Susceptibilites in $N_f = 2$ QCD

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Introduction

- Susceptibilities are derivatives of the pressure with respect to the quark chemical potential.

$$\chi_{n_u\, n_d} = \frac{\partial^{n_u+n_d} P}{\partial \mu_u^{n_u}\, \partial \mu_d^{n_d}}$$

  We restrict ourselves to 2-flavor QCD with degenerate u, d.

- $\chi_{n_u\, n_d}$ have been widely studied in the lattice for understanding properties of QGP, and for calculations at finite μ_q.

  Gottlieb et al, PRL 59 ('87) 2247.
  Gavai & Gupta, PRD 68 ('03) 034506; Allton et al., *ibid*, 014507.

- χ_{20} connected to event-by-event fluctuations of baryon number in heavy ion collisions.
- Similarly, 4th order susceptibilities have been used to study event-by-event distributions of conserved quantum numbers.

  Gavai & Gupta, PLB 696 ('11) 459. Gupta et al., Science, 2011.
  Bazavov et al., PRD 86('12) 034509. Borsanyi et al., JHEP 1201 ('12) 138. ...

Details

- Here we present results for 2nd and 4th order susceptibilities in $N_f = 2$ QCD in the temperature range $0.9 - 2.1\ T_c$.
- We use $N_\tau = 8$ lattices and an aspect ratio $LT = 4$. Staggered quarks with degenerate u, d are used such that $m_\pi \sim 220$ MeV.
- Configurations generated using R algorithm. Near T_c an additional run with finer stepsize was made to cross-check.
- The temperature scale is set using T_c, determined from peak of the susceptibility of the Polyakov loop, and two-loop running.
- 50000-70000 configurations generated at each temperature near T_c, and 25000 at higher temperatures.

Details

- Susceptibilities calculated on every 250th configuration using Gaussian random vectors.
- The number of vectors varied from 2000 in the lower temperatures to 400 at high temperatures.

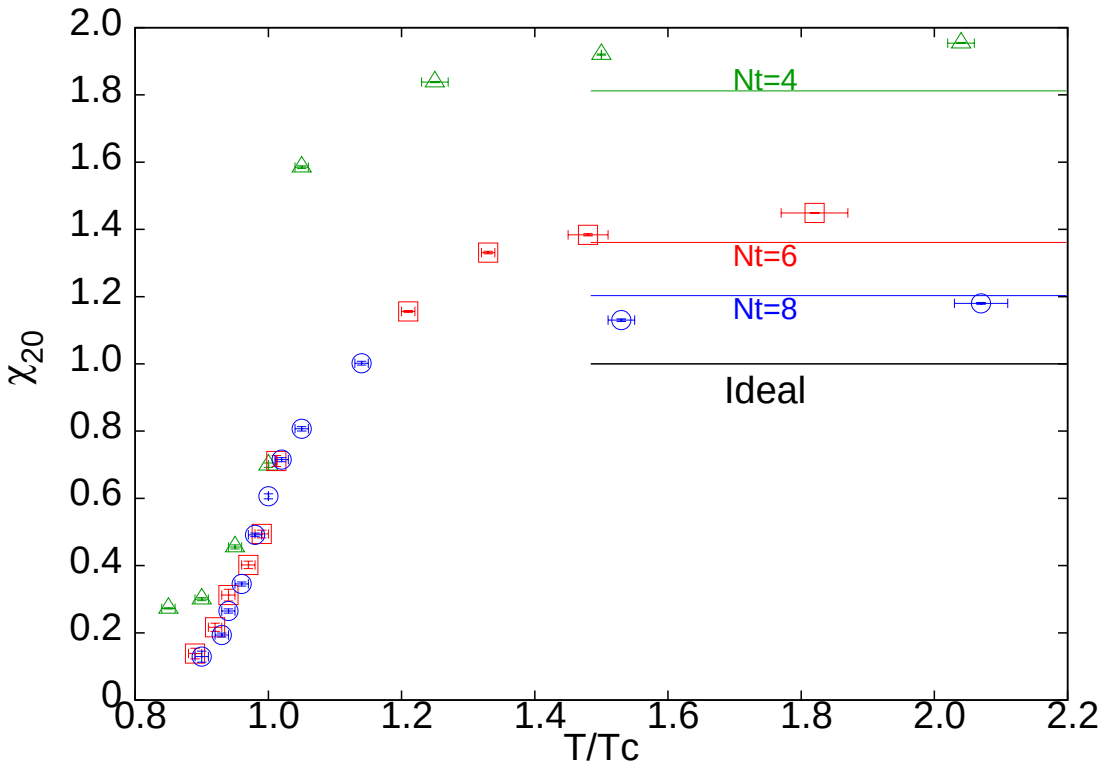
Talk by S. Gupta, session 5A.

- Cutoff effects estimated by comparing to earlier ILGTI results at coarser lattices.
- Broad features agree with results on susceptibilities with other discretizations.

Petreczky, Borsanyi, Wagner, session 1A; Schmidt, session 3A; ...

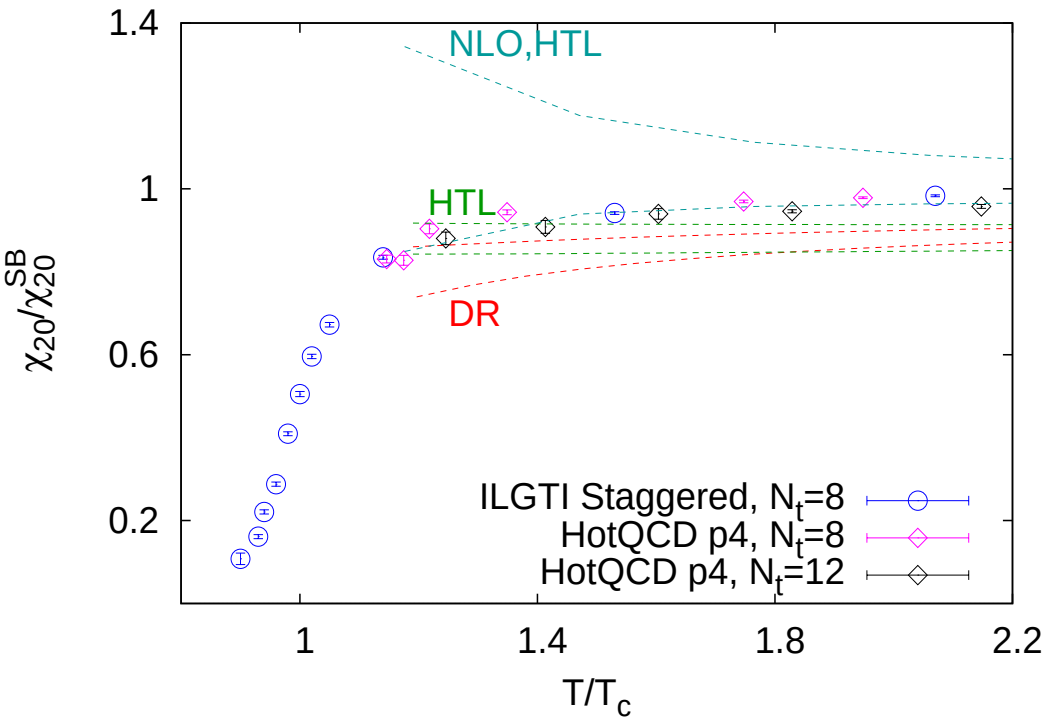
- The susceptibilites can also be used to estimate the location of the QCD critical point in the $\mu_B - T$ plane.

Talk by R. Gavai, session 5A.



The data at $N_\tau = 6$ and 8 agree quite well in the transition regime.
At higher temperatures, cutoff effects more pronounced, and stronger than the free theory.
In particular, the $N_\tau = 8$ results approach SB limit from below.

Comparison with weak coupling theory

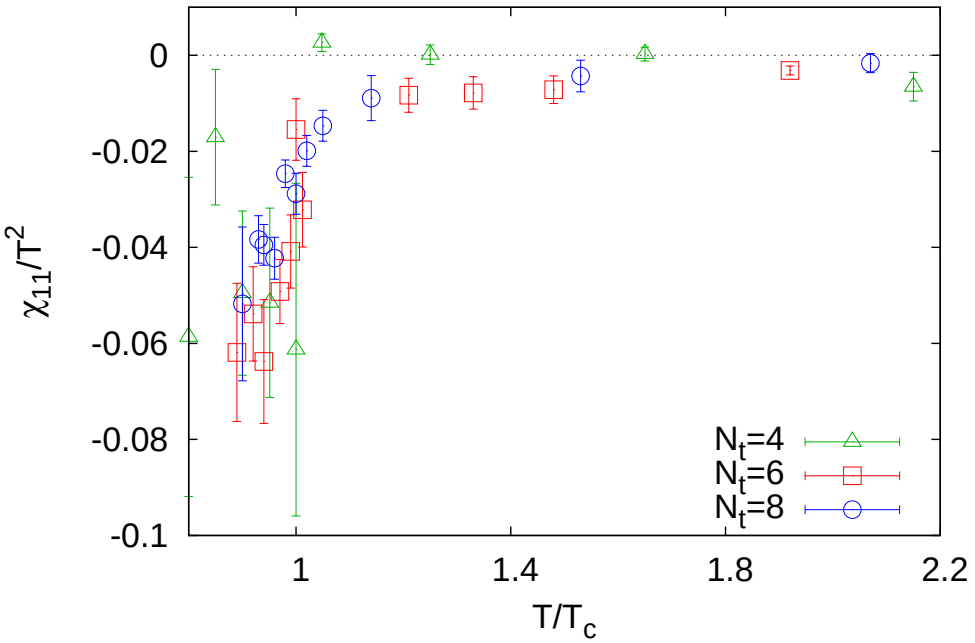


LO HTL, DR: Andersen,Mogliacci,Strickland,Su,Vuorinen, 1307.8098.
NLO HTL: Haque, Mustafa, Strickland, 1302.3228. p4: Petreczky, session 5A

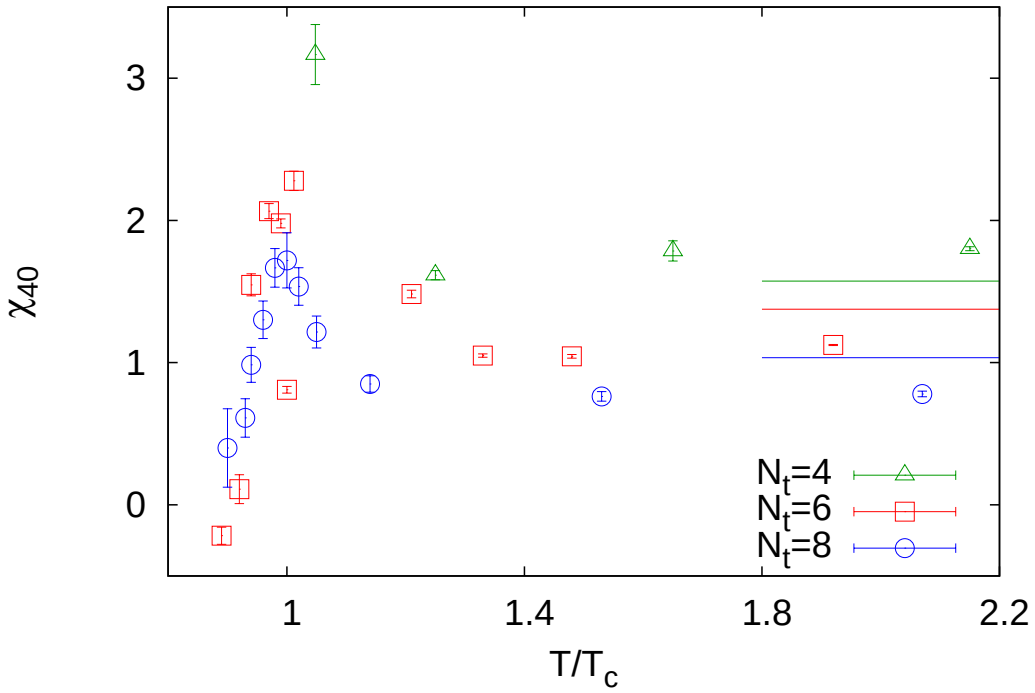
Used $T_c = 170$ MeV to convert T [MeV] to T/T_c (180 MeV for N_t=8 p4).

Second order off-diagonal susceptibilities like χ_{11} measure correlations between different quantum numbers, and explore what are the degrees of freedom carrying the quantum numbers.

Koch, Majumdar, Randrup, PRL 95('05) 182301.

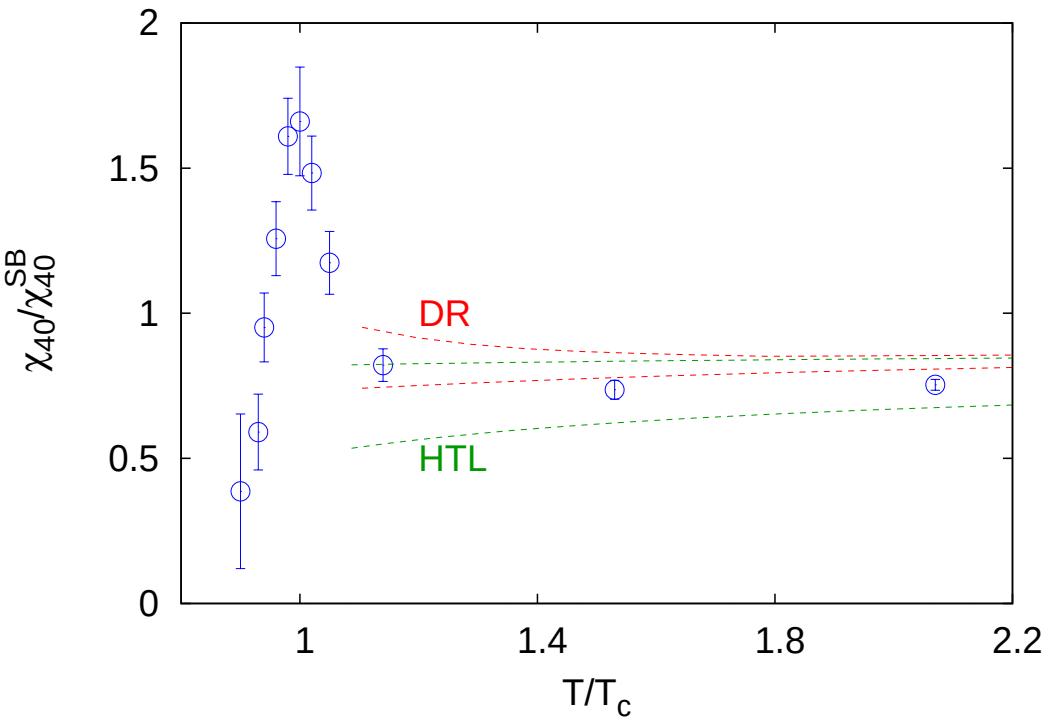


$N_\tau = 8$ data further confirm earlier indication: very little correlation between u and d carriers above $\sim 1.5\,T_c$.



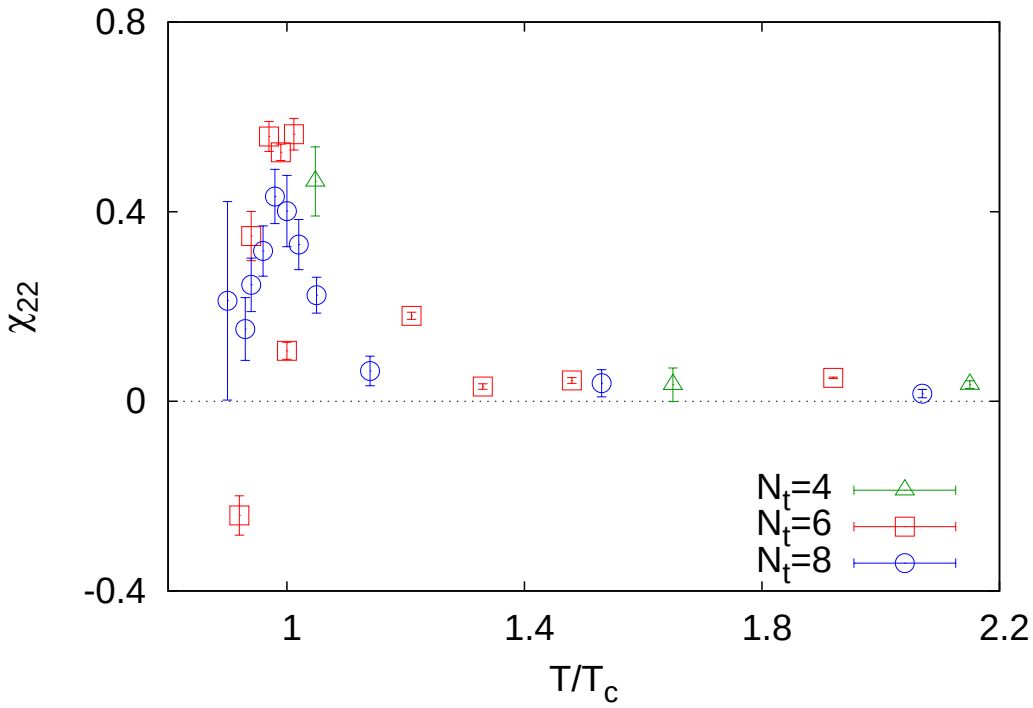
χ_{40} shows a peak near T_c, where cutoff effect is pronounced.

χ_{40} and perturbation theory



Pert.: Andersen,Mogliacci,Strickland,Su,Vuorinen, 1307.8098.

χ_{22}



χ_{22} shows a peak near T_c but at higher temperatures approaches SB value of 0 fast.

Using susceptibilities to calculate observables at finite μ_B

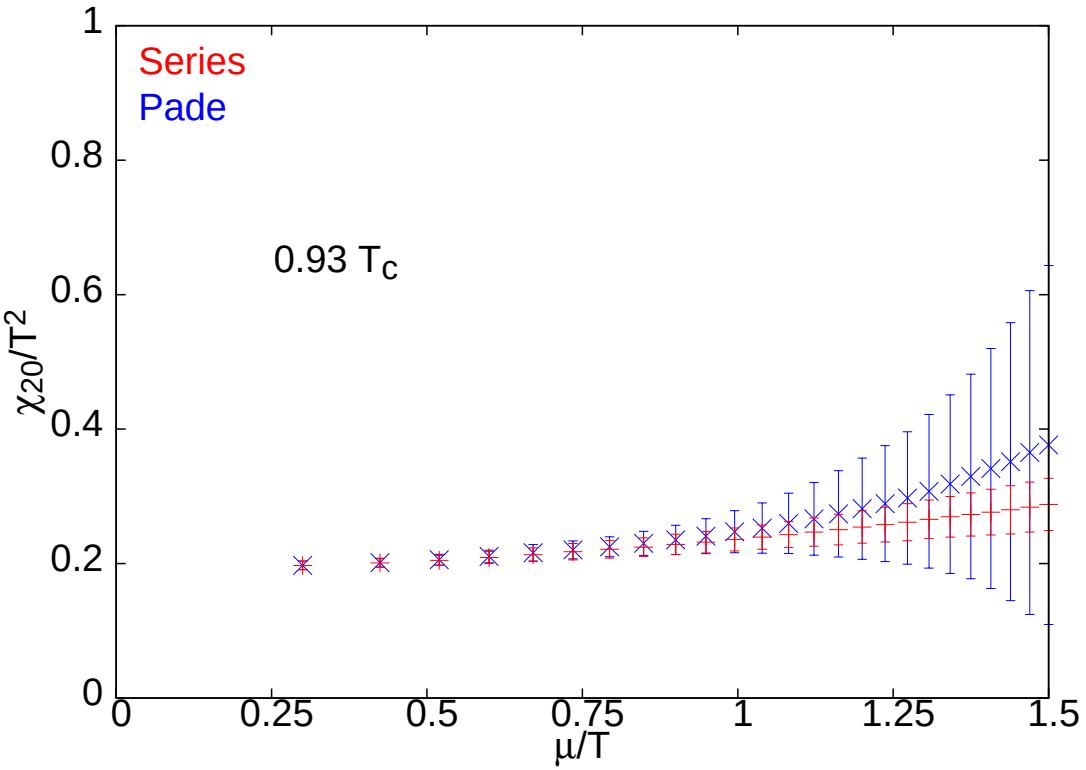
The susceptibilities are expansion coefficients in a series expansion of pressure:

$$P(T,\mu_u,\mu_d) \;=\; P(T,\mu_u=\mu_d=0) \;+\; \sum_{n_u,n_d} \chi_{n_u n_d} \frac{\mu_u^{n_u}}{n_d!} \frac{\mu_d^{n_d}}{n_d!}$$

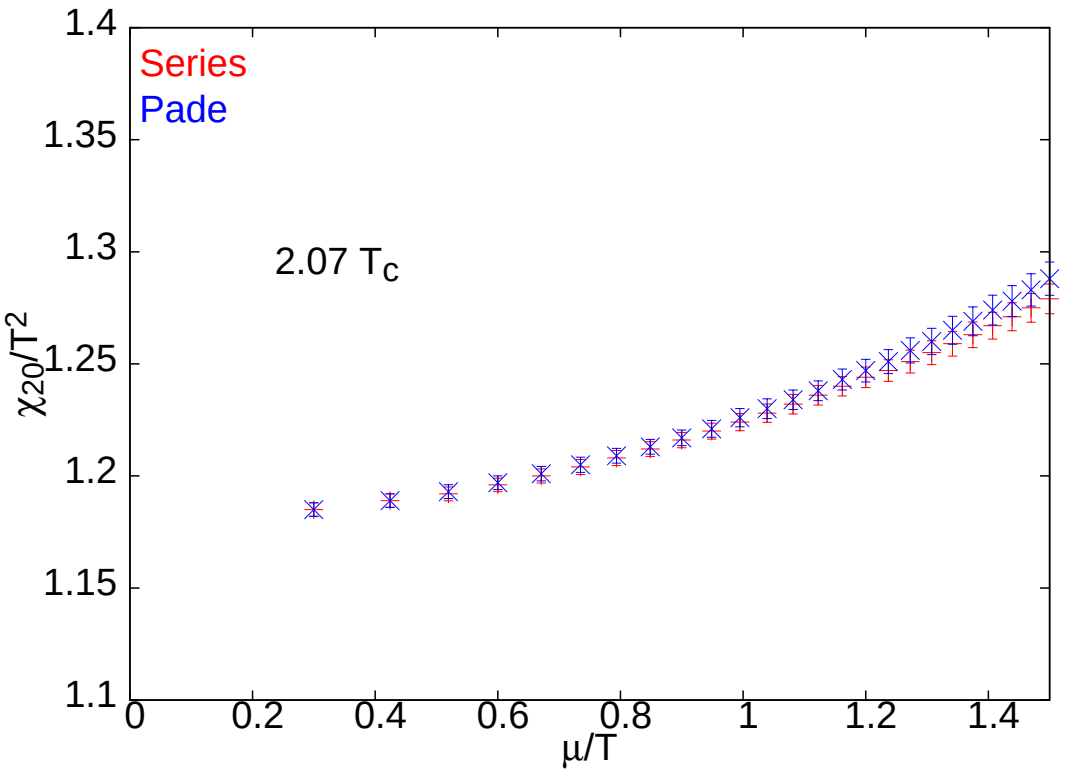
Similar expansions for other quantities can be worked out from this. In particular, we look at the quark number susceptibility.

A Pade approximant can be used to improve the convergence of the series.

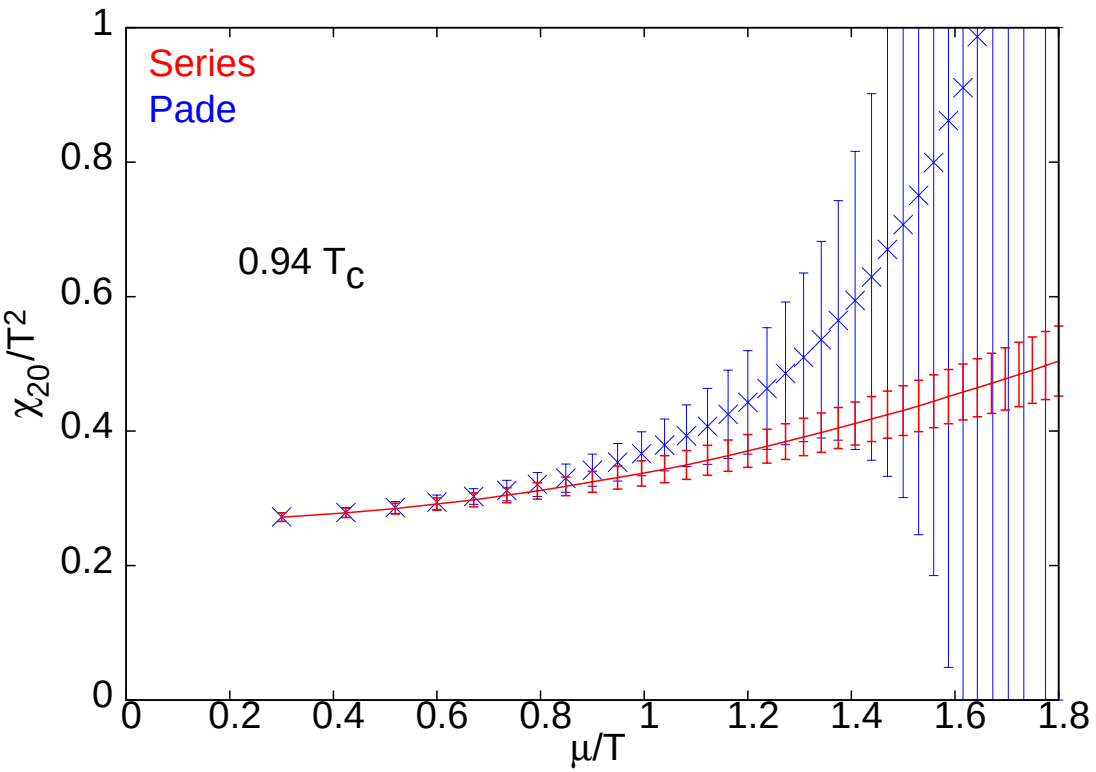
$\chi_{20}(\mu)$


Series
Pade
0.93 T_c
χ_{20}/T^2
μ/T
0
0.2
0.4
0.6
0.8
1
0
0.25
0.5
0.75
1
1.25
1.5

$\chi_{20}(\mu)$


Series
Pade
2.07 T_c
χ_{20}/T^2
μ/T
1.4
1.35
1.3
1.25
1.2
1.15
1.1
0
0.2
0.4
0.6
0.8
1
1.2
1.4

$\chi_{20}(\mu)$

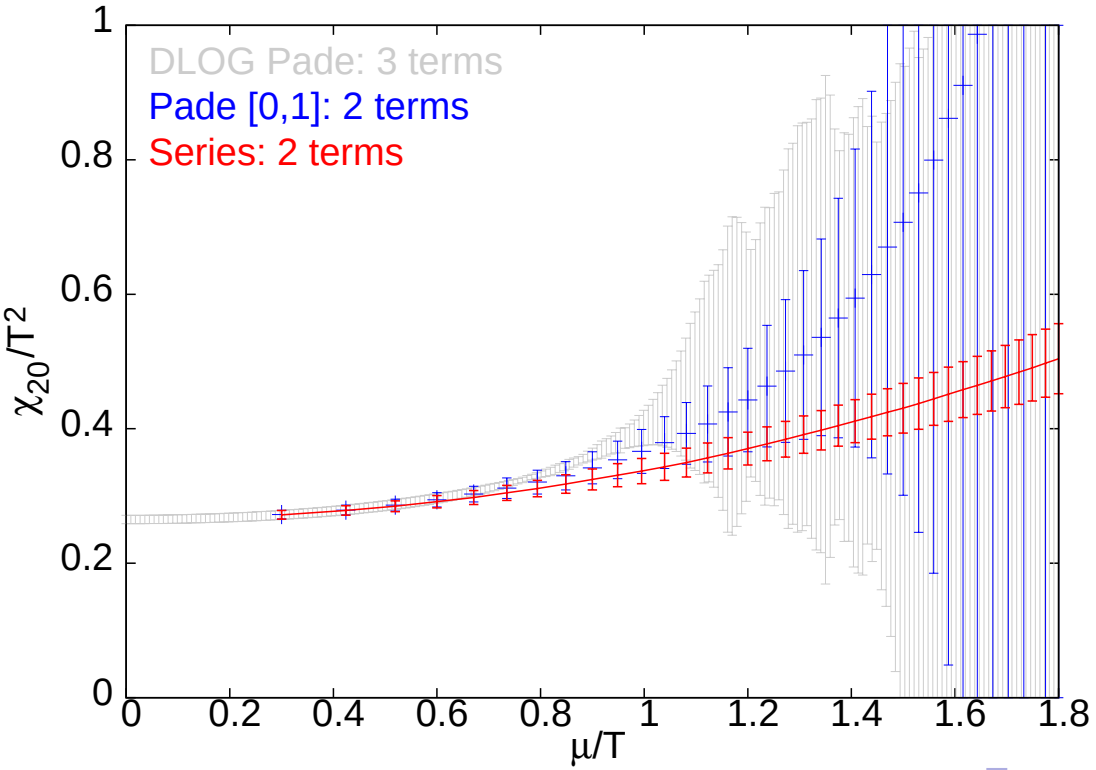


$\chi_{20}(\mu)$

When one has a critical point, in the critical region

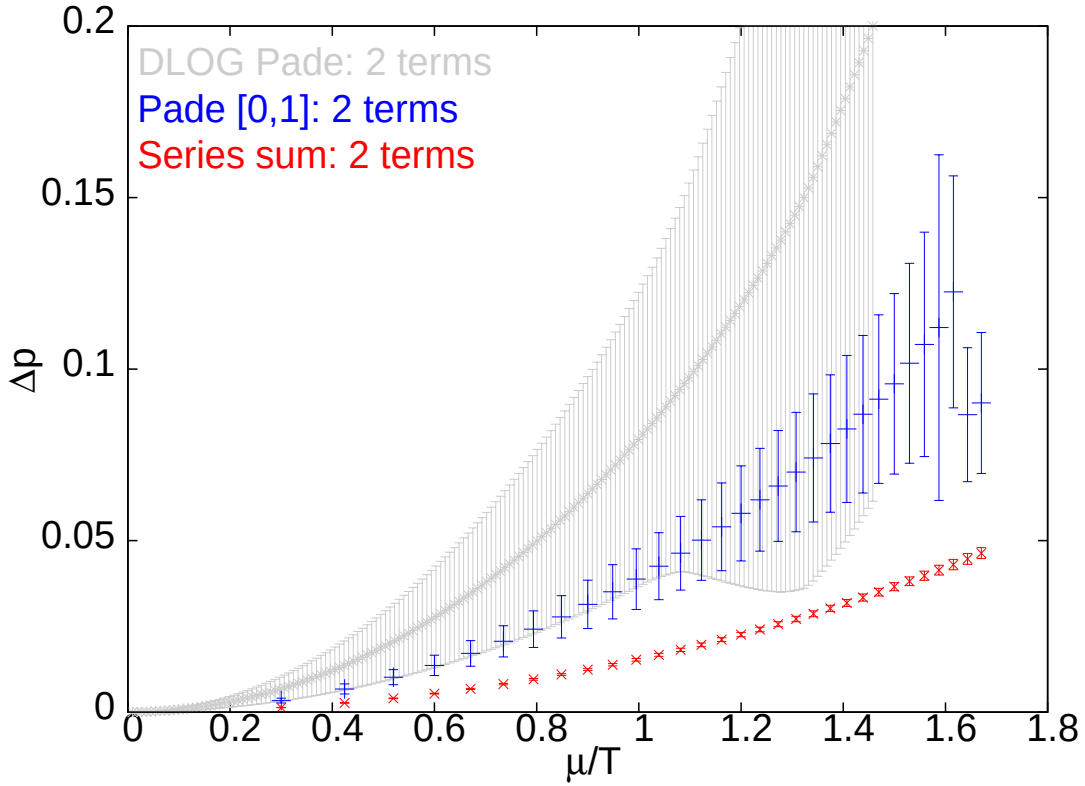
$$\chi_2^B \sim A(\mu_B^2 - \mu_c^2)^\psi \Longrightarrow m_1 = \frac{d}{d\mu_B^2} \log \chi_2^B \sim \frac{A\psi}{\mu_B^2 - \mu_c^2}$$

Can resum series for m_1 with Pade.



$\Delta P(\mu)$

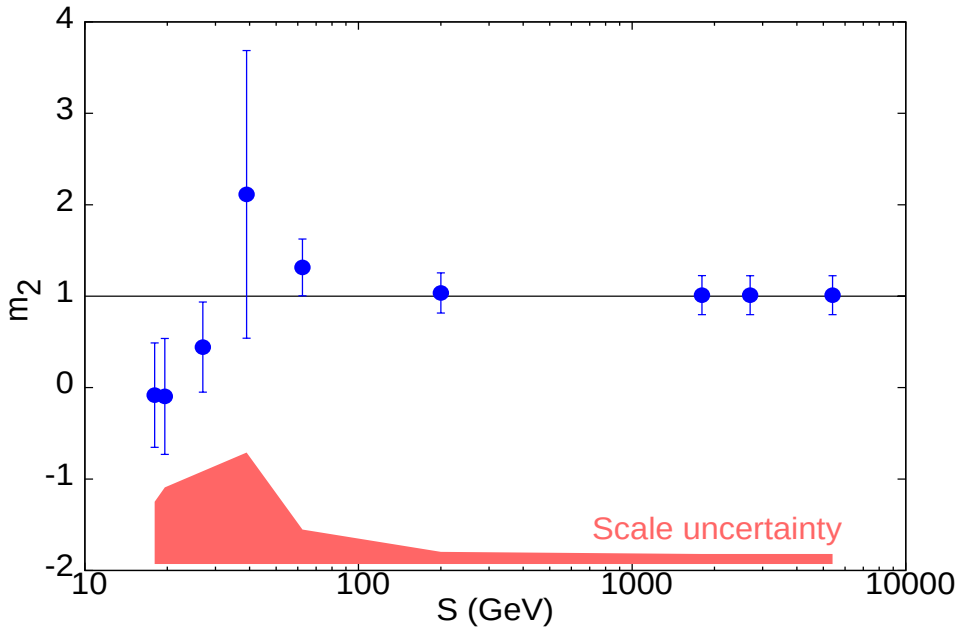
$\Delta P(\mu)$ can be obtained by integrating the result for χ_2.



Event shape distribution and lattice

Ratios of susceptibilities can be connected to the moments of baryon number distribution, which are calculable experimentally. e.g., $m_2 = \frac{T^2\chi^4(T,\mu_B)}{\chi^2(T,\mu_B)} = \kappa\sigma^2$, κ, σ^2 kurtosis and variance of event-by-event baryon distribution, if T, μ_B for the experimental event is known.

Gavai & Gupta, PL B 696 ('11) 459. Gupta et al, Science 332('11) 1525



Summary

- We presented results for susceptibilities of order 2 and 4 in QCD with 2 degenerate light flavors, for lattices with $a = 1/8T$.
- Comparison with earlier results from coarser lattices indicate that the cutoff error is controllable, though a continuum extrapolation is not possible at the moment.
- The results are in good agreement with results with p4 action at similar lattice spacing, and with perturbation theory.
- For $\chi_2^B(\mu)$, series summation and Pade to lowest orders are in agreement away from T^E.
- We suggest a new way of resummation of the series for quark number susceptibility in the critical region.

Thanks: S. Mogliacci, M.G. Mustafa, P. Petreczky, A. Vuorinen.