More effects of Dirac low-mode removal

Mario Schröck, M. Denissenya, L. Ya. Glozman, C. B. Lang

Institut für Physik, FB Theoretische Physik

Universität Graz

Lattice2013 Mainz

August 01, 2013

KARL-FRANZENS-UNIVERSITÄT GRAZ
UNIVERSITY OF GRAZ

Motivation

- in previous studies we removed low lying Dirac eigenmodes from the valence quark sector and subsequently performed a hadron spectroscopy
- we found persistence of exponentially decaying states with essentially improved signal-to-noise ratios
- approximately degenerate masses of chiral partners, e.g., the vector and axial vector currents
- loss of dynamically generated mass in the Landau gauge quark propagator

“Unbreaking” chiral symmetry

[C.B. Lang, M.S., PRD **84** (2011) 087704]

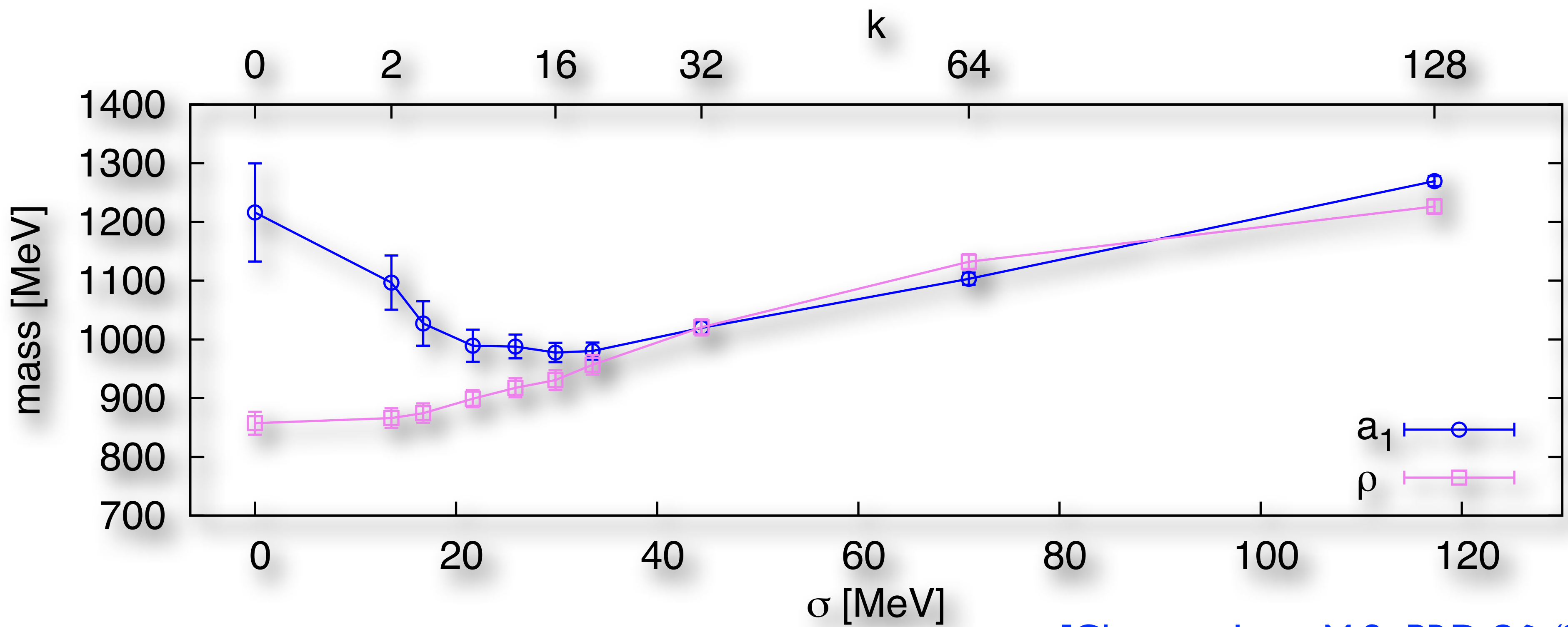
- we subtract the Dirac low-mode contribution from the valence quark propagators

$$S_{\mathrm{red}(k)} = S_{\mathrm{full}} - \sum_{i=1}^{k} \mu_i^{-1} \left|w_i\right\rangle \left\langle w_i\right| \gamma_5$$

- $\mu_i, \ |w_i\rangle$ are the eigenvalues and vectors of the hermitian Dirac operator $D_5 = \gamma_5 D$ and k denotes the truncation level

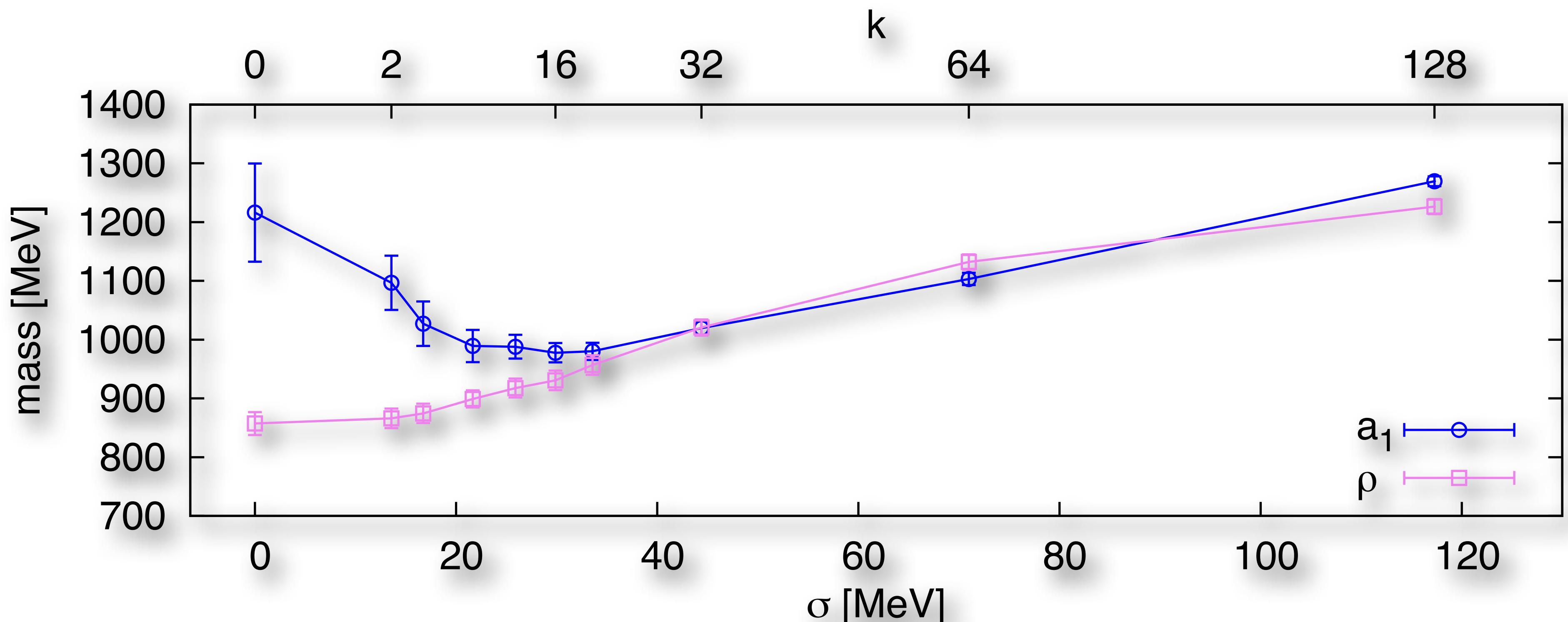
- this truncation corresponds to removing the chiral condensate of the valence quark sector by hand

Open questions



[Glozman, Lang, M.S., PRD **86** (2012) 014507]

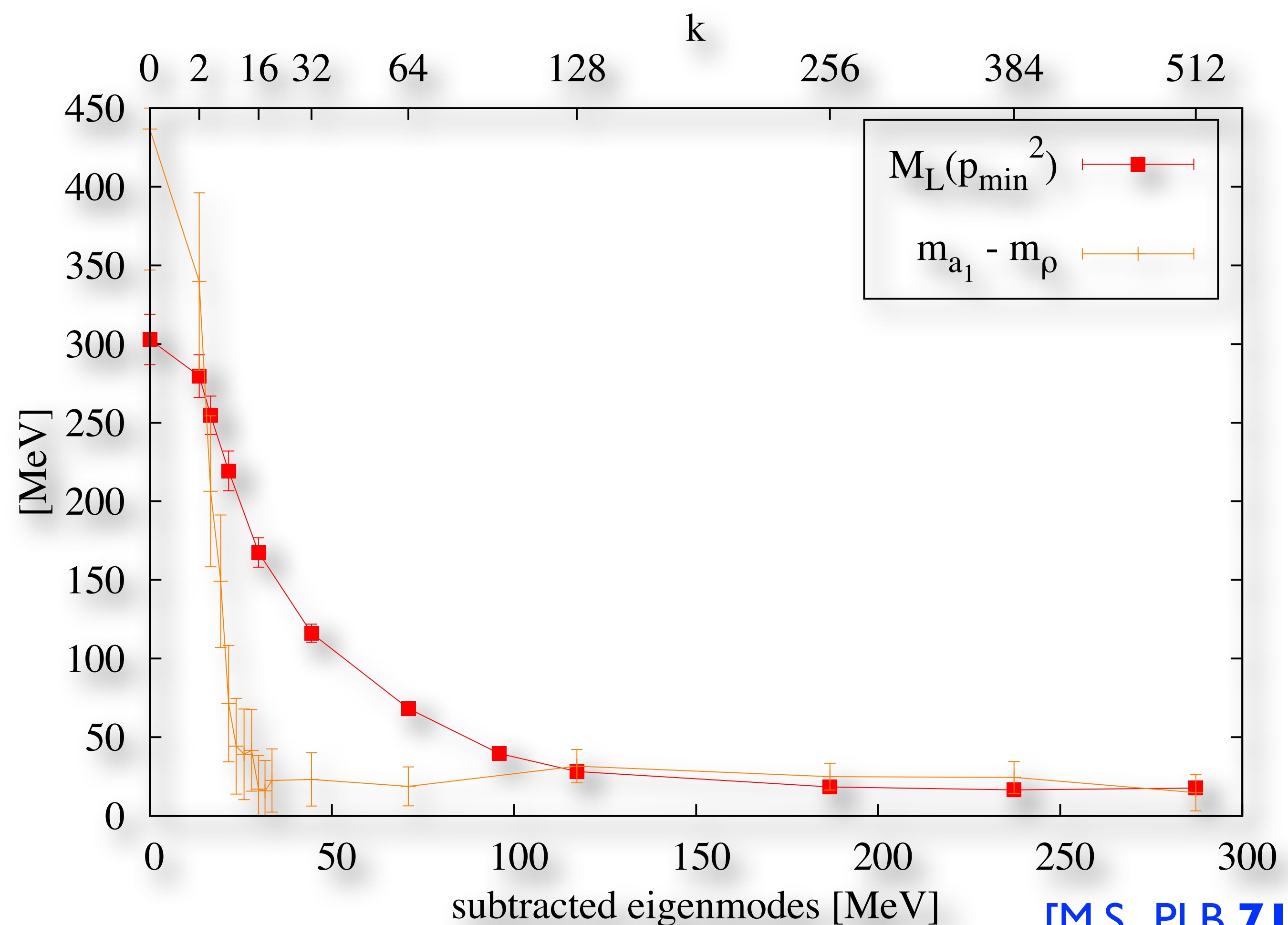
Open questions



[Glozman, Lang, M.S., PRD **86** (2012) 014507]

- how crucially is the locality of the Dirac operator violated when we remove the low lying spectrum?
- why do the hadron masses increase upon low-mode truncation?

Open questions cont'd

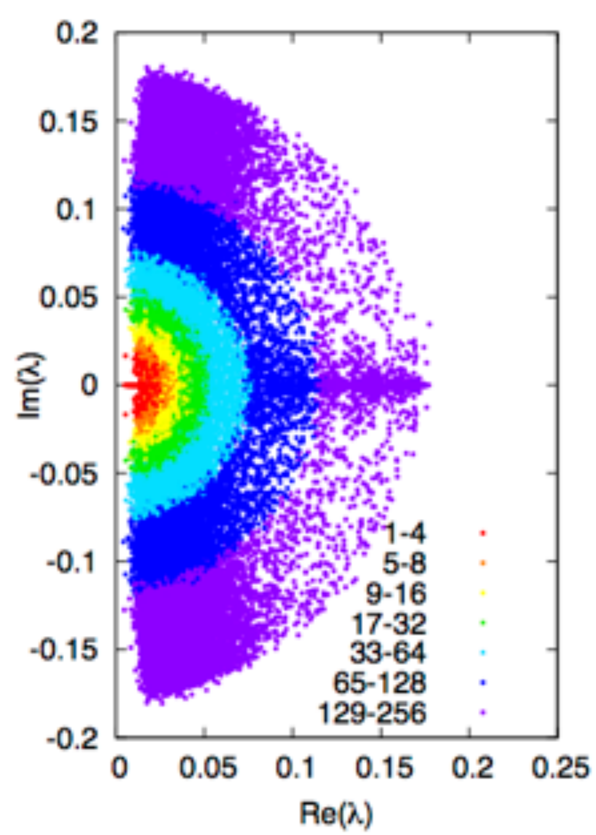


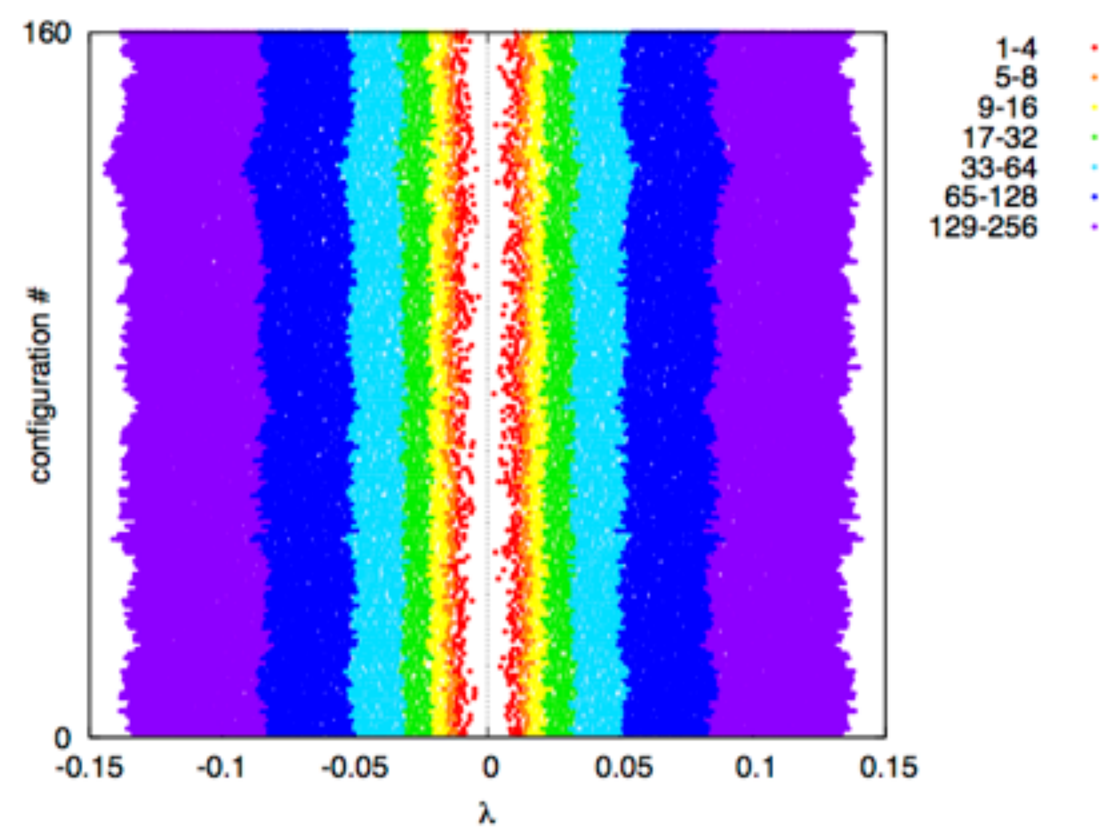
[M.S., PLB **711** (2012) 217-224]

- why do the vector and axial vector appear degenerate from truncation level ~30 MeV on, whereas the quark mass function appears flat only after subtracting ~150 MeV?

The setup

- we adopt 161 gauge field configurations with two flavors of degenerate CI fermions [Gattringer et al., PRD **79** (2009) 054501]
- pion mass $m_\pi = 322(5)\,\mathrm{MeV}$
- lattice size $16^3 \times 32$, lattice spacing $a = 0.144(1)\,\mathrm{fm}$
- $L \cdot m_\pi \approx 3.75$



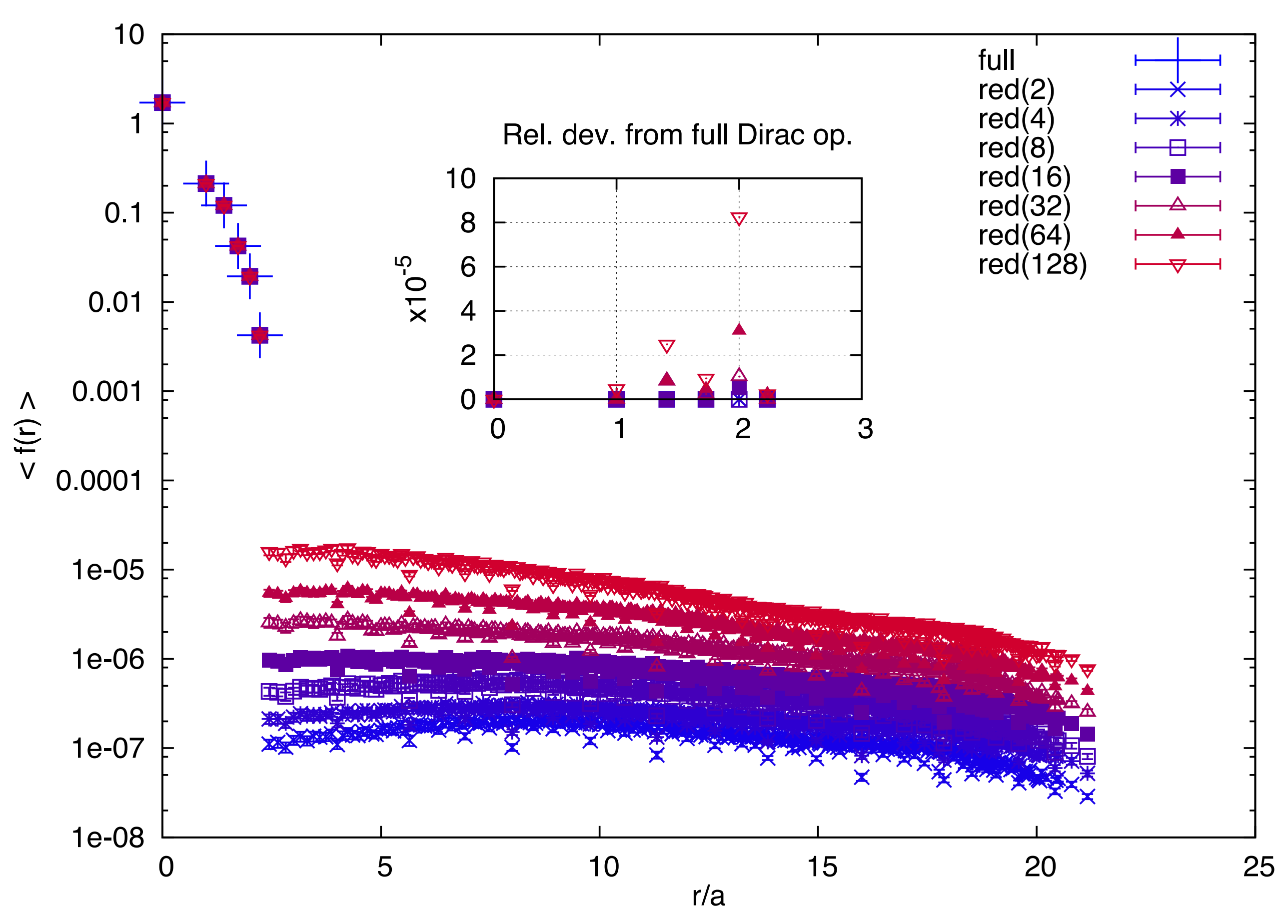


Locality properties

- to what extent is the locality of the low-mode truncated Dirac operator violated?

$$\psi(x)^{[x_0,\alpha_0,a_0]} = \sum_y D_5(x,y)\,\eta(y)^{[x_0,\alpha_0,a_0]}$$

$$f(r) = \max_{x,\alpha_0,a_0} \{\|\psi(x)\| \mid |x| = r\}$$



Landau gauge quark propagator

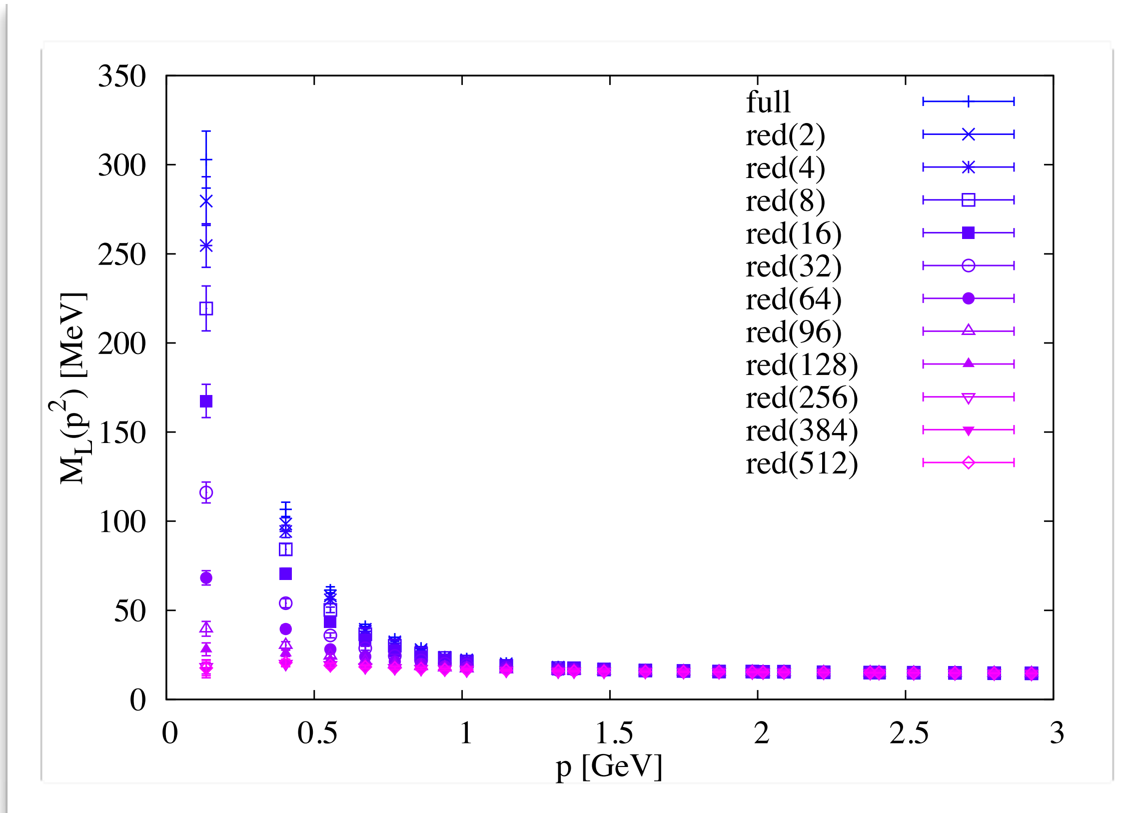
- we study the quark propagator to shed light on the origin of the large meson mass upon Dirac low-mode reduction

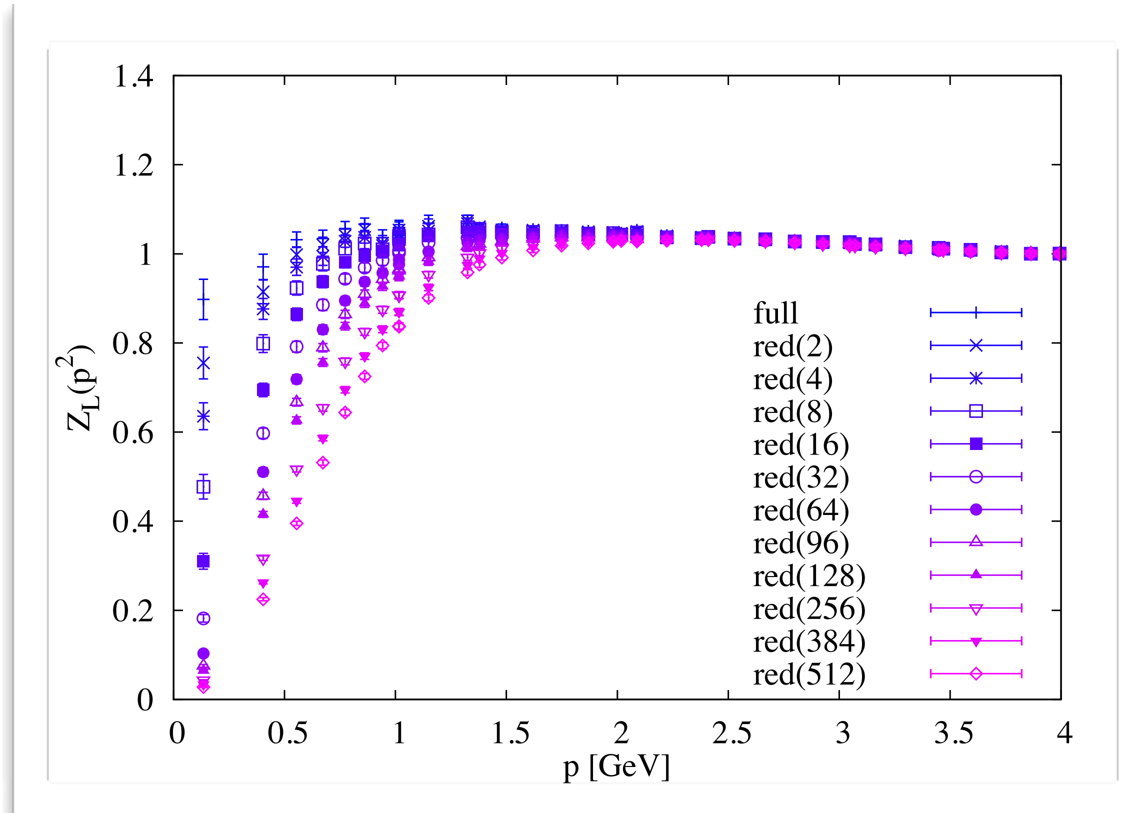
- the renormalized quark propagator has the form

$$S(\mu;p^2) = \left(i\not{p}A(\mu;p^2) + B(\mu;p^2)\right)^{-1} = \frac{Z(\mu;p^2)}{i\not{p} + M(p^2)}$$

- we extract the wavefunction renormalization function $Z(\mu;p^2)$ and the mass function $M(p^2)$ from the lattice and study their evolution under low-mode truncation

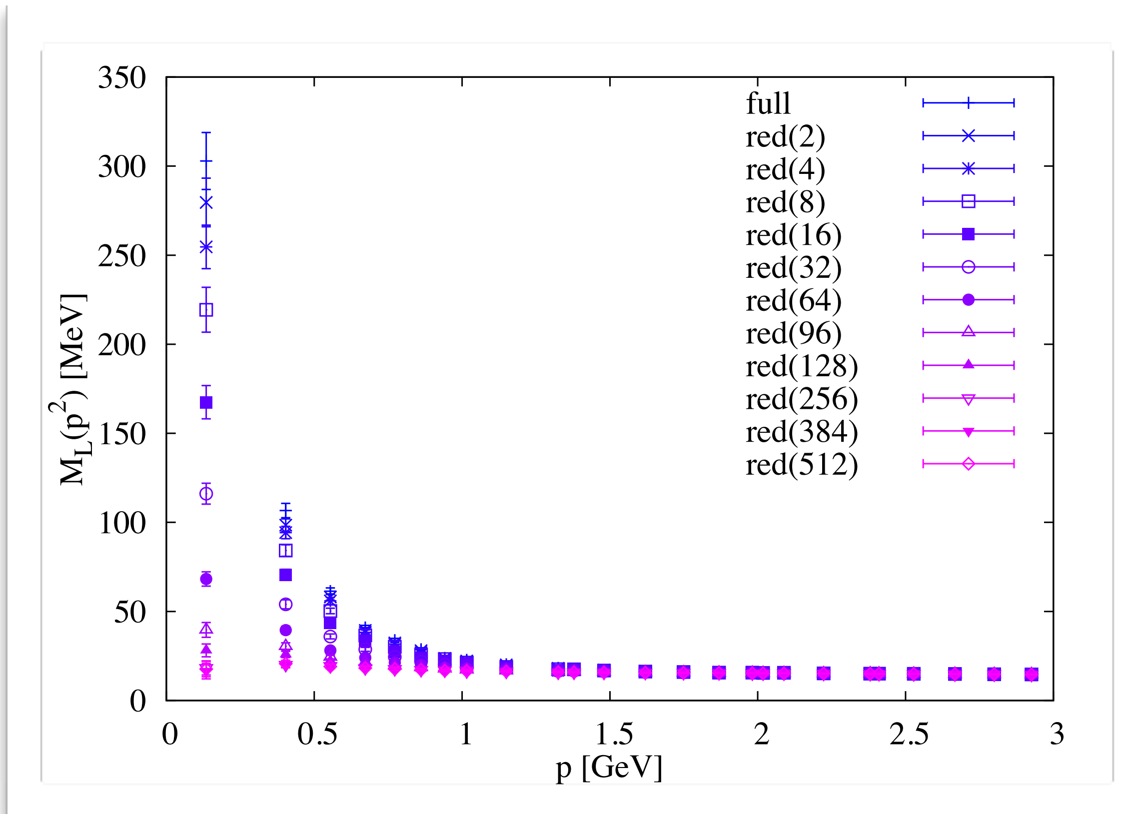
Truncated quark propagator

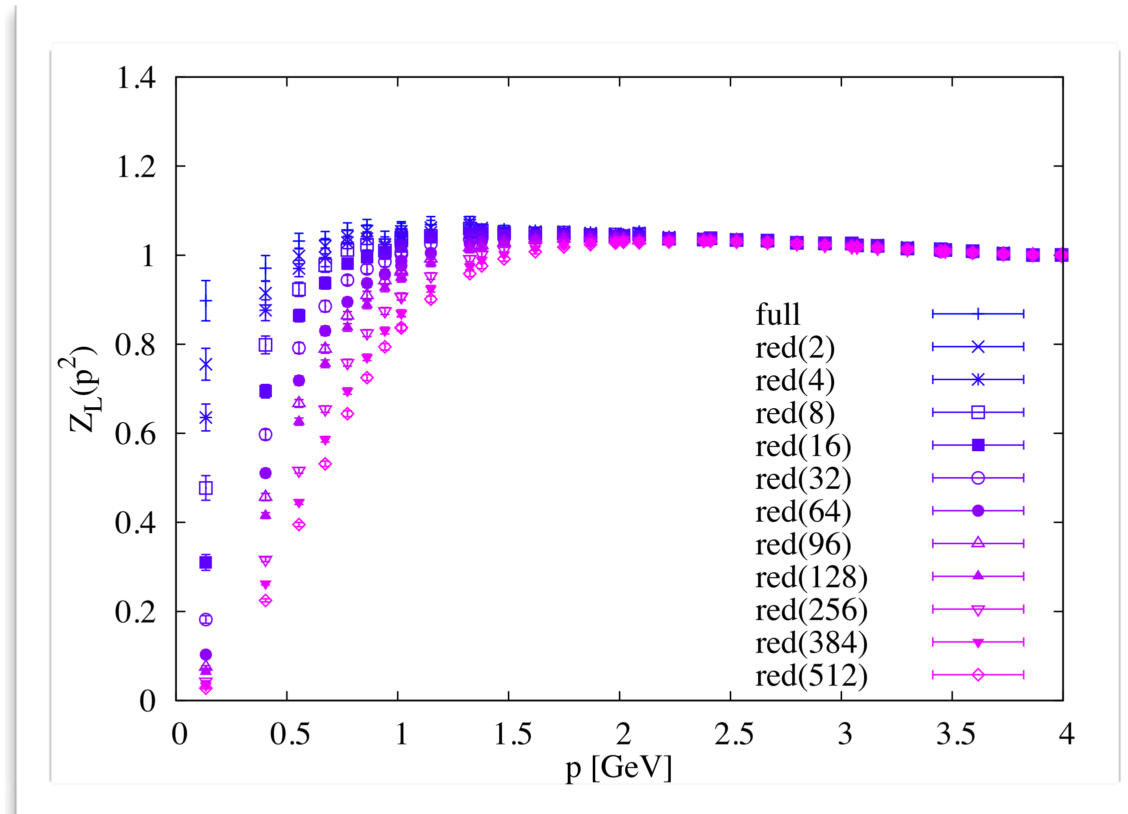




[M.S., PLB **711** (2012) 217-224]

Truncated quark propagator

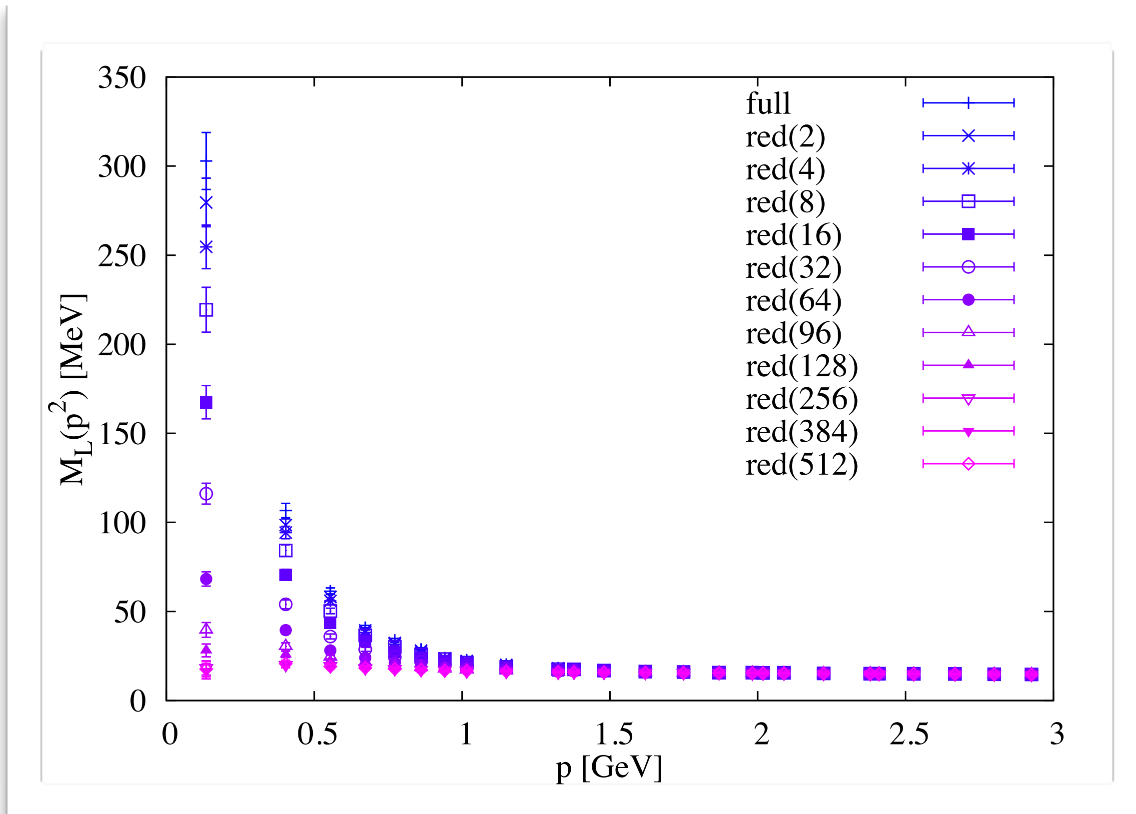


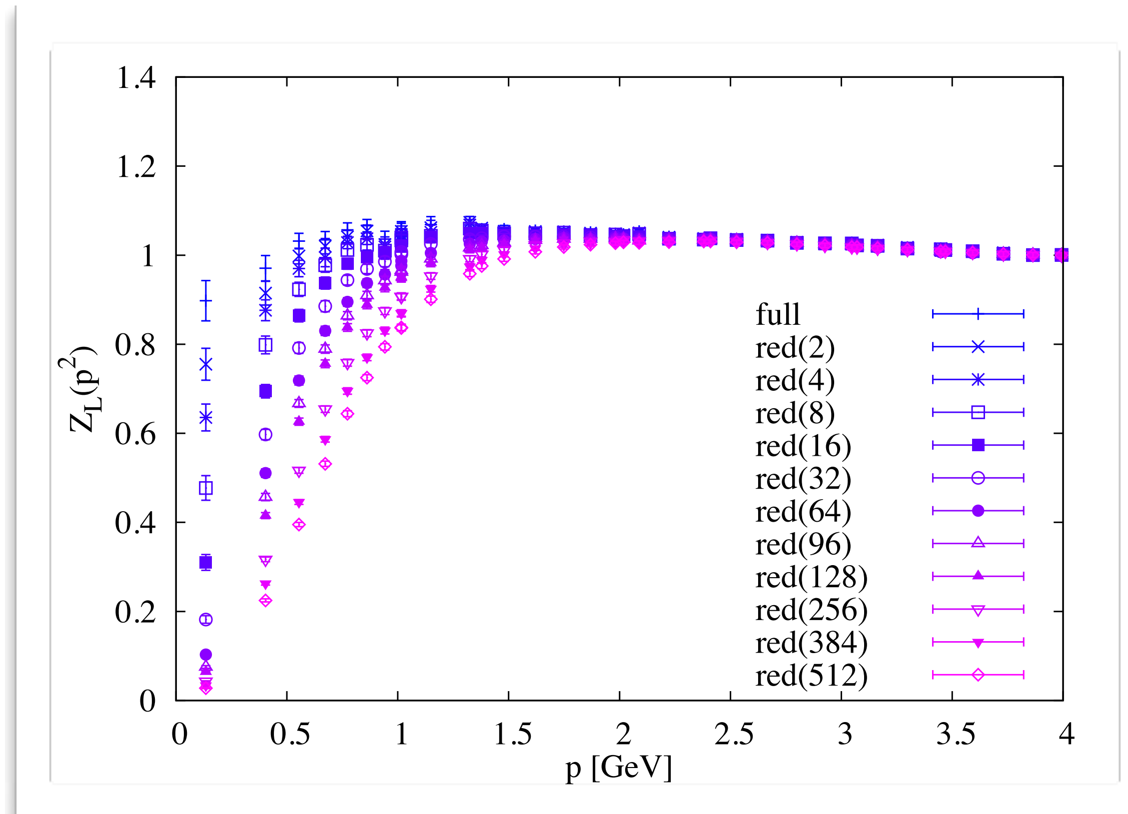


[M.S., PLB **711** (2012) 217-224]

- flattening of $M(p^2)$ $\iff$ vanishing of $\langle \overline{\psi}\psi \rangle$

Truncated quark propagator





[M.S., PLB **711** (2012) 217-224]

- flattening of $M(p^2)$ $\Longleftrightarrow$ vanishing of $\langle\, \overline{\psi}\psi \,\rangle$
- $Z(p^2)\big|_{p\ll 1} \to 0 \;\Longleftrightarrow\; S(p^2)\big|_{p\ll 1} \to 0$:

  suppression of low momentum quarks

Dirac modes and quark momenta

- the eigenvalues of the free Dirac operator can be derived analytically

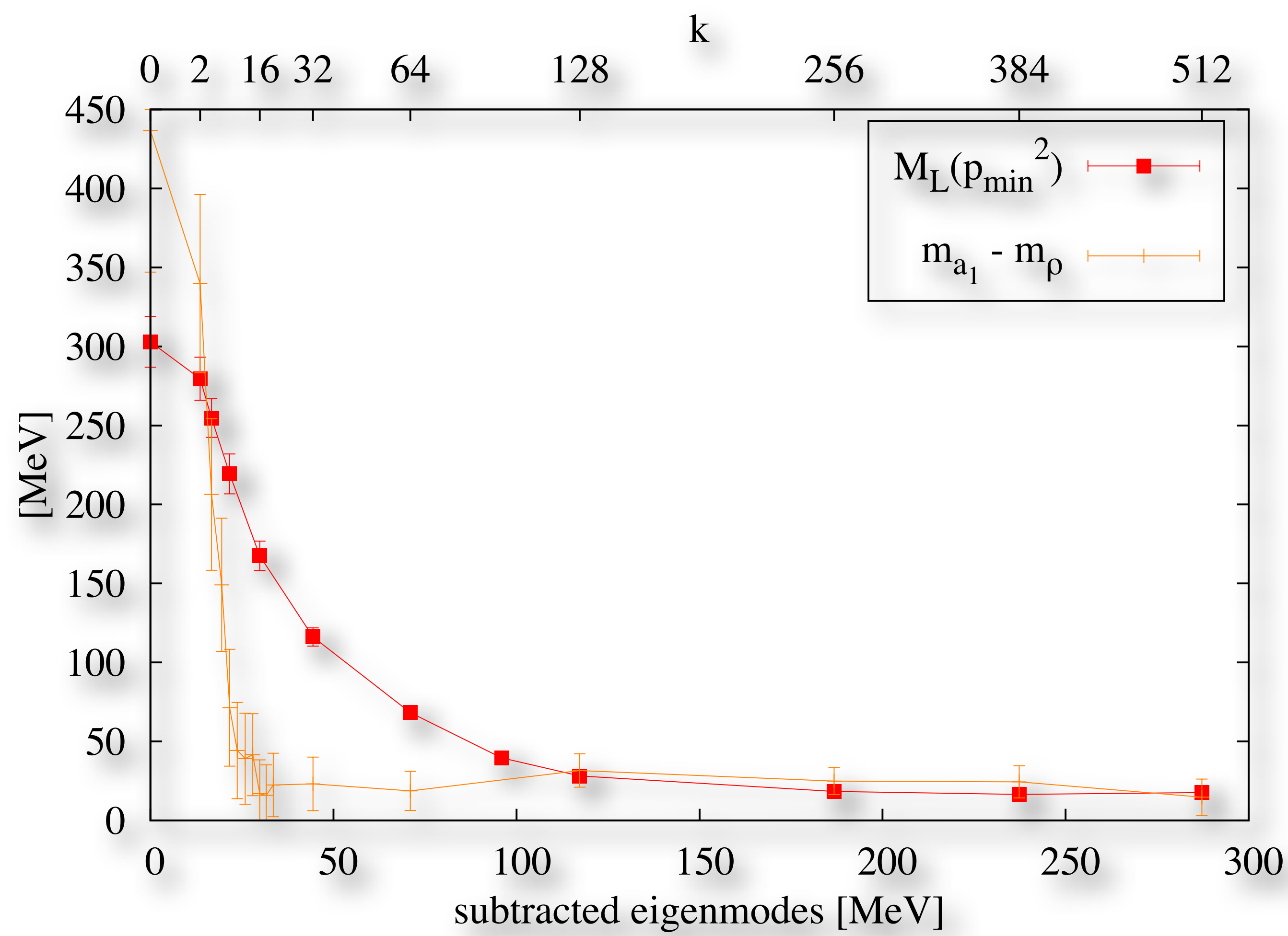
$$\lambda = s \pm i\,|k|$$

- where $s(p)$ denotes the scalar part of the Dirac operator and $k(p)$ are the lattice momenta
- setting the small eigenvalues to zero makes the low momentum states imaginary and thus unphysical

Increased quark momenta

i. explains growing of meson masses

ii. chiral restoration in mesons is partially *effective*: compare chiral restoration in mesons with vanishing of the chiral condensate:



[M.S., Phys. Lett. B **711** (2012) 217-224]

The sea quark sector

- sea quarks enter via the fermion determinant

$$\langle \mathcal{O}[U] \rangle = \frac{\int \mathcal{D}U \;\; \mathrm{e}^{-S_G[U]} \, \det[D]^2 \, \mathcal{O}[U]}{\int \mathcal{D}U \;\; \mathrm{e}^{-S_G[U]} \, \det[D]^2}$$

- which can be divided into low- and high-mode parts

$$\det[D] = \prod_i \lambda_i = \prod_{i \le k} \lambda_i \cdot \prod_{i > k} \lambda_i$$

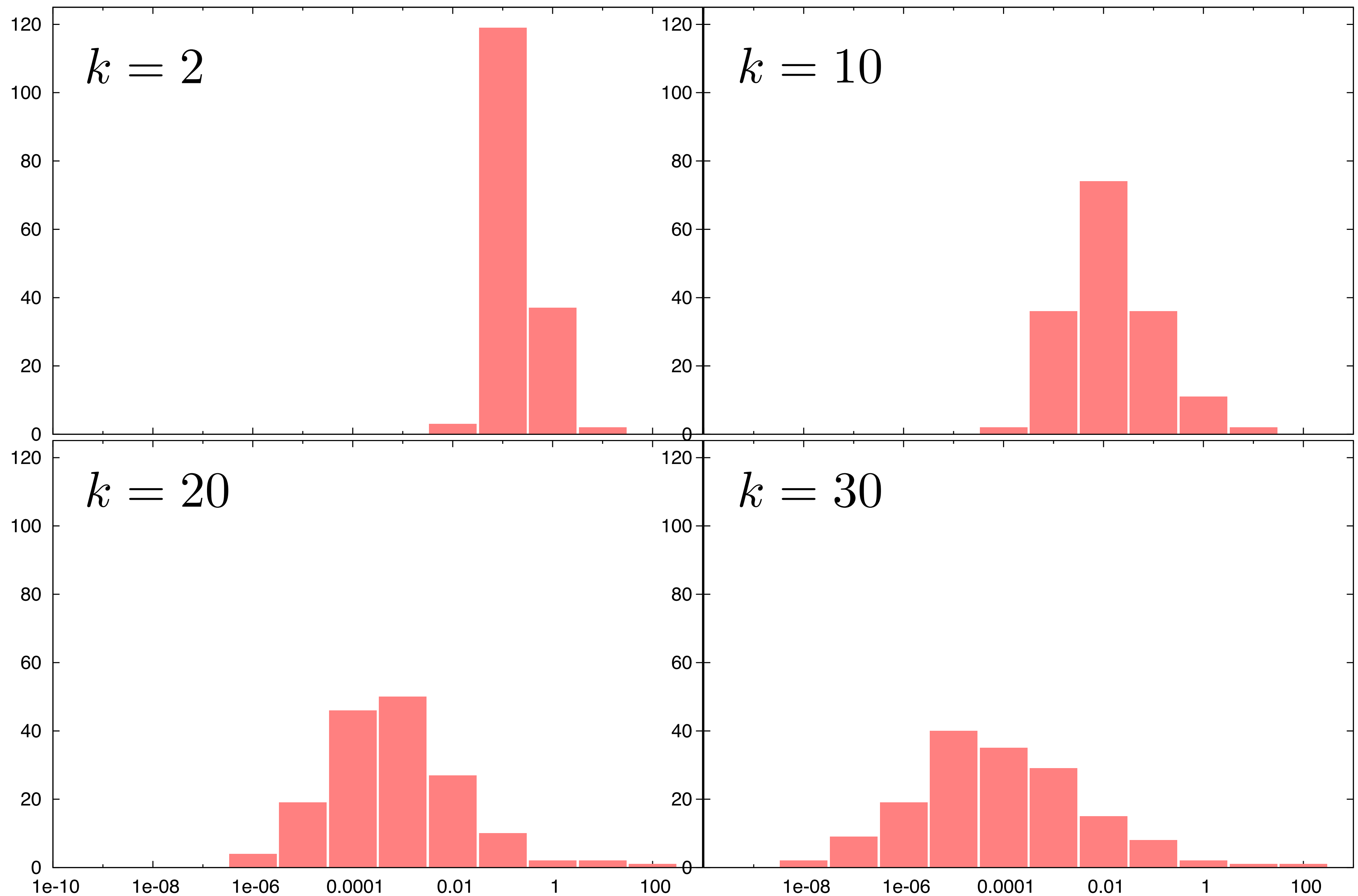
- we can define a weight factor to cancel the *LM* part

$$w_k \equiv \Big(\det[D]_{\mathrm{lm}(k)} \Big)^{-2}, \quad \overline{w}_k[U_n] \equiv \frac{w_k[U_n]}{\sum_n w_k[U_n]} \cdot N$$

The sea quark sector II

- the low-mode truncated path integral is then

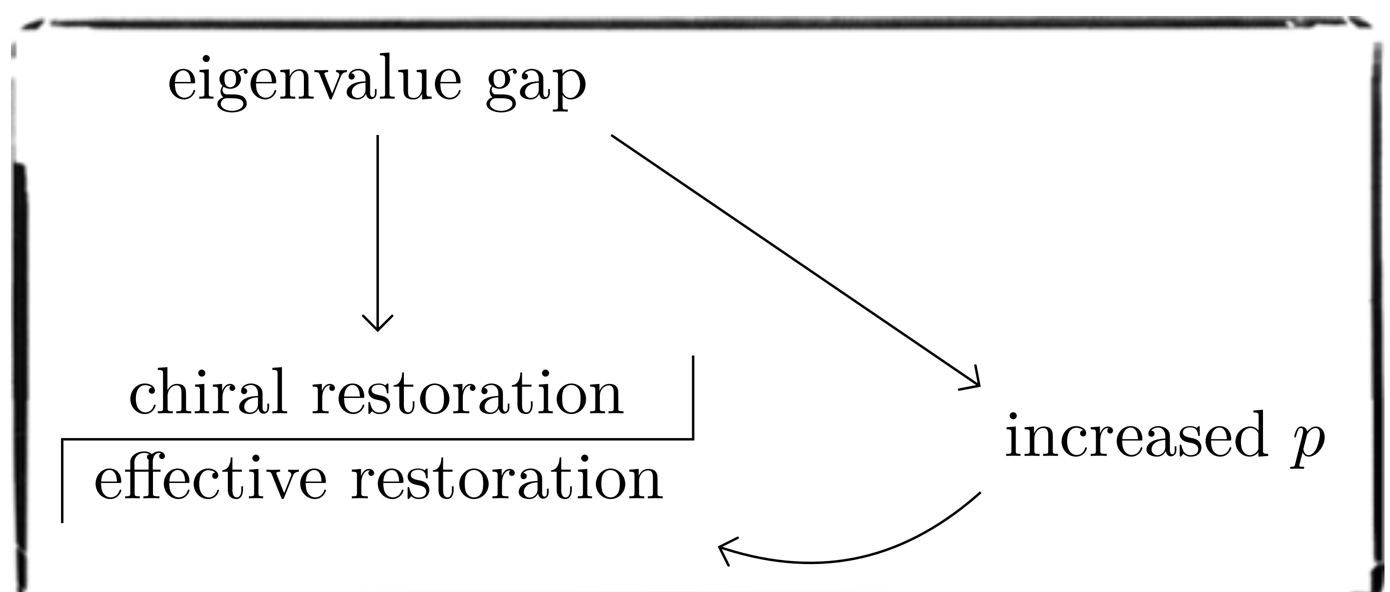
$$\langle \mathcal{O}\left[U\right]\rangle_{w_k} \approx \frac{1}{N}\sum_n \mathcal{O}\left[U_n\right]\overline{w}_k\left[U_n\right]$$



Summary

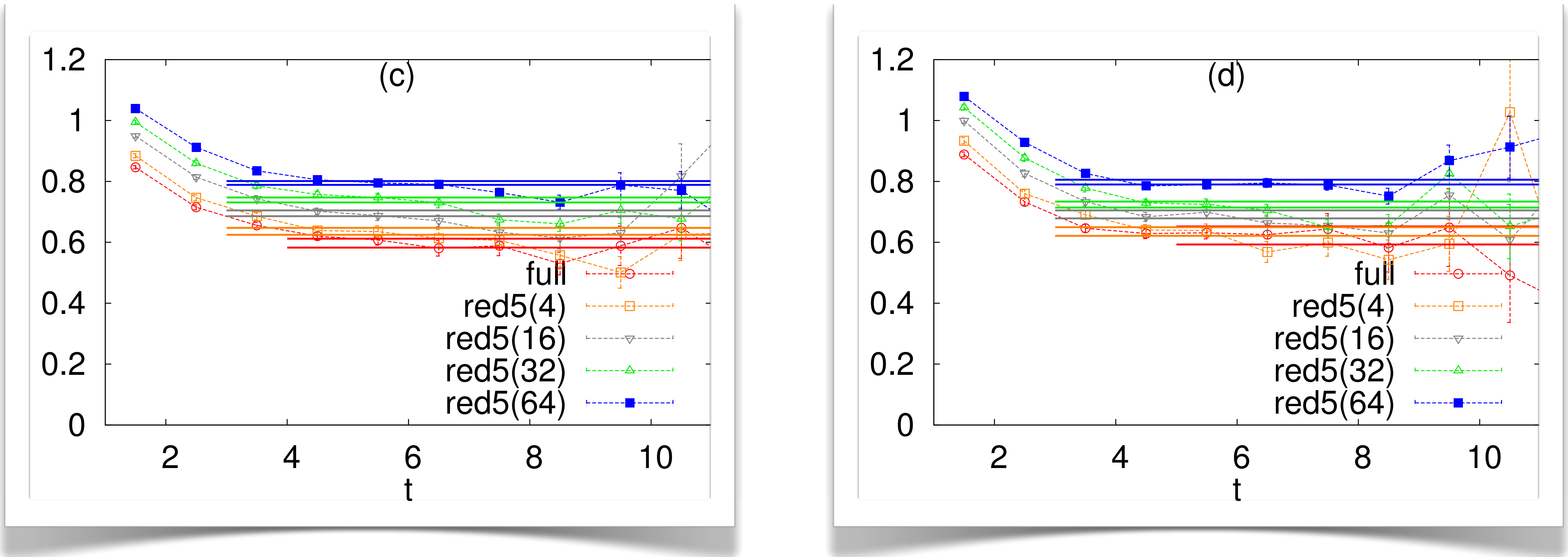
Removing the lowest lying part of the Dirac spectrum:

- does not have severe effects on the locality of the theory (at finite cutoff)
- suppresses quarks with low momenta, i.e., artificially increases the average momenta of the quarks
- the latter increases the energy of the hadrons
- chiral restoration in the hadron spectrum is partially effective



Appendix

Rho without low-modes



[C.B. Lang, M.S., Phys. Rev. D **84** (2011) 087704]

- Low-mode truncated effective masses of the $J^{PC} = 1^{--}$ sector in comparison to the eff. masses from full propagators

- interpolators: (c) $\bar{u}\gamma_i d$ (d) $\bar{u}\gamma_4\gamma_i d$