Spectroscopy of doubly and triply-charmed baryons from lattice QCD

Padmanath M.

Department of Theoretical Physics,
Tata Institute of Fundamental Research,
Mumbai. India.

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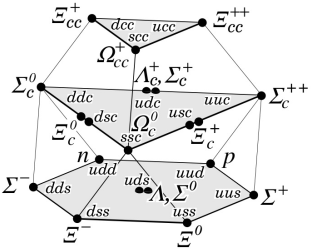
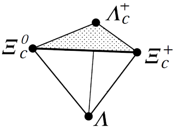
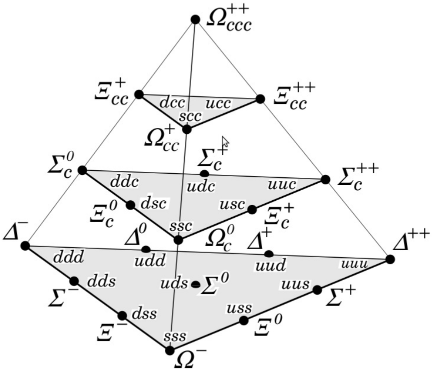
- arXiv:1307.7022 [hep-lat] .
- In collaboration with R. G. Edwards, N. Mathur and M. Peardon.
- Computations performed on computational facilities at DTP, TIFR, Mumbai, Jefferson Laboratory and TCHPC, Trinity College, Dublin.

4 (u, d, s, c) degenerate flavors







We have one heavy and 2+1 light flavor quarks.

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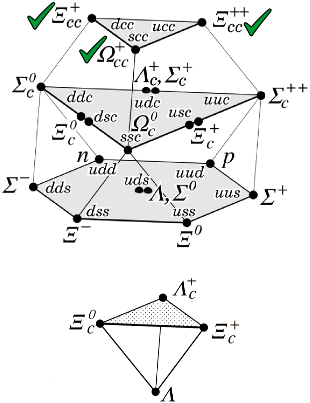

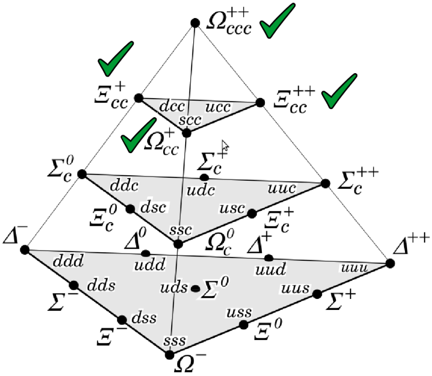

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Ensemble details

Calculations performed on lattices generated by
Hadron Spectrum Collaboration.

- Dynamical configurations ($N_f = 2 + 1$).
- Anisotropic lattices with $\xi = a_s/a_t \sim 3.5$.
- Scale set via m_Ω : $a_s = 0.12$ fm
- Lattice size : $16^3 \times 128$.
- Statistics : 96 cfgs and 4 time sources.
- Clover fermions : Non-perturbtive O(a) improvement.
- Spatial links are stout smeared.
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- Spatial links are stout smeared.
- Quark fields are distilled.
- Caveat : Pion mass $\sim$ 391 MeV.

Interpolating operators

Ω_{ccc}

Non-Rel: $SU(6) \otimes O(3)$

D \ J	1/2	3/2	5/2	7/2
0	**0**	**1**	**0**	**0**
1	**1**	**1**	**0**	**0**
2_{hybrid}	**1**	**1**	**0**	**0**
2	**2**	**3**	**2**	**1**

Ω_{cc} and Ξ_{cc}

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Whole operator set

Ω_{ccc}	G_1		H		G_2	
	g	u	g	u	g	u
Total	20	20	33	33	12	12
Hybrid	4	4	5	5	1	1
NR	4	1	8	1	3	0

g $\rightarrow +$ u $\rightarrow$ -

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Whole operator set

Ξ_{cc} Ω_{cc}	G_1		H		G_2	
	g	u	g	u	g	u
Total	55	55	90	90	35	35
Hybrid	12	12	16	16	4	4
NR	11	3	19	4	8	1

Generalized eigenvalue problem

Using this large operator basis, with definite J^P in the continuum limit, to build the correlation matrix

$$\mathcal{C}_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} \exp^{-E_n t}$$

Solving the generalized eigenvalue problem for this correlation matrix

$$\mathcal{C}_{ij}(t) v_j^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0) C_{ij}(t_0) v_j^{(n)}(t,t_0)$$

- Principal correlators given by eigenvalues
  $\lambda_n(t,t_0) \sim (1-A_n)\exp^{-m_n(t-t_0)} + A_n \exp^{-m'_n(t-t_0)}$
- Eigenvectors related to the overlap factors
  $Z_i^{(n)} = \langle 0|\mathcal{O}_i|n\rangle = \sqrt{2E_n}\exp^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

Spin identification

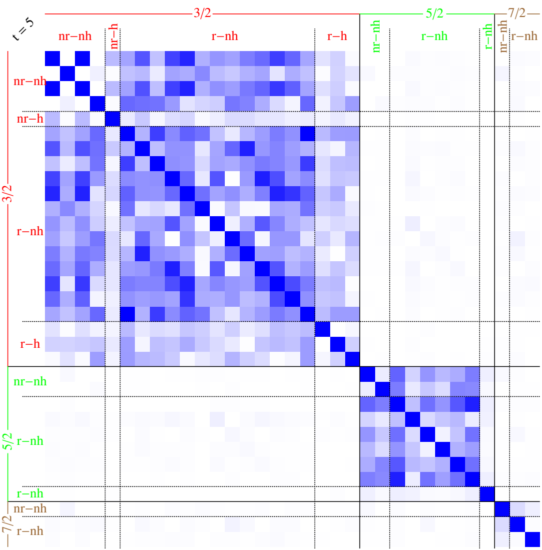
Discretized space-time breaks rotational symmetry down to octahedral symmetry.

- Continuum spin operators subduced to lattice irreps.
- G_1, H and G_2 : O_h irreps representing half spin.

Λ	d_Λ	J
G_1	2	1/2, 7/2, 9/2, ...
H	4	3/2, 5/2, 7/2, ...
G_2	2	5/2, 7/2, 9/2, ...

- Subduced operators carry a memory of the continuum spin J.
- An operator of spin J overlaps mainly with states of spin J.
  Overlap factors to identify spin of states.

ccc correlation matrix plot (H^g; at t=5) : $\mathcal{C}_{ij}/\sqrt{\mathcal{C}_{ii}\mathcal{C}_{jj}}$



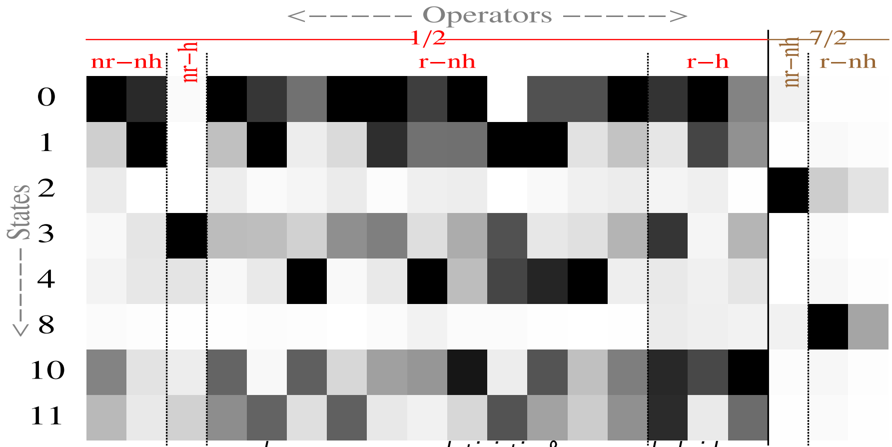
$$nr - nh = non - relativistic \ \& \ non - hybrid$$

$$nr - h = non - relativistic \ \& \ hybrid$$

$$r - nh = relativistic \ \& \ non - hybrid$$

$$r - h = relativistic \ \& \ hybrid$$

Spin identification using overlap factors : (ccc, G_1^g)



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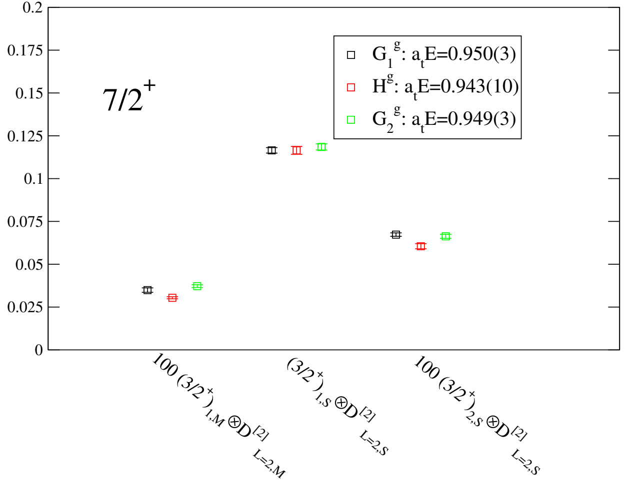
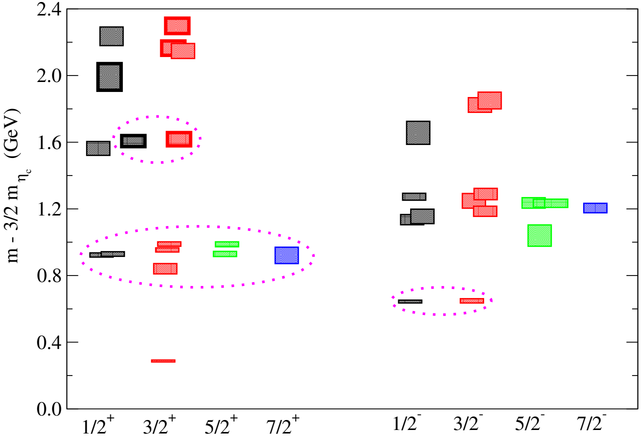
Spin identification across multiple irreps : $7/2^+$



Ω_{ccc} spectrum



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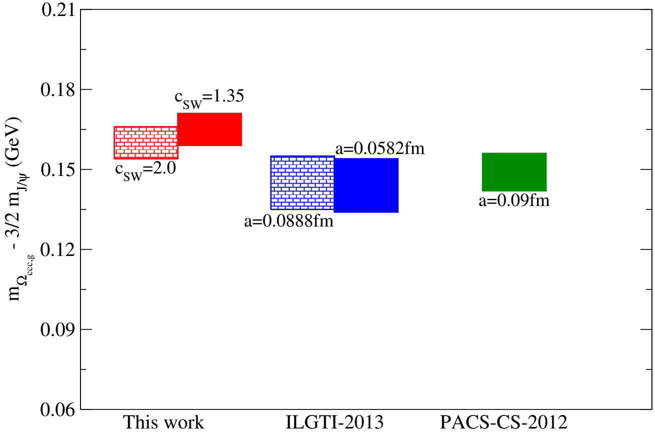
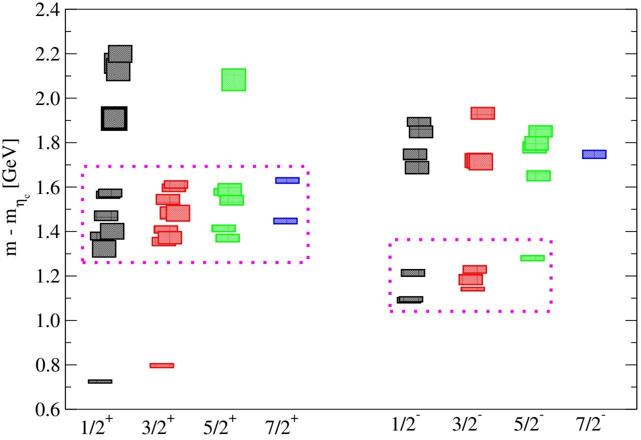
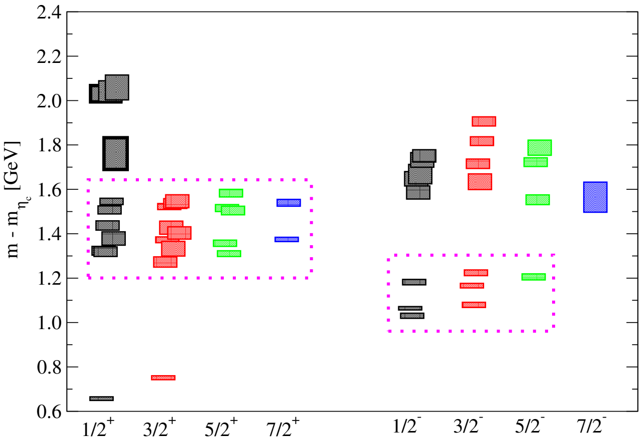
Ω_{ccc} $(3/2^+)$ ground state : discretization errors



Ω_{cc} spectrum



Ξ_{cc} spectrum



Interpolating operators

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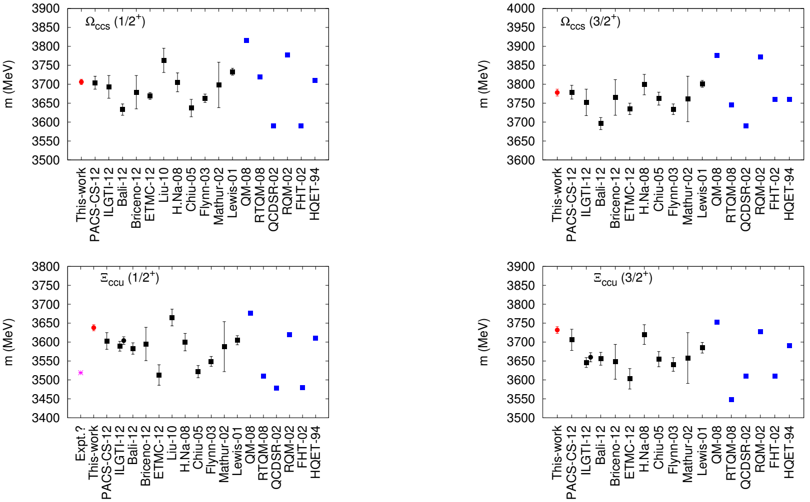
$cc(q)$ ground states



● → This work

■ → *Other lattice results* ■ → *EFT and model calculations*

⋆ → *Experiment*

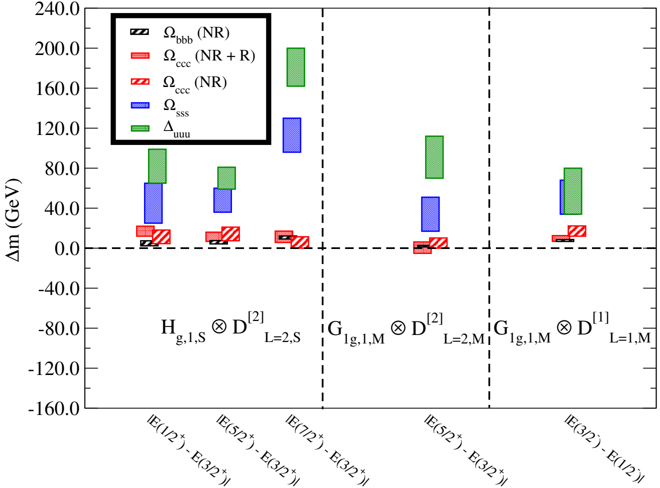
m_q dependence of energy splittings

- Spin-Orbit interactions inversly proportional to m_q^2.
  Vanishes in the heavy quark limit.
  Degeneracy lifts : a measure of heavyness of the quark mass.

- Binding energy quark mass dependence.
  Mass of a hadron with n heavy quarks: $M_{H_{nq}} = nm_Q + A + B/m_Q + \mathcal{O}(1/m_Q^2)$.
  Energy splittings : $a + b/m_Q + \mathcal{O}(1/m_Q^2)$.
  Fits with heavy quark inspired functional forms.

- From energy splittings ($\Xi_{cc}^* - D_c$, $\Omega_{cc}^* - D_s$ and $\Omega_{ccc} - \eta_c$) and ($\Xi_{cc}^* - D_c^*$, $\Omega_{cc}^* - D_s^*$ and $\Omega_{ccc} - J/\psi$), we extrapolate to bottom mass and get $B_c^* - B_c = 80 \pm 8$ MeV and $\Omega_{ccb}^* = 8050 \pm 10$ MeV.

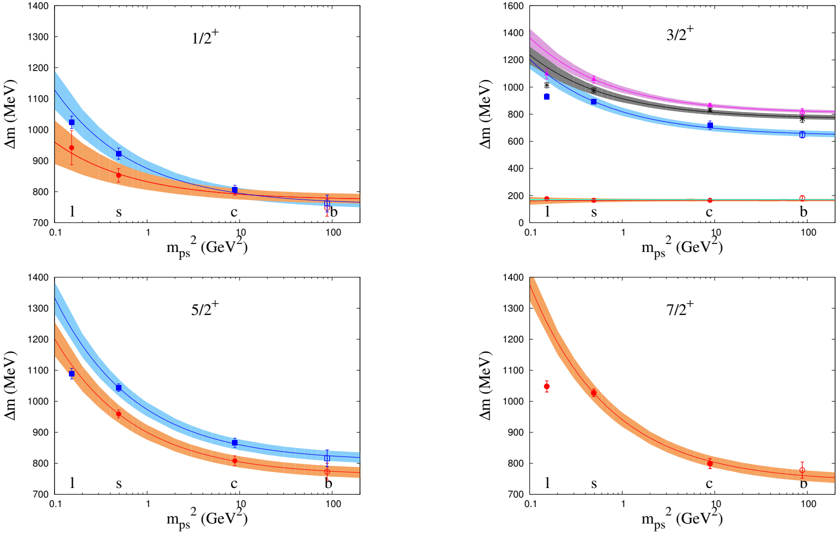
Spin-Orbit splittings in Ω like baryons



u and s → Edwards, et. al., Phys. Rev. D **87**, 054506 (2013)

b → S. Meinel, Phys. Rev. D **85**, 114510 (2012)

Quark mass dependence of Ω like baryons



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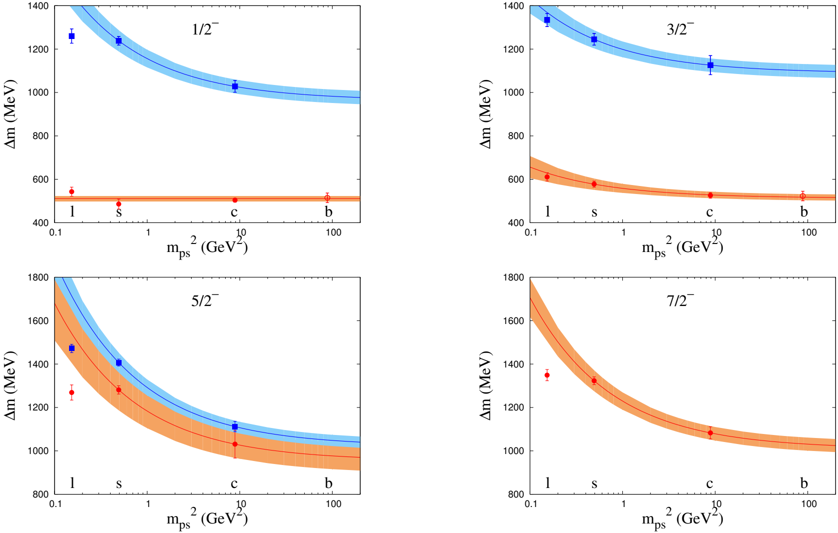

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Summary and conclusions

- Non-perturbative calculation for excited state spectroscopy of Ω_{ccc}, Ω_{cc} and Ξ_{cc}..
- Non-relativistic spectrum pattern observed up to the second energy band.
- Identification of the spin and spatial structure of the states using the overlap factors.
- SO splittings : The degeneracy more or less satisfied for m_c.
- Energy splittings : Heavy quark inspired form gives good fit with m_b, m_c as well as m_s. For some, the fits even pass through m_l also.
- Extrapolations to bottom sector : $B_c^* - B_c = 80 \pm 8$ MeV and $\Omega_{ccb}^* = 8050 \pm 10$ MeV.
- No multi hadron operators being used : Further works required to see their effects.
- Singly charm baryons under inverstigation.