Euclidean 4D quantum gravity with a non-trivial measure term

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Mainz, August 1st, 2013

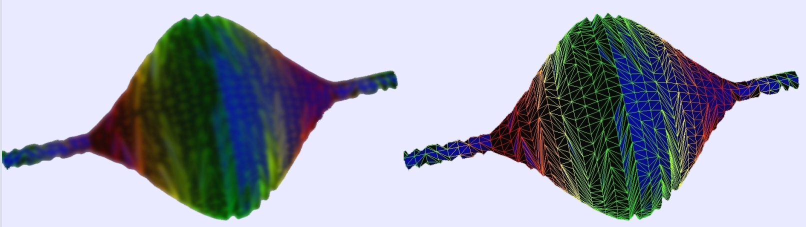
Outline

Dynamical Triangulations (DT) is a background independent approach to quantum gravity.

It provides a lattice regularization of the formal gravitational path integral via a sum over simplicial manifolds

$$Z = \int \mathrm{D}[g] e^{-S^E[g]} \quad \rightarrow \quad \sum_{\mathcal{T}} \frac{1}{C(\mathcal{T})} e^{-S^R[\mathcal{T}]}.$$

The Einstein-Hilbert action, has a natural realization on simplicial



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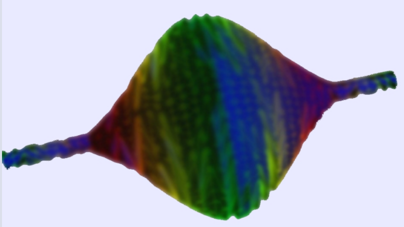

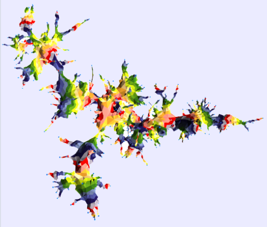

Euclidean Dynamical Triangulations in four dimensions

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The Einstein-Hilbert action, has a natural realization on simplicial manifolds called Regge action,

$$S^E[g] = -\frac{1}{G} \int \mathrm{d}t \int \mathrm{d}^D x \sqrt{g} (R - 2\Lambda) \quad \rightarrow \quad S^R[\mathcal{T}] = -\kappa_2 N_2 + \kappa_4 N_4.$$

- $N_2, \; N_4$ - number of triangles, four-simplices
- $\kappa_2, \; \kappa_4$ - bare coupling constants related to the Newton's constant G and the cosmological constant Λ

The measure term

The pure model has two coupling constants $(\kappa_4 \approx \kappa_4^{\mathrm{crit}})$ and there exist only two phases separated by first order transition.

No unique choice of the measure $\mathrm{D}[g]$

Pseudo-canonical ensemble of combinatorial triangulations (S^4):

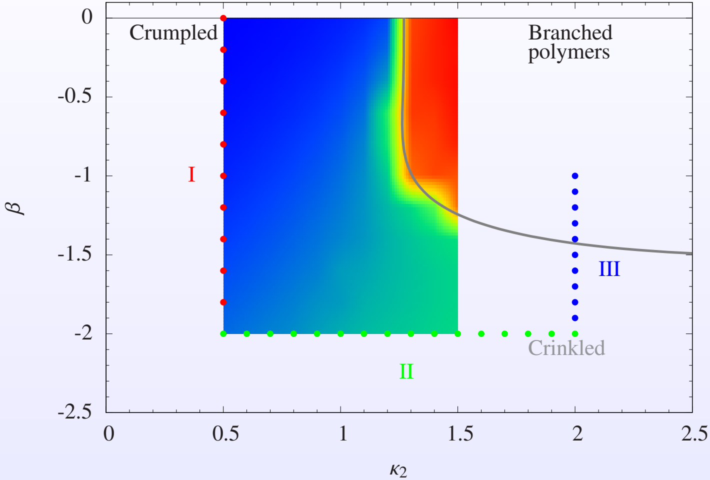
$$Z(\kappa_2, \kappa_4, \beta) = \sum_{\mathcal{T}} \frac{1}{C(\mathcal{T})} \cdot \prod_{t=1}^{N_2} o_t^{\beta} \cdot e^{-\left[-\kappa_2 N_2 + \kappa_4 N_4 + \varepsilon (N_4 - \bar{N}_4)^2\right]},$$

where o_t is the order of triangle t.

- Placing gauge field on triangulation $\rightarrow$ dual lattice.
- The additional coupling constant β may introduce new phase(s) and **higher order** transition.

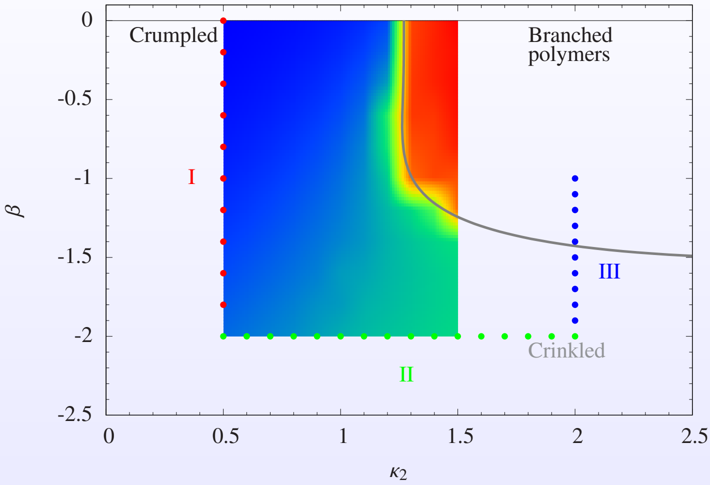
Ambjørn, Bilke, Brugmann, Burda, Frohlich, Jurkiewicz, Krzywicki, Marinari, Petersson, Tabaczek, Thorleifsson

The phase diagram



For $\beta = 0$ the **branched polymer** phase is sharply separated from the **crumpled phase** by a jump of $\langle r \rangle$ and a peak of $\chi(N_0)$.
For $\beta < 0$ we observe the **crinkled region**.

The phase diagram



- Need to study various total volumes
- Path consisting of segments I, II, III:

  crumpled phase → crinkled region → branched polymers

The phases

The crumpled phase

- Collapsed geometry. Small extension $\langle r \rangle$. $d_h \approx \infty$, $d_s \approx \infty$.
- Two singular vertices $o_v \propto N_4$, sub-singular link $o_l \propto N_4^{2/3}$.
- No baby universes.

The branched polymer phase

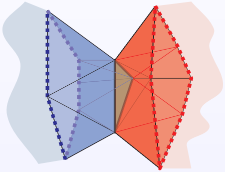
- Elongated geometry, $\langle r \rangle \propto N_4^{1/2}$. $d_h = 2$, $d_s = 4/3$
- Dominated by *minimal necks* separating baby universes
- Tree-like structure. Large baby universes.

The crinkled region

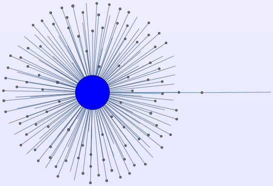
- Properties interpolate between crumpled and branched polymer
- Slow grow of extension $\langle r \rangle$ with N_4. $d_h \approx \infty$, $d_s \approx \infty$.
- Triangles of high order, $\mathrm{Max}\, o_t \propto N_4^{1/6}$. Not present in other phases
- Many minimal necks, but no large baby universes.
- Loops in minimal neck graph structure related to triangles of high order.

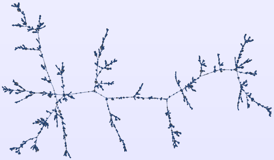
Baby Universes

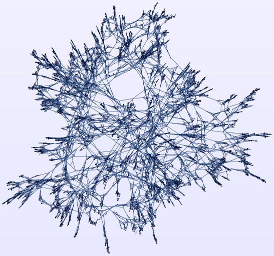
A **minimal neck** is a set of *five* tetrahedra forming a 4-simplex not present in the triangulation. Minimal necks equip triangulations with a graph structure. A **baby universe** is separated by a *miniml neck*.



Crumpled **Branched polymers** **Crinkled**

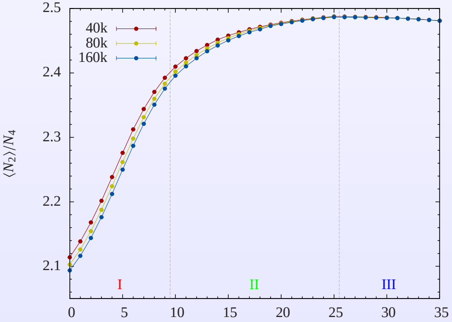


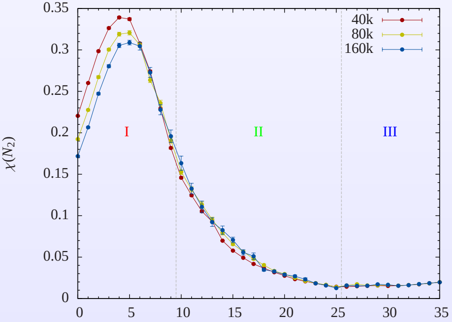




The path - N_2 observable

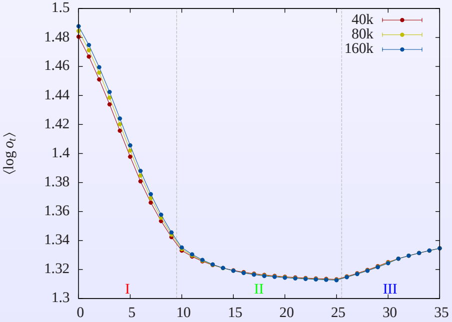
- N_2 is conjugate to κ_2.
- No jump of $\langle N_2 \rangle$, but different scaling with N_4.
- Peak of susceptibility $\chi(N_2)$ in segment I decreases with N_4.

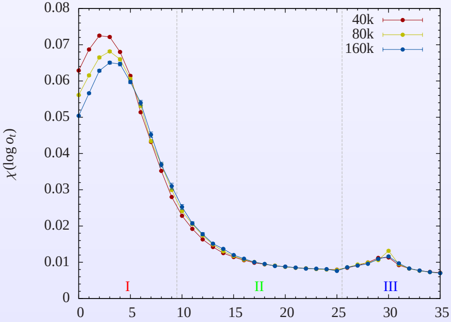




The path - $\log o_t$ observable

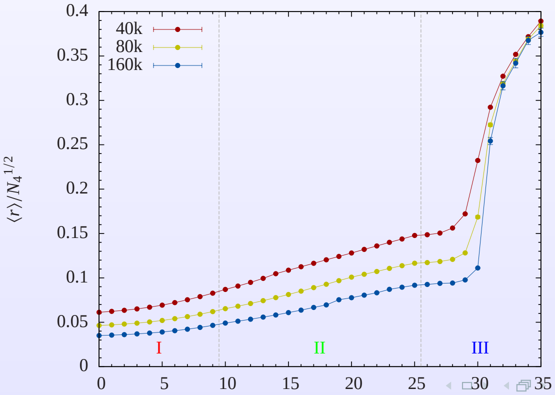
- $\log o_t$ is conjugate to β. $\langle \log o_t \rangle$ increases when β increases.
- Peak of $\chi(\log o_t)$ in segment I decreases with N_4.
- There is a peak of susceptibility at BP-crinkled transition.





The path - $\langle r \rangle$ observable

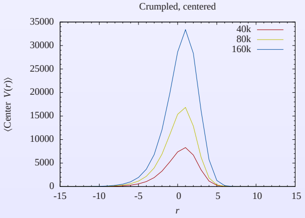
- In BP phase $\langle r \rangle$ is large and scales as $N_4^{1/2}$.
- Jump of $\langle r \rangle$ at the boundary of BP phase.
- No sign of any transition between the crumpled phase and a possible crinkled phase.



$V(r)$ - average number of simplices at a geodesic distance r.
For Hausdorff dimension d_h we expect scaling,

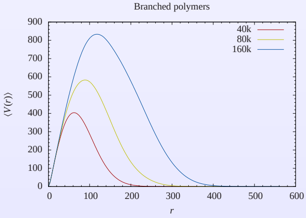
$$x = N_4^{-1/d_h} \cdot r, \quad v(x) = N_4^{-1+1/d_h} \cdot V(r).$$

Crumpled



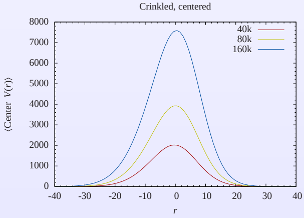
Large d_h

Branched polymers



$d_h = 2$

Crinkled



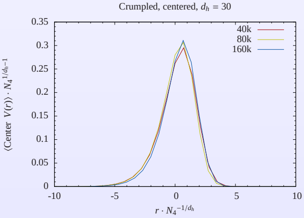
Large d_h

Hausdorff dimension

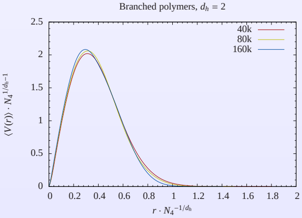
$V(r)$ - average number of simplices at a geodesic distance r.
For Hausdorff dimension d_h we expect scaling,

$$x = N_4^{-1/d_h} \cdot r, \quad v(x) = N_4^{-1+1/d_h} \cdot V(r).$$

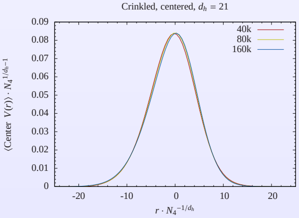
Crumpled | **Branched polymers** | **Crinkled**



Large d_h



$d_h = 2$

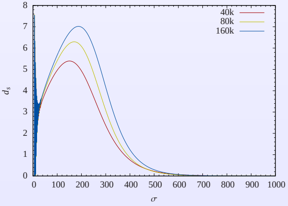


Large d_h

Spectral dimension

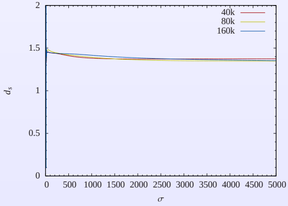
- The spectral dimension d_s describes the effective dimension seen by a diffusing particle.
- It may depend on diffusion time σ.
- Similar scale dependence of $d_s(\sigma)$ in crinkled region and CDT de Sitter phase, but here it grows with N_4.

Crumpled



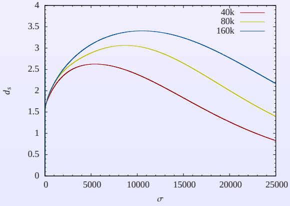
Grows with N_4

Branched polymers



$$d_s = \frac{4}{3}$$

Crinkled



Grows with N_4

Conclusions

- Monte Carlo simulations of four-dimensional Euclidean Dynamical Triangulations with a measure term using combinatorial triangulations suggest that the transformation from crumpled to crinkled triangulations is gradual.
- There is no signal, growing with the total volume, of a phase transition between the crumpled phase and the crinkled phase.
- Following the path from the crumpled phase to the crinkled region, the singular structure dissolves gradually and breaks into smaller pieces.
- Configurations in the crinkled region look less "*crumpled*" but it seems to be a finite size effect.
- Branch polymer phase is visibly separated from other phases.

Thank you for your attention!