Perturbative analysis of twisted volume reduced theories

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Objective

- Investigate the volume and large N dependence of Yang-Mills theories

- Revival of large N volume reduction:

  - Twisted Eguchi-Kawai — González-Arroyo & Okawa
  - Continuum reduction — Narayanan & Neuberger
  - Adjoint fermions — Kotvun, Unsal & Yaffe

- Use the volume dependence to control the onset of non-perturbative effects
  ('t Hooft, Lüscher, González-Arroyo, van Baal, ...)

In this talk

SU(N) pure gauge Yang-Mills with twisted boundary conditions

Volume effects depend on $l_{\rm eff}$

- 2-d torus x R - Effective size $l_{\rm eff} = N\, l$
- 4-d torus - Effective size $l_{\rm eff} = \sqrt{N}\, l$

✦ Perturbative analysis

✦ SU(N) non-perturbative results in 2+1 dimensions

Twisted Boundary Conditions ('t Hooft) d torus size l

$$A_\mu(x + l\,\hat{\nu}) = \Gamma_\nu A_\mu(x) \Gamma_\nu^\dagger \qquad \Gamma_\mu \Gamma_\nu = e^{\frac{2\pi i n_{\mu\nu}}{N}} \Gamma_\nu \Gamma_\mu$$

Irreducible twists - $N^2 - 1$ linearly independent traceless $\hat{\Gamma}(p)$

$$T^a A_\nu^a(x) = \mathcal{N} \sum_p{}' e^{ip\cdot x} \hat{A}_\nu(p)\, \hat{\Gamma}(p) \qquad \mathcal{N} \equiv \frac{1}{\sqrt{V}}$$

$$p_\mu = p_\mu^s + p_\mu^c$$

- 2-torus $n_{ij} = \epsilon_{ij} k$

$$\vec{p}^{\,c} = \frac{2\pi \vec{n}}{N l}$$

- 4-torus $n_{\mu\nu} = \epsilon_{\mu\nu}\, k \sqrt{N}$

$$p_\mu^c = \frac{2\pi n_\mu}{\sqrt{N}\, l}$$

Effective box - size

$$p_\mu = \frac{2\pi n_\mu}{l_{\text{eff}}}$$

$$n_\mu \in \mathbb{Z}$$

$$n \neq 0 \,(\text{mod}\, N_{\text{eff}})$$

Perturbation theory

Group structure constants $$F(-p,q,\tilde{q}) = -2i\,\mathrm{Tr}(\hat{\Gamma}^{\dagger}(p)\,[\hat{\Gamma}(q),\hat{\Gamma}(\tilde{q})])$$

$$F(p,q,-p-q) = -\sqrt{\frac{2}{N}}\sin\left(\frac{\theta_{\mu\nu}}{2}p_{\mu}q_{\nu}\right)$$

$$\theta_{\mu\nu} = \left(\frac{l_{\mathrm{eff}}}{2\pi}\right)^2 \times \tilde{\epsilon}_{\mu\nu}\,\tilde{\theta}$$

in 2-d

$$\tilde{\theta} = \frac{2\pi\bar{k}}{N}$$

$$n_{ij} = \epsilon_{ij}k$$

$$k\bar{k} = 1\,(\mathrm{mod}\,N)$$

in 4-d

$$\tilde{\theta} = \frac{2\pi\bar{k}}{\sqrt{N}}$$

$$n_{\mu\nu} = \epsilon_{\mu\nu}\,k\,\sqrt{N}$$

$$k\bar{k} = 1\,(\mathrm{mod}\,\sqrt{N})$$

Feynman rules   González-Arroyo, Okawa, Korthals-Altes

Momenta quantized in units of l_{eff}	$N\, l$	2-d
	$\sqrt{N}\, l$	4-d

Vertices

$$g\,\mathcal{N}\,F(p,q,\tilde{q}) = -\sqrt{\frac{2\lambda}{V_{\text{eff}}}}\,\sin(\theta_{\mu\nu}\,q_\mu \tilde{q}_\nu)$$

$$\theta_{\mu\nu} = \left(\frac{l_{\text{eff}}}{2\pi}\right)^2 \times\ \tilde{\epsilon}_{\mu\nu}\,\tilde{\theta}$$

Non-commutativity

In perturbation theory, physics depends on $\tilde{\theta}\,,\ \lambda\,,\ l_{\text{eff}}$

Volume independence or Reduction

Possible caveats

- Perturbative instabilities in the large N limit

Negative self-enegy $\longrightarrow$ Tachyonic instabilities

Hayakawa, Guralnik e.a., Bietenholz e.a., ...

- Non-perturbative effects ?

1- point lattice TEK $n_{\mu\nu} = \epsilon_{\mu\nu}\, k \sqrt{N}$

Symmetry breaking Ishikawa&Okawa, Teper&Vairinhos, e.a., Azeyanagi e.a.

Avoided if

$$k \text{ and } \bar{k} \propto N \quad \text{as} \quad N \to \infty$$

González-Arroyo & Okawa

Results in 2+1 d

$T^2 \times R$ with $n_{12} = k$

Look at the energy of electric flux

Mass Gap in PT

$$\frac{2\pi|\vec{n}|}{Nl} \qquad \vec{n} \neq \vec{0} \pmod N$$

$$e_i = -\bar{k}\epsilon_{ij}n_j, \quad \text{with} \quad k\bar{k} = 1 \quad (\text{mod} N)$$

Generated by Polyakov loop operators (winding = e)

Perturbation theory

4-d SU(2)
Daniel, González-Arroyo, Korthals-Altes

Lattice
Lüscher & Weisz, Snippe

Gluon dispersion relation

$$\mathcal{E}^2(p) = \vec{p}^{\,2} - \sum_\mu \Pi_{\mu\mu}(p)|_{\text{on}-\text{shell}}$$

$$\vec{p} = \frac{2\pi\vec{n}}{NL}$$

$$\frac{\mathcal{E}^2}{\lambda^2} = \vec{p}^{\,2} - \boxed{\frac{4\pi}{Nl\lambda} G\Big(\frac{\vec{e}}{N}\Big)}$$

$$G\Big(\frac{\vec{e}}{N}\Big) = \frac{1}{16\pi^2}\int_0^\infty \frac{dx}{\sqrt{x}}\Big(\theta_3^2(0,x) - \prod_{i=1}^{2}\theta_3(\frac{e_i}{N}, ix) - \frac{1}{x}\Big)$$

Remarks

$$\frac{\mathcal{E}^2}{\lambda^2} = \frac{|\vec{n}|^2}{4x^2} - \frac{1}{x} G(\frac{\vec{e}}{N})$$

- λ dimensionful + Nl dependence

  Dimensionless quantities depend on $x = \frac{Nl\lambda}{4\pi}$

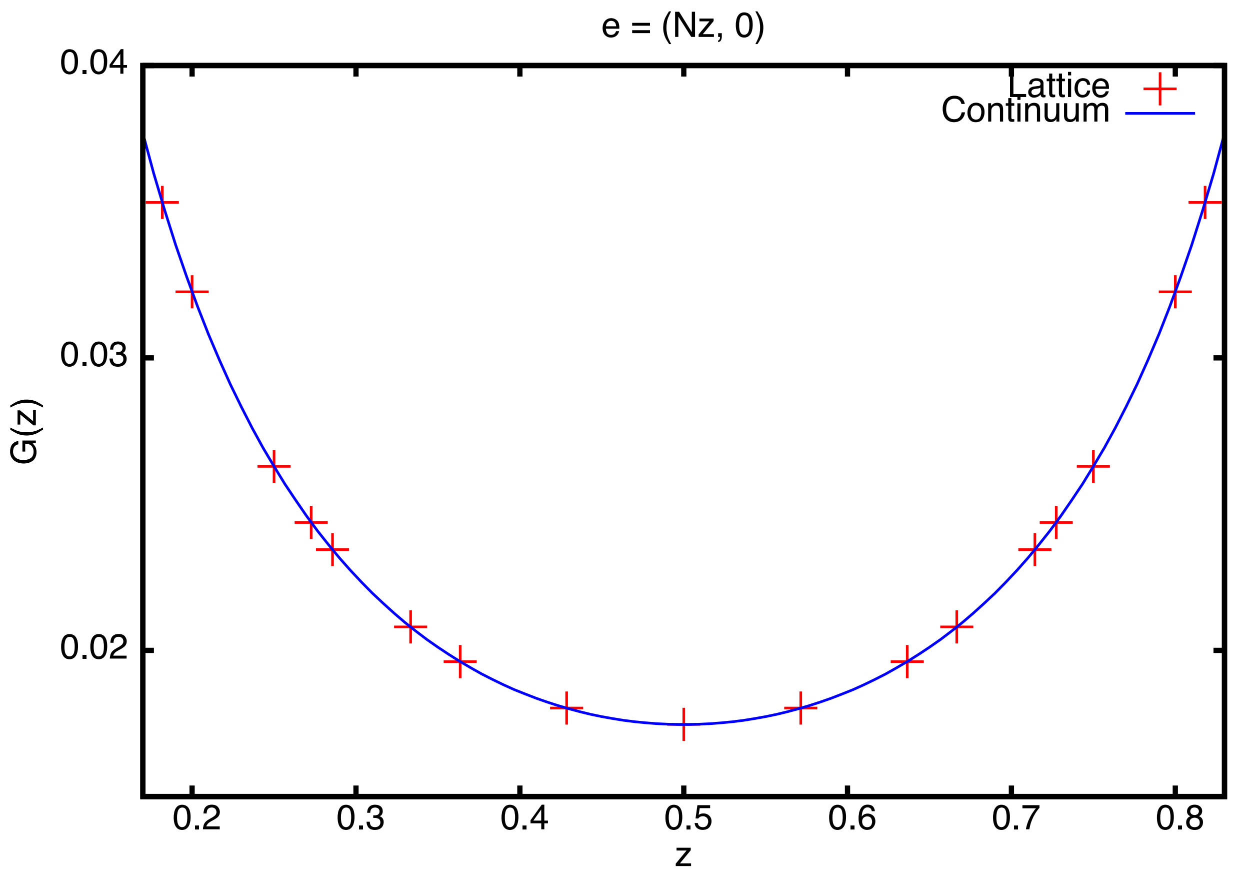
- Tachyonic instability

  $$x_T(\vec{e}) = \frac{|\vec{n}|^2}{4G(\frac{\vec{e}}{N})}$$

  Non-commutative geometry, Guralnik et al.

✦ Tachyonic instability

$$x_T(\vec{e}) = \frac{|\vec{n}|^2}{4G(\frac{\vec{e}}{N})}$$



$$G\left(\frac{\vec{e}}{N}\right) \propto \frac{N}{|\vec{e}|}$$

$$x_T(\vec{e}) = \frac{|\vec{e}||\vec{n}|^2}{4N}$$

$$e_i = -\bar{k}\epsilon_{ij}n_j$$

$$x_T = \frac{4\pi^2 k^2}{N} \qquad |\vec{e}| = 1$$

$$x_T = \frac{4\pi^2 \bar{k}}{N} \qquad |\vec{n}| = 1$$

k and $\bar{k} \propto N$ as $N \to \infty$

✦ If $x_T > 1$ PT not enough to claim instability

Non-perturbative effects

Electric-flux energies grow linearly with l

$$\frac{\mathcal{E}}{\lambda} = \frac{\sigma_{\vec{e}}\, l}{\lambda}$$

$$\sigma_{\vec{e}} = N\sigma' \phi\Big(\frac{\vec{e}}{N}\Big) \qquad \phi(z) = \phi(1-z)$$

$$\phi(z) = z(1-z) \qquad \phi(z) = \sin(\pi z)/\pi$$

- Compatible with reduction
- The linear growth can overcome the tachyonic behaviour

$$\frac{\mathcal{E}(z)}{\lambda} = 4\pi x\, \frac{\sigma'}{\lambda^2}\, \phi(z)$$

Subleading $1/l$ effects

Nambu-Goto with winding $\vec{e}$ on Kalb-Ramond B-field background

$$\frac{\mathcal{E}^2(\vec{e})}{\lambda^2} = \left(\frac{\sigma|\vec{e}|\,l}{\lambda}\right)^2 - \frac{\pi\sigma}{3\lambda^2} + \boxed{\sum_i \left(\frac{\epsilon_{ij} e_j\, B}{\lambda l}\right)^2}$$

Low energy - non-commutative field theory Susskind & Witten

$$\theta_{ij} = -\epsilon_{ij}\, l^2 \,\frac{1}{B} \quad \longleftrightarrow \quad \theta_{ij} = -\epsilon_{ij}\, l^2\, \frac{N\bar{k}}{2\pi}$$

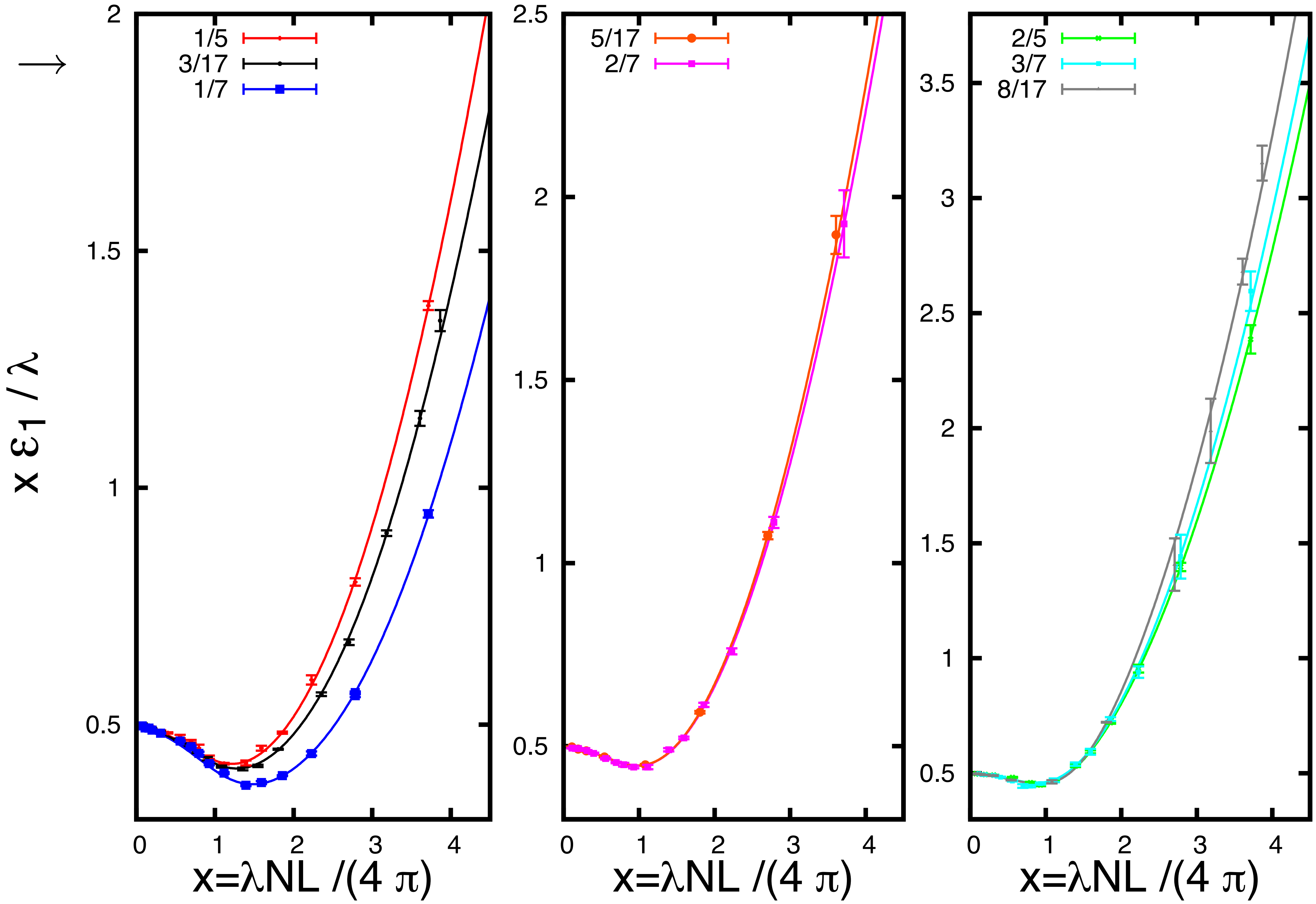
$$B = \frac{2\pi k}{N} \quad \longrightarrow \quad \left(\frac{2\pi|\vec{n}|}{Nl\lambda}\right)^2$$

tree-level PT

$$\boxed{\frac{\mathcal{E}_1^2}{\lambda^2} = \gamma^2 x^2 + \beta + \frac{1}{4x^2}}$$

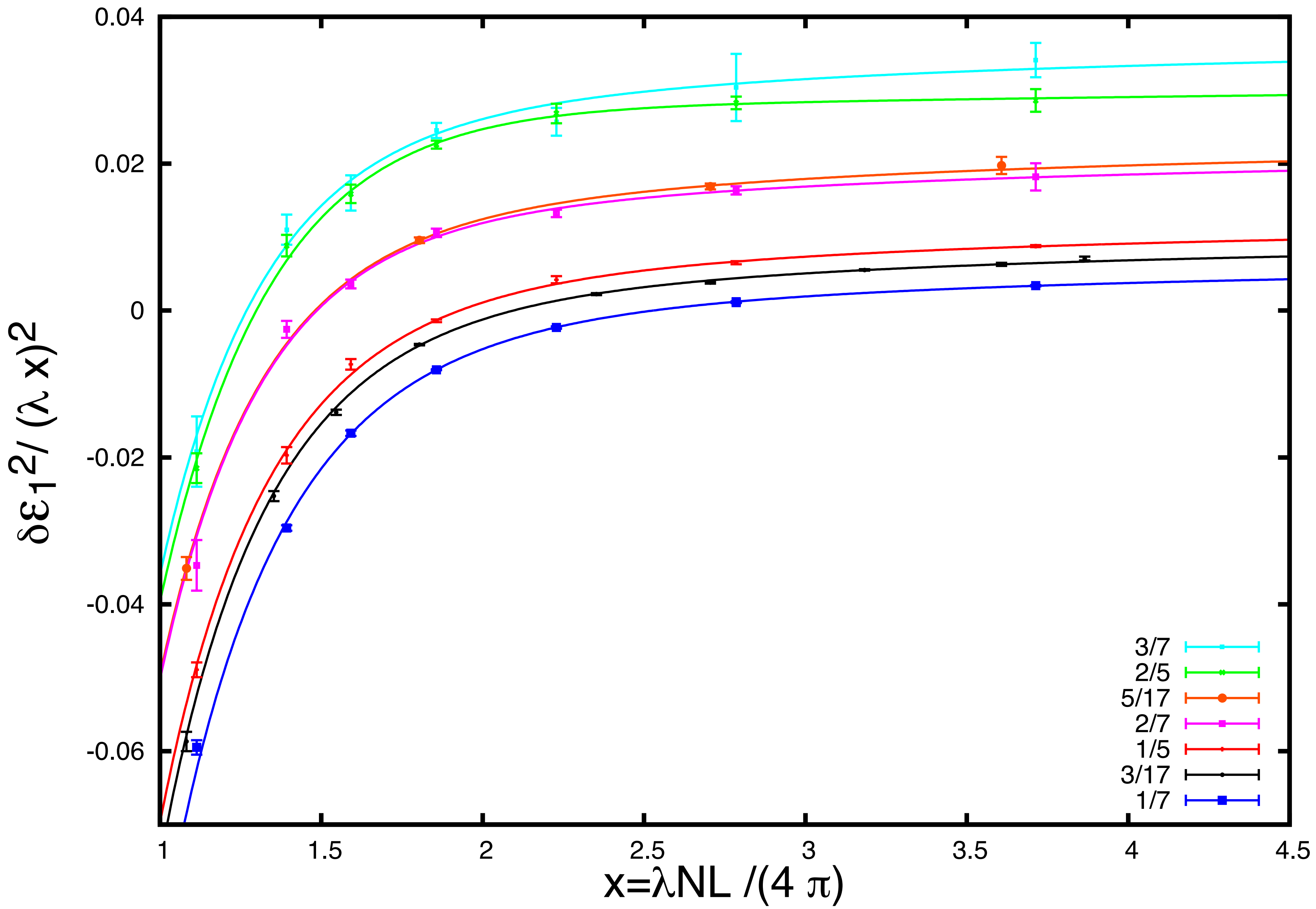
$$\frac{\mathcal{E}_1^2}{\lambda^2} = \frac{1}{4x^2} + \frac{\alpha}{x} + \frac{\mathcal{A}}{x^3\sqrt{x}} e^{-\frac{S_0}{x}} + \beta + \gamma^2\, x^2 \qquad \vec{n} = (1,0)$$

$$\frac{\tilde{\theta}}{2\pi} \longrightarrow$$



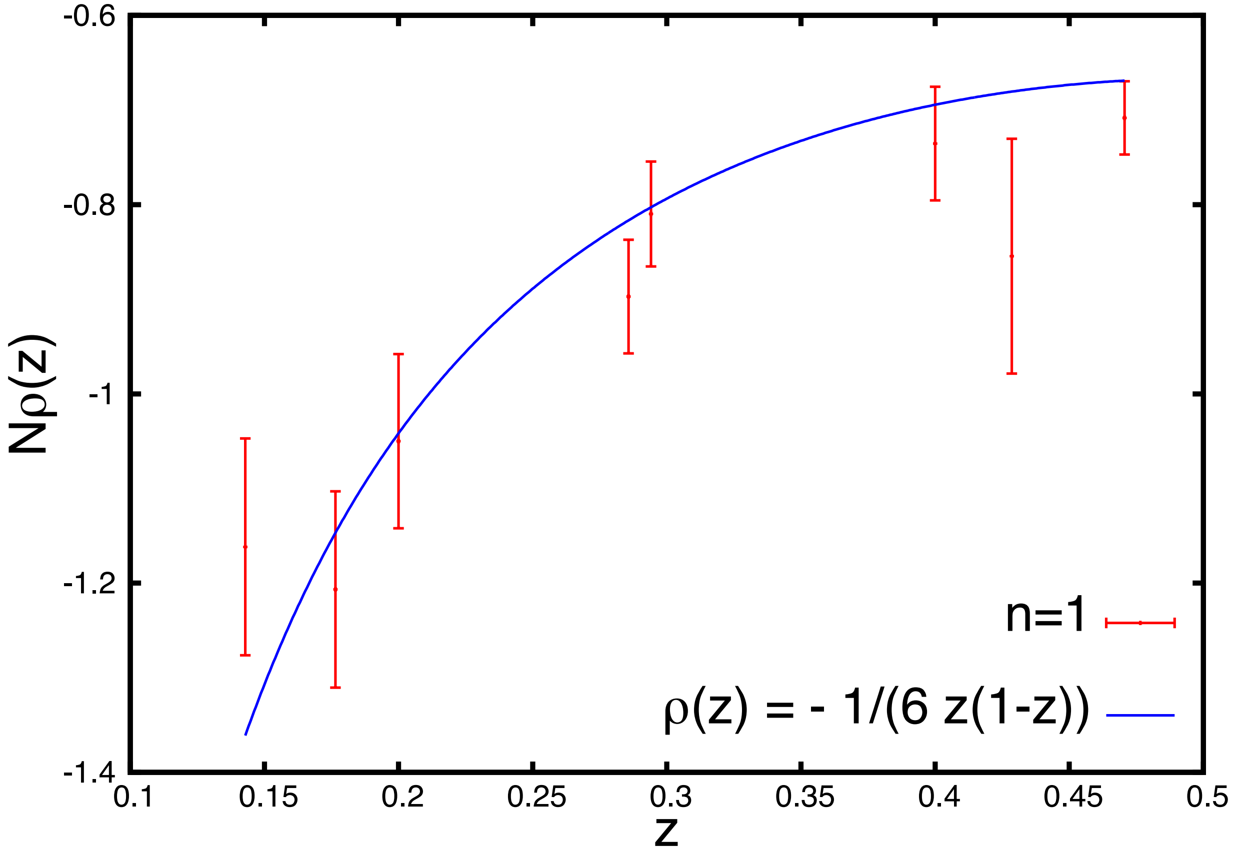
String tension

$$\frac{\mathcal{E}_1^2}{\lambda^2 x^2} - \frac{1}{4x^4} = \left(\frac{4\pi\sigma'}{\lambda^2}\phi(z)\right)^2 + \cdots$$



$\leftarrow \left(\frac{4\pi\sigma'}{\lambda^2}\phi(z)\right)^2$

$\leftarrow \frac{\tilde{\theta}}{2\pi}$



Lüscher term

$$\mathcal{E} = \sigma l + \frac{\pi \rho}{l} + \cdots$$

$$N\rho(z) \equiv \frac{2\beta}{\gamma} = \frac{\mathcal{C}}{z(1-z)}$$

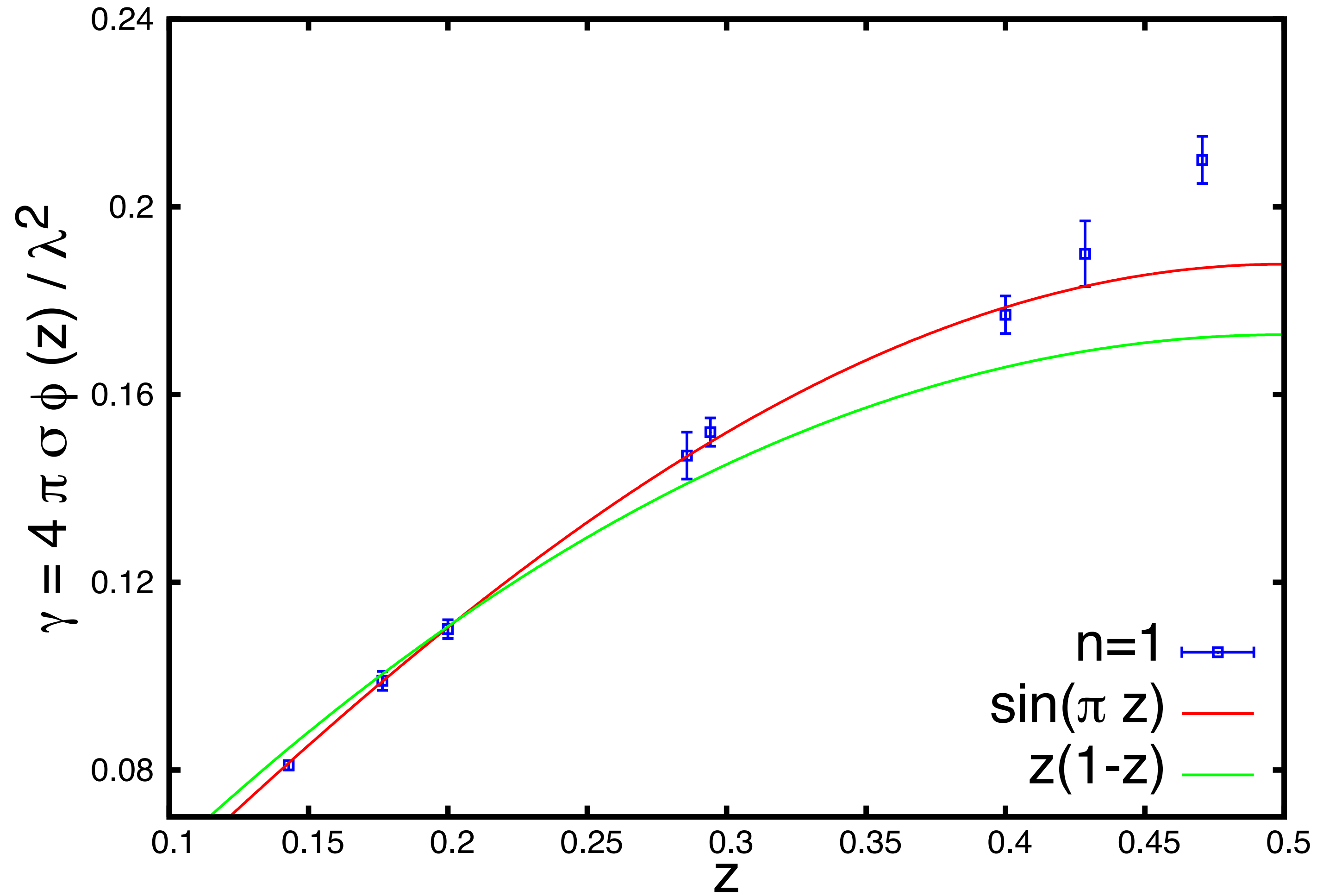
$$\mathcal{C} = -\frac{1}{6}$$

k-string tension

$$\sqrt{\sigma'} = 0.217(1)\lambda$$

$$\gamma(z) = 4\pi \frac{\sigma'}{\lambda^2}\phi(z)$$

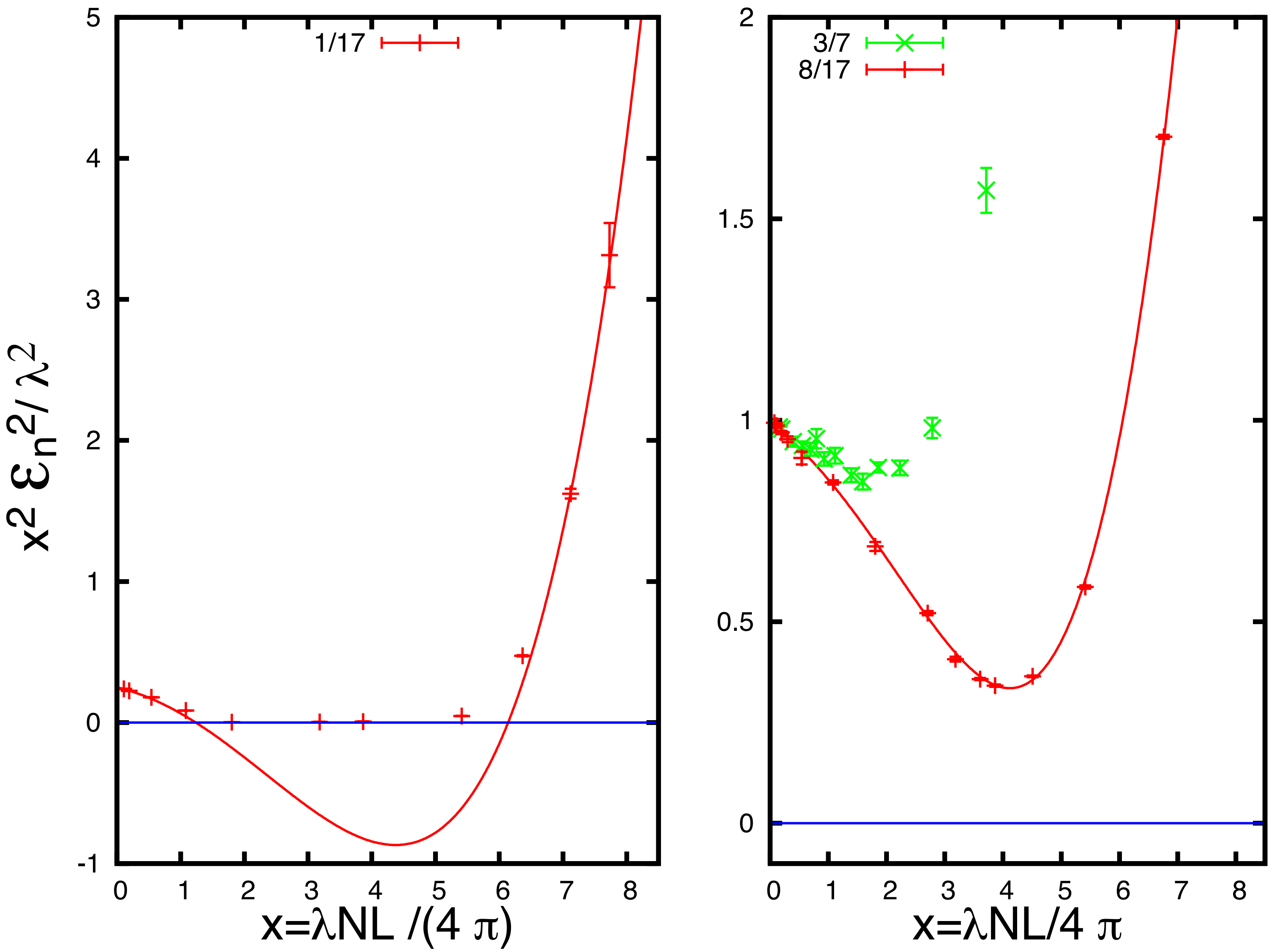
$$\phi(z) = \sin(\pi z)/\pi$$



Tachyonic instabilities

$$\frac{|\vec{n}|^2}{4} + \alpha\, x + \beta\, x^2 + \gamma^2\, x^4 \geq 0$$

$k = \bar{k} = 1$ $\qquad$ $k = 2\,,\ \bar{k} = \dfrac{N-1}{2}$



$|\vec{n}| = |\vec{e}| = 1$ $\qquad$ $|\vec{n}| = 2\,, \quad |\vec{e}| = 1$

$$\frac{k}{N} > \frac{1}{12}$$

$$\frac{\bar{k}}{N}\left(1 - \frac{\bar{k}}{N}\right) > \frac{1}{12}$$

Summary

- Perturbation theory indicates physical quantities depend on

$$\tilde{\theta}\,,\;\lambda\,,\;l_{\rm eff}$$

- $l_{\rm eff}$ combines N and l dependence
- Tachyonic instabilities can be avoided for appropriate choices of the twist
- In 2+1 dimensions non-perturbative effects preserve volume reduction