BARYON PROPERTIES IN MESON MEDIUMS FROM LATTICE QCD

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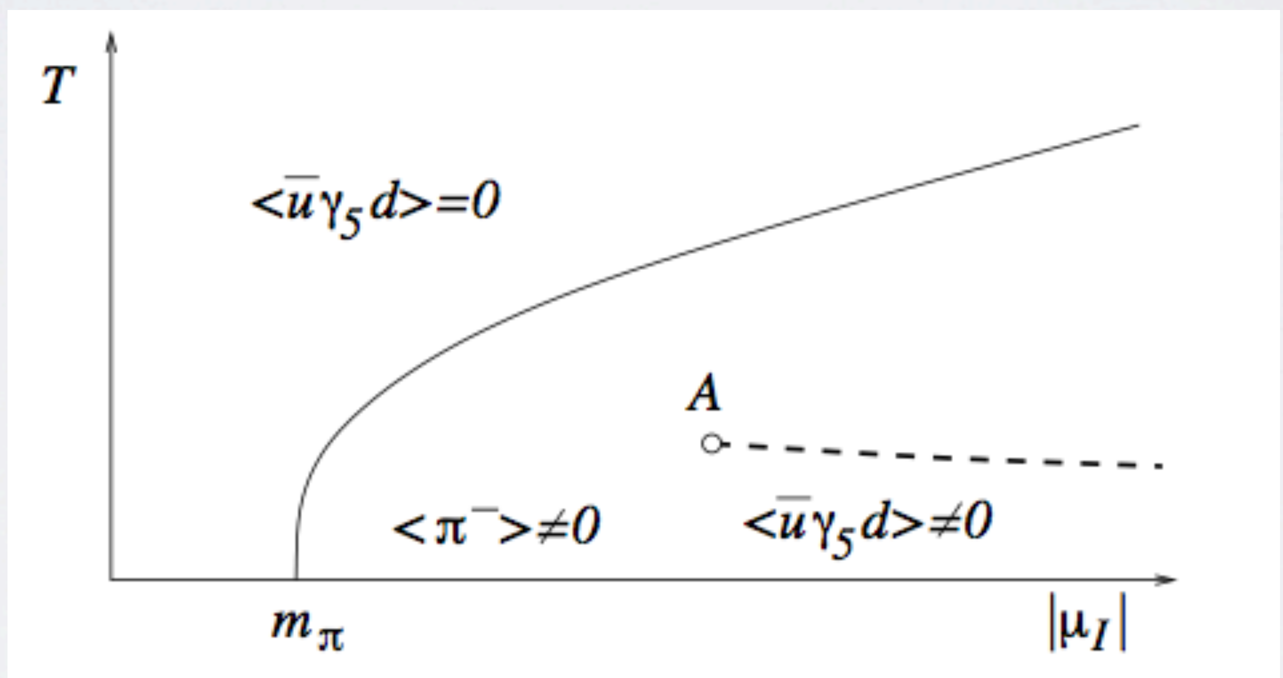
in collaboration with W. Detmold (MIT)

Lattice 2013, Mainz, Germany, Thursday, August 1, 2013

MANY MESON SYSTEMS

- SNR $\sim \sqrt{N_{\text{cfg}}}$
- Explore lattice methods for complex hadronic systems
- Interesting phase diagram (BEC)
- Possibly relevant in very dense matter (e.g. neutron stars)



Son & Stephanov (2001)

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- SNR ~ $\sqrt{N_{\mathrm{cfg}}}$
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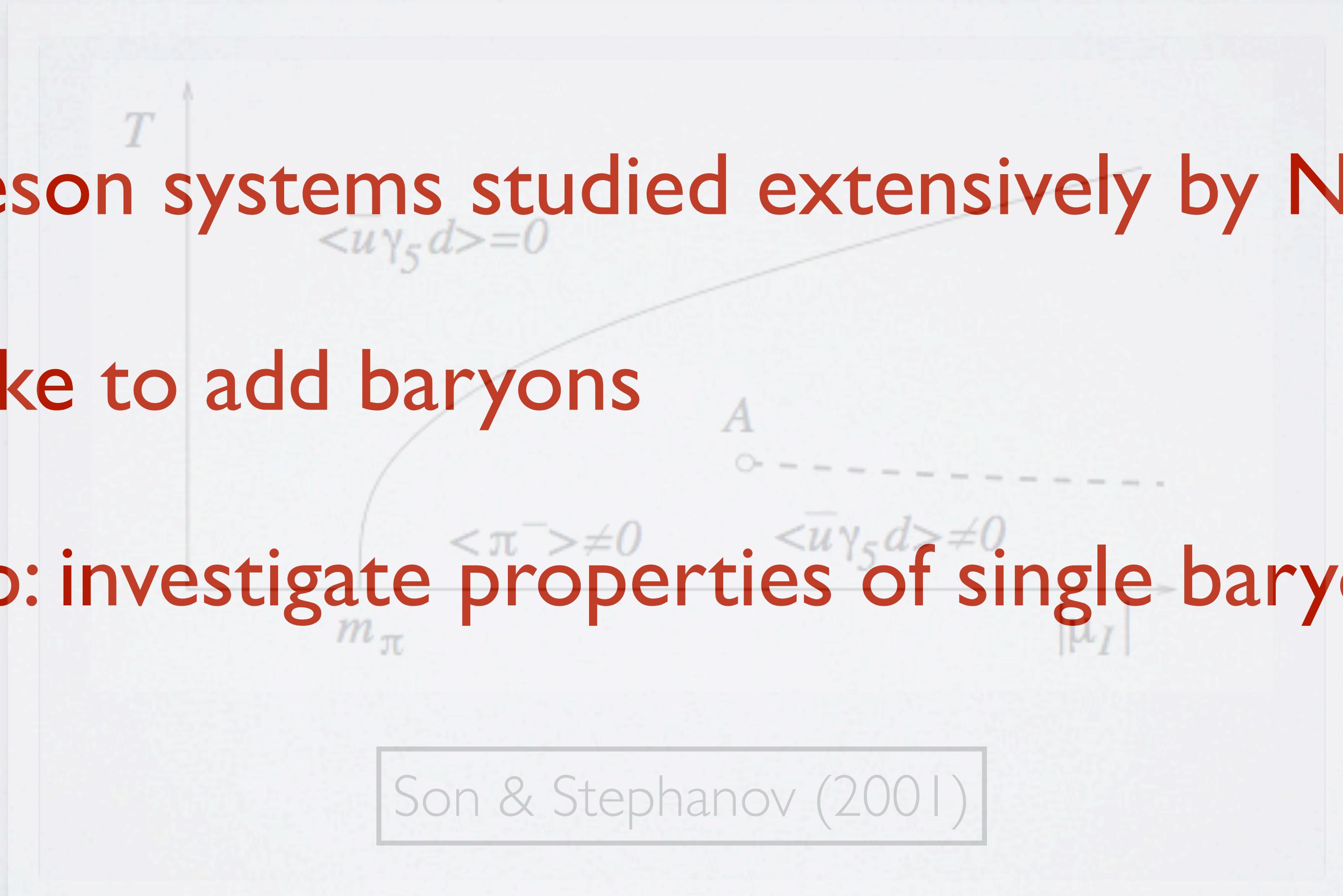
- Multi-meson systems studied extensively by NPLQCD
- Would like to add baryons
- First step: investigate properties of single baryon in meson medium



Son & Stephanov (2001)

THIS WORK:

System	Quark content
$\Sigma^+(\pi^+)^n$	$uus(u\bar{d})^n$
$\Xi^0(\pi^+)^n$	$uss(u\bar{d})^n$
$p(K^+)^n$	$uud(u\bar{s})^n$
$n(K^+)^n$	$udd(u\bar{s})^n$

Will calculate:

- Ground-state energies
- 2- and 3-body interaction parameters
- LECs - tree-level ChiPT

CONTRACTIONS

NPLQCD (2007)

First let's look at the simpler case for n mesons*

$$\Pi_{a,\alpha}^{b,\beta} \equiv \sum_{c,\gamma} \sum_{\mathbf{x}} \left[S_d(\mathbf{x},t;\mathbf{0},0)\gamma_5\right]^{b,\beta,c,\gamma} \left[S_u^\dagger(\mathbf{x},t;\mathbf{0},0)\gamma_5\right]_{a,\alpha,c,\gamma}$$

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$$\longrightarrow \quad \Pi_a^b$$

12x12 matrix for 12 dof

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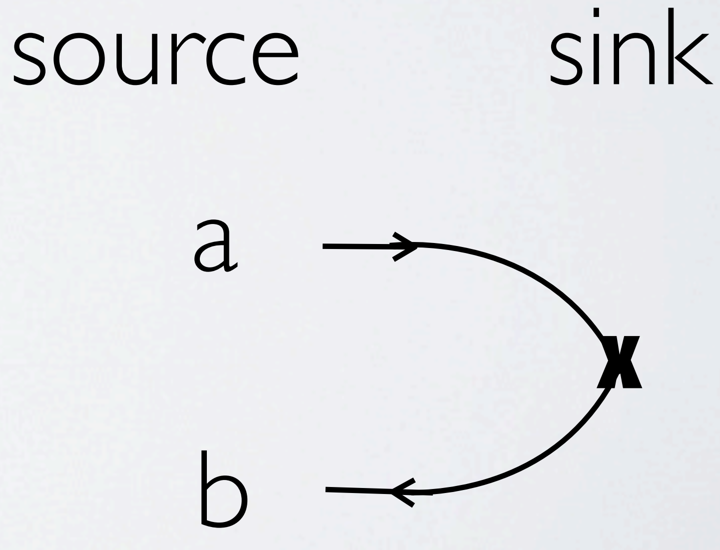
$$\longrightarrow \quad \Pi_a^b$$

Graphically:



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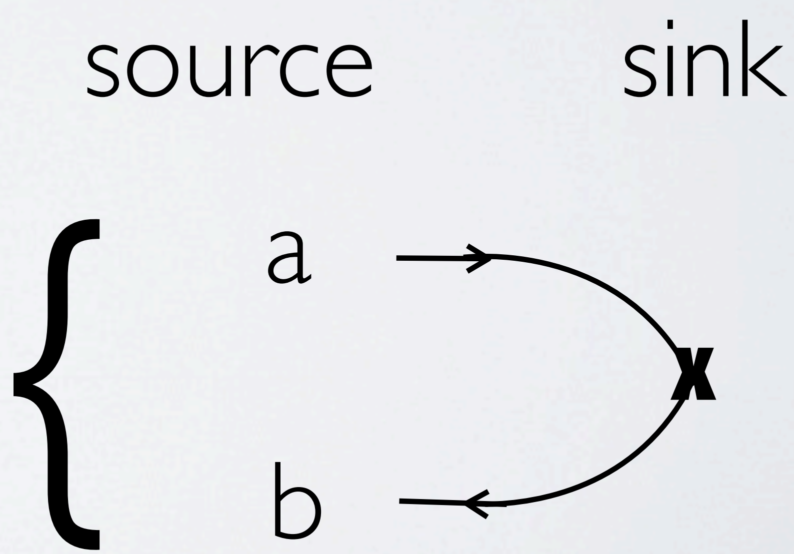
$$\longrightarrow \quad \Pi_a^b$$

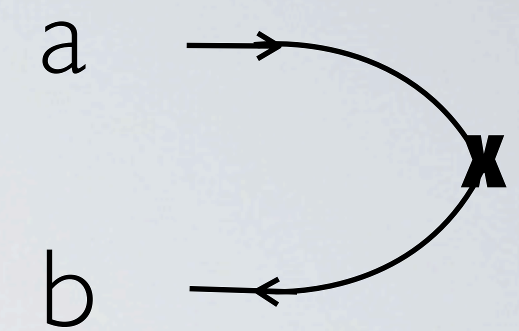
Graphically: need to tie up source indices



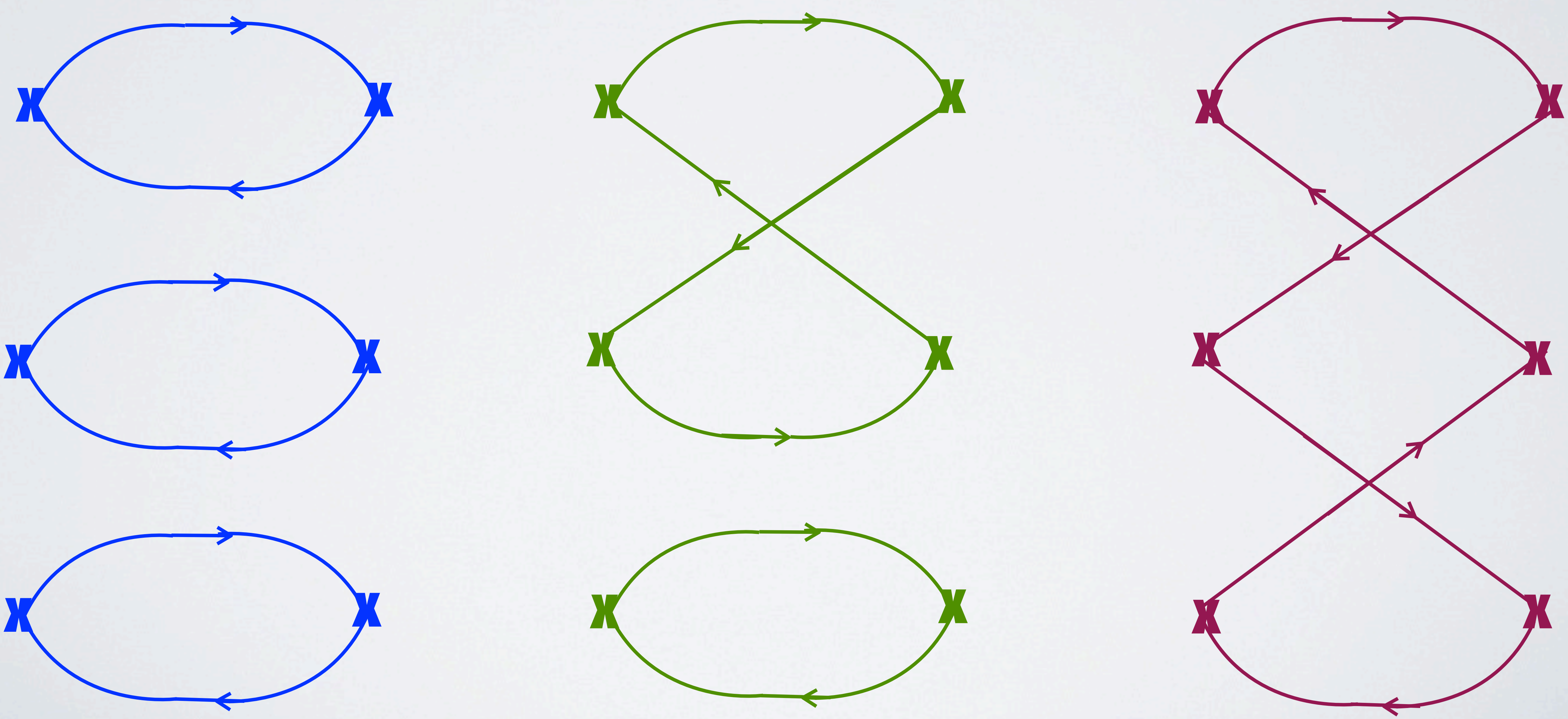
$$\det(1+\lambda\Pi) = \frac{1}{12!}\sum_{m=1}^{12} \lambda^m C_m(t)$$

$$= e^{\mathrm{Tr}\,\ln(1+\lambda\Pi)}$$

$$\Pi_a^b \;=$$



$$\mathcal{O}(\lambda^3) : C_3(t) = (\mathrm{tr}[\Pi])^3 - 3\,\mathrm{tr}[\Pi^2]\mathrm{tr}[\Pi] + 2\,\mathrm{tr}[\Pi^3]$$



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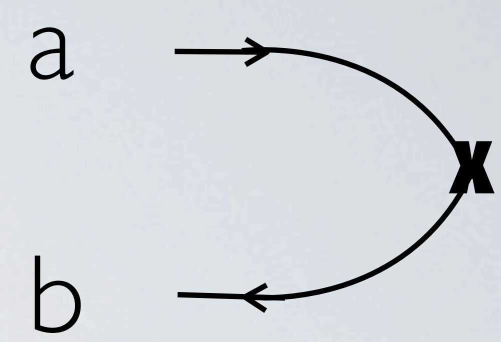


$$\mathcal{O}(\lambda^3) : C_3(t) = (\mathrm{tr}[\Pi])^3 - 3\,\mathrm{tr}[\Pi^2]\mathrm{tr}[\Pi] + 2\,\mathrm{tr}[\Pi^3]$$

Easily extended for multiple species of mesons, e.g. pions and kaons

$$\det(1+\lambda\Pi) \longrightarrow \det(1+\lambda\Pi+\kappa K)$$

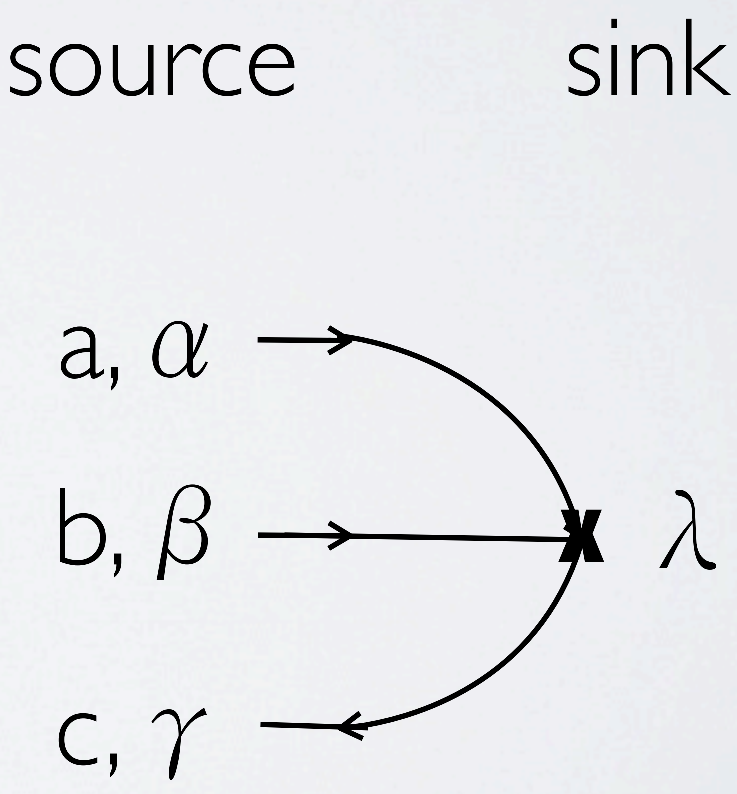
Detmold & Smigielski (2011)

ADDING A BARYON

Baryon block

$$B_{a,\alpha,b,\beta,c,\gamma,\lambda} \equiv \sum_{\sigma,h,i,j} [S_{q_1} C\gamma_5]_{a,\alpha,h,\sigma} [S_{q_2}]_{b,\beta,i,\sigma} [S_{q_3}]_{c,\gamma,j,\lambda} \epsilon_{h,i,j}$$

Graphically:



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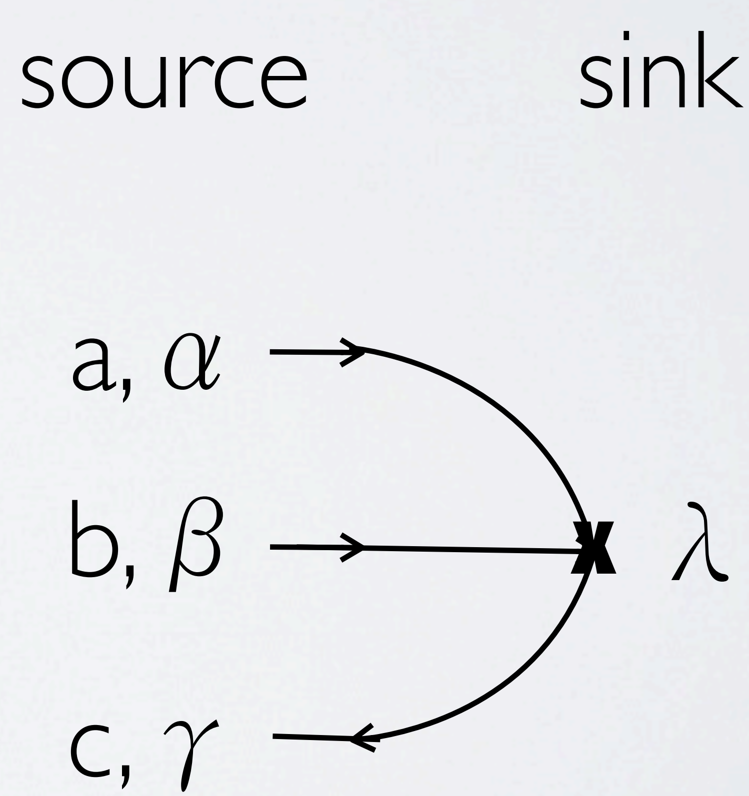

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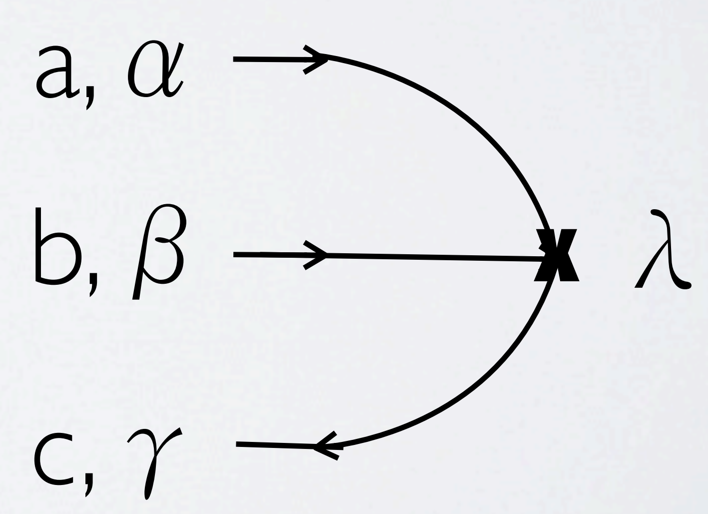
source sink

Graphically:



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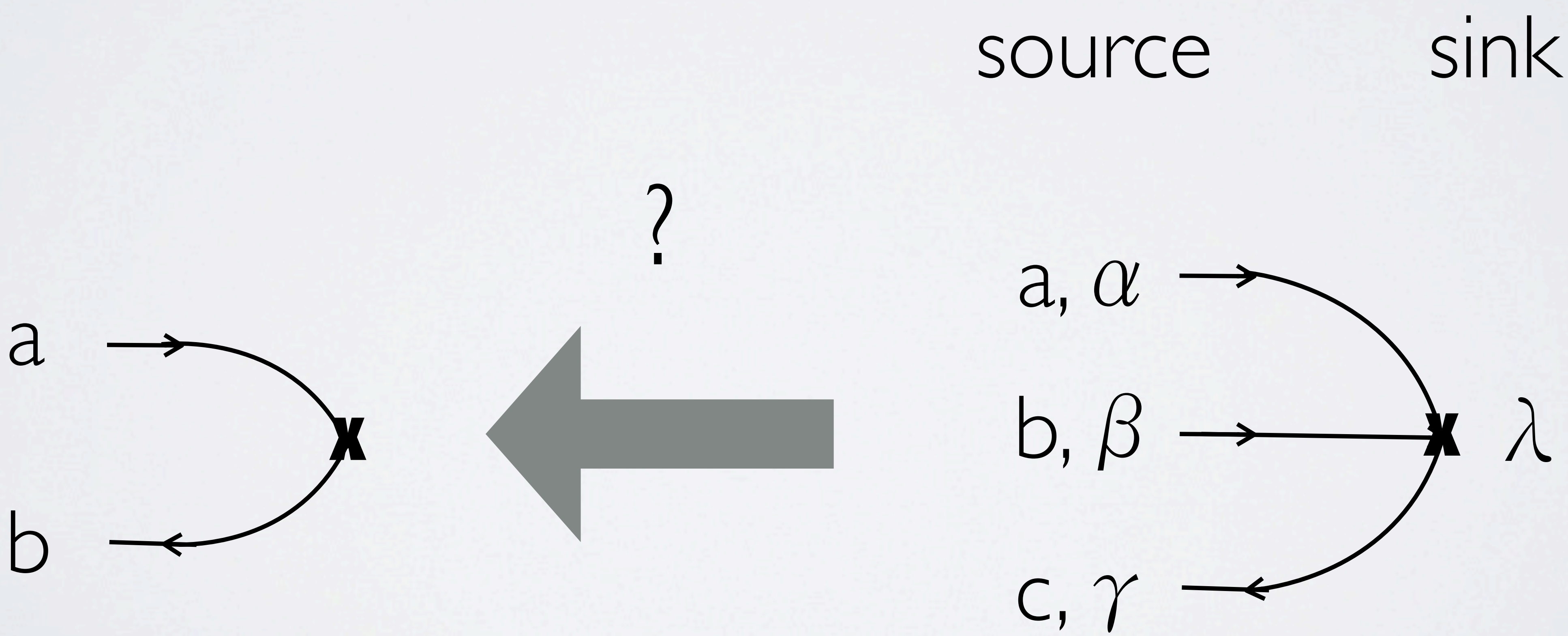
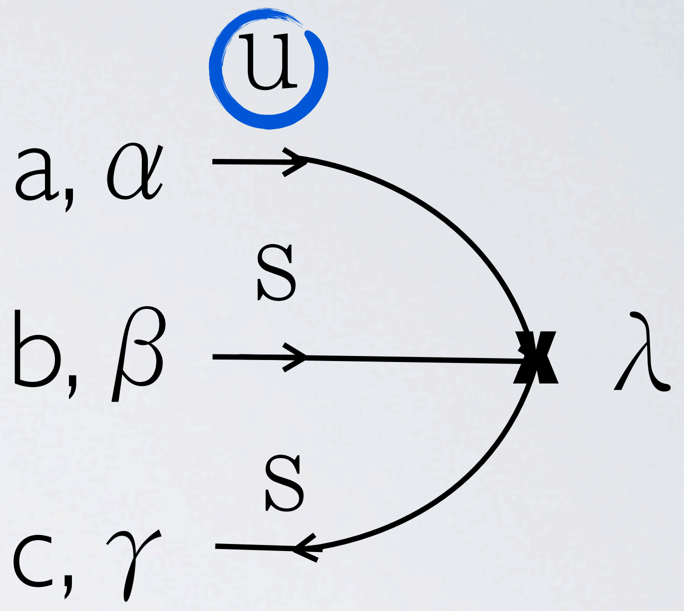


Ξ +PIONS, N+KAONS

Ξ^0 +pions

only u quarks need to be contracted with pions



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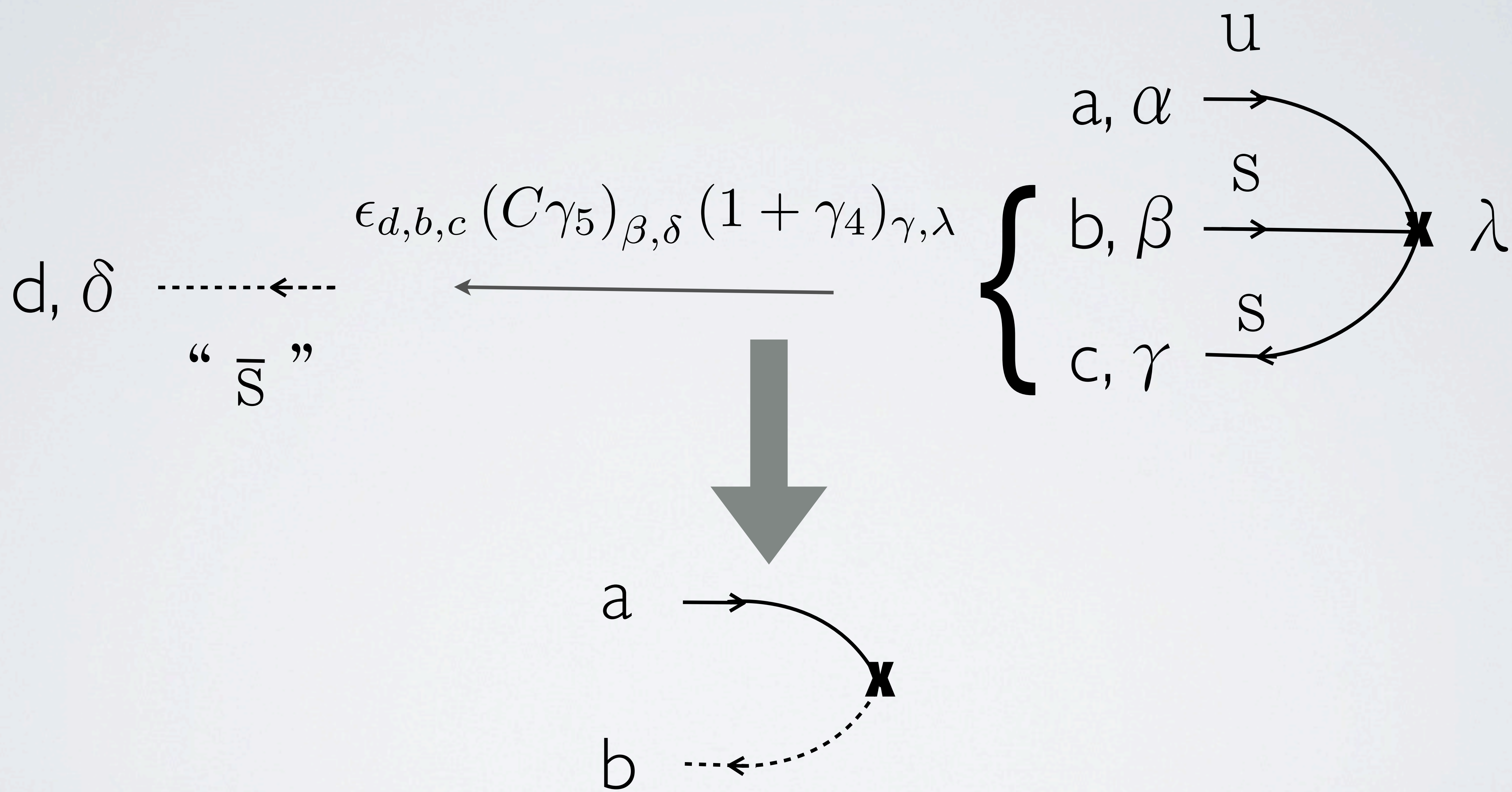

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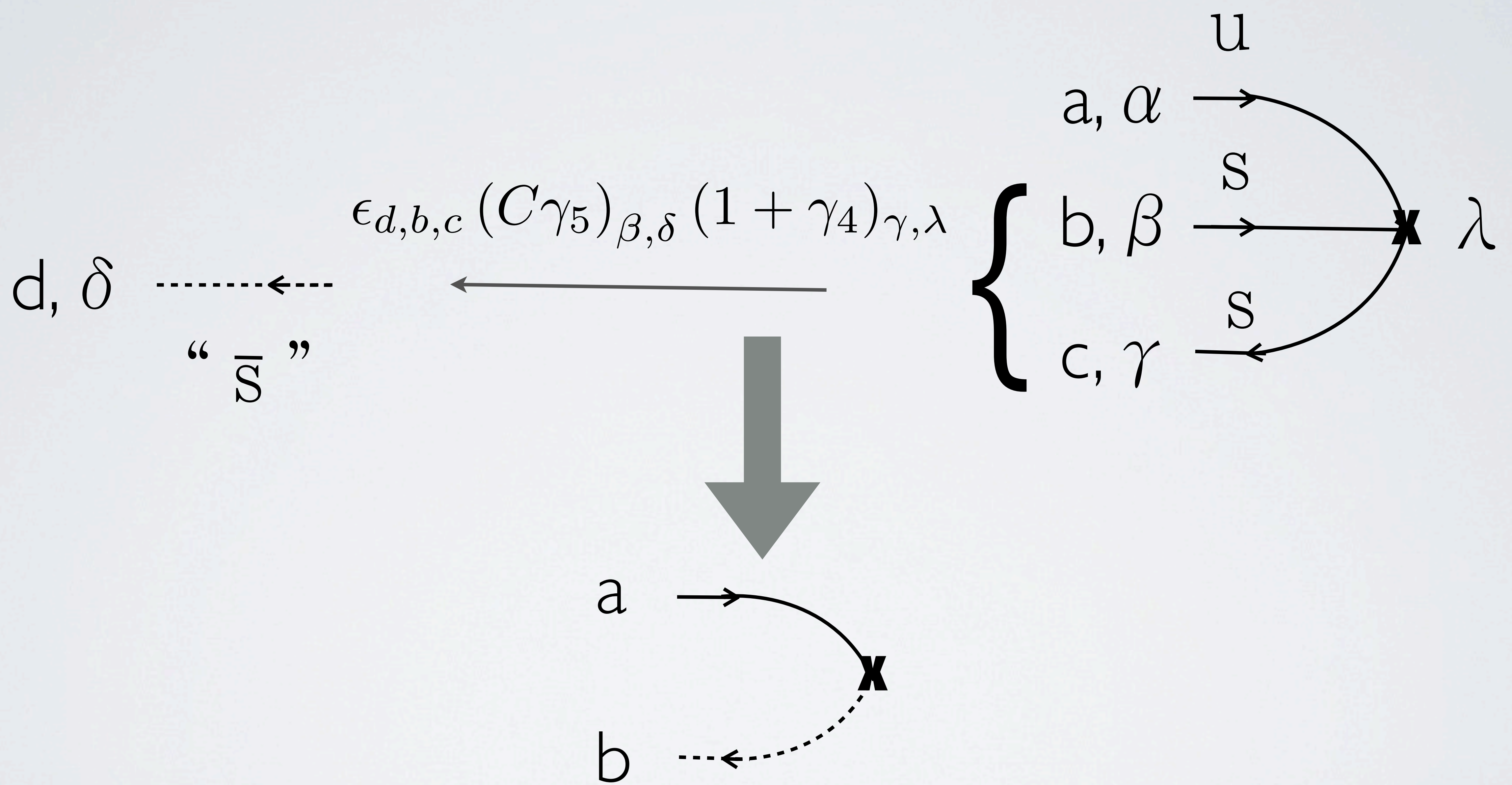
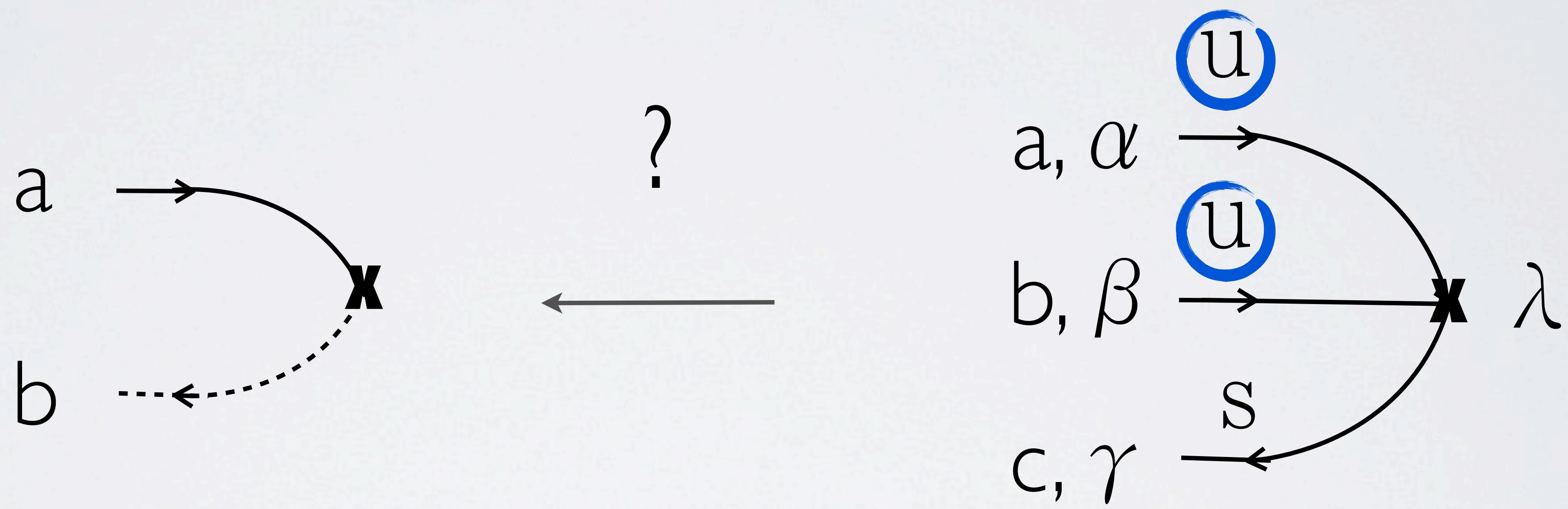


Plug in to formula for mixed species

Σ +PIONS, P+KAONS

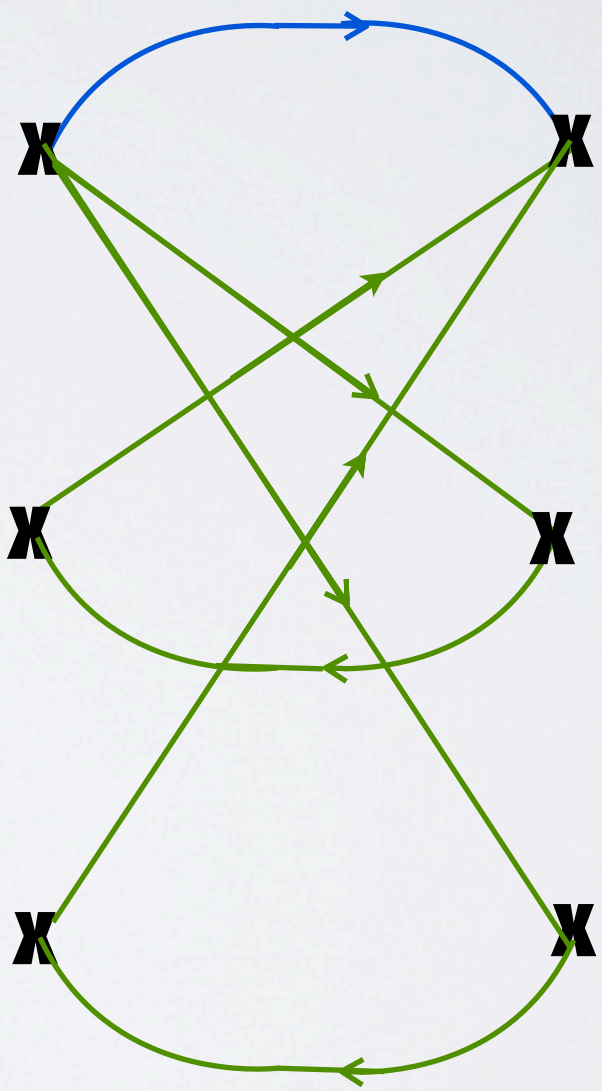
Σ^+ +pions



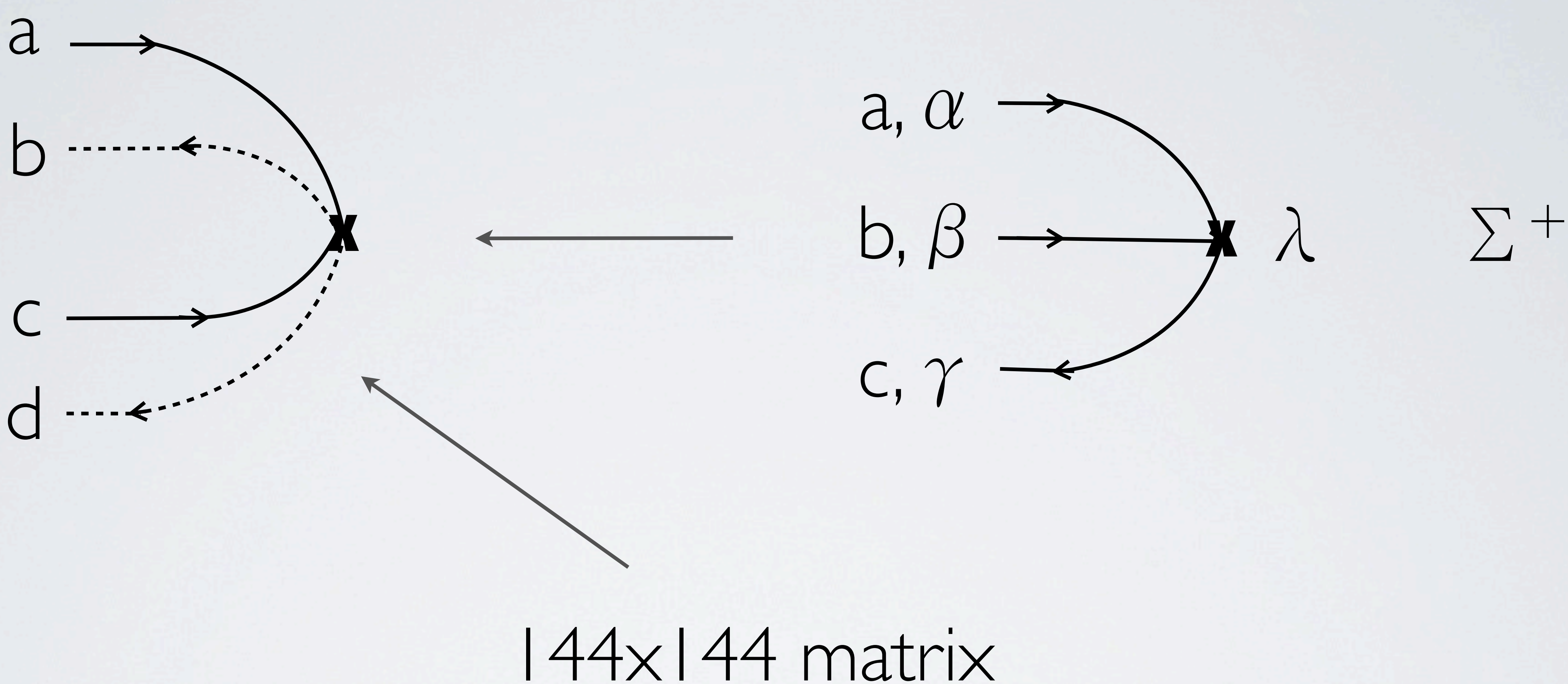
Σ +PIONS, P+KAONS

Σ^+ +pions

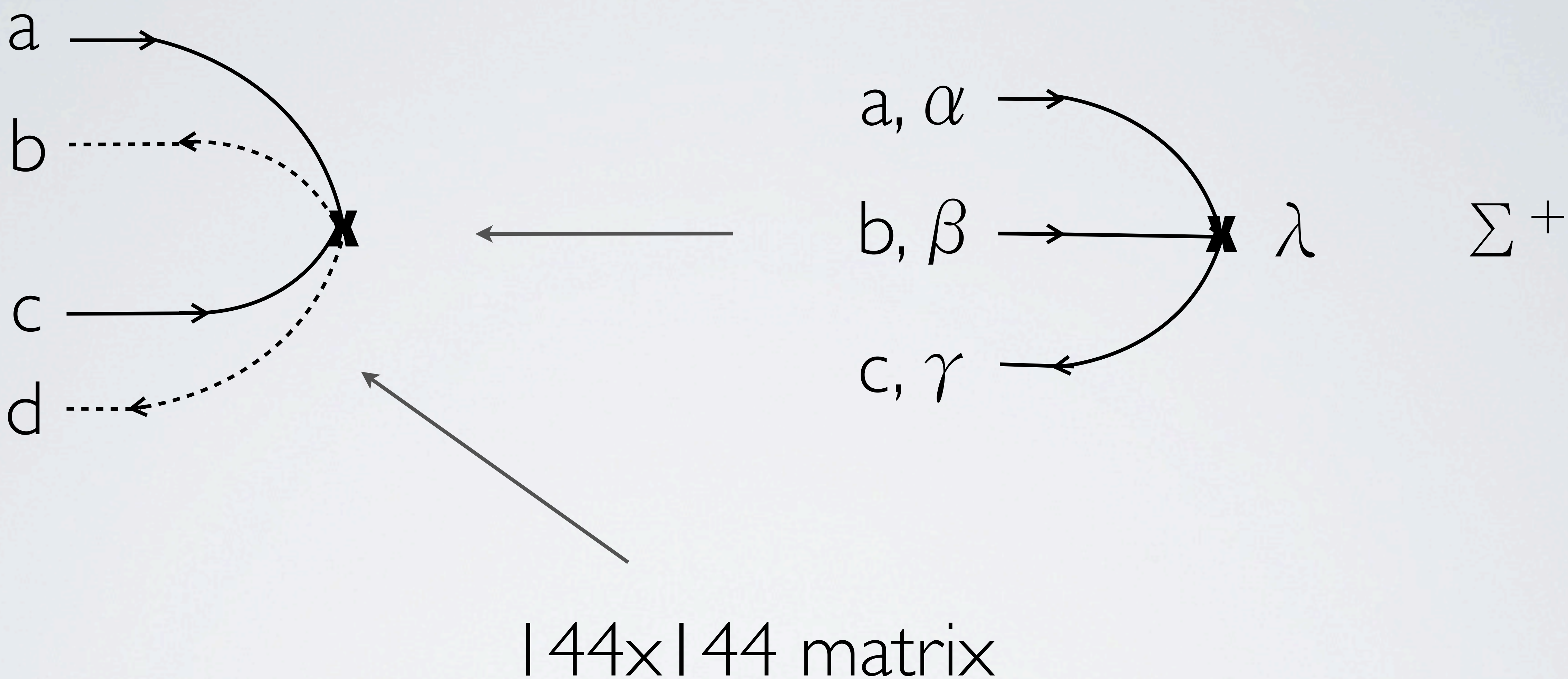
Miss diagrams where baryon exchanges both quarks



Σ +PIONS, P+KAONS



Σ +PIONS, P+KAONS



$$\Pi \otimes \Pi, \qquad 1 \otimes \Pi, \qquad \Pi \otimes 1$$

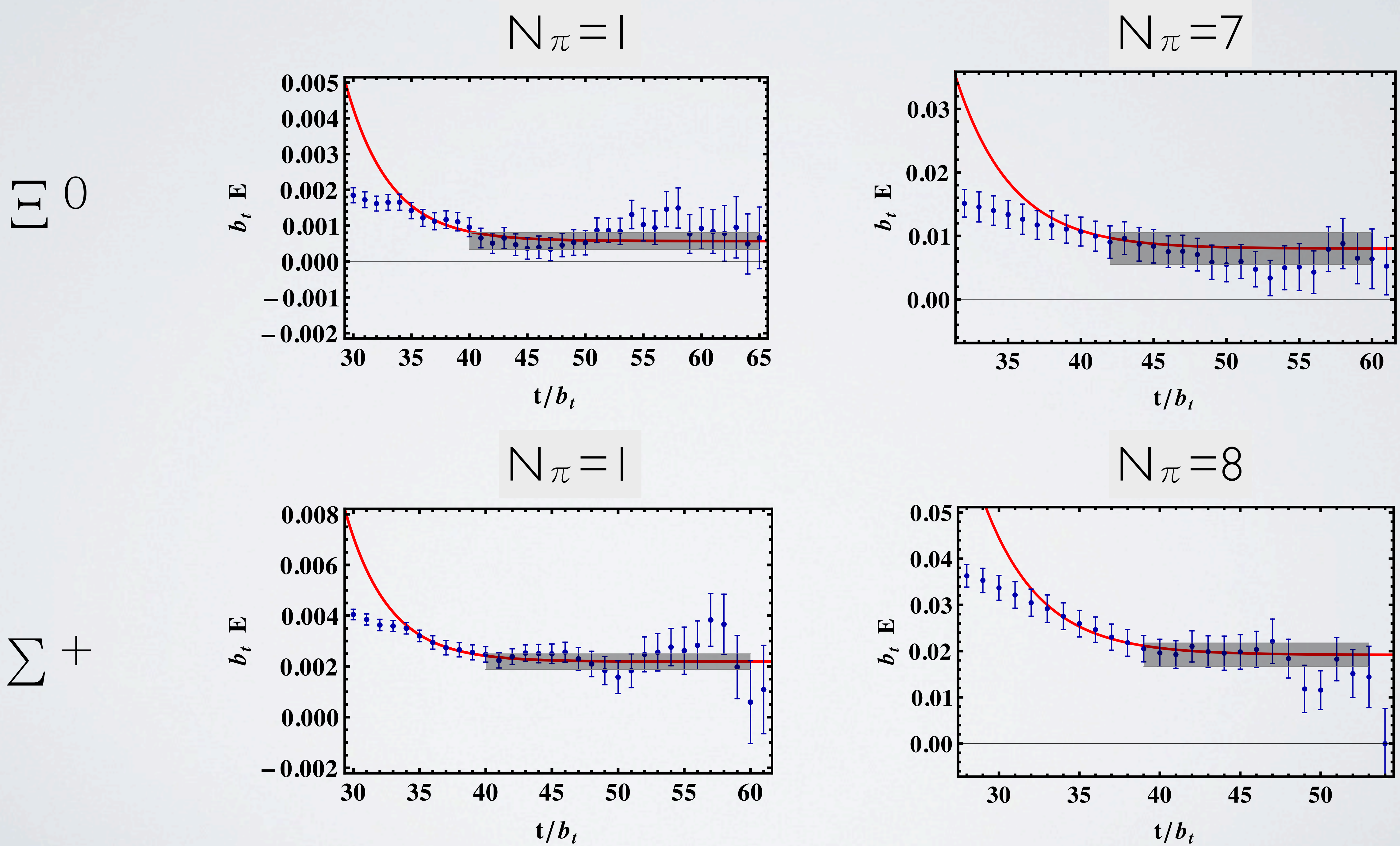
LATTICE DETAILS

- HSC lattices
  - clover, tadpole improved
  - a_s~0.125 fm, a_t~a_s/3.5,

    m_π~390 MeV, 32^3x256
- NPLQCD propagators
  - same discretization as gauge fields
  - ~ 200 per configuration

ENERGY SPLITTINGS

$$\Delta M_{\mathrm{eff}}^{(n)}(t) = \ln\left(\frac{C_{B,n}(t)/C_{B,n}(t+1)}{[C_B(t)/C_B(t+1)][C_n(t)/C_n(t+1)]}\right)$$



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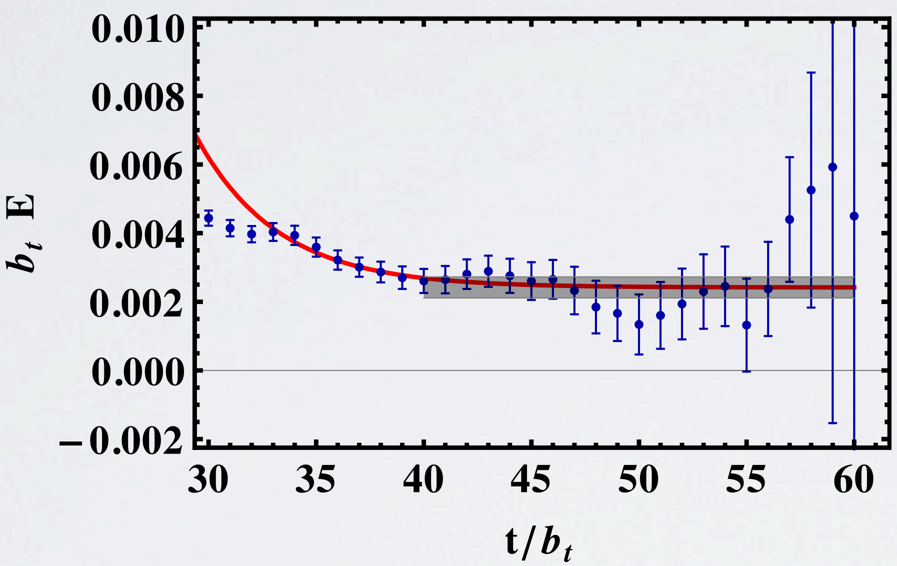
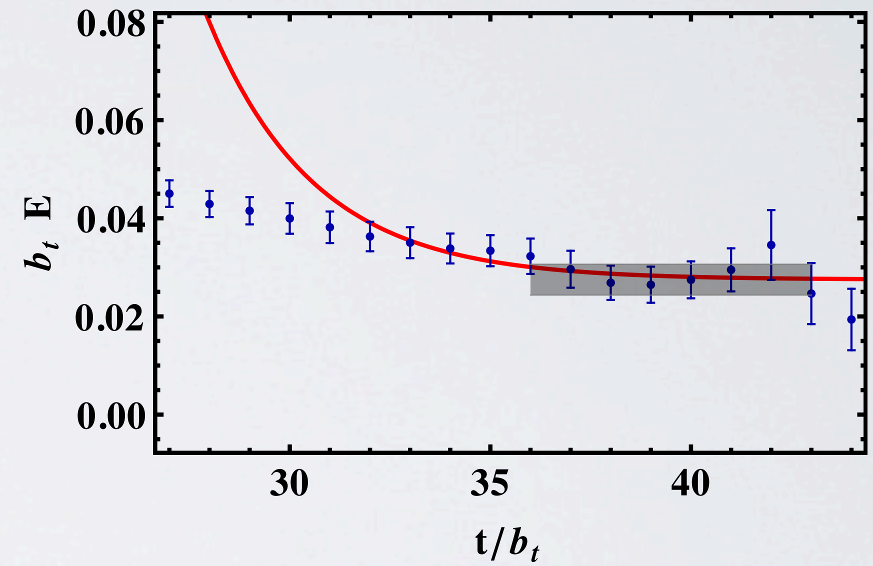
proton

N_K=1

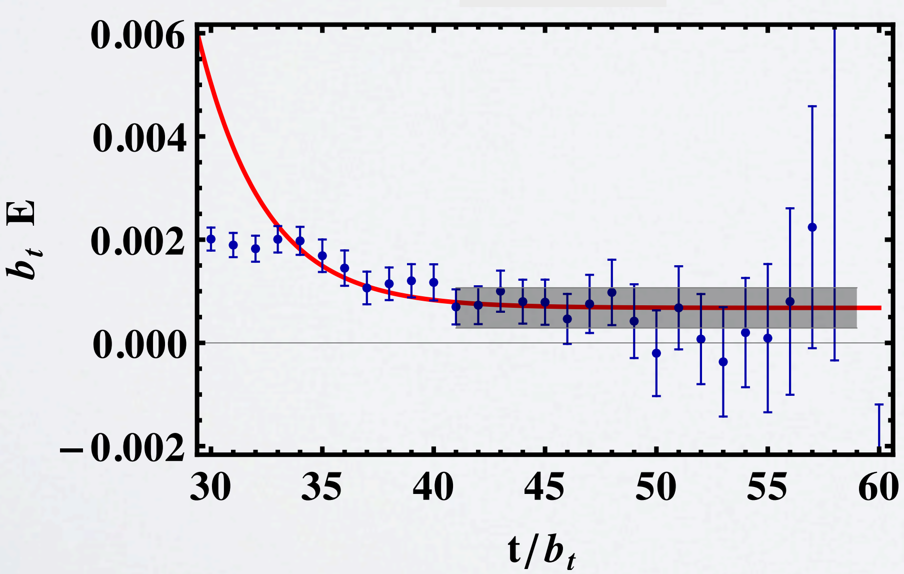


N_K=9

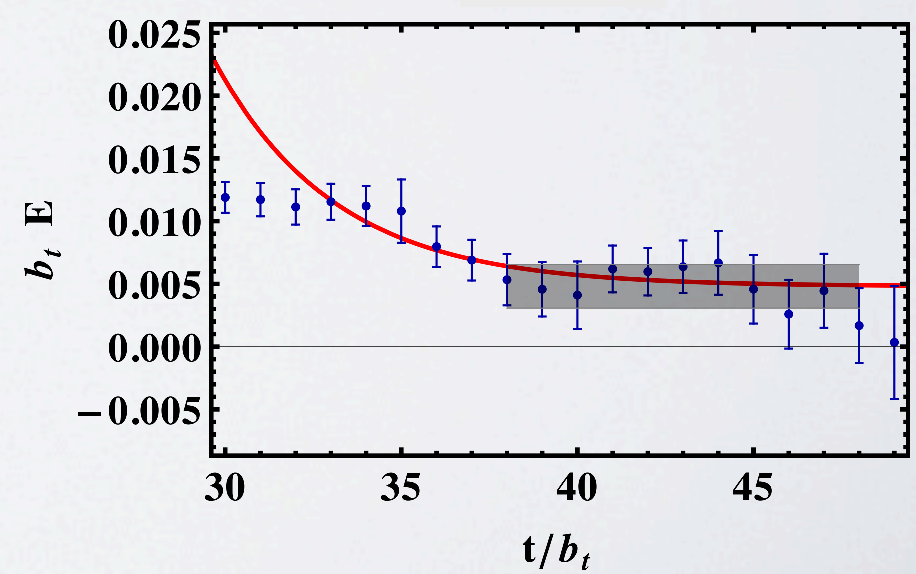


neutron

N_K=1



N_K=5



ENERGIES IN A BOX

Beane, Detmold & Savage (2007)
Smigielski & Wasem (2008)

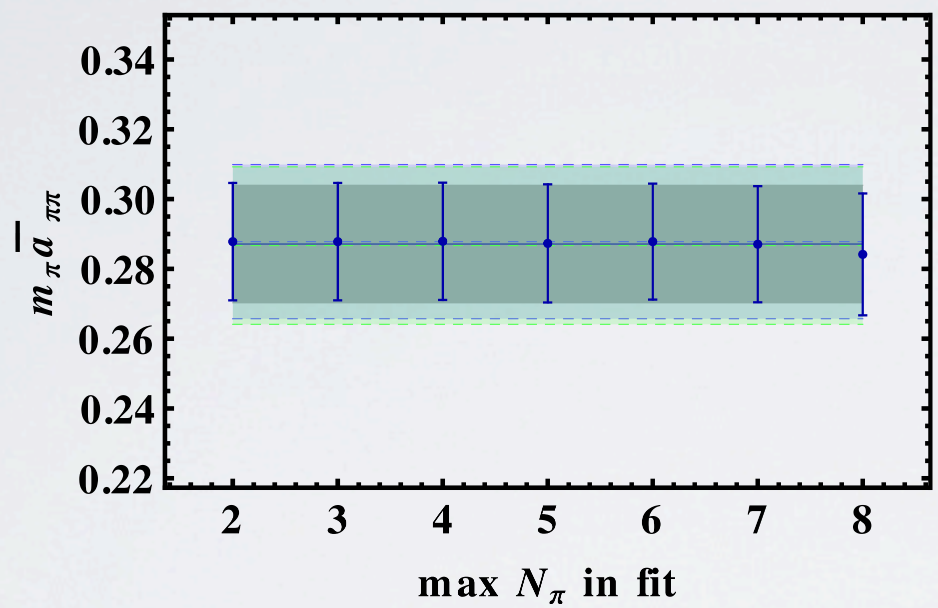
- Large volume expansion of g.s. energy for two species of bosons in a box to $O(L^{-6})$
    - extension of Lüscher's relation for 2 particles in a box
    - includes 2- and 3-body parameters, $\bar{a}_{MB}$, $\bar{a}_{MM}$, $\eta_{3,MMB}(L)$
- Since single baryon carries the spin for the entire system, can treat like different species of boson

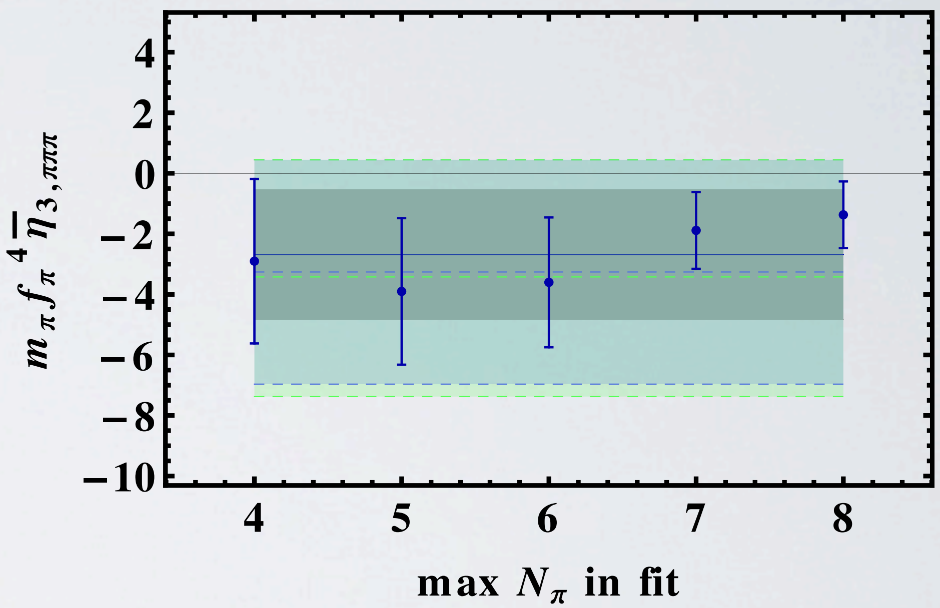
MESON SCATTERING PARAMETERS

2-body

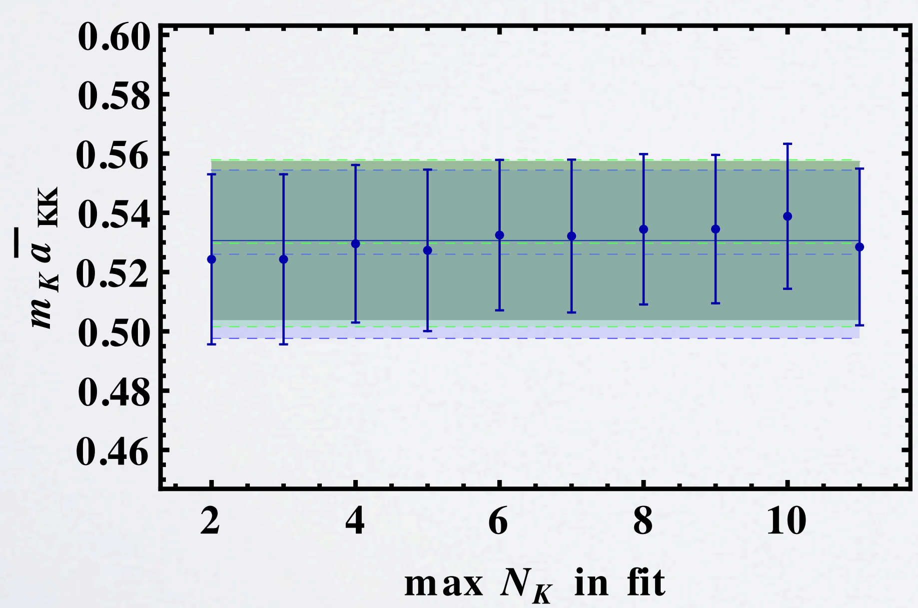
3-body

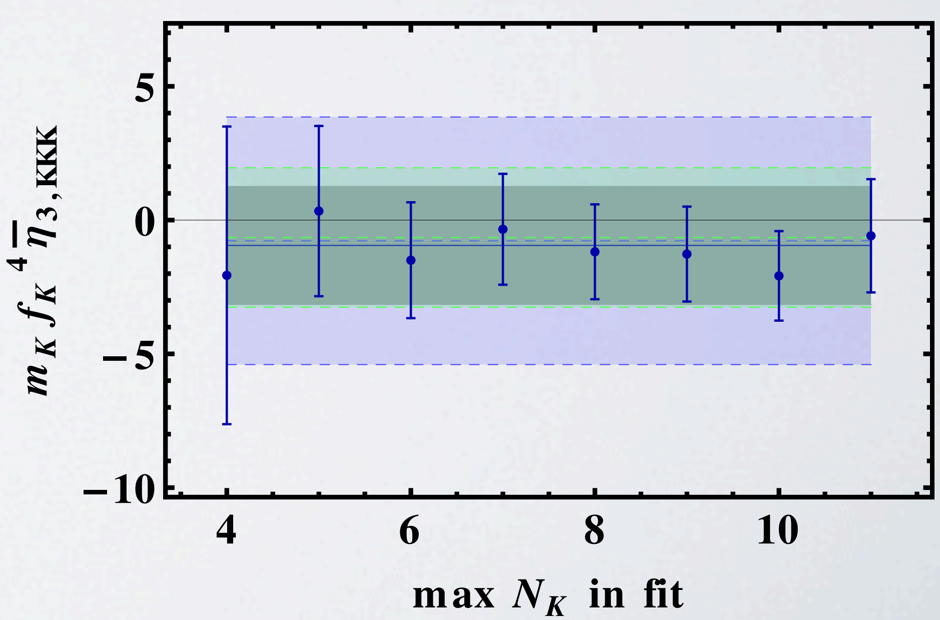
pions



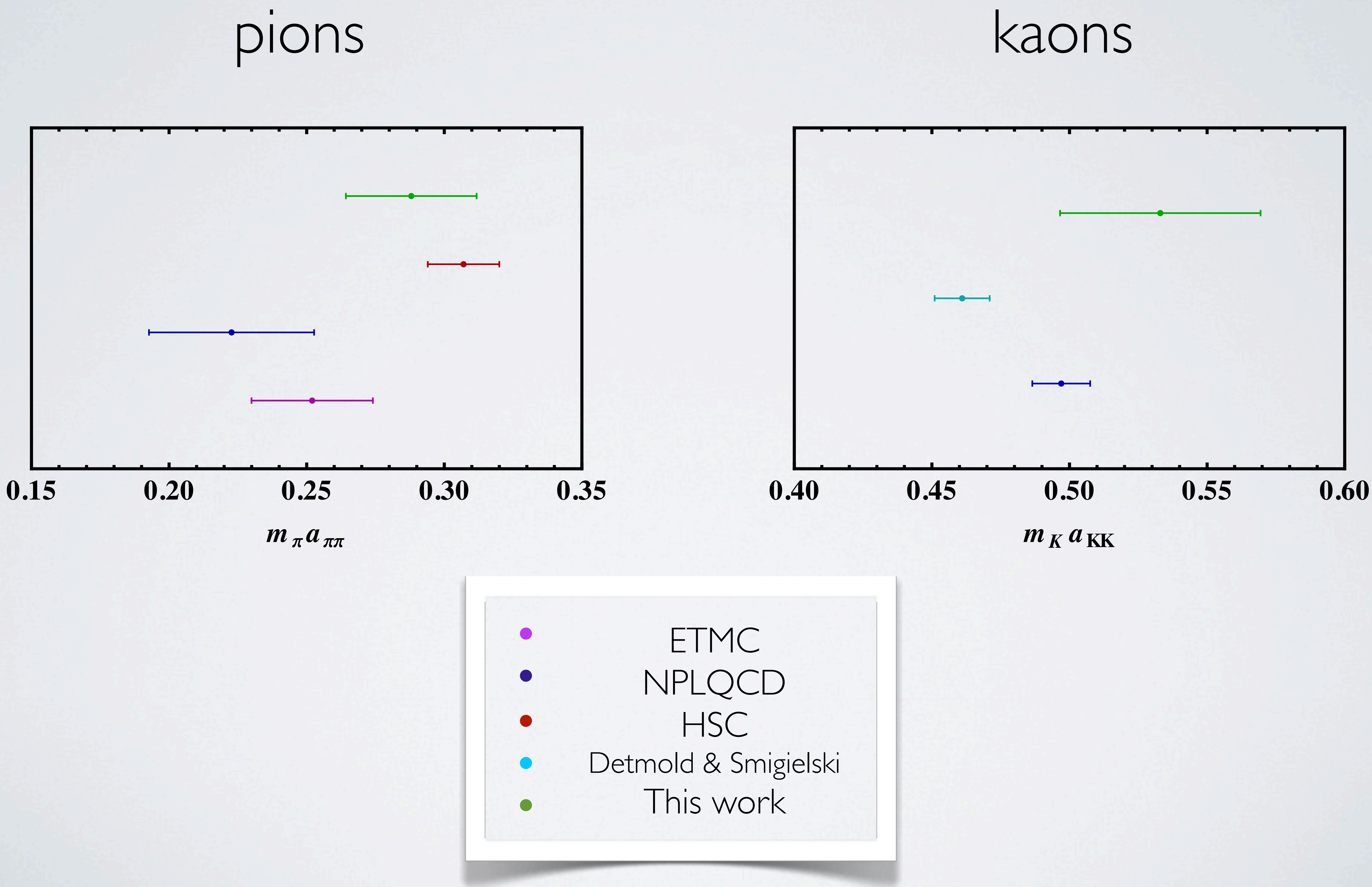


kaons



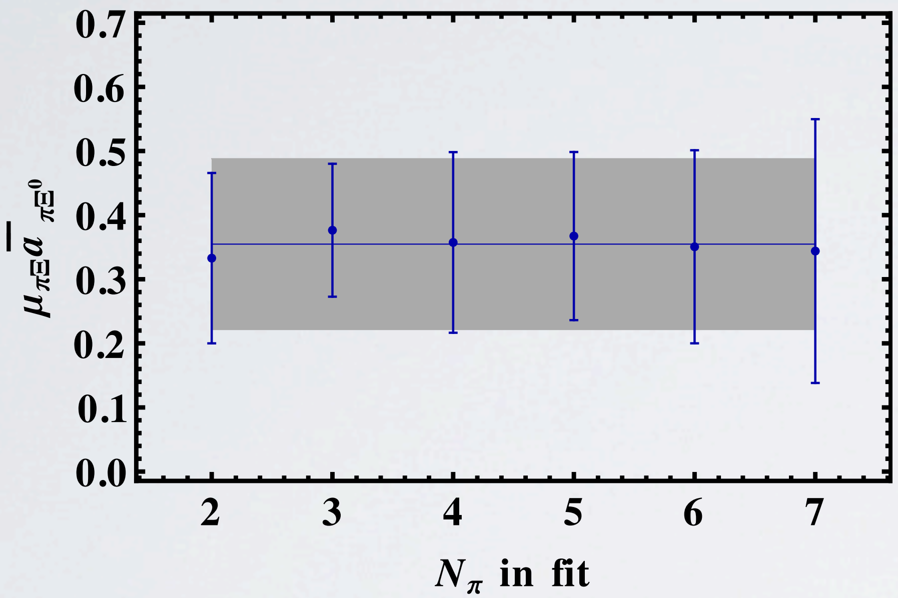


MESON SCATTERING PARAMETERS

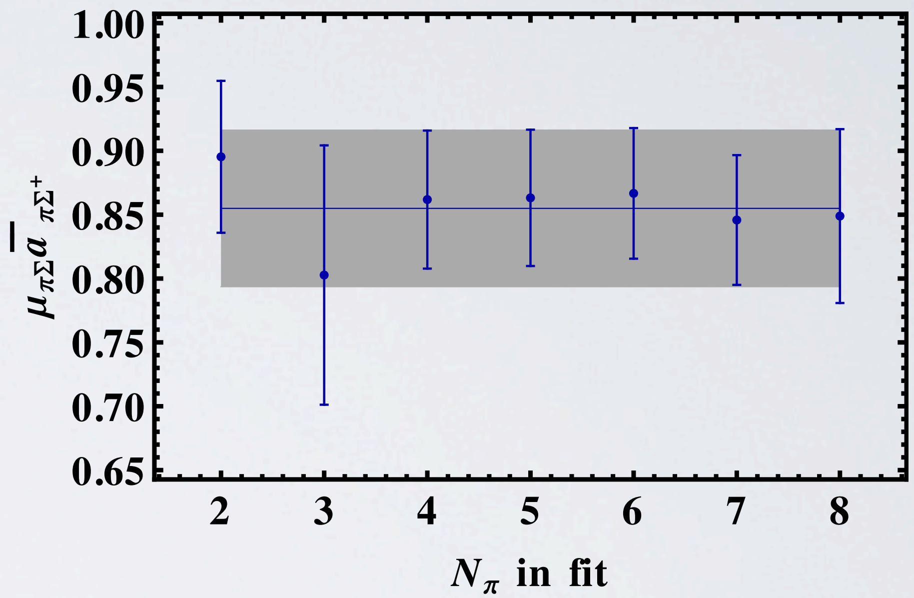


MESON-BARYON 2-BODY PARAMETERS

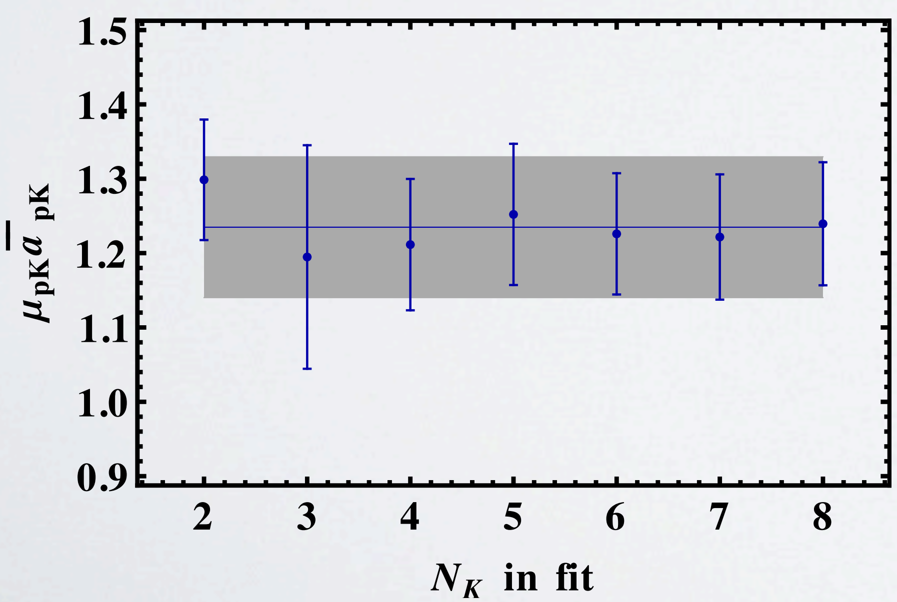
Ξ^0, π^+



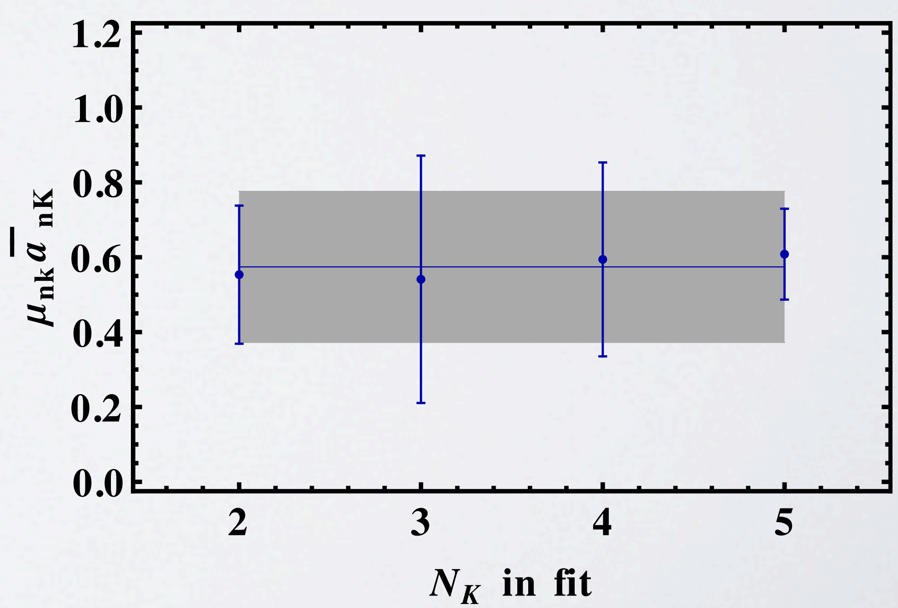
Σ^+, π^+

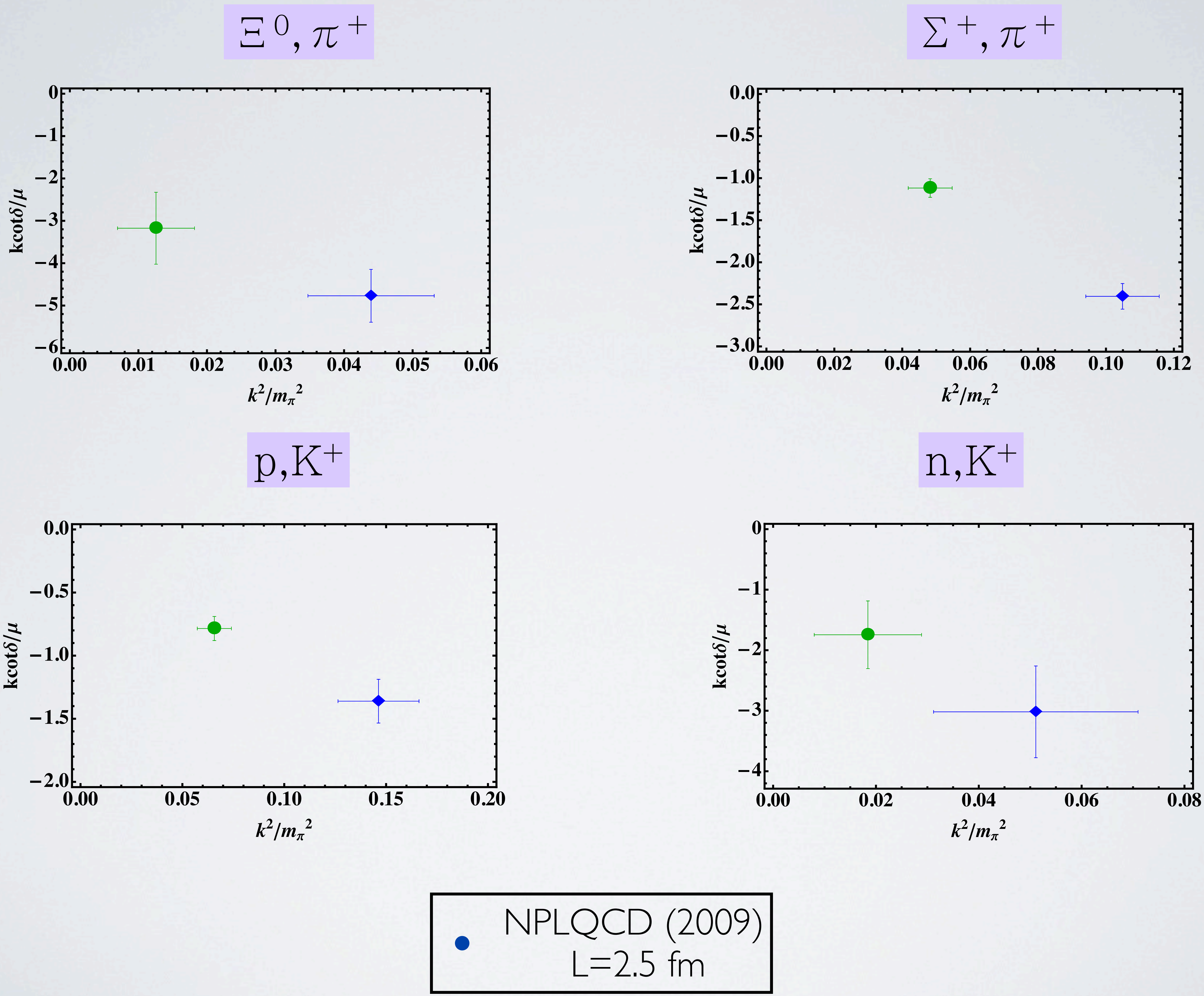


p,K$^+$



n,K$^+$

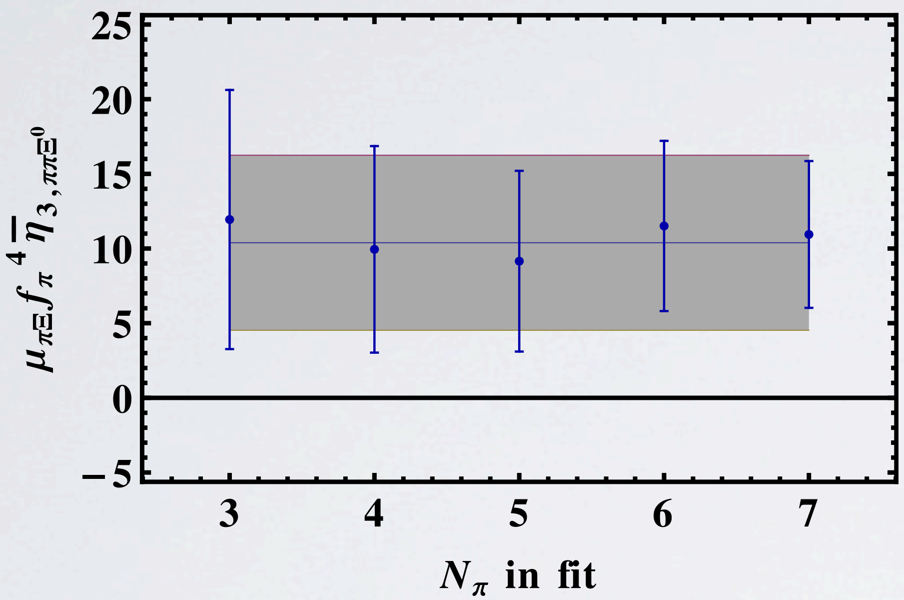




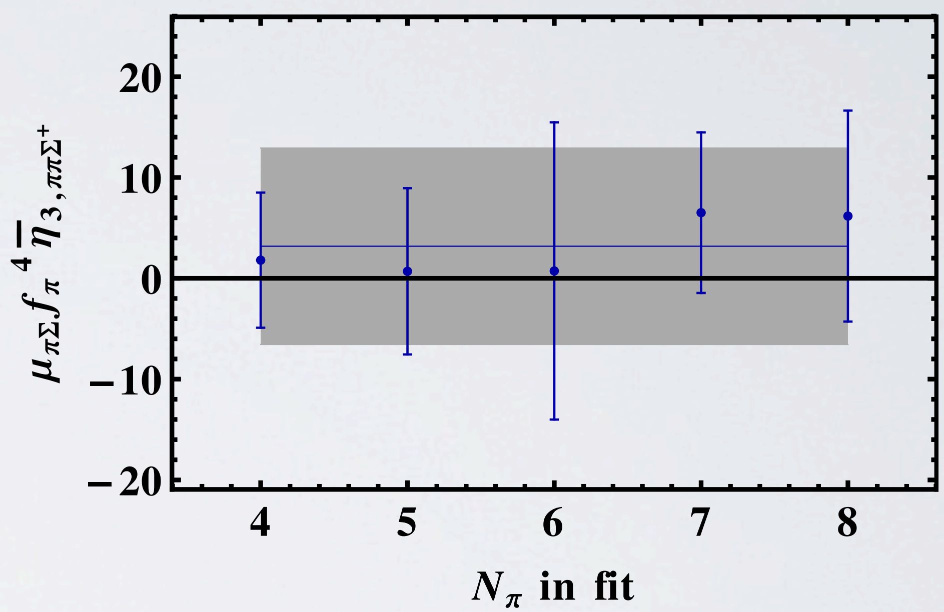
Ξ^0, π^+
Σ^+, π^+
p,K+
n,K+
kcotδ/μ
k^2/m_π^2
0
−1
−2
−3
−4
−5
−6
0.00 0.01 0.02 0.03 0.04 0.05 0.06
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
0.00 0.02 0.04 0.06 0.08 0.10 0.12
0.00 0.05 0.10 0.15 0.20
0.00 0.02 0.04 0.06 0.08
NPLQCD (2009)
L=2.5 fm

MESON-BARYON 3-BODY PARAMETERS

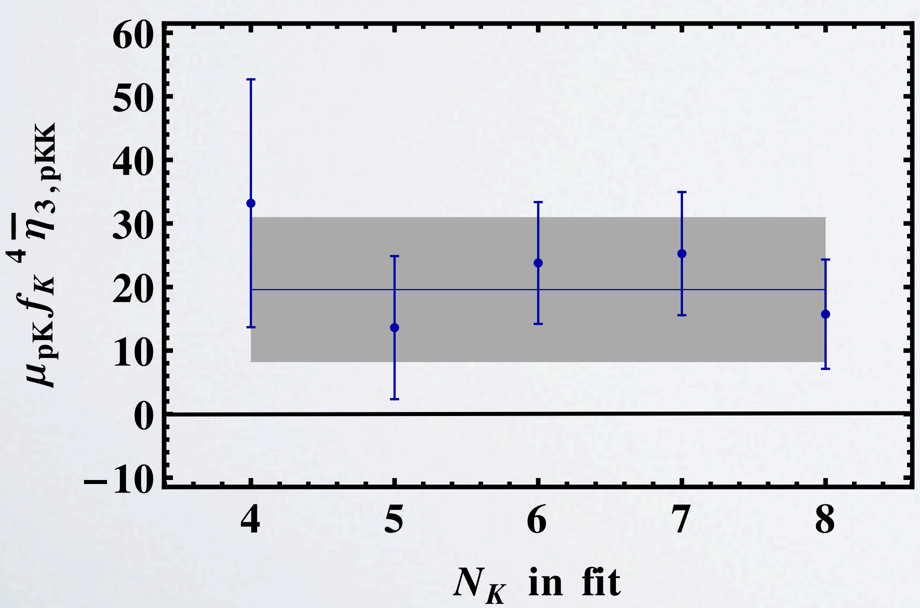
Ξ^0, π^+



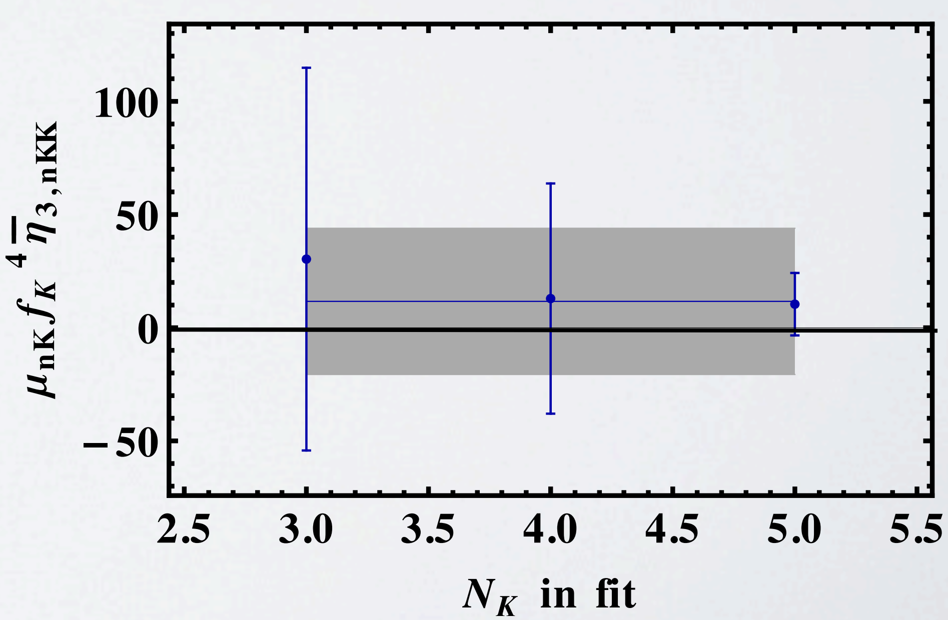
Σ^+, π^+



p,K^+



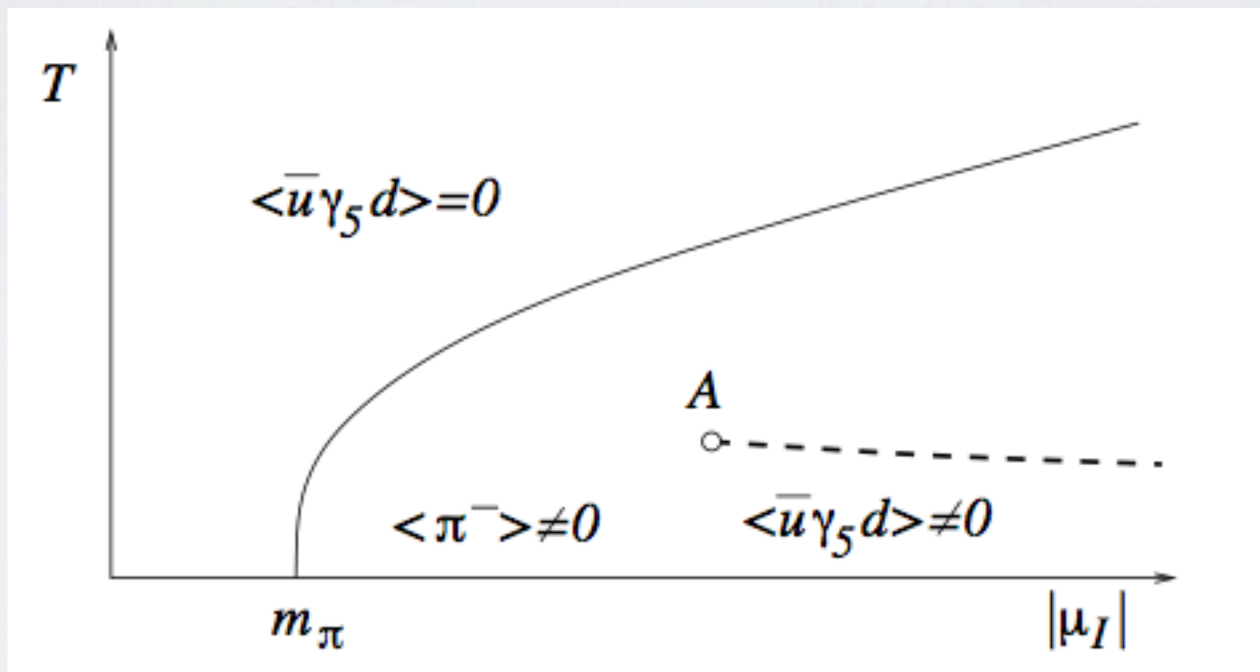
n,K^+



TREE-LEVEL χPT

*Son & Stephanov (2001)
†Bedaque, Buchoff, Tiburzi (2009)

SU(2)†: $M_{\Xi^0}(\mu_I, \cos\alpha)\,,\ M_{\Sigma^+}(\mu_I, \cos\alpha)$



$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \dfrac{m_\pi^2}{\mu_I^2} & (\langle\pi\rangle \neq 0) \end{cases}$$ *

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$$-M_{\Xi^0}(\mu_I, 1)\,,\ -M_{\Sigma^+}(\mu_I, 1)$$

Subtract mass in vacuum to give LECs corresponding to pion interactions

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_\pi^2}{\mu_I^2} & (\langle\pi\rangle \neq 0) \end{cases} \quad *$$

TREE-LEVEL χPT

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SU(2)†: $M_{\Xi^0}(\mu_I, \cos\alpha)\,,\ M_{\Sigma^+}(\mu_I, \cos\alpha)$

SU(3): $M_{\Xi^0}(\mu_I, m_\pi) \Longleftrightarrow M_n(\mu_K, m_K)$

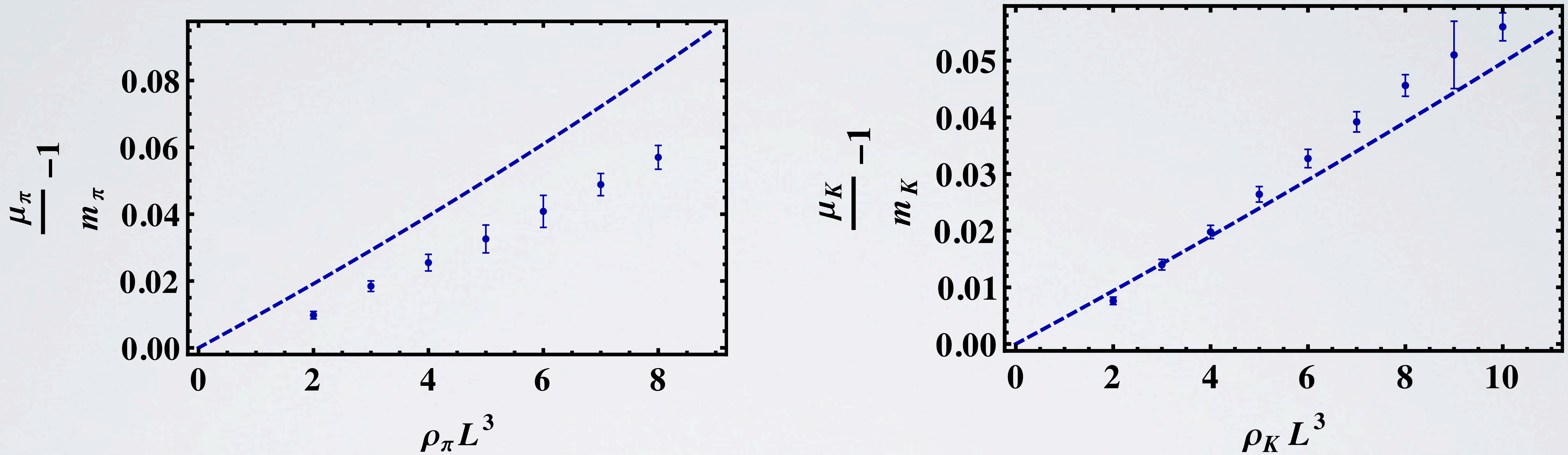
$M_{\Sigma^+}(\mu_I, m_\pi) \Longleftrightarrow M_p(\mu_K, m_K)$

+ term ∝ $m_K^2 - m_\pi^2$

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_K^2}{\mu_K^2} & (\langle K \rangle \neq 0) \end{cases}$$

CHEMICAL POTENTIAL

$$\rho_{\pi,K} = -\frac{\partial \mathcal{L}_{stat}}{\partial \mu_{\pi,K}} = f^2_{\pi,K}\mu_{\pi,K}\left(1 - \frac{m^4_{\pi,K}}{\mu^4_{\pi,K}}\right)^*$$



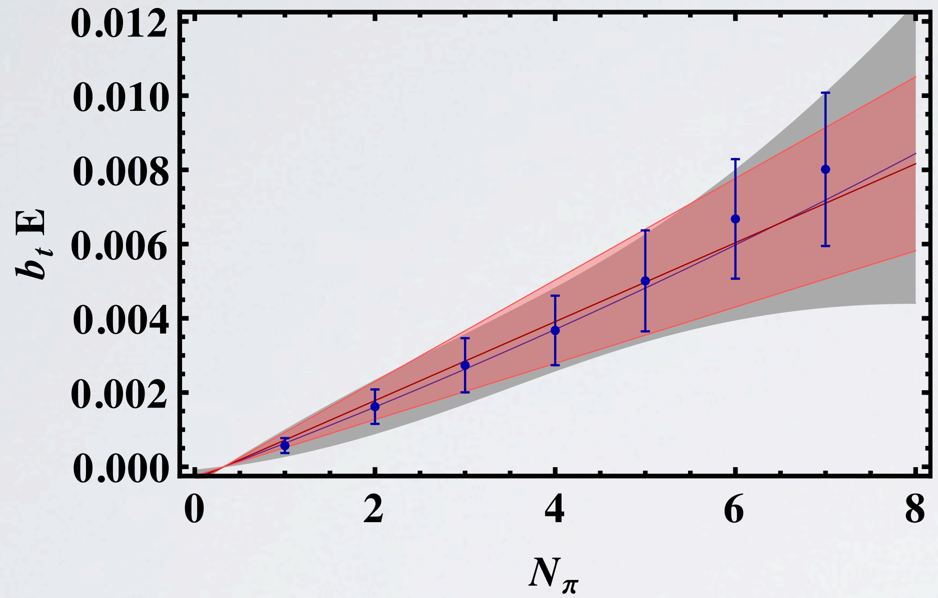
- $\mu_{\pi,K}/m_{\pi,K}$-1 very small
- Expanding mass relations around $\mu_{\pi,K}=m_{\pi,K}$ gives different linear combinations of LECs
    - fits much more stable

*Son & Stephanov (2001)

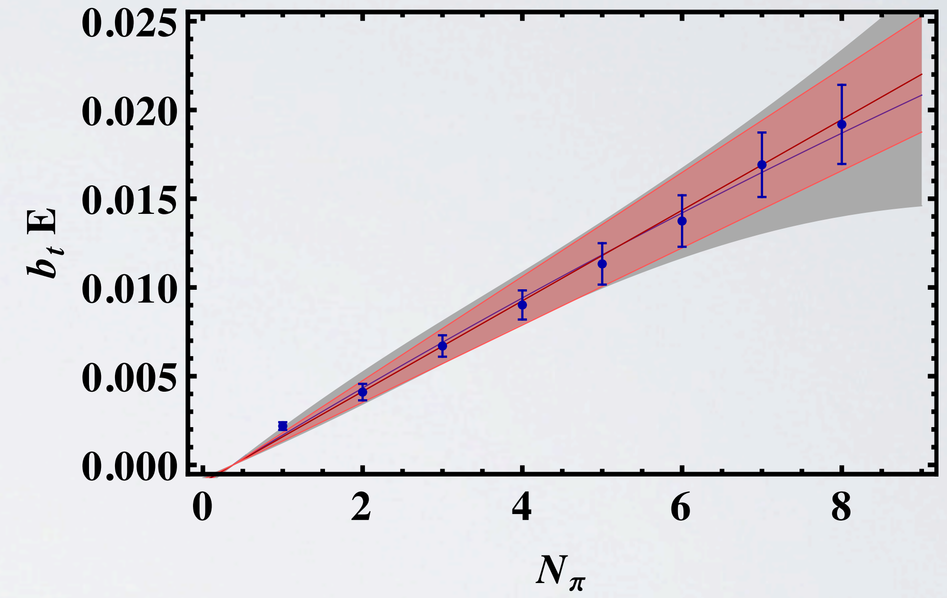
LOW-ENERGY CONSTANTS

$$M(\mu_{I,K}) \approx a\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right) + b\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right)^2 + \cdots$$

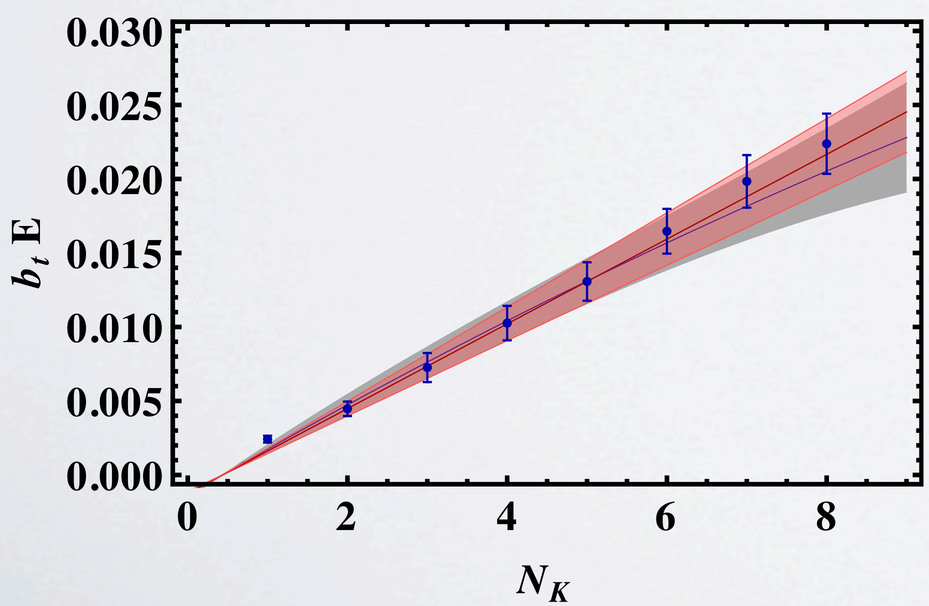
Ξ^0, π^+



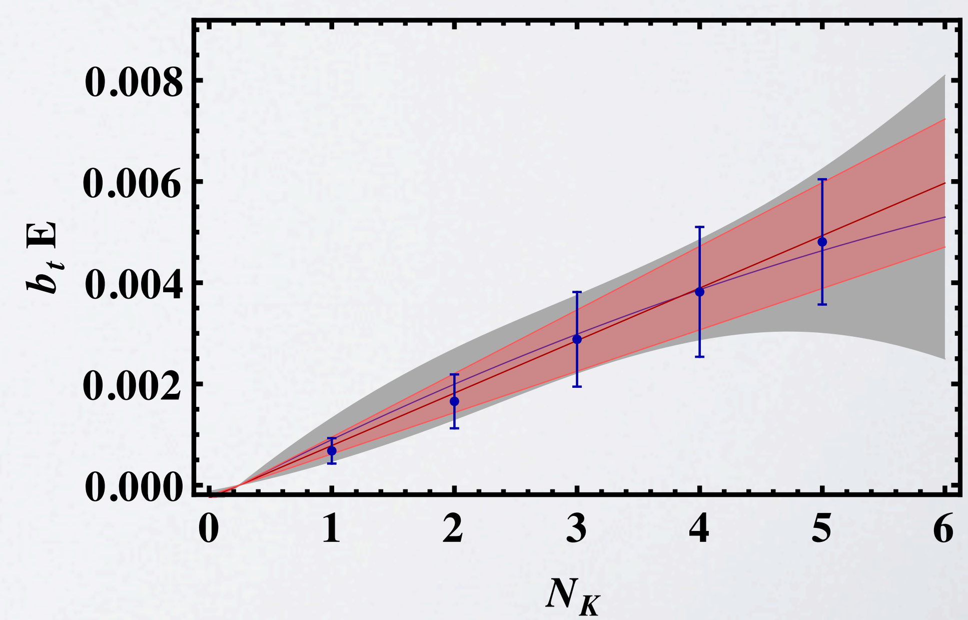
Σ^+, π^+



p, K^+



n, K^+



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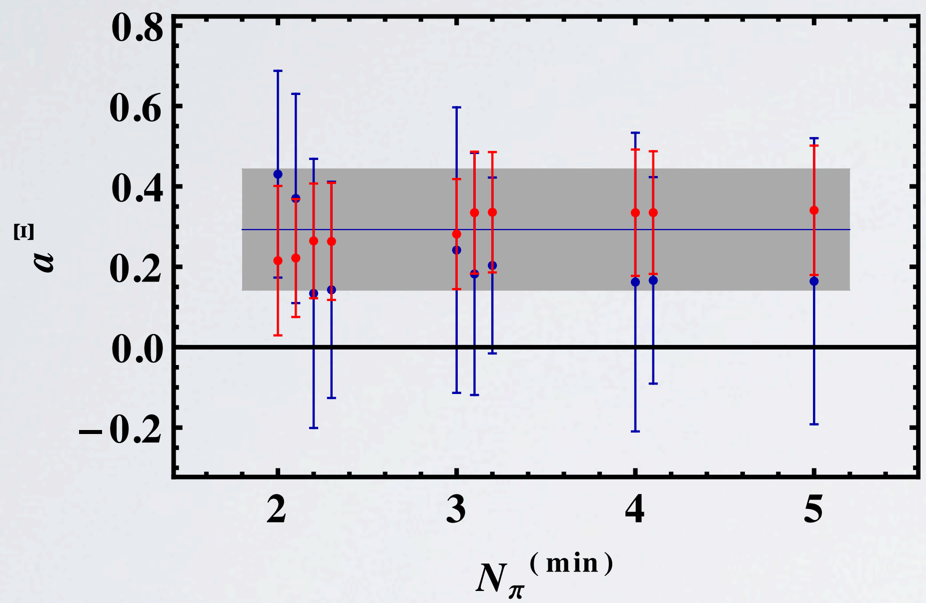
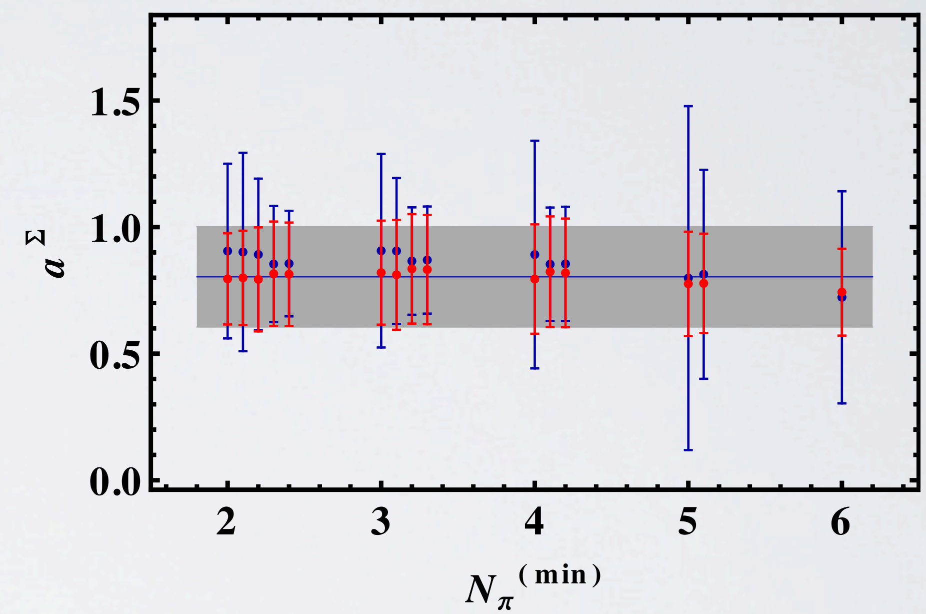
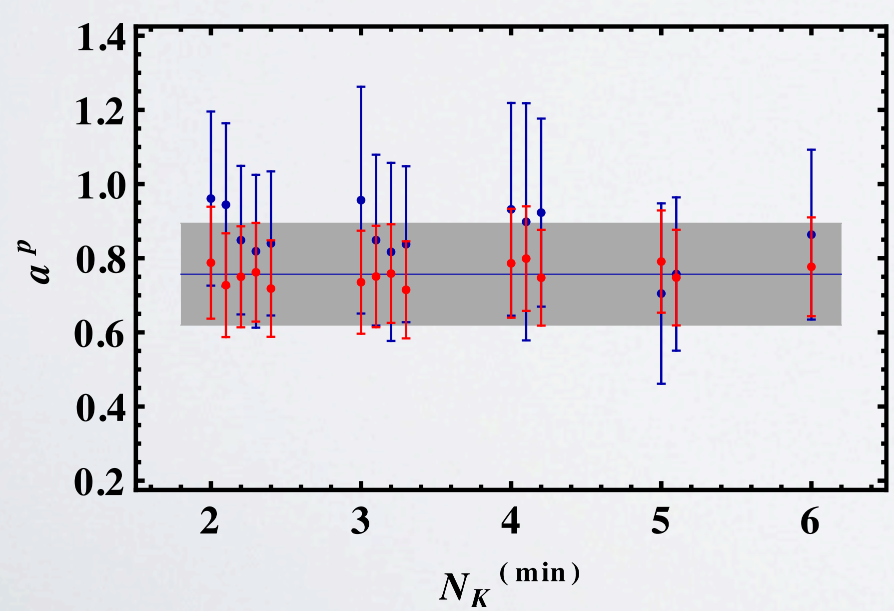
Ξ^0, π^+



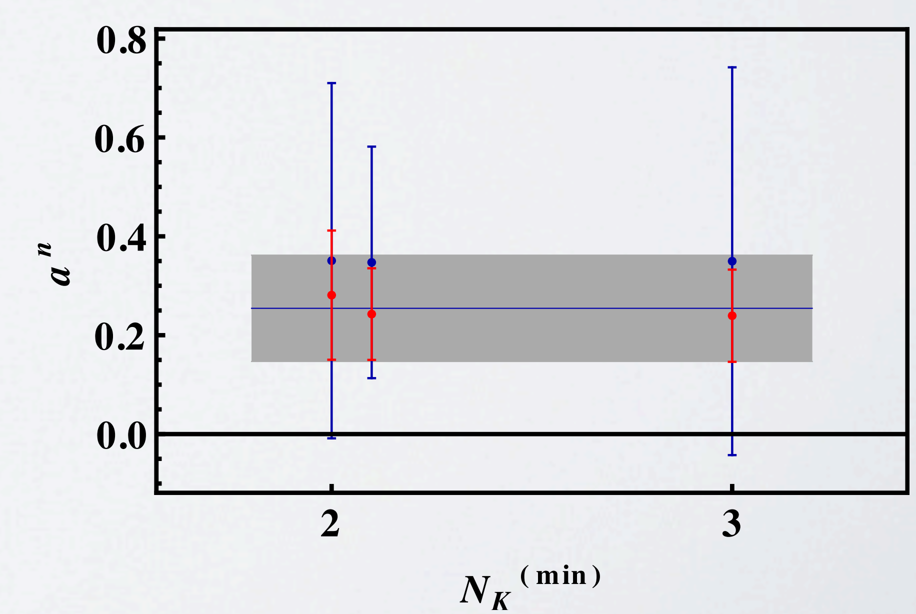
Σ^+, π^+



p,K+

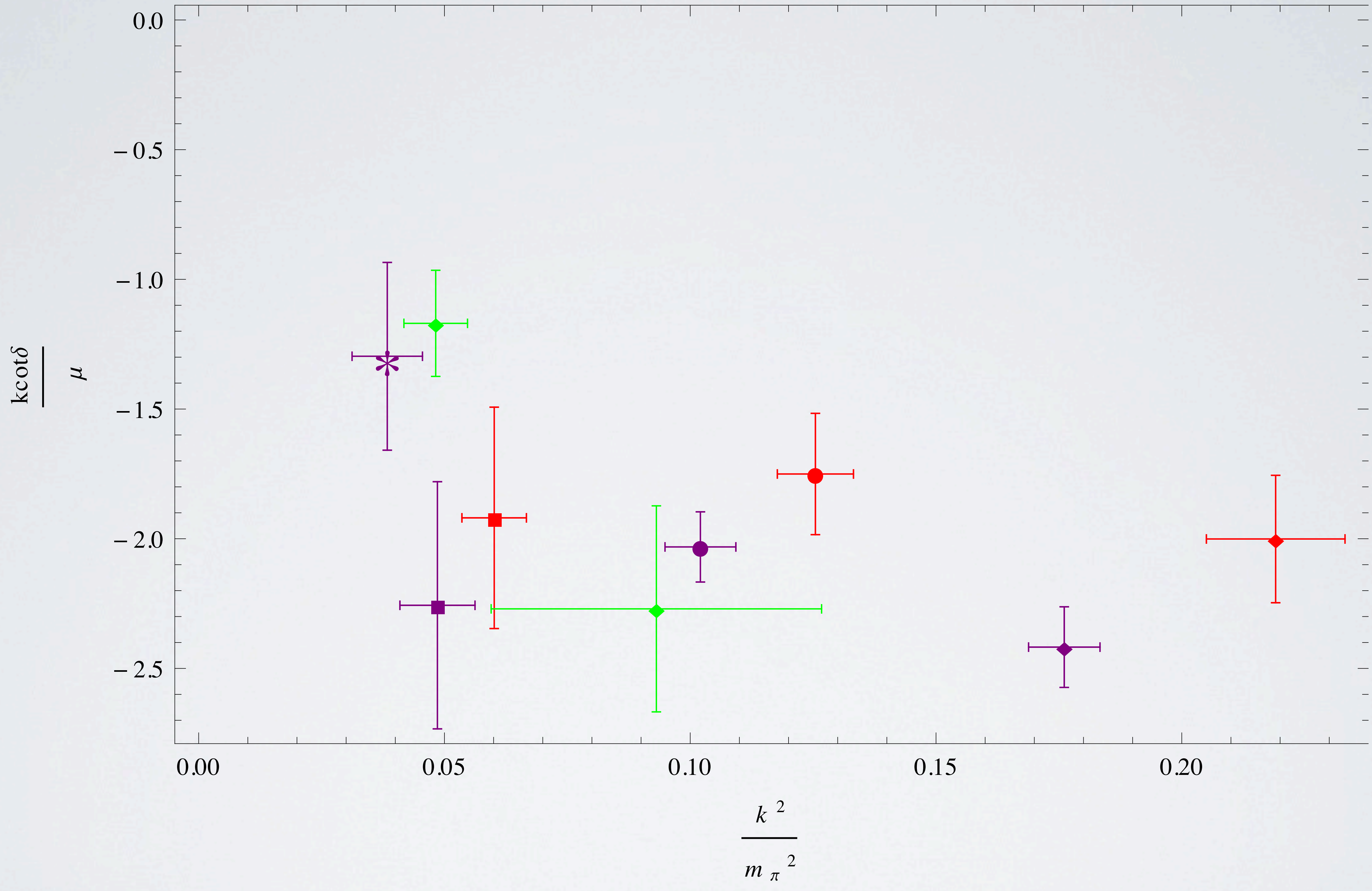


n,K+



SUMMARY

- Investigated systems of up to 9 mesons + 1 baryon
  - Some volume-dependence of meson-baryon scattering phase shifts found when compared to previous work by NPLQCD
    - may indicate significant effective range contribution and/or inelasticities
  - First calculation of meson-meson-baryon 3-body interaction
  - Some combinations of LECs accessible
- Thermal effects & noise current limitation to system size
- Would like to study attractive channels (need to deal with disconnected contributions)



0.0
−0.5
−1.0
−1.5
−2.0
−2.5
kcotδ / μ
0.00
0.05
0.10
0.15
0.20
k 2 / m π 2