SU(3) flavour symmetry breaking and charmed states


– QCDSF-UKQCD Collaboration –

[Lattice 2013, Mainz, Germany]
QCDSF related talks with 2 + 1 flavours:

• James Zanotti
  \textit{SU}(3) flavour breaking and baryon structure

• Ashley Cooke
  Flavour Symmetry Breaking in Octet Hyperon Matrix Elements

• Paul Rakow
  The Hadronic Decays of Decuplet Baryons

• Holger Perlt
  Perturbatively improving renormalization constants

• Gerrit Schierholz
  Dynamical 2 + 1 flavor QCD + QED
Introduction

- Background:
  - Given 2 + 1 simulations (at quark masses larger than physical quark masses), how can we usefully approach the physical point?
  - Possibility: $SU(3)$ flavour expansion about flavour symmetric line
  - Mass ‘fan’ plots

- Extend expansion to PQ quark masses (ie valence quarks $\neq$ sea quarks)

- (quenched) charm quark

- Open charm masses

- Conclusions
Many paths to approach the physical point

eg \( m_u = m_d = m_l \)

QCDSF strategy: extrapolate from a point on the \( SU(3)_F \) flavour symmetry line to the physical point

\[(m_0, m_0) \rightarrow (m_l^*, m_s^*)\]

Choice here: keep the singlet quark mass \( \bar{m} \) constant

\[\bar{m} = m_0 = \frac{1}{3} (2m_l + m_s)\]
QCDSF strategy

- develop $SU(3)$ flavour symmetry breaking expansion for hadron masses
- expansion in:

  $\delta m_q = m_q - \bar{m}, \quad \bar{m} = \frac{1}{3}(m_u + m_d + m_s) = m_0$

  - expansion coefficients are functions of $\bar{m}$
  - trivial constraint

  $\delta m_u + \delta m_d + \delta m_s = 0$

  - path called ‘unitary line’ as expand in both sea and valence quarks
**SU(3) flavour symmetry breaking expansions**

- **octet pseudoscalar mesons:**

\[
M^2(ab) = M_{0\pi}^2 + \alpha(\delta m_a + \delta m_b) \\
+ \beta_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
+ \beta_1(\delta m_a^2 + \delta m_b^2) + \beta_2(\delta m_a - \delta m_b)^2 \\
+ \ldots \quad [a, b = u, d, s \text{ (outer ring)}]
\]

- **octet baryons:**

\[
M^N_2(aab) = M_{0N}^2 + A_1(2\delta m_a + \delta m_b) + A_2(\delta m_b - \delta m_a) \\
+ B_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
+ B_1(2\delta m_a^2 + \delta m_b^2) + B_2(\delta m_b^2 - \delta m_a^2) + B_3(\delta m_b - \delta m_a)^2 \\
+ \ldots \quad [a, b = u, d, s \text{ (outer ring)}]
\]

\[
M^\Lambda_2(aab) = M_{0\Lambda}^2 + A_1(2\delta m_a + \delta m_b) - A_2(\delta m_b - \delta m_a) \\
+ B_0 \delta m_f^2 \\
+ B_1(2\delta m_a^2 + \delta m_b^2) - B_2(\delta m_b^2 - \delta m_a^2) + B_4(\delta m_b - \delta m_a)^2 \\
+ \ldots \quad [a, b = l, s \text{ (when no } \Lambda^0 - \Sigma^0 \text{ mixing)}]
\]

stable under strong ints.
Main observation:

- Provided $\overline{m}$ kept constant, then expansion coefficients remain unaltered whether
  - $1 + 1 + 1$
  - $2 + 1$
- Opens possibility of determining quantities that depend on $1 + 1 + 1$ flavours (ie pure QCD isospin breaking effects) from just $2 + 1$ simulations
Defining the scale – using singlet quantities

- pseudoscalar mesons (centre of mass):
  \[
  X^2_\pi = \frac{1}{6}(M^2_{K^+} + M^2_{K^0} + M^2_{\pi^+} + M^2_{\pi^-} + M^2_{K^0} + M^2_{K^-}) = (0.4116 \text{ GeV})^2 \\
  = M^2_0 + \left(\frac{1}{6}\beta_0 + \frac{2}{3}\beta_1 + \beta_2\right)(\delta m^2_u + \delta m^2_d + \delta m^2_s) = M^2_0 + O(\delta m^2_q)
  \]

- octet baryons (centre of mass):
  \[
  X^2_N = \frac{1}{6}(M^2_p + M^2_n + M^2_{\Sigma^+} + M^2_{\Sigma^-} + M^2_{\Xi^0} + M^2_{\Xi^-}) = (1.160 \text{ GeV})^2 \\
  = M^2_0 + \frac{1}{6}(B_0 + B_1 + B_3)(\delta m^2_u + \delta m^2_d + \delta m^2_s) = M^2_0 + O(\delta m^2_q)
  \]

- gluonic quantities: \( X^2_{t_0} = 1/t_0, \ldots \)

- other possibilities:
  \[
  X^2_\Lambda = \frac{1}{2}(M^2_{\Xi} + M^2_{\Lambda}), \quad X^2_\rho = \frac{1}{6}(M^2_{K^{*+}} + M^2_{K^{*0}} + M^2_{\rho^+} + M^2_{\rho^-} + M^2_{K^{*0}} + M^2_{K^{*-}}), \ldots
  \]

- all singlet quantities
  \[
  X^2_S = \# + \#(\delta m^2_q)
  \]

(almost) constant

- form dimensionless ratios (within a multiplet):
  \[
  \tilde{M}^2 \equiv \frac{M^2}{X_S^2}, \quad S = \pi, N, \ldots, \quad \tilde{A}_i \equiv \frac{A_i}{M^2_0}, \ldots \text{ in expansions}
  \]

stable under strong ints.
Lattice

- $O(a)$ NP improved clover action
  - tree level Symanzik glue
  - mildly stout smeared 2 + 1 clover fermion
  - $\beta = 5.50 [5.80]$, $32^3 \times 64$

$$m_q = \frac{1}{2} \left( \frac{1}{\kappa_q} - \frac{1}{\kappa_{0c}} \right)$$

$k_{0c}$ is chiral limit along symmetric line

giving

$$m_0 = \frac{1}{2} \left( \frac{1}{\kappa_0} - \frac{1}{\kappa_{0c}} \right) = \bar{m} = \frac{1}{3} (2m_l + m_s) = \frac{1}{2} \left( \frac{2}{\kappa_l} + \frac{1}{\kappa_s} - \frac{1}{\kappa_{0c}} \right)$$

So $1/\kappa_{0c}$ cancels: given $\kappa_0$ and $\kappa_l$ gives $\kappa_s$

$$\delta m_q = m_q - m_0 = \frac{1}{2} \left( \frac{1}{\kappa_q} - \frac{1}{\kappa_0} \right)$$
Use the pseudoscalar fan plot to determine the physical quark mass: $\delta m_i^*$

- $2 + 1, \ q = l, \ s,$
  $\delta m_u = \delta m_d = \delta m_l$
  $\delta m_s = -2\delta m_l$

- $O(a)$-improved clover fermions;
  $32^3 \times 64$ lattices
  [fitted, filled pts]

- $\delta m_l = m_l - \overline{m}$

- $\overline{m} = \text{const.}$
  [to find need to tune]

- $M_N = M^N(lll'''),$
  $M_\Sigma = M^N(lls),$
  $M_\Xi = M^N(ssl),$
  $M_{Ns} = M^N(sss'')$
  [PQ]
Scale determination

- \( X^2_{t_0}, X^2_{w_0}, X^2_\pi, X^2_\rho, X^2_N \approx X^2_\Lambda \)
along the unitary line
  \( [M_\pi \sim 410 \text{ MeV} - 260 \text{ MeV}] \)
- as constant down to physical point use \( X^\text{exp}_N \) to determine scale

\[
a^2_S = \left(\frac{aX_S}{X^\text{exp}_S}\right)^2
\]

- Goal: vary \( m_0 \) – when the \( a_S \) cross (ie independent of \( S \)) gives common scale \( a \)
- at this ‘magic’ point find

\[
a \approx 0.074 \text{ fm}
\]
\[
\sqrt{t_0^\text{exp}} \approx 0.153 \text{ fm}
\]
\[
w_0^\text{exp} \approx 0.179 \text{ fm}
\]
Reaching the charm quark mass range

- unitary range rather small so introduce PQ partially quenching (ie valence quark masses $\neq$ sea quark masses) and NNLO
- eg pseudoscalar meson octet

\[ M^2(ab) = M_{0\pi}^2 + \alpha(\delta \mu_a + \delta \mu_b) \]
\[ + \beta_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \beta_1(\delta \mu_a^2 + \delta \mu_b^2) + \beta_2(\delta \mu_a - \delta \mu_b)^2 \]
\[ + \gamma_0 \delta m_u \delta m_d \delta m_s + \gamma_1(\delta \mu_a + \delta \mu_b)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \]
\[ + \gamma_2(\delta \mu_a + \delta \mu_b)^3 + \gamma_3(\delta \mu_a + \delta \mu_b)(\delta \mu_a - \delta \mu_b)^2 \]

- $\delta \mu_q = \mu_q - \bar{m}$ $q \in \{a, b, \ldots\}$; valence quarks of arbitrary mass, $\mu_q$
- expansion coefficients: $M_{0\pi}^2(\bar{m}), \alpha(\bar{m}), \ldots$
- mixed sea/valence mass terms
- unitary limit: $\delta \mu_q \rightarrow \delta m_q$
2 + 1 joint fits

- unitary line data
  \[ [\mu_q \rightarrow m_q] \]
- no visible curvature

- PQ data
  \[ [\delta m_I = 0] \]
- illustration, to avoid 3-dim plot
  \[ a' \text{ distinct quark but same mass as } a \]
  \[
  \hat{M}^2(aa') = 1 + 2\delta \mu_a \bar{\alpha}_1 + 2\bar{\beta}_1 \delta \mu_a^2 + 8\gamma_2 \delta \mu_a^3
  \]

Very different \( x \)-scales involved
Octet baryon expansion coefficients

\[ M_N^{2(aab)} = M_{0N}^2 + A_1(2\delta \mu_a + \delta \mu_b) + A_2(\delta \mu_b - \delta \mu_a) \]
\[ + B_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + B_1(2\delta \mu_a^2 + \delta \mu_b^2) + B_2(\delta \mu_b^2 - \delta \mu_a^2) + B_3(\delta \mu_b - \delta \mu_a)^2 \]
\[ + C_0 \delta m_u \delta m_d \delta m_s + [C_1(2\delta \mu_a + \delta \mu_b) + C_2(\delta \mu_b - \delta \mu_a)](\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \]
\[ + C_3(\delta \mu_a + \delta \mu_b)^3 + C_4(\delta \mu_a + \delta \mu_b)^2(\delta \mu_a - \delta \mu_b) \]
\[ + C_5(\delta \mu_a + \delta \mu_b)(\delta \mu_a - \delta \mu_b)^2 + C_6(\delta \mu_a - \delta \mu_b)^3 \]

\[ M_{\Lambda}^{2(aa'b)} = M_{0\Lambda}^2 + A_1(2\delta \mu_a + \delta \mu_b) - A_2(\delta \mu_b - \delta \mu_a) \]
\[ + B_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + B_1(2\delta \mu_a^2 + \delta \mu_b^2) - B_2(\delta \mu_b^2 - \delta \mu_a^2) + B_4(\delta \mu_b - \delta \mu_a)^2 \]
\[ + C_0 \delta m_u \delta m_d \delta m_s + [C_1(2\delta \mu_a + \delta \mu_b) - C_2(\delta \mu_b - \delta \mu_a)](\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \]
\[ + C_3(\delta \mu_a + \delta \mu_b)^3 + (C_4 - 2C_3)(\delta \mu_a + \delta \mu_b)^2(\delta \mu_a - \delta \mu_b) \]
\[ + C_7(\delta \mu_a + \delta \mu_b)(\delta \mu_a - \delta \mu_b)^2 + C_8(\delta \mu_a - \delta \mu_b)^3 \]

- similar procedure
**2 + 1 joint fits**

- **unitary line data**
  
  \[ [\mu_q \rightarrow m_q] \]

- **no visible curvature**

- **PQ data (both N and Λ)**
  
  \[ [\delta m_l = 0] \]

- **illustration, to avoid 3-dim plot**
  
  \[ a' \text{ distinct quark but same mass as } a \]

\[
\tilde{M}^2(aaa'') = 1 + 3 \tilde{A}_1 \delta \mu_a + 3 \tilde{B}_1 \delta \mu_a^2 + 8 \tilde{C}_3 \delta \mu_a^3
\]

Very different x-scales involved
Method

- Use PQ data to determine expansion coefficients
  - $\alpha, \beta, \gamma$ – pseudoscalar octet
  - $A, B, C$ – baryon octet
- Determine physical quark masses

\[ \delta m_u^*, \quad \delta m_d^*, \quad \delta m_s^*, \quad \delta \mu_c^* \]

by fitting to (eg)

\[ M_{\pi^+}^{\text{exp}}(u\bar{d}), \quad M_{K^+}^{\text{exp}}(u\bar{s}), \quad M_{\eta_c}^{\text{exp}}(c\bar{c}) \]

[together with $\kappa_0$, so 4 inputs]
Open Charm masses

Can describe states with same wavefunction (and hence expansion) as previously used

- **pseudoscalar mesons**
  \[ D^0(c\bar{u}), \quad D^+(c\bar{d}), \quad D^+_s(c\bar{s}) \]

  which all have the wavefunction
  \[ \mathcal{M} = q\gamma_5 c \quad q = u, d, s \]

- **baryons**
  - single open charm \((C = 1)\) states
    \[ \Sigma_{c}^{++}(uuc), \quad \Sigma_{c}^{0}(ddc), \quad \Omega_{c}^{0}(ssc) \]

    which all have the wavefunction
    \[ \mathcal{B} = \epsilon(q^T C \gamma_5 c)q \quad q = u, d, s \]

    [also if \(m_u = m_d = m_s\), then in addition as no mixing \(\Sigma_{c}^{++}(ll'l') = \Sigma_{c}^{++}(ll'c) = \Sigma_{c}^{0}(ll'c)\) and \(\Lambda_{c}^{++}(ll'c)\)]

  - double open charm \((C = 2)\) states
    \[ \Xi_{cc}^{++}(ccu), \quad \Xi_{cc}^{+}(ccd), \quad \Omega_{cc}^{+}(ccs) \]

    which all have the wavefunction
    \[ \mathcal{B} = \epsilon(c^T C \gamma_5 c)q \quad q = u, d, s \]
### SU(4) 20-plet

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Charmed pseudoscalar mesons

- $D^0(c\bar{u})$, $D^+(c\bar{d})$, $D_s^+(c\bar{s})$,
- small lattice artifacts

- splittings:
  - $D^+(c\bar{d}) - D^0(c\bar{u})$,
  - $D_s^+(c\bar{s}) - D^0(c\bar{u})$,
  - $D_s^+(c\bar{s}) - D^+(c\bar{d})$
Charmed $C = 1$ baryons

- $\Sigma_{c}^{++}(uuc)$, $\Sigma_{c}^{0}(ddc)$, $\Omega_{c}^{0}(ssc)$
- some lattice artifacts (?)

- splittings:
  - $\Sigma_{c}^{0}(ddc) - \Sigma_{c}^{++}(uuc)$,
  - $\Omega_{c}^{0}(ssc) - \Sigma_{c}^{++}(uuc)$,
  - $\Omega_{c}^{0}(ssc) - \Sigma_{c}^{0}(ddc)$
Charmed $C = 2$ baryons

- $\Xi_{cc}^{++}(ccu)$, $\Xi_{cc}^{+}(ccd)$, $\Omega_{cc}^{+}(ccs)$
- some lattice artifacts (?)
- [*] SELEX

- splittings:
  $\Xi_{cc}^{+}(ccd) - \Xi_{cc}^{++}(ccu)$,
  $\Omega_{cc}^{+}(ccs) - \Omega_{cc}^{++}(ccu)$,
  $\Omega_{cc}^{+}(ccs) - \Xi_{cc}^{+}(ccd)$
Conclusions

• For $u$, $d$, $s$ quarks, have developed a method to approach the physical point

• Precise $SU(3)$ flavour symmetry breaking expansions – nothing ad-hoc

• Extend expansions – PQ (mass valence quarks $\neq$ mass sea quarks) to
  • better determine expansion coefficients
  • determine $c$ quark mass

• Applied method to determine some open charm states

• Future:
  • need to better check $O(a^2)$ effects
  • mixing: in a $2 + 1$ world no $\Sigma^0 - \Lambda^0$ mixing, but determined coefficients can be used to determine $\Sigma^0(uds) - \Lambda^0(uds)$ mixing

  generalise to eg $\Sigma_c^+ - \Lambda_c^+$, $\Xi_c^0 - \Xi_c'^0$ mixing

  • baryon decuplet
  • QED effects