ 2.9
One-end tck Dev	Charm	$\approx$ 3.3s	$\approx$ 0.3s	$\approx$ 8.0s	$\approx$ 1.7

- **$R_{HP/LP}$ is the number of LP sources equivalent to one HP source in terms of computer time**
- **If HP inversion expensive, TSM always pays off**
- **If HP inversion not so expensive, overhead plays key role**
- **Results depend on implementation!! Our implementation: GPUs**

Results: One-end trick vs time-dilution + HPE @ strange quark mass



- Plots at fixed insertion time
- One-end trick for the difference σ_s clearly superior
- In contrast, the sum g_A^s seems to lag behind time-dilution + HPE
- The one-end trick gives all time-slices, plateau fit with reduced correlations possible
- In the end, same performance with one-end trick and time-dilution + HPE for g_A^s

Conclusions

- **TSM highly reduces the variance while keeping the same computer cost**
- **The one-end trick gives great results and all time-slices at low cost, but shows reduced performance for heavy quark with TSM**
- **Time-dilution + HPE has roughly the same cost as time-dilution alone and improves results greatly**
- **Time-dilution requires several inversions for one-derivative insertions**
- **HPE expected to work better with larger quark masses**
- **Rules of thumb:**
  - Light/strange quarks with one-end trick + TSM
  - Heavy quarks with time-dilution + HPE + TSM
  - For one-derivative contractions, one-end trick (+ TSM with light/strange quarks)

This presentation has been funded by the Research Promotion Fundation under project ΠΡΟΣΕΛΚΥΣΗ/ΝΕΟΣ/0609/16