Investigating the Sharpe-Singleton scenario on the lattice by direct eigenvalue computation

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Outline of the talk

- Motivation for work
- Setup
- Results
- Conclusions and outlook

Motivation for work

- Understand low energy behavior of QCD
- Understand chiral symmetry breaking on the lattice
- Qualify the chiral properties of different fermion discretizations
- Implementation of a fully parallel Arnoldi algorithm expected to have significant performance improvement over existing implementations

WχPT

- Chiral Perturbation Theory with added terms that describe discretization effects[1]
- The action for WχPT:

$$S = \frac{m}{2}\Sigma V \mathrm{Tr}(U + U^\dagger) + \frac{\zeta}{2}\Sigma V \mathrm{Tr}(U - U^\dagger) - a^2 V \Delta$$
$$\Delta = W_6[\mathrm{Tr}(U + U^\dagger)]^2 + W_7[\mathrm{Tr}(U - U^\dagger)]^2 + W_8 \mathrm{Tr}(U^2 + U^{2^\dagger})$$

- $\gamma_5 D_W \gamma_5 = D_W^\dagger$
- Constraints for LECs[2,3]

$$W_8 > 0, W_6 < 0 \text{ and } W_7 < 0$$

- Sign of $W_8 + 2W_6$ determines the phase of the theory

[1] S.R. Sharpe, R.L. Singleton, arXiv:hep-lat/9804028
[2] P.H. Damgaard, K. Splittorff, J.J.M. Verbaarschot, arXiv:1001.2937
[3] M.T.Hansen, S.R.Sharpe, arXiv:1111.2404

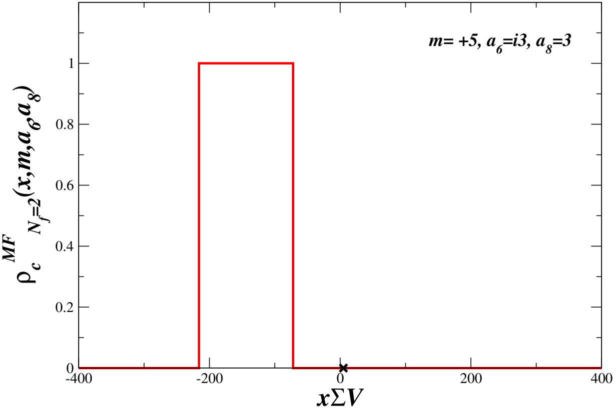
Sharpe-Singleton scenario

- A phase present in QCD with Wilson fermions, with no continuum analogue[4]
- Realized for

$$W_8 + 2W_6 < 0$$

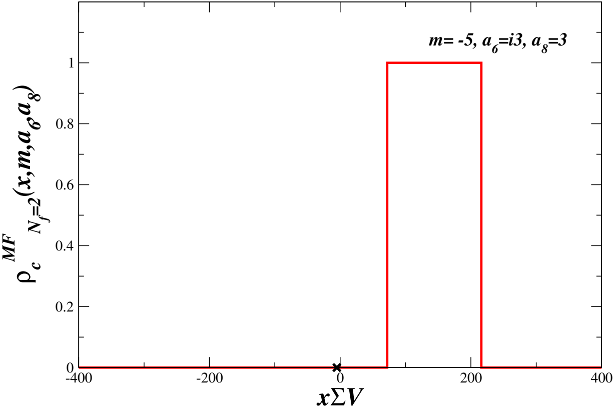
[4] S.R. Sharpe, R.L. Singleton, arXiv:hep-lat/9804028

Sharpe-Singleton scenario



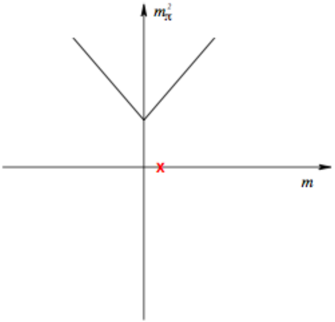
- Quark mass has a collective effect on the eigenvalue distribution, an effect not present in either quenched simulations or in the Aoki phase
- Figure from M. Kieburg, K. Splittorff, J.J.M. Verbaarschot, arXiv:1202.0620

Sharpe-Singleton scenario



- Quark mass has a collective effect on the eigenvalue distribution, an effect not present in either quenched simulations or in the Aoki phase
- This is what we try to see
- Figure from M. Kieburg, K. Splittorff, J.J.M. Verbaarschot, arXiv:1202.0620

Sharpe-Singleton scenario



- Pion remains massive even when quark mass goes to zero, where as in the Aoki phase you get massless pions
- Another way to tell the phases apart
- Figure from K. Splittorff, arXiv:1211.1803

Setup

- SU(3) with $N_f = 2$
- Wilson fermions with clover improvement
- n-HYP smearing
- $V = 12^4$, $\beta = 5.47$, corresponding to $a = 0.16$ fm, and $m_0 = 0.2, \ldots, -0.4$
- Eigenvalues calculated using a parallelized Arnoldi algorithm running on GPUs

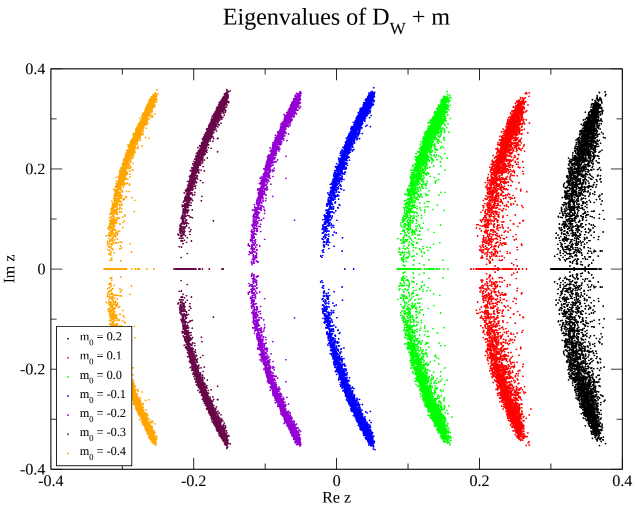
Arnoldi algorithm

- Eigenvalue algorithm based on Krylov subspace projection
- Obtain information on the eigenspectrum of the operator based on how it rotates a random vector after multiple applications
- Krylov subspace: $K_n(A, v_0) = \mathrm{span}[v_0, Av_0, A^2 v_0, \ldots, A^{n-1} v_0]$
- Creates n vectors q_i, the Arnoldi vectors, that span the Krylov subspace K_n
- Denote the matrix formed by the Arnoldi vectors $q_1 \ldots q_n$ by Q_n, then have $H_n = Q_n^\dagger A Q_n$
- H_n is upper Hessenberg, actually formed element by element by specific inner products between q_i
- Calculate the eigenvalues of H_n to obtain Ritz eigenvalues, which in practice converge to eigenvalues of A
- We adopted the widely used implicitly restarted version of this algorithm to get a robust numerical tool

Parallelized Arnoldi algorithm on the GPU

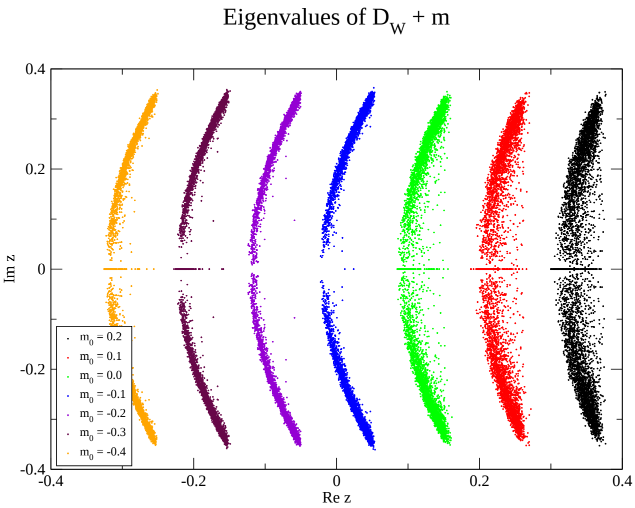
- Everything done on GPUs except for diagonalization of the Hessenberg matrix
- Complexity of the problem: $O(10^{12})$ FLOPS to obtain 50 eigenvalues on our setup
- Performance of GPU (Tesla M2070) vs. CPU (AMD Opteron) (seconds per Arnoldi iteration):
  - 2 GPUs: 2.5 - 8.9
  - 4 GPUs: 1.3 - 3.3
  - 8 GPUs: 0.8 - 2.6
  - 2 CPUs (8 cores): 27.3 - 69
  - 16 CPUs (64 cores): 2.4 - 4.8
- About 6x performance gain
- Still room for fine tuning of the algorithm

Results



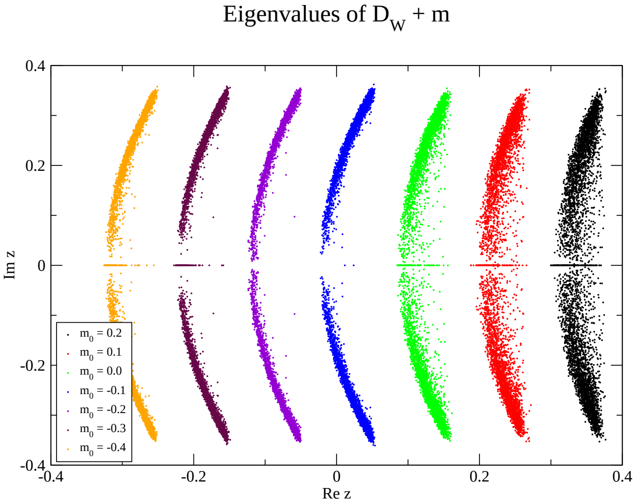
- 50 eigenvalues calculated from 100 configurations

Results



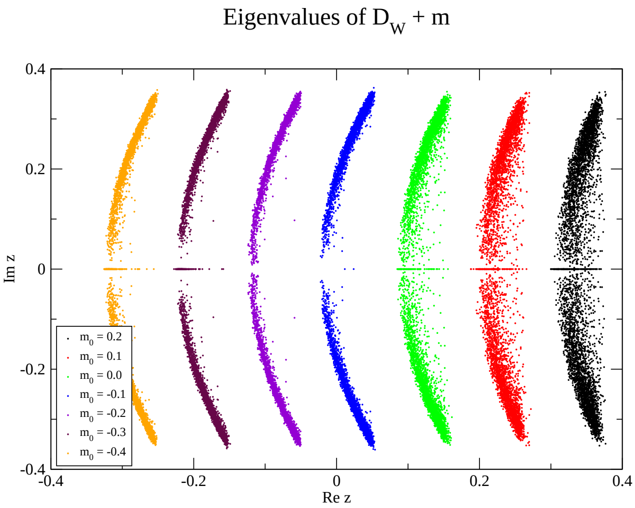
- The simulations took two weeks of wall clock time, with most time spent on the negative mass distributions

Results



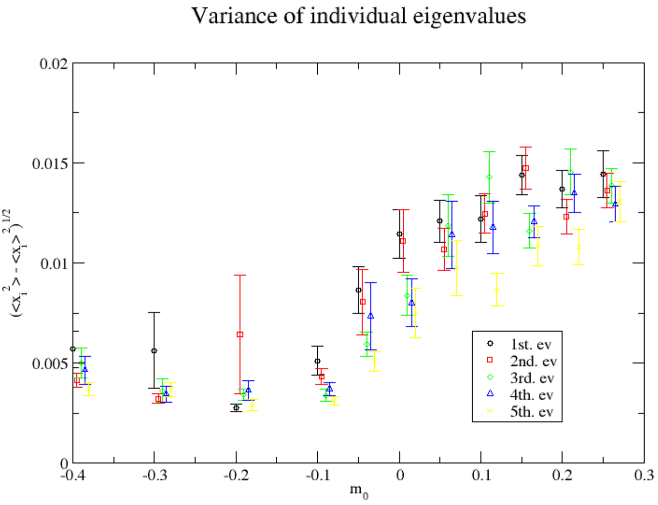
- Shows mass dependency in the shape of the distribution
- Cutoff effect because of sorting criterion

Results



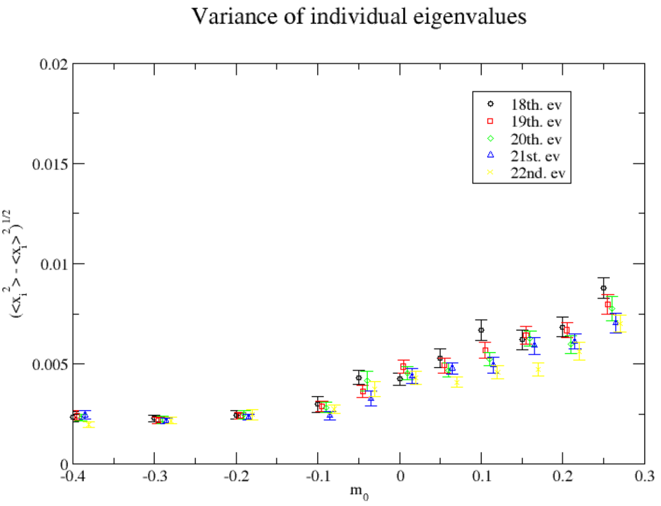
- You can see a hole forming as you approach zero quark mass, not sure about a collective jump

Results



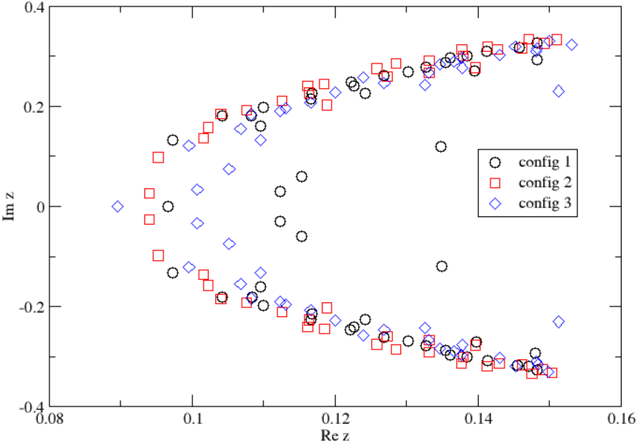
- In order to quantify mass dependence, calculate variance
- Effect slightly dampened because of cutoff

Results



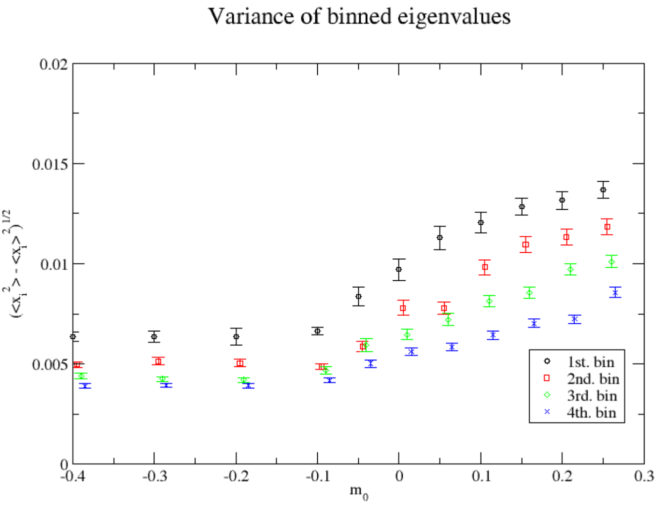
- For the largest eigenvalues looks similar, but more subdued

Results



- Eigenvalues from three separate configurations illustrating overlap between eigenvalues

Results



- Binning the eigenvalues smoothens the effect

Sharpe-Singleton scenario?

- We can not conclusively say in what phase we were in
- The distributions show mass dependence, but not sure about the dramatic collective jump described by WχPT
- Pion mass measurements showed that mass was nonzero for all values of quark mass, but measurements did not look reliable

Conclusions and outlook

- Our implementation of the Arnoldi algorithm performs nicely
- Eigenvalue distributions show mass-dependent behavior
- Eigenvalues lie quite close to a circle
- No conclusive evidence for being in the Sharpe-Singleton scenario
- Measuring LECs from the distribution not straightforward, need to understand the mass dependence first
- Future work: bigger lattice, focusing polynomials, Sharpe-Singleton-scenario, larger N_f