The Roper Puzzle

- Discrepancy in various lattice calculations
- Fitting methods: variation vs. sequential Bayesian fitting
- $\pi N$ state and $S_{11}$

\textbf{QCD Collaboration:}

Y. Chen, M. Gong, K.F. Liu, M. Sun, R. Suffian

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Many Facets of Roper Resonance

Theory:

- Quark potential model prediction is 100-200 MeV too high (Liu and Wong, 1983, Capstick and Isgur, 1986)
- Skyrmion can accommodate it as a radial excitation (J. Breit and C. Nappi, 1984, Liu, Zhang, Black, 1984; U. Kaulfuss and U. Meissner, 1985)
- Suggestion as a pentaquark (Krewald 2000); as a member of the antidecuplet (Jaffe, Wilczek, 2003)
- Perhaps a hybrid (Barnes, Close, etc. 1983)
- Lattice calculations

(PDG--1440 MeV)
Quenched Lattice Calculations of Roper
Roper on the lattice

4 issues about lattice calculations:

- Radial excitation or pentaquark state?
- Dynamical fermions
- Variation vs Bayesian fitting
- Dynamical effect
Roper is seen on the lattice with three-quark interpolation field.

Weight:

\[ |\langle 0 | O_N | R \rangle|^2 > |\langle 0 | O_N | N \rangle|^2 > 0 \] (point source, point sink)

\[ \sum O_N(x) \]

Point sink

Wall source

\[ \langle 0 | O_N(0) | N \rangle \langle N | \sum \psi(x) \sum \psi(y) \sum \psi(z) | 0 \rangle > 0 \]

However,

\[ \langle 0 | O_N(0) | R \rangle \langle R | \sum \psi(x) \sum \psi(y) | \sum \psi(z) | 0 \rangle < 0 \]
Nucleon and Roper wavefunctions for $m_\pi = 633$ MeV

$O_{RN} = 0.30$
Bethe-Salpeter Wavefunction

\[ \rho_r + \rho_{\vec{r}_N} = \int d\vec{r} \Psi^*_R(r)\Psi_N(r) = 0 \text{ at non-relativistic limit,} \]

\[ O_{RN} = \int dr \Psi^*_R(r)\Psi_N(r) \uparrow \text{ as } m_q \downarrow \]
Roper and Nucleon Wavefunctions at $m_\pi = 438$ MeV

$O_{RN} = 0.59$
Dynamical Fermions (Overlap on DWF Configurations)

- Improvement of nucleon correlator with low-mode substitution

24^3 \times 64 \text{ lattice with } m_\pi = 331 \text{ MeV, } a = 1.73 \text{ GeV}^{-1}

47 configurations

Point source: \( m_N = 1.13(14) \text{ GeV} \);
Z_3 \text{ grid source: } m_N = 1.08(5) \text{ GeV};
Z_3 \text{ grid smeared source: } m_N = 1.14(2) \text{ GeV};
Variation: \( m_N = 1.16(1) \text{ GeV} \)
Roper state from Coulomb wall source

$\epsilon = 1.73$ GeV

$m_{\pi} = 331$ MeV (sea), $a = 1.73$ GeV$^{-1}$

$24^3 \times 64$ lattice with $m_{\pi} = 331$ MeV (sea), $a = 1.73$ GeV$^{-1}$

$m_N = 999(46)$ MeV

$m_R = 1404(112)$ MeV
Roper and Nucleon Wavefunctions at $m_\pi = 438$ MeV

$O_{RN} = 0.59$
Variation with 2 operators
(10 – operator 1, no smearing, 23 – operator 2, 3 smearing)

Variation with wall and point sources?
\( \chi_1, J^P = \frac{1}{2}^+ \)

\( \chi_2, J^P = \frac{1}{2}^+ \)

\( \times 2 \text{ (n=0)}, J^P = \frac{1}{2}^- \)

\( \times 2 \text{ (n=1)}, J^P = \frac{1}{2}^- \)

\( N, J^P = 1^+ \)

\( N^+ (1440) \)

\( N^-(1535) \)

\( N + \pi \)

\( O(p^3) \)

S-wave

\( m_N \text{ [GeV]} \)

\( m_{\pi}^2 \text{ [GeV}^2\)
Evidence of $\eta'$N GHOST State in $S_{11}$ (1535) Channel

\begin{align*}
\eta \rightarrow \pi \pi
\end{align*}
Dynamical Fermions

η + η + Π

η and Π
Negative Parity Channel

Overlap on $32^3 \times 64$ DWF lattice, $L_a \sim 4.5$ fm, sea pion mass $\sim 170$ MeV with Coulomb wall source

$J^P=(1/2)^-$ Energy $M_N + M_{\pi}^{(sv)}$ $M_{\pi}^{(sv)}=171.0(4)$ MeV $M_N+M_{\pi}^{(sv)}$ $M_{\pi}^{(sv)}=171(1)$ MeV

$J^P=(1/2)^-$ Energy $M_N + M_{\pi}^{(sv)}$ $M_{\pi}^{(sv)}=241.3(4)$ MeV $M_N+M_{\pi}^{(sv)}$ $M_{\pi}^{(sv)}=171(1)$ MeV
Quenched Vs Dynamical $N^{1-}_{1/2}$ (1535) (Sommer scale)
N* spectrum in LQCD & dynamical coupling

Lattice N* states ($m_\pi=396$MeV)

Dynamics of $P_{11}$-states:
The bare state at ~1750 MeV through coupling to inelastic channels generates 2 poles below 1400 MeV. They are identified with the “Roper” resonance.

LQCD finds states as predicted in SU(6)xO(3)


Dynamical Effect

\[ \eta \eta + \eta N(p - \text{wave}) \equiv \Delta E \]

\[ = \left| \left\langle 0 \mid \chi_{3q} \mid \pi N, \eta N(p - \text{wave}) \right\rangle \right|^2 \]

\( \Delta E \)
Summary

• Part of the discrepancy between the variational method and sequential Bayenes fitting is attributable to the size of the interpolation field.
• Roper is the radial excitation of nucleon with possible large couplings to N\(\eta\) and N\(\pi\).
• To understand the remaining difference:
  – Use Coulomb wall source/sink in the variation.
  – Compare the following ratios

\[
\frac{\langle 0 | \chi_{3q} | \pi N(1/2^\pm) \rangle}{\langle 0 | \chi_{3q} | N(1/2^\pm) \rangle}
\]