QCD thermodynamics with dynamical overlap fermions

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Motivation & Aims

- Staggered fermions
  - cheap, well studied
  - continuum results at physical quark masses
  - rooting: validity is still debated
  - taste breaking $\longrightarrow$ large $m_{\pi,\mathrm{rms}}$
- Wilson fermions
  - theoretically sound
  - no taste breaking
  - explicit chiral symmetry breaking

Motivation & Aims

- Chiral properties at finite T $\longrightarrow$ chiral fermions are needed
- Domain-wall fermions
  - exact chiral symmetry only in $L_5 \to \infty$ limit
- Overlap fermions
  - exact lattice chiral symmetry
- Aims of this study:
  - $a \to 0$ with dynamical overlap fermions
  - cross check of staggered fermions

Outline

Details of action

- Tree level Symanzik improved gauge action
- $N_f = 2$ overlap fermions

$$D_{\text{ov}} = \left(m_0 - \frac{m}{2}\right)\left(1 + \gamma_5 \operatorname{sgn}(H_W)\right) + m, \qquad H_W = \gamma_5 D_W(-m_0)$$

  - multi-shift inverter, Zolotarev rational approximation
  - lowest eigenvalues of H_W separately $\longleftarrow$ Krylov–Schur algorithm

- D_W Wilson kernel:
  - $-m_0 = -1.3$
  - 2 steps of HEX smearing, $\alpha_1 = 0.72, \alpha_2 = 0.60, \alpha_3 = 0.44$

Topology fixing

- HMC trajectories $\longrightarrow$ difficulties at topological sector boundaries
- fix topology:

$$S_E = \sum_x \left\{ \bar{\psi}_E(x) D_W(-m_0) \psi_E(x) + \phi^\dagger(x) [D_W(-m_0) + i m_B \gamma_5 \tau_3] \phi(x) \right\}$$

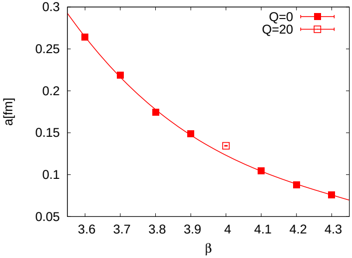
- equivalent to adding $\det \left(\frac{H_W^2(-m_0)}{H_W^2(-m_0) + m_B^2} \right)$ to $S_{g,\mathrm{eff.}}$

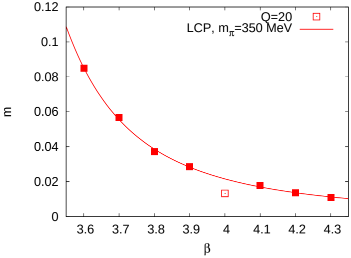
Fukaya et al., *Phys.Rev.* **D74** (2006) 094505

- $m_B = 0.54, \qquad -m_0 = -1.3$
- m_0 and m_B are fixed in lattice units $\longrightarrow$ infinitely large masses in the continuum limit
- in $V \to \infty$ limit physics is topology independent
- power-like corrections at finite V may arise

Line of constant physics

- Lattices:
  - $12^3 \cdot 24$ for $\beta = 3.6, 3.7, 3.8, 3.9$
  - $16^3 \cdot 32$ for $\beta = 4.0, 4.1$
  - $32^3 \cdot 32$ for $\beta = 4.2, 4.3$
- $m = 0.015 - 0.06$
- a is set using $w_0 = 0.1755\,\text{fm}$
- chiral symmetry $\longrightarrow$ $m_\pi^2 \propto m$
- m is set via $m_\pi \cdot w_0 = 0.312$ $\longrightarrow$ $m_\pi = 350\,\text{MeV}$



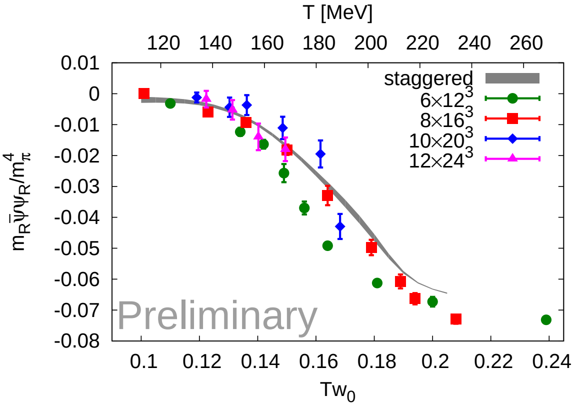


Staggered reference calculations

- Tree level Symanzik improved gauge action (same as with overlap)
- $N_f = 2$ staggered fermions
- 4 steps of stout smearing, $\rho = 0.125$
- LCP analogous to overlap
  - scale via w_0
  - quark mass via $m_\pi \cdot w_0 = 0.312$
  - 16 ensembles in the range $\beta = 3.8 - 4.1$
- $N_s/N_t = 2 \longrightarrow m_\pi \cdot L \approx 3.5 - 5$ in transition regime (same as overlap)
- $N_t = 6, 8, 10$ simulations

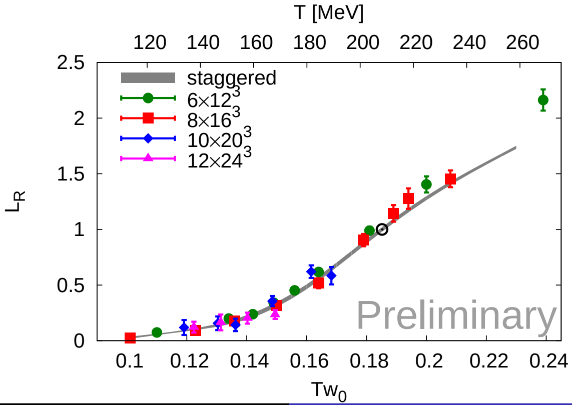
Chiral condensate

- $\bar{\psi}\psi = \frac{T}{V}\frac{\partial\left(\log Z\right)}{\partial m}$
- Renormalization: $\frac{m_R \bar{\psi}\psi_R}{m_\pi^4} = \frac{m\left(\bar{\psi}\psi - \bar{\psi}\psi_0\right)}{m_\pi^4}$



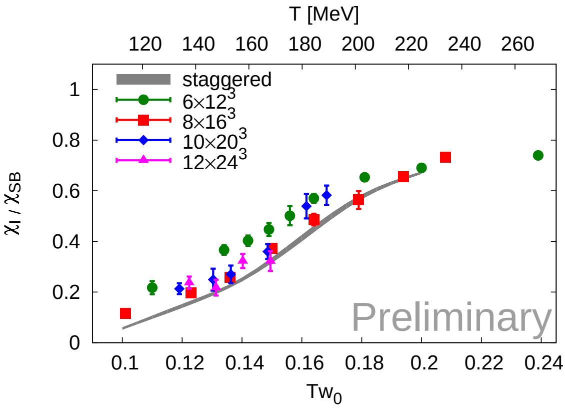
Polyakov loop

- L_0: multiplicative divergence of the form $\exp\left[F_0(\beta)/T\right]$
- renormalization condition: $L_R\,(T = 208\,\text{MeV}) = 1$
- $F_0(\beta) = \frac{1}{N_t}\log L$ such that (N_t, β) corresponds to $T = 208\,\text{MeV}$
- $L_R = L_0 \cdot \exp\left[-N_t \cdot F_0(\beta)\right]$



Isospin susceptibility

- $$\chi_\text{I} = \frac{T}{V}\frac{\partial^2(\log Z)}{\partial\mu_\text{I}^2}\bigg|_{\mu_\text{I}=0}$$
- $\mu_\text{u} = \mu_\text{I}/2, \quad \mu_\text{d} = -\mu_\text{I}/2$



Conclusions & outlook

- Conclusions
  - Not conclusive yet $\longrightarrow$ need more statistics
  - Continuum limit looks feasible
- Outlook
  - Collect more statistics
  - Larger volumes to check finite volume effects
  - Include strange quark, reach for lower pion mass

Stefan–Boltzmann limits of χ_{I}

N_t	4	6	8	10	12
$\xi = 2$ overlap	1.700	1.588	1.362	1.241	1.186
$\xi = \infty$ overlap	1.619	1.513	1.290	1.170	1.117
$\xi = \infty$ staggered	2.235	1.861	1.473	1.266	1.164
$\xi = \infty$ Wilson	4.168	2.258	1.521	1.265	1.161