Monte Carlo studies on the expanding behavior of the early universe in the Lorentzian type IIB matrix model

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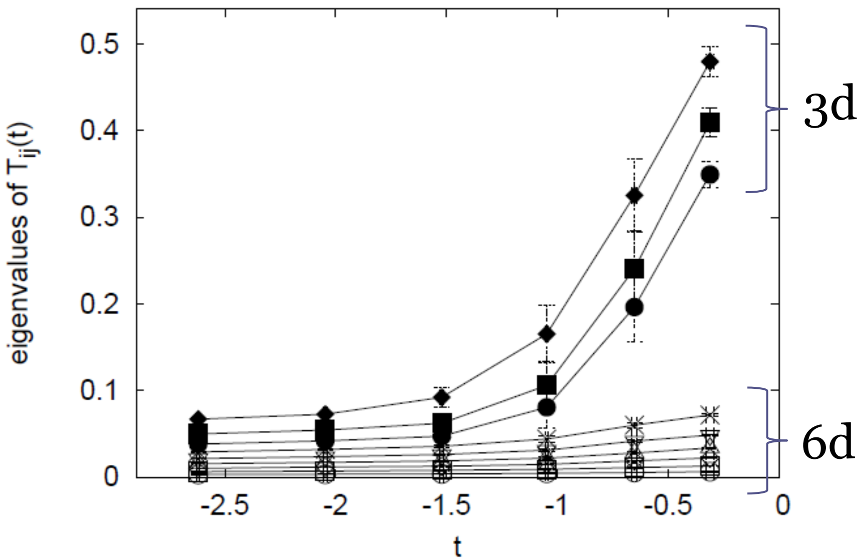
Introduction

Theory	QCD	String theory
Non-perturbative formulation	Lattice gauge theory	Matrix model

IIB matrix model

- Euclidean version — Ishibashi-Kawai-Kitazawa-Tsuchiya ('97)
- Lorentzian version — Kim-Nishimura-Tsuchiya ('11) — We can study with "real time".



Birth of 4d universe

In this works

How does the universe expand?

Inflation, power-law expansion…

Plan

- ✓ Introduction
- Definition of the Lorentzian IIB matrix model
- Exponential expansion at early time
- Power-law ($t^{1/2}$) expansion at late time
- Summary

Lorentzian IIB matrix model

- The action

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}\,[A_\mu, A_\nu]^2$$

metric $\eta = diag(-1,1,\cdots,1)$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}\,\Psi_\alpha\,(\mathcal{C}\Gamma^\mu)_{\alpha\beta}\,[A_\mu, \Psi_\beta]$$

$\left.\begin{array}{ll} A_\mu \ (\mu = 0,\ldots,9) & \text{:Lorentz vector} \\ \Psi_\alpha \ (\alpha = 1,\ldots 16) & \text{:Majorana-Weyl spinor} \end{array}\right\}$ $N \times N$ Hermitian matrices

- We deal with "real time" instead of imaginary time
- SO(9,1) symmetry
- The time coordinate is represented by the eigenvalue distribution of A_0

Diagonalize A_0

- SU(N) transformation

$$\begin{cases} A_\mu \to U A_\mu U^\dagger \\ \Psi^\alpha \to U \Psi^\alpha U^\dagger \end{cases}$$

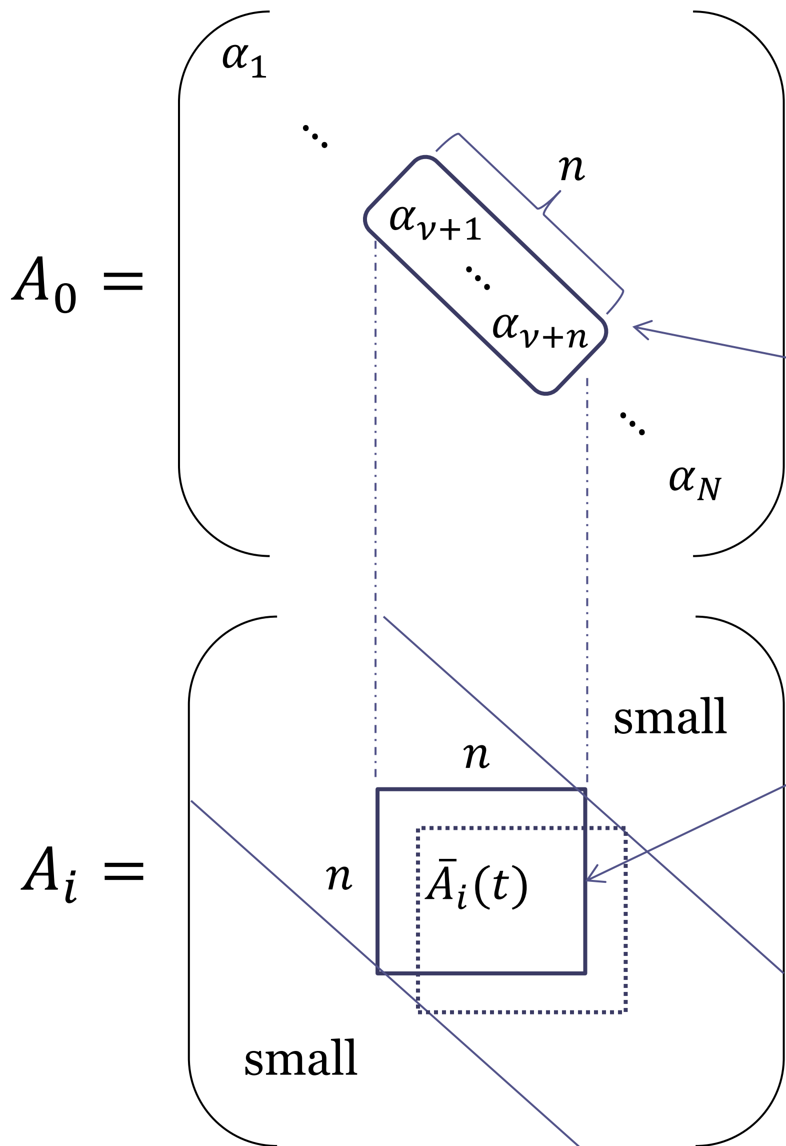
- To study time-evolution
  - diagonalize A_0

$$A_0 = diag \overbrace{(\alpha_1, \alpha_2, \ldots \alpha_N)}^{\text{interpreted as discrete time}}$$

  - order eigenvalues in this way

$$\alpha_1 < \alpha_2 < \cdots < \alpha_N$$

Definition of time-evolution



- Diagonalize A_0
- Order eigenvalues
  $\alpha_1 < \alpha_2 < \cdots < \alpha_N$

SU(N) transformation

The time "t" is defined by

$$t = \frac{1}{n}\sum_{i=1}^{n} \alpha_{\nu+i}$$

The state of the universe $\bar{A}_i(t)$ at time t

A_i has band diagonal structure.

Non-trivial dynamical property

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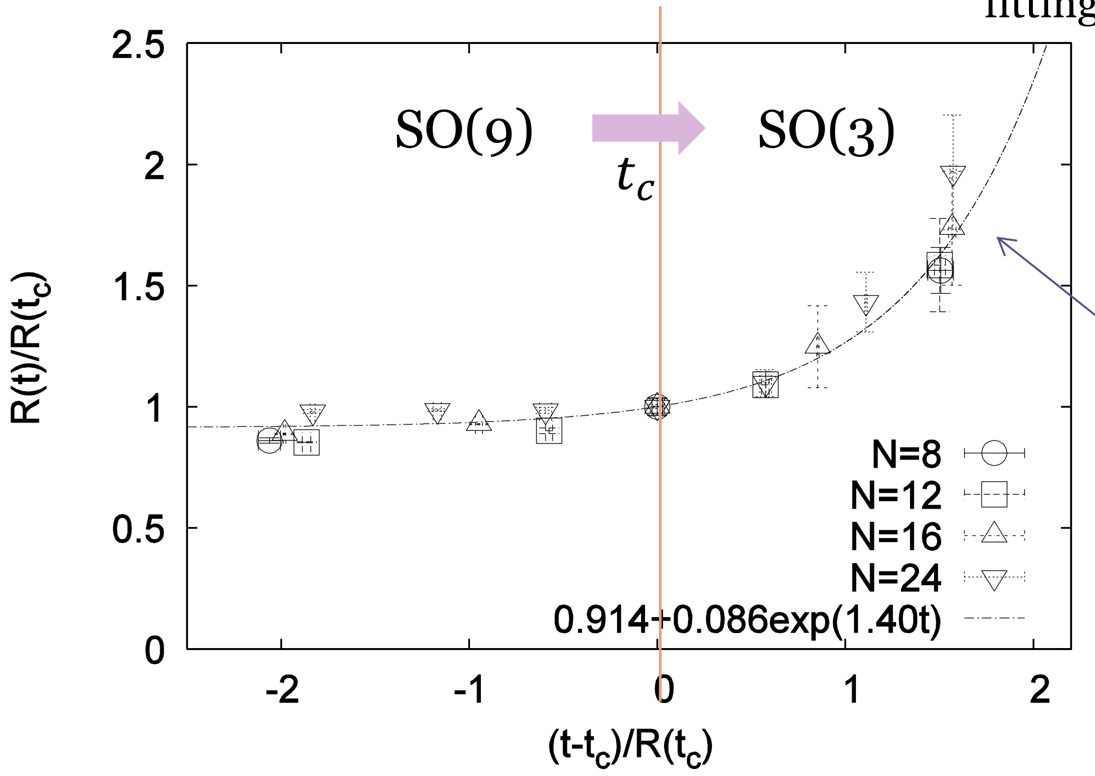
Exponential expansion

- In order to see expanding of 9d space for each time t, we define

$$R(t)^2 = \frac{1}{n} tr \bar{A}_i^2(t)$$

fitting with $f(x) = a + (1-a) e^{bx}$



・3 out of 9 directions start to expand after a critical time.

Exponential expansion

Inflation

Effects of fermionic action

$$S_{\mathrm{f}} = \mathrm{tr}\left(\Psi_\alpha \left(\Gamma^\mu\right)_{\alpha\beta}\left[A_\mu, \Psi_\beta\right]\right)$$

$$= \mathrm{tr}\left(\Psi_\alpha \left(\Gamma^0\right)_{\alpha\beta}\left[A_0, \Psi_\beta\right]\right) + \mathrm{tr}\left(\Psi_\alpha \left(\Gamma^i\right)_{\alpha\beta}\left[A_i, \Psi_\beta\right]\right)$$

The important term for early time

The important term for late time

Keep the first team only

Toy model for early time

$$\boxed{\mathrm{Pf(M(A))} = \Delta^{d-1} = \prod_{i>j}\left(\alpha_i - \alpha_j\right)^{2(d-1)}}$$

Repulsive force between eigenvalues of A_0

Toy model for late time

$$\boxed{\mathrm{Pf(M(A))} = 1}$$

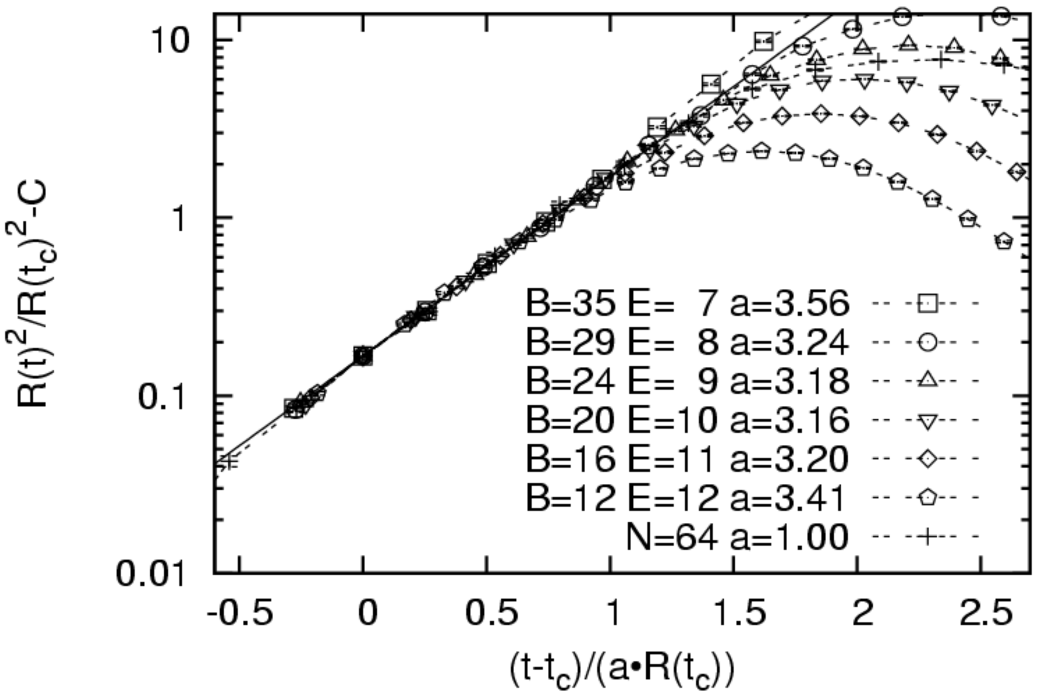
Quench fermions

Exponential expansion for early time in the toy model

- approximate fermionic action in early time

$$\mathrm{Pf}(M(A)) = \Delta^{d-1} = \prod_{i>j} (\alpha_i - \alpha_j)^{2(d-1)}$$

fitting with $a + (1-a)e^{bx}$



exponential expansion

The first term is important to expand exponentially.

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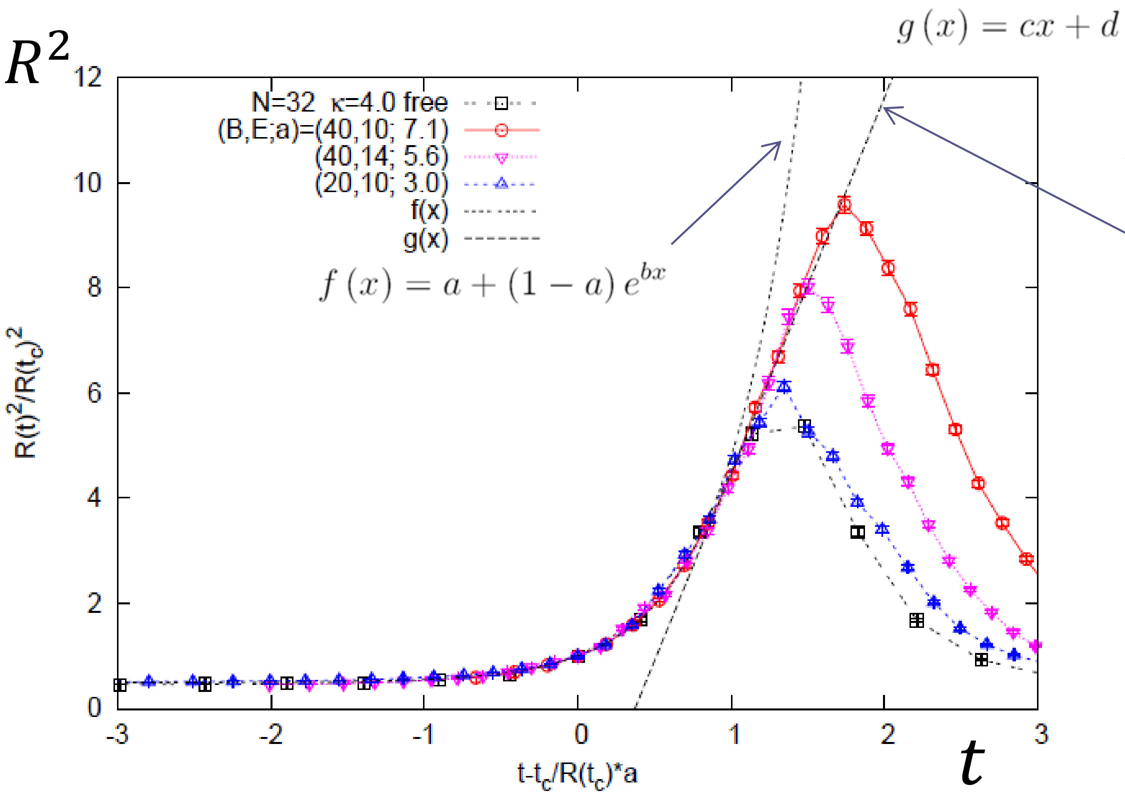
Power-law expansion for late time in the toy model

- Toy model with quenching fermions

$$\boxed{\mathrm{Pf}(M(A)) = 1}$$



$R(t)^2 \sim t \Rightarrow R(t) \sim t^{1/2}$

$t^{1/2}$behavior

Radiation dominated FRW universe

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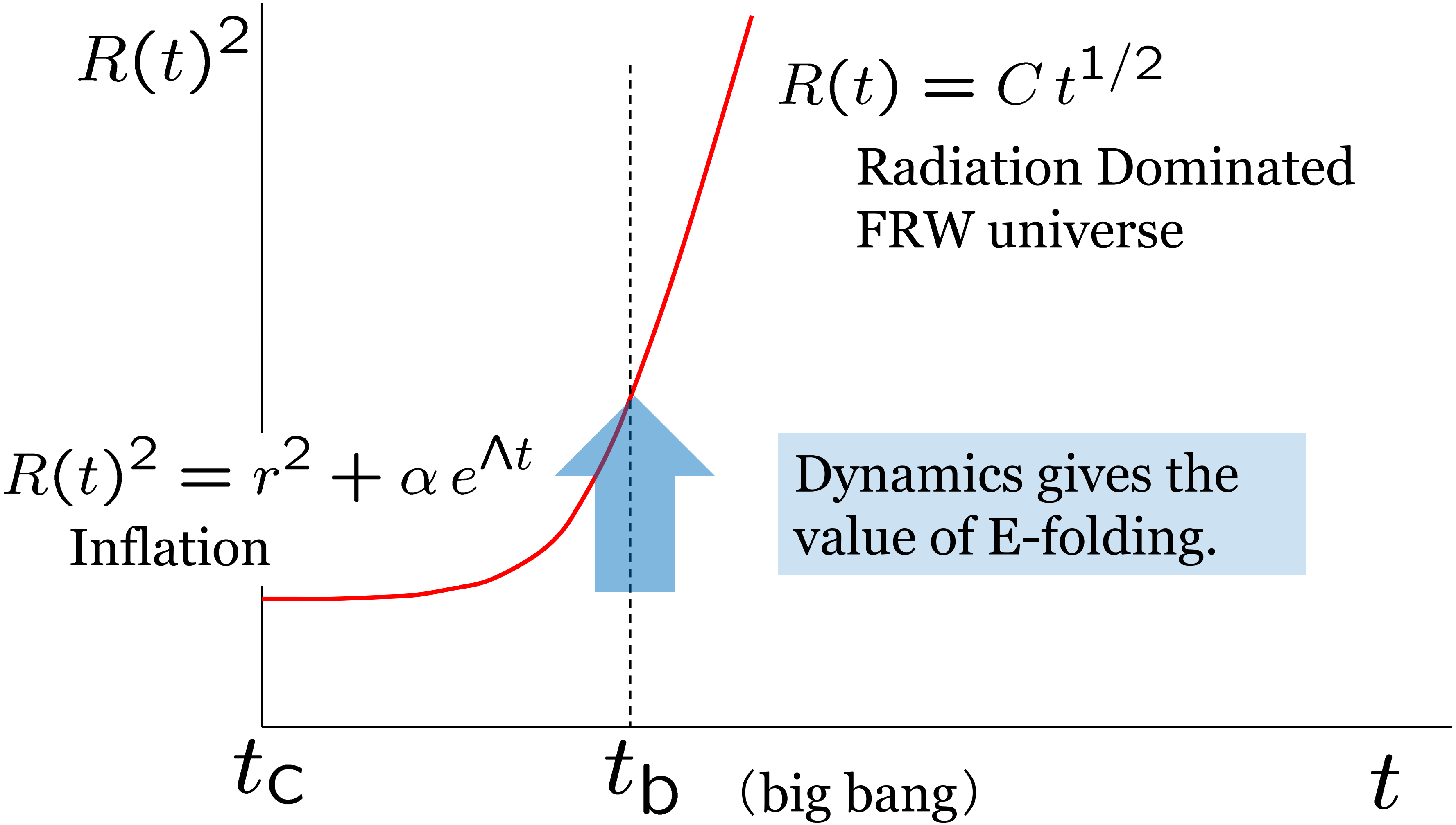
Summary

- ☐ The Lorentzian IIB MM
  - ・non-perturbative formulation of string theory.
- ☐ Previous work - Kim-Nishimura-Tsuchiya('11)
  - ・3 out of 9 spatial directions start to expand.
- ☐ In this work
  - 1) exponential expansion is observed. ➡ Inflation
  - 2) Toy model $\rightarrow$ we can study longer time.
    - ☐ early time
      exponential expansion

      $$\Delta^{d-1} = \prod_{i>j} (\alpha_i - \alpha_j)^{2(d-1)}$$
    - ☐ late time
      power-law ($t^{1/2}$) expansion

      Pf(M(A)) = 1

Expected scenario for full Lorentzian IIB MM



□ Future works

- ➢ Study the late time behaviors directly
  (instead of using toy model)
- ➢ Renormalization group method
  (integrate out early time d.o.f.)

⇨ Investigate the transition

$R(t)\sim e^{\Lambda t}$	$\Rightarrow$	$R(t)\sim t^{1/2}$
inflation	Big bang?	Radiation dominated FRW universe

□ Related works

- ➢ Studies of classical eq. of motion in Lorentzian IIB MM
  Steinacker('11)
  Kim-Nishimura-Tsuchiya('11)
- ➢ Realization of the standard model particles
  C-S-Z ('11), Aoki ('11)
  Nishimura-Tsuchiya ('11)