# Kaon semileptonic decay from the SU(3)-symmetric point down to physical quark masses

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IDEAS

# RBC+UKQCD Collaborations


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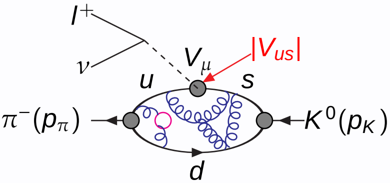
# Motivation

- computation of non-perturbative physics contribution to SM process $K \to \pi l \nu$
- tests and constraints for the SM, e.g. $|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2 + |V_{\mathrm{ub}}|^2 = 1 + \delta$
- needed in a model-independent way
- kaons and pions rather clean in lattice QCD (no large scale separations)
- KLOE-2 promised to provide new data and reduce exp. error
  *Eur.Phys.J. C68(2010)*

# $|V_{us}|$ from $K_{l3}$ decay




$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW}(1+\Delta_{SU(2)}+\Delta_{\rm EM})^2\, I\, |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

- $I$ phase space integral (via FF shape from experiment)
- $S_{\rm EW}$ short distance EW corrections
- $\Delta_{SU(2)}$ iso-spin breaking corrections ($\chi$PT)
- $\Delta_{\rm EM}$ long distance EM corrections ($\chi$PT)

$f_+^{K\pi}(0)|V_{us}| = 0.2163(5) \to$ 0.3%-precision for $f_+^{K\pi}(0)$ required

*FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)*

# $K \to \pi$ form factor in $\chi$PT

$$f_+^{K\pi}(0) = 1 + f_2(f, m_\pi, m_K, m_\eta) + f_4(m_\pi, m_K, m_\eta, f, \mathrm{LEC}) + \ldots$$

$f_2$ known function of the meson masses (SU(3) $\chi$PT):

- $f_2(f_\pi, m_\pi^2, m_K^2)|_{\rm phys} = -0.023$ $(K^0 \to \pi^- l\nu)$ *Gasser, Leutwyler, Nucl.Phys.B250,1985*
- $f_2$ depends on the expansion parameter $f$: $f_2 \propto \frac{1}{f}(H_{\pi K} + H_{\eta K})$
  *cf. study in RBC/UKQCD Eur.Phys.J. C69 (2010) 159-167*
- $f_2 \propto (m_K^2 - m_\pi^2)^2/m_K^2 + \ldots$
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- cutoff- and finite size effects SU(3) symmetry suppressed: corrections only on $1 - f_+^{K\pi}(0)$

# Definition of the form factor in terms of correlators

$$\langle\pi(p_\pi)|V_\mu|K(p_K)\rangle|_{q^2=0} = f_+^{K\pi}(0)(p_\pi+p_K)_\mu + f_-^{K\pi}(0)(p_\pi-p_K)_\mu$$

$$\langle\pi(p_\pi)|S\ \ |K(p_K)\rangle|_{q^2=0} = f_0^{K\pi}(0)\frac{m_K^2-m_\pi^2}{m_s-m_u}$$ *HPQCD PRD 82, 114506 (2010)*

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$$R^{(2)}_{\mu,K\pi}(t_{\rm snk},\vec{p}_K,\vec{p}_\pi) = 2\sqrt{E_K E_\pi}\sqrt{\frac{C_{\mu,K\pi}(t,t_{\rm snk},\vec{p}_K,\vec{p}_\pi)\,C_{\mu,\pi K}(t,t_{\rm snk},\vec{p}_\pi,\vec{p}_K)}{C_{0,KK}(t,t_{\rm snk},\vec{p}_K,\vec{p}_K)\,C_{0,\pi\pi}(t,t_{\rm snk},\vec{p}_\pi,\vec{p}_\pi)}},$$

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- basic technique: define ratios of correlation functions with good signal/noise properties Hashimoto et al., Phys.Rev. D61 (1999) 014502
- $R^{(2)}$ renormalised, similar ratio $R^{(1)}$ cf. Boyle et al. JHEP 0705 (2007) 016

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- $R^{(2)}$ renormalised, similar ratio $R^{(1)}$ cf. Boyle et al. JHEP 0705 (2007) 016
- analyse all four components of vector current $\rightarrow$ system of linear equations $\rightarrow f_+^{K\pi}(0)$ ($f_-^{K\pi}(0)$)
- SU(3)-symmetry: $f_+^{K\pi}(0) \stackrel{\hat{m}_q\rightarrow m_s}{=} 1$ exact when using above $V$-ratio

# Lattice setup

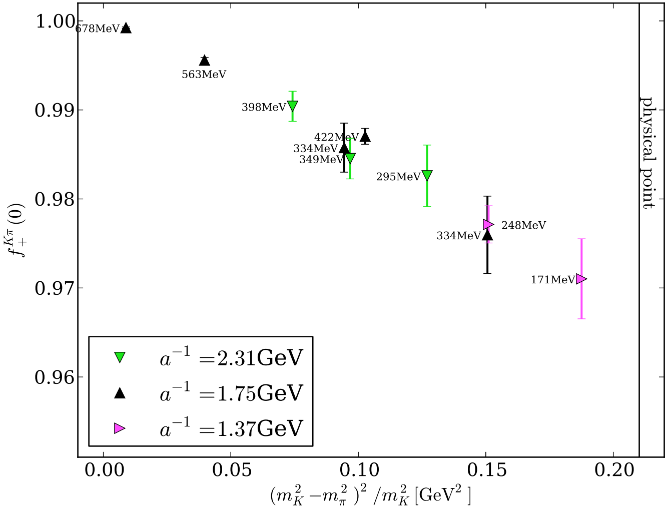
| $L/a$ | $a$/fm | $m_\pi$/MeV | $m_\pi L$ | $L/a$ | $a$/fm | $m_\pi$/MeV | $m_\pi L$ |
|---|---|---|---|---|---|---|---|
| 24 | 0.11 | 678 | 9.3 | 32 | 0.09 | 398 | 5.5 |
| 24 | 0.11 | 563 | 7.7 | 32 | 0.09 | 349 | 4.8 |
| 24 | 0.11 | 422 | 5.8 | 32 | 0.09 | 295 | 4.1 |
| 24 | 0.11 | 334 | 4.6 | 32 | 0.14 | 248 | 5.7 |
| 24 | 0.11 | 334 | 4.6 | 32 | 0.14 | 171 | 3.9 |
| 48 | 0.11 | 141 | 3.9 | preliminary | | | |
| 64 | 0.09 | 137 | 3.8 | | | | |

accepted by JHEP
arXiv:1305.7217

$a = 0.09$, 0.11fm Iwasaki+Shamir/Moebius,
$a = 0.14$ Iwasaki+DSDR+Shamir

- RBC+UKQCD $N_f = 2 + 1$ domain wall configurations
- only unitary light mass points
- blue: all-mode-averaging *Blum et al. arXiv:1208.4349*
- partially twisted boundary conditions $\to$ all valence computations directly for $q^2 = 0$ *Boyle et al. Eur.Phys.J. C69 (2010) 159-167*

# Bare data




- mistuning in $m_s$ nicely absorbed by rescaling of $x$-axis

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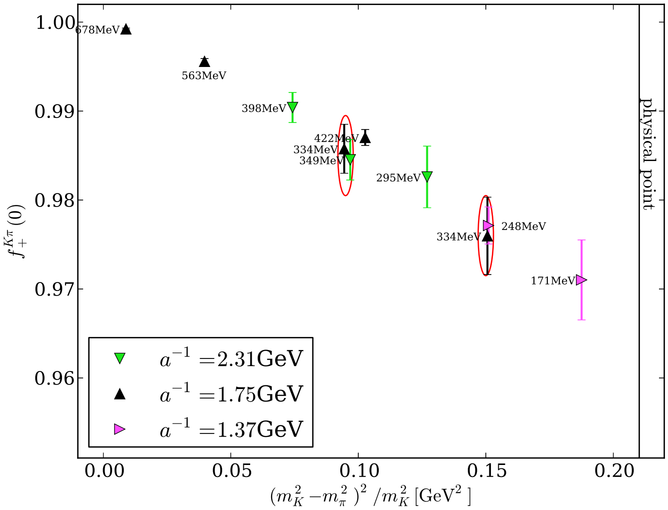


- mistuning in $m_s$ nicely absorbed by rescaling of $x$-axis
- cut-off effects well below stat. accuracy (even for DSDR)

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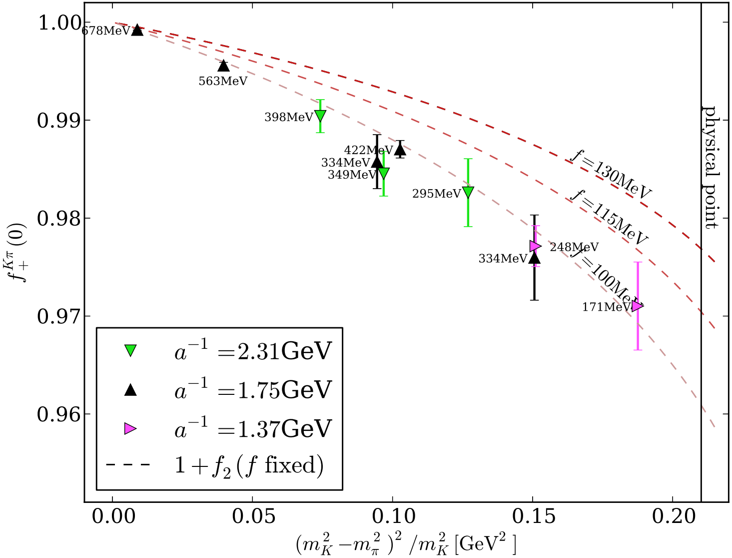

1.00
0.99
0.98
0.97
0.96
0.00
0.05
0.10
0.15
0.20
$f_+^{K\pi}(0)$
$(m_K^2 - m_\pi^2)^2 / m_K^2$ [GeV$^2$]
678MeV
563MeV
398MeV
422MeV
334MeV
349MeV
295MeV
248MeV
334MeV
171MeV
$f$=130MeV
$f$=115MeV
$f$=100MeV
physical point
$a^{-1}$ =2.31GeV
$a^{-1}$ =1.75GeV
$a^{-1}$ =1.37GeV
$1+f_2$ ($f$ fixed)

# Ansätze for chiral extrapolation

A) $f_+^{K\pi}(0) = A + \frac{(m_K^2 - m_\pi^2)^2}{m_K^2} A_0$

B) $f_+^{K\pi}(0) = 1 + f_2(f, m_\pi^2, m_K^2, m_\eta^2) + (m_K^2 - m_\pi^2)^2 \left(A_0 + A_1(m_K^2 + m_\pi^2)\right)$

C) $f_+^{K\pi}(0) = 1 + \frac{(m_K^2 - m_\pi^2)^2}{m_K^2} \left(A_0 + A_1(m_K^2 + m_\pi^2)\right)$

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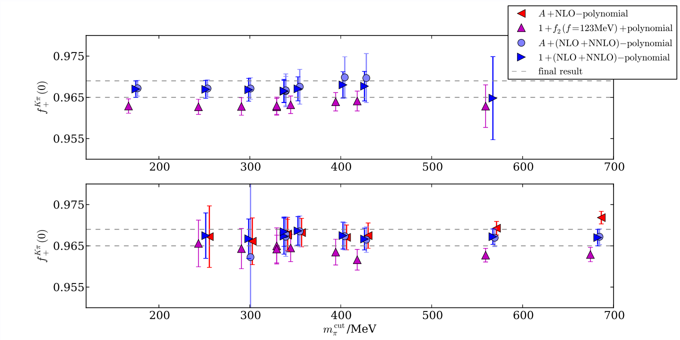


upper(lower) panel: variation of smallest(largest) included pion mass

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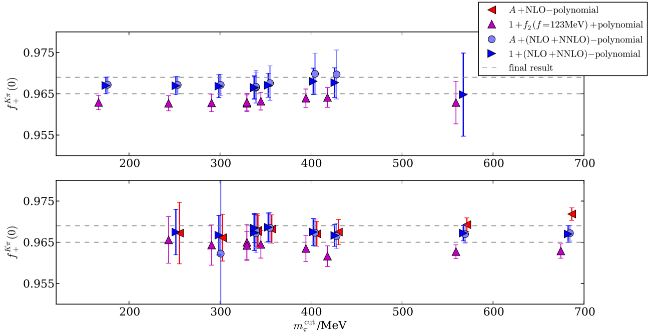
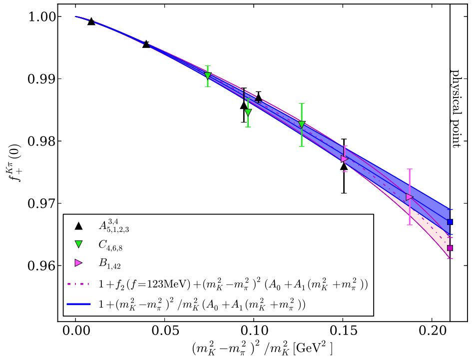



upper(lower) panel: variation of smallest(largest) included pion mass

- all fits agree when including only data from smallest pion masses
- polynomial fits show surprising indpendence on the mass-cut

# Result and error budget




- use central value of NLO+NNLO polynomial fit (fit C)
- take difference to fit with $f_2(f=123\text{MeV})$ (fit B) and polynomial for NNLO as estimate for systematic due to model

*2× Ghorbani arXiv:1301.0919*

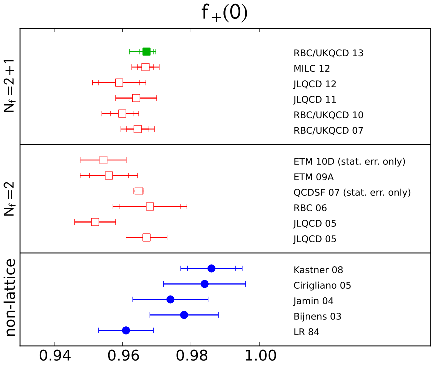
$$f_+^{K\pi}(0) = 0.9670(20)_{\text{stat}}\left({}^{+\ 0}_{-42}\right)_{\text{model}}(7)_{\text{FSE}}(17)_{\text{cutoff}}$$

0.2% 0.4% 0.07% 0.2%

$$= 0.9670(20)\left({}^{+18}_{-46}\right)$$

*JHEP, arXiv:1305.7217*

# Comparison with other results and SM-unitarity



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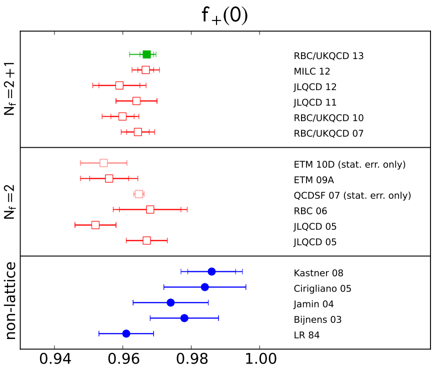


- experimental result $|V_{us} f_+^{K\pi}(0)| = 0.2163(5)$ *FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)*

$$|V_{us}| = 0.2237(^{+13}_{-\ 8})$$

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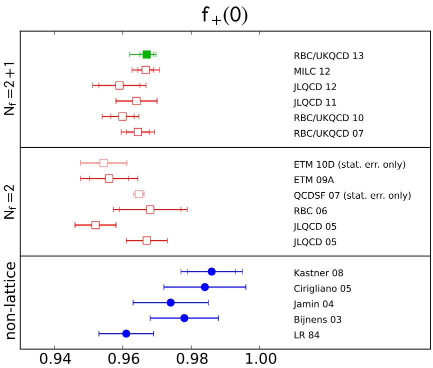


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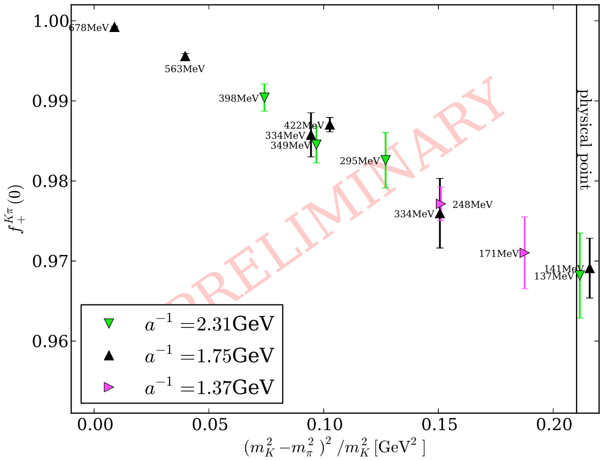
- using $|V_{ud}| = 0.97425(22)$ *Hardy & Towner, Phys.Rev. C79 (2009)* , $|V_{ub}| = 4.15(49)$ *PDG*:

$$V_u^2 - 1 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008(^{+7}_{-6})$$

# Including new physical point results




- allows to reduce model dependence in mass-extrapolation to a minimum
- further confirmation that cutoff effects are tiny

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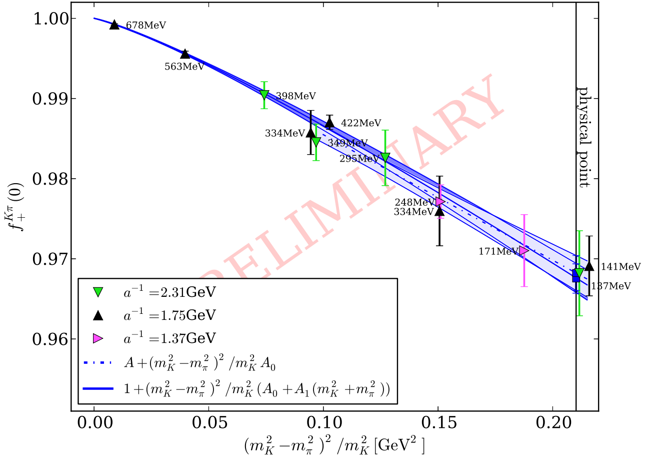


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# Excited state study

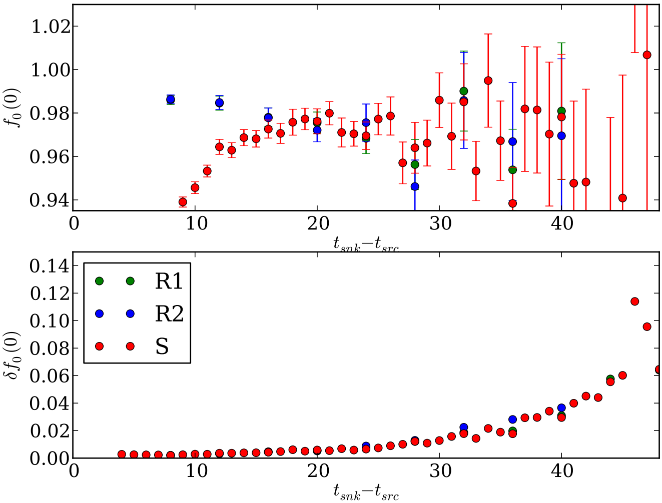
$$C^{(\mu)}_{P_i P_f}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) \equiv Z_V \sum_{\vec{x}_f, \vec{x}} e^{i\vec{p}_f\cdot(\vec{x}_f - \vec{x})} e^{i\vec{p}_i\cdot\vec{x}} \langle\, O_f(t_f, \vec{x}_f)\, V_\mu(t, \vec{x})\, O_i^\dagger(t_i, \vec{0})\,\rangle$$

- solved for large range of $|t_f - t_i|$ (twist in $\pi$ and only strange in extended leg $\rightarrow$ cheap)
- allows for study of excited state contribution in two ways:
  - a) study fit to ratio $R$ for different source-sink separations
  - b) summation method *Maiani et al. NPB293(1987), Gusken et al. PLB227(1989), Bulava et al. JHEP 1201(2013), Capitani et al. PRD86(2012)*:

  preliminary study

# Excited state study a)

result for the form factor determined from ratios for $V$ and $S$ for various source-sink separations and relative error



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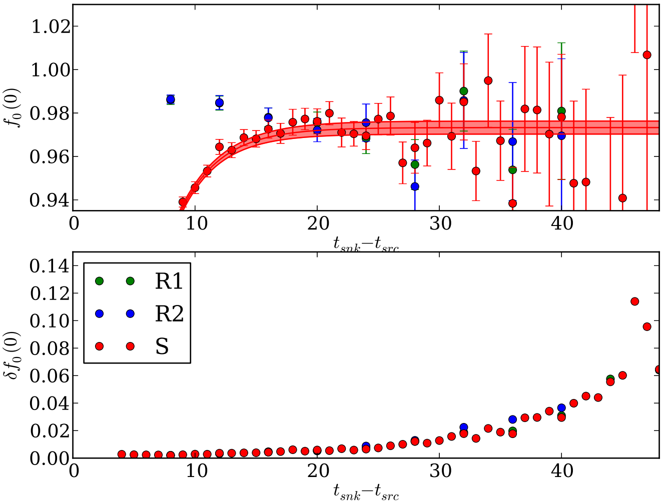

# Excited state study b) Summation method

$$\left(R^{(2)}(t_f,t)\right)^2 = \frac{\langle\pi|V|K\rangle^2}{\langle\pi|V|\pi\rangle\langle K|V|K\rangle}\left(1+e^{-(t_f-t)\Delta_\pi}\frac{\langle K|V|\pi^{(1)}\rangle}{\langle\pi|V|K\rangle}+e^{-t\Delta_K}\frac{\langle K^{(1)}|V|\pi\rangle}{\langle\pi|V|K\rangle}+\dots\right)$$
$$\times\left(1+e^{-(t_f-t)\Delta_K}\frac{\langle\pi|V|K^{(1)}\rangle}{\langle\pi|V|K\rangle}+e^{-t\Delta_\pi}\frac{\langle\pi^{(1)}|V|K\rangle}{\langle\pi|V|K\rangle}+\dots\right).$$

$$\sum_{t=0}^{t_f} e^{-\alpha t} = \frac{e^{-(t_f+1)\alpha}-1}{e^{-\alpha}-1},$$

# Excited state study b) Summation method

$$
\begin{aligned}
\Sigma_{R^{(2)}}(t_f) &\equiv \sum_{t=0}^{t_f} (R^{(2)}(t_f,t))^2 \\
&= a_0 + t_f \langle \pi | V | K \rangle^2 \\
&\quad + \mathcal{O}\left( t_f e^{-t_f \Delta_K}, t_f e^{-t_f \Delta_\pi}, \frac{1-e^{-(t_f+1)\Delta_K}}{1-e^{-\Delta_K}}, \frac{1-e^{-(t_f+1)\Delta_\pi}}{1-e^{-\Delta_\pi}}, \frac{1-e^{-(t_f+1)(\Delta_\pi+\Delta_K)}}{1-e^{-(\Delta_\pi+\Delta_K)}} \right)
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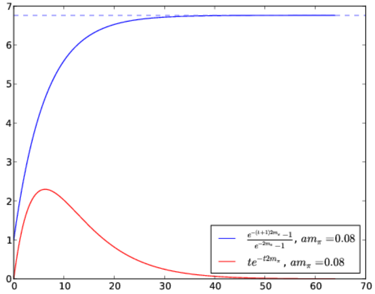

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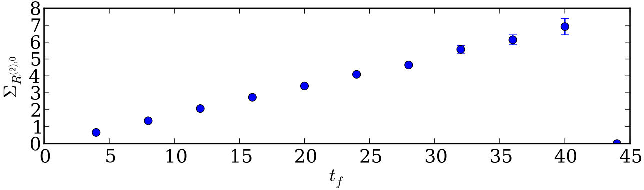
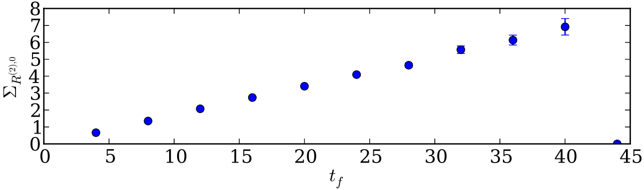
$$= a_0 + t_f\langle\pi|V|K\rangle^2$$
$$+\mathcal{O}\left(t_f e^{-t_f\Delta_K}, t_f e^{-t_f\Delta_\pi}, \frac{1-e^{-(t_f+1)\Delta_K}}{1-e^{-\Delta_K}}, \frac{1-e^{-(t_f+1)\Delta_\pi}}{1-e^{-\Delta_\pi}}, \frac{1-e^{-(t_f+1)(\Delta_\pi+\Delta_K)}}{1-e^{-(\Delta_\pi+\Delta_K)}}\right)$$



# Excited state study b) Summation method

$$\begin{aligned}\Sigma_{R^{(2)}}(t_f) &\equiv \sum_{t=0}^{t_f}(R^{(2)}(t_f,t))^2\\ &= a_0 + t_f\langle\pi|V|K\rangle^2\\ &\quad+\mathcal{O}\left(t_f e^{-t_f\Delta_K}, t_f e^{-t_f\Delta_\pi}, \frac{1-e^{-(t_f+1)\Delta_K}}{1-e^{-\Delta_K}}, \frac{1-e^{-(t_f+1)\Delta_\pi}}{1-e^{-\Delta_\pi}}, \frac{1-e^{-(t_f+1)(\Delta_\pi+\Delta_K)}}{1-e^{-(\Delta_\pi+\Delta_K)}}\right)\end{aligned}$$




results for $f_+^{K\pi}(0)$ from slope agree with results from fits to ratio but stat. error larger

# Outlook

JHEP-accepted arXiv:1305.7217-result (without physical point)

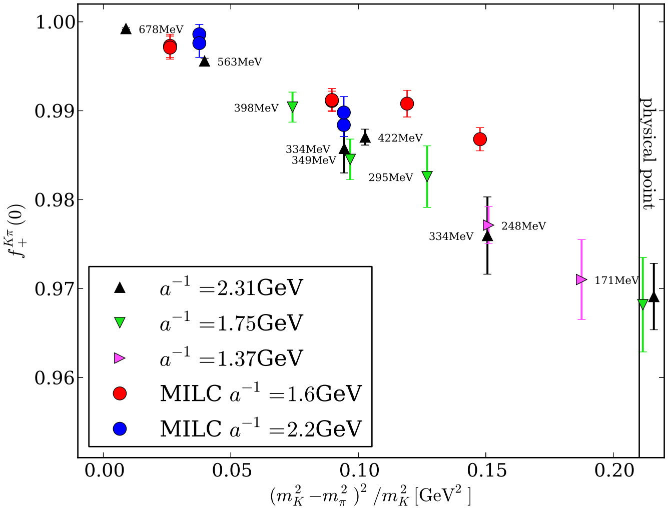
$$f_+^{K\pi}(0) \;=\; 0.9670(20)_{\rm stat}\;\left({}^{+\;0}_{-42}\right)_{\rm model}\;(7)_{\rm FSE}(17)_{\rm cutoff}$$

$$\qquad\qquad 0.2\% \qquad \underbrace{0.4\%} \qquad 0.07\% \quad 0.2\%$$

→ 0ish once phys. pt. included

- model-dependence should become tiny once analysis with physical point finalised
- current cut-off effect estimate based on power counting, we are looking into more elaborate estimates
- it is now time to think about EM and isospin breaking effects

The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) ERC grant agreement No 279757



678MeV
563MeV
398MeV
422MeV
334MeV
349MeV
295MeV
248MeV
334MeV
171MeV
physical point
$f_+^{K\pi}(0)$
$(m_K^2 - m_\pi^2)^2/m_K^2$ [GeV$^2$]
$a^{-1}$ =2.31GeV
$a^{-1}$ =1.75GeV
$a^{-1}$ =1.37GeV
MILC $a^{-1}$ =1.6GeV
MILC $a^{-1}$ =2.2GeV