The spectrum of supersymmetric Yang Mills theory – new results and recent measurements

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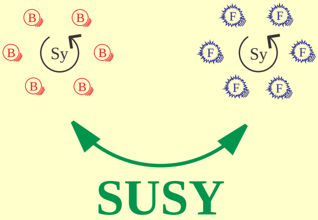


$\mathcal{N} = 1$ SUSY Yang-Mills Theory

Lagrangean

$$\mathcal{L} = \int d^2\theta \, \mathrm{Tr}(W^A W_A) + \mathrm{h.\,c.}$$
$$= \frac{1}{4} F_{\mu\nu}^{\;a} F_{\mu\nu}^{\;a} + \frac{1}{2} \overline{\lambda}^a \gamma_\mu (\mathcal{D}_\mu \lambda)^a + \frac{1}{2} D^a D^a$$

Vector supermultiplet:

- Gauge field $A_\mu^a(x)$, $\quad a = 1, \ldots, N_c^2 - 1\,,$ "Gluon"
  Gauge group $\mathrm{SU}(N_c)$
- Majorana-spinor field $\lambda^a(x)$, $\quad \overline{\lambda} = \lambda^T C\,,$ "Gluino"
  adjoint representation: $\mathcal{D}_\mu \lambda^a = \partial_\mu \lambda^a + g\, f_{abc} A_\mu^b \lambda^c$
- (auxiliary field $D^a(x)$)

SUSY: (on-shell) $\quad \delta A_\mu^a = -2\,\mathrm{i}\, \overline{\lambda}^a \gamma_\mu \varepsilon\,, \quad \delta \lambda^a = -\sigma_{\mu\nu} F_{\mu\nu}^{\;a} \varepsilon$

$\mathcal{N} = 1$ SUSY Yang-Mills Theory

- Simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended standard model

$$\mathcal{N} = 1$$

- Similar to QCD

  Differences: λ : 1.) Majorana, "$N_f = \frac{1}{2}$"
  2.) adjoint representation of SU(N_c)

- Gluino mass term

$$m_{\tilde{g}} \; \overline{\lambda}^a \lambda^a$$

breaks SUSY softly.

Non-perturbative Problems

- Spontaneous breaking of chiral symmetry $Z_{2N_c} \rightarrow Z_2$
  $\updownarrow$
  Gluino condensate $< \lambda\lambda > \neq 0$
- Spectrum of bound states
  $\rightarrow$ Supermultiplets
- Confinement of static quarks.
- Spontaneous breaking of SUSY?
- SUSY restauration on the lattice
- Check predictions from
  effective Lagrangeans
  (Veneziano, Yankielowicz, ...)

Spontaneous breaking of chiral symmetry

$\mathrm{U}(1)_\lambda$: $\quad \lambda' = \mathrm{e}^{-\mathrm{i}\varphi\gamma_5}\lambda\,, \quad \overline{\lambda}' = \overline{\lambda}\,\mathrm{e}^{-\mathrm{i}\varphi\gamma_5} \quad \leftrightarrow \quad$ R-symmetry

$$J_\mu = \overline{\lambda}\gamma_\mu\gamma_5\lambda$$

Anomaly: $\quad \partial^\mu J_\mu = \frac{N_c g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$

breaks $\quad \mathrm{U}(1)_\lambda \quad \rightarrow \quad Z_{2Nc}$

Spontaneous breaking $\quad Z_{2N_c} \quad \rightarrow \quad Z_2$

by Gluino condensate $\quad < \lambda\lambda > \neq 0$

$\leftrightarrow \quad$ first order phase transition at $m_{\tilde{g}} = 0$

$N_c = 2: \quad < \lambda\lambda > = \pm C\Lambda^3$

Spectrum of bound states

Expect colour neutral bound states of gluons and gluinos
$\rightarrow$ Supermultiplets

Predictions from effective Lagrangeans:
chiral supermultiplet (Veneziano, Yankielowicz)

- 0^- gluinoball a–η' $\sim$ $\overline{\lambda}\gamma_5\lambda$
- 0^+ gluinoball a–f_0 $\sim$ $\overline{\lambda}\lambda$
- spin $\frac{1}{2}$ gluino-glueball $\sim$ $\sigma_{\mu\nu}\,\mathrm{Tr}\,(F_{\mu\nu}\lambda)$

Generalization (Farrar, Gabadadze, Schwetz):
additional chiral supermultiplet

- 0^- glueball
- 0^+ glueball
- gluino-glueball

possible mixing

SUSY on the Lattice

Lattice breaks SUSY.

Restauration in the continuum limit?

Curci, Veneziano:

use Wilson action, search for continuum limit with SUSY

$$S = -\frac{\beta}{N_c} \sum_p \mathrm{Re}\,\mathrm{Tr}\; U_p$$

$$+\frac{1}{2}\sum_x \left\{ \overline{\lambda}^a_x \lambda^a_x - \kappa \sum_{\mu=1}^{4} \left[\overline{\lambda}^a_{x+\hat{\mu}} V_{ab,x\mu} (1+\gamma_\mu) \lambda^b_x + \overline{\lambda}^a_x V^t_{ab,x\mu} (1-\gamma_\mu) \lambda^b_{x+\hat{\mu}} \right] \right\}$$

$$\beta = \frac{2N_c}{g^2}$$

$$\kappa = \frac{1}{2m_0 + 8}$$ hopping parameter, $m_0 =$ bare gluino mass

$$V_{ab,x\mu} = 2\,\mathrm{Tr}\,(U^\dagger_{x\mu} T_a U_{x\mu} T_b)$$

SUSY on the Lattice

$$S_f = \frac{1}{2}\overline{\lambda} Q \lambda = \frac{1}{2}\lambda M \lambda \,, \qquad M \equiv CQ$$

$$\int [d\lambda]\, \mathrm{e}^{-S_f} = \mathrm{Pf}(M) = \pm\sqrt{\det Q}$$

Effective gauge field action

$$S_{\mathrm{eff}} = -\frac{\beta}{N_c}\sum_p \mathrm{Re}\,\mathrm{Tr}\; U_p - \frac{1}{2}\log\det Q[U]$$

Include sign $\mathrm{Pf}(M)$ in the observables.

Gauge group SU(2) in most of our work.

Monte Carlo Algorithm

- Two-Step Polynomial Hybrid Monte Carlo algorithm
  (TS-PHMC) Frezzotti, Jansen; Montvay, Scholz
  very efficient, $\tau < 10$ at smallest $m_{\tilde{g}}$
- Rational Hybrid Monte Carlo algorithm (RHMC)

Sign Problem

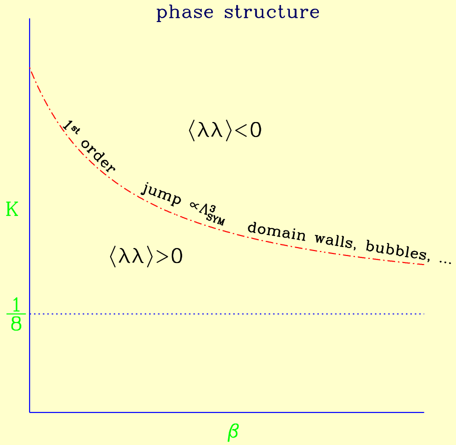
monitoring of $\mathrm{sign}\,\mathrm{Pf}(M)$

- through spectral flow
- by calculation of real negative eigenvalues of Q with Arnoldi

$\rightarrow$ negative Pfaffians occur in our simulations near κ_c, but rarely.

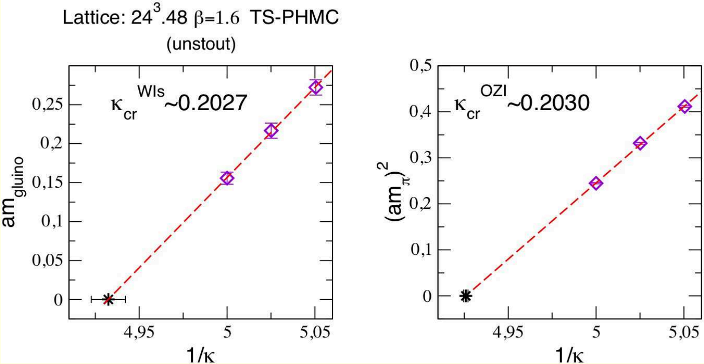
Phase transition for SU(2)

Expectation:



The dashed-dotted line $\kappa = \kappa_c(\beta)$ is a first order phase transition at zero gluino mass.

Phase transition point



SUSY Ward identities

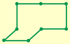
renormalized gluino mass

$$am_{\tilde{g}} Z_S = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

OZI-arguments

$$(am_{a-\pi})^2 \simeq A\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

Bound states

Glueballs: 0^+, $0^- \cong$ 

Gluino-glueballs, Spin $\frac{1}{2}$ Majorana

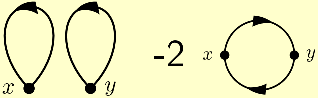
$$\chi_\alpha \simeq \frac{1}{2} F^a_{\mu\nu} (\sigma_{\mu\nu})_{\alpha\beta} \lambda^a_\beta$$

Gluino-balls

$$\overline{\lambda}\gamma_5\lambda \quad : \quad \text{a–}\eta' \quad , \quad 0^-$$

$$\overline{\lambda}\lambda \quad : \quad \text{a–}f_0 \quad , \quad 0^+$$

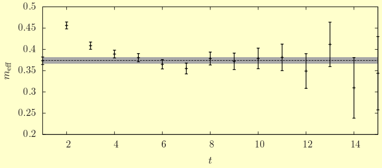
Correlators of mesons have disconnected pieces



Examples

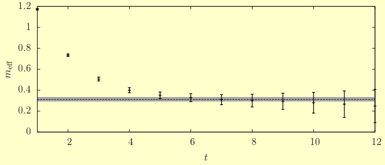
gluino-glue

Lattice $24^3 \times 48$ $\kappa = 0.1492$



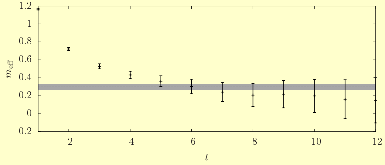
a–η'

Lattice $24^3 \times 48$ $\kappa = 0.1492$



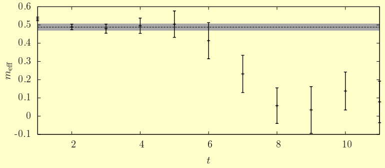
a–f_0

Lattice $24^3 \times 48$ $\kappa = 0.1492$

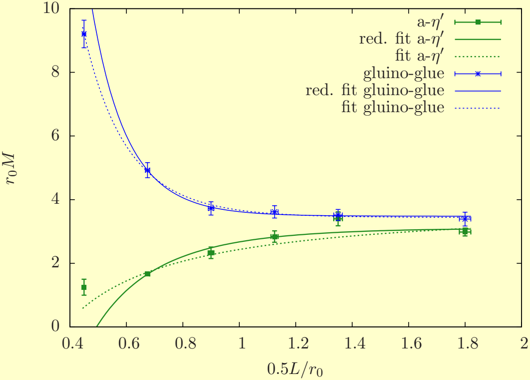


0^+ glueball

Lattice $24^3 \times 48$ $\kappa = 0.1490$



Finite size effects



Masses of gluino-glue and the a - η' meson as a function of L

$\beta = 1.75$, $\kappa = 0.1490$

Finite size effects are sufficiently small for $L > 1.2$ fm.
(QCD units: $r_0 = 0.5$ fm)

Spectrum

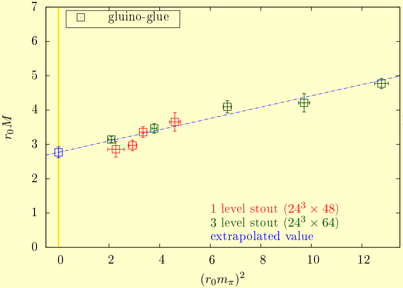
Lattices $16^3 \cdot 32$, $24^3 \cdot 48$, $(32^3 \cdot 64)$, Stout links

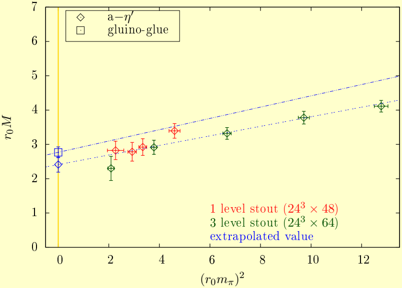
$\beta = 1.6$, $a \sim 0.088$ fm, $L \geq 2$ fm
$\beta = 1.75$, $a \sim 0.058$fm, $L = 1.39$ fm, (1.86 fm)
$m_{a-\pi} \sim 570$ MeV (461 MeV) (QCD units: $r_0 = 0.5$ fm)

Previous results:

gap between gluino-glue and its supposed superpartners

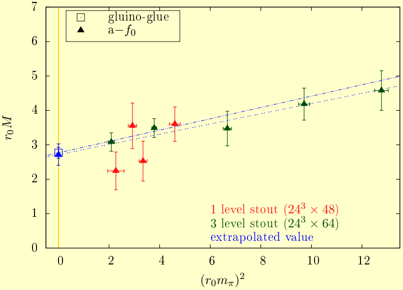
Extrapolations to $m_{\tilde{g}} = 0$

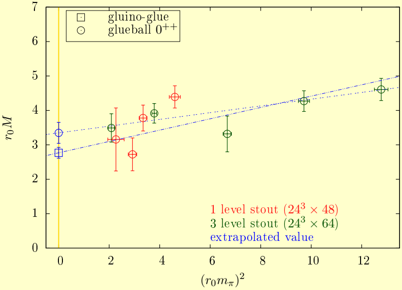




Spectrum

Extrapolations to $m_{\tilde{g}} = 0$





β	a–η'	a–f_0	$\tilde{g}g$	glueball 0^{++}
1.6	670(63)	571(181)	1386(39)	721(165)
1.75	950(87)	1070(123)	1091(62)	1319(120)

Comparison of the bound state masses in units of MeV

Summary

Status:

- Finite size effects are sufficiently small for $L > 1.2$ fm
- Efficient algorithms: TS-PHMC, RHMC
- Consistency with SUSY Ward identities
- Quantitative results about the low-energy spectrum
- Better statistics
- Extrapolations towards vanishing gluino mass
- The previously seen considerable gap between the spin-1/2 gluino-glue bound state and its expected super-partners is not any longer seen.
- Results are consistent with the formation of degenerate supermultiplets

Summary

Goals:

- Scaling, extrapolation to continuum
- Refined methods for spectrum analysis

Work in progress:

- Same statistics and analysis at $\beta = 1.9$
- Clover improvement (see Stefano Piemonte's talk)
- Smaller gluino mass

Recent publications:

- G. Bergner, T. Berheide, I. Montvay, G. Münster, U. D. Özugurel, D. Sandbrink, JHEP **1209** (2012) 108 [arXiv:1206.2341 [hep-lat]]
- G. Bergner, I. Montvay, G. Münster, U. D. Özugurel, D. Sandbrink, [arXiv:1304.2168 [hep-lat]]
- S. Musberg, G. Münster, S. Piemonte, JHEP **1305** (2013) 143 [arXiv:1304.5741 [hep-lat]]