Nucleon form factors with light Wilson quarks

Jeremy Green[a]

Michael Engelhardt[b] Stefan Krieg[c,d] Stefan Meinel[a]
John Negele[a] Andrew Pochinsky[a] Sergey Syritsyn[e]

[a]Center for Theoretical Physics, Massachusetts Institute of Technology

[b]Physics Department, New Mexico State University

[c]Bergische Universität Wuppertal

[d]IAS, Jülich Supercomputing Centre, Forschungszentrum Jülich

[e]Lawrence Berkeley National Laboratory

The 31st International Symposium on Lattice Field Theory
July 29–August 3, 2012

Dirac and Pauli form factors

Dirac and Pauli form factors:

$$\langle p(P', s')|\bar{q}\gamma^\mu q|p(P, s)\rangle = \bar{u}(p', s')\left(\gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu}\frac{\Delta_\nu}{2m_p}F_2^q(Q^2)\right)u(p, s),$$

where $\Delta = P' - P$, $Q^2 = -\Delta^2$.

- Isovector combination:

$$F_{1,2}^v = F_{1,2}^u - F_{1,2}^d = F_{1,2}^p - F_{1,2}^n,$$

  where $F_{1,2}^{p,n}$ are form factors of the electromagnetic current in a proton and in a neutron.
- Dirac and Pauli radii defined via slope at $Q^2 = 0$:

$$F_{1,2}(Q^2) = F_{1,2}(0)(1 - \tfrac{1}{6}r_{1,2}^2 Q^2 + O(Q^4));$$

  $F_2(0) = \kappa$, the anomalous magnetic moment.
- Proton charge radius, $(r_E^2)^p = (r_1^2)^p + \frac{3\kappa^p}{2m_p^2}$, has 7σ discrepancy between measurements from e–p interactions and from Lamb shift in muonic hydrogen.

High-precision study of excited states

- USQCD ensemble with $N_f = 2 + 1$ Wilson-clover quarks coupled to stout-smeared gauge fields.
- $a \approx 0.114$ fm, $32^3 \times 96$, $m_\pi \approx 317$ MeV $\longrightarrow m_\pi L = 5.9$
- Five source-sink separations: $T/a \in \{6, 8, 10, 12, 14\}$, $T \sim 0.7$–1.6 fm;
- Source tuned to optimize ground state overlap.
- ~ 24000 measurements yields reasonably precise results.
- Renormalization factors not yet computed, but this does not affect excited-states study.

Ground-state matrix elements from multiple T

- Standard ratio-plateau method: compute ratio

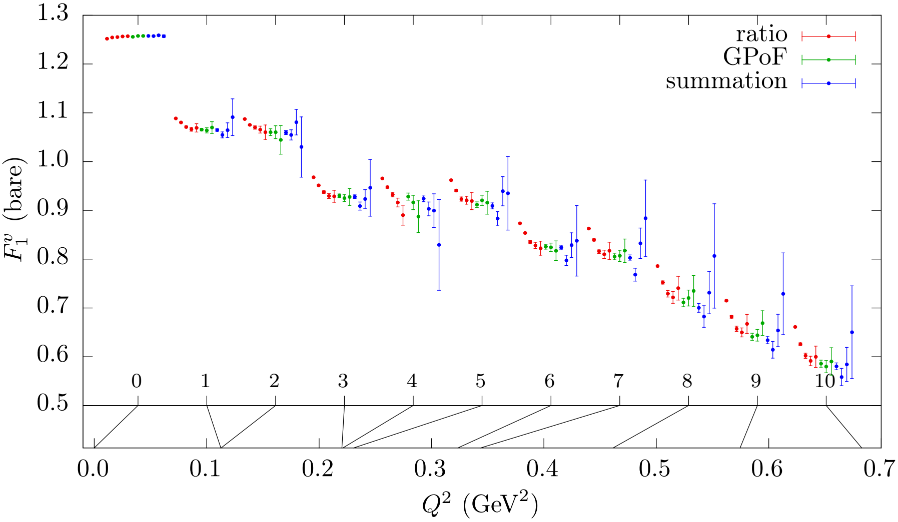
$$
\begin{aligned}
R(T,\tau) &= C_{3\text{pt}}(T,\tau)/C_{2\text{pt}}(T) \\
&= c_{00} + c_{10}e^{-\Delta E\tau} + c_{01}e^{-\Delta E(T-\tau)} + c_{11}e^{-\Delta ET} + \dots,
\end{aligned}
$$

  where c_{00} is the desired ground-state matrix element. Then average a fixed number of points around $\tau = T/2$, yielding asymptotic errors that fall off as $e^{-\Delta E_{10}T/2}$.
- Summation method (PoS(Lattice 2010) 147 [1011.1358]; *ibid.* 303 [1011.4393]): compute sums

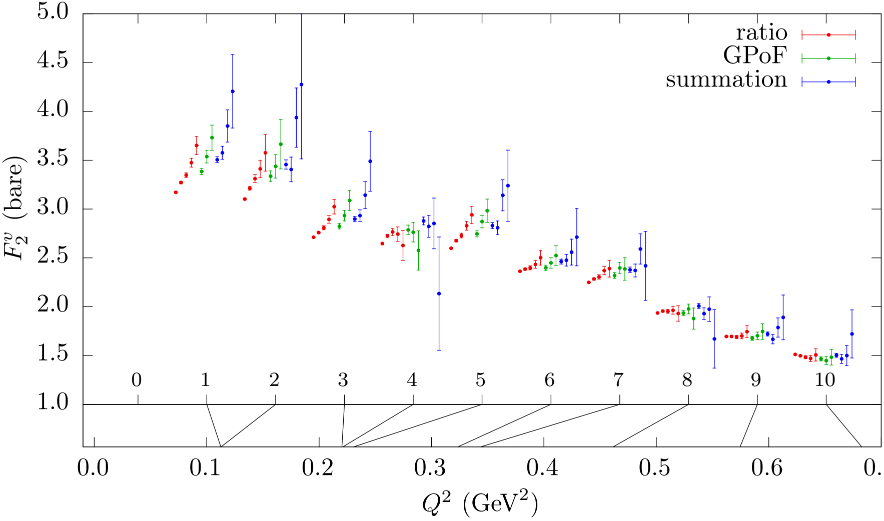
$$
S(T) = \sum_\tau R(T,\tau) = b + c_{00}T + dTe^{-\Delta ET} + \dots,
$$

  then find their slope, which gives c_{00} with errors that fall off as $Te^{-\Delta E_{10}T}$.
- Generalized pencil-of-function (GPoF) method (AIP Conf. Proc. **1374**, 621 [1010.0202]): recognize time-displaced operator $N^\tau(t) \equiv N(t+\tau)$ as linearly independent from $N(t)$. Use the variational method to find a linear combination of N and N^τ that eliminates the first excited state. Applying the ratio-plateau method yields the ground-state with errors $e^{-\Delta E_{20}T/2}$.

Isovector Dirac form factor $F_1^v(Q^2)$



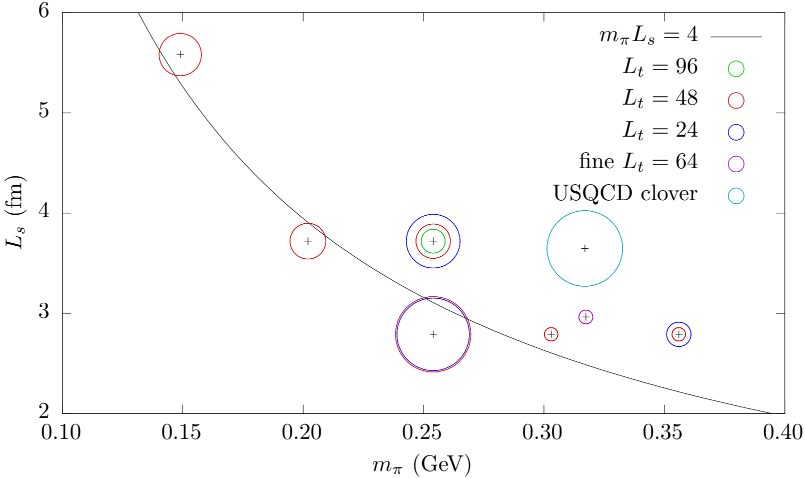
Isovector Pauli form factor $F_2^v(Q^2)$



BMW action and ensembles

- $N_f = 2 + 1$ tree-level clover-improved Wilson fermions coupled to double-HEX-smeared gauge fields.
- Pion mass ranging from 149 MeV to 356 MeV.
- Ten coarse lattices with $a = 0.116$ fm;
  one fine lattice with $a = 0.09$ fm.
- No disconnected diagrams, so we focus on isovector observables.
- Three source-sink separations for controlling excited-state contributions: $T \in \{0.9, 1.2, 1.4\}$ fm; use summation method for main results.

Ensembles



Areas of circles scale with number of measurements: largest is 24,000.

Chiral extrapolation

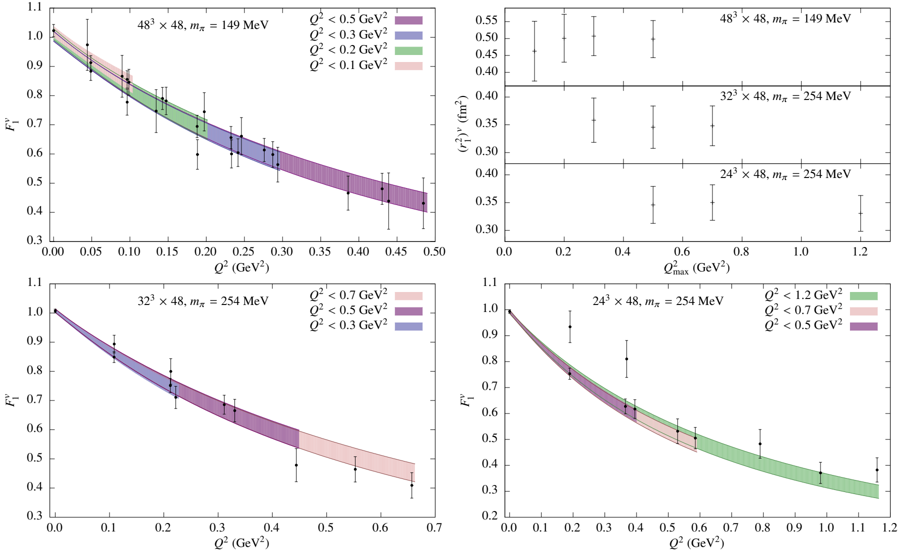
Use SU(2) heavy baryon ChPT, to order ϵ^3 in SSE. Inputs:

- F_π^0, pion decay constant
- Δ, delta-nucleon mass difference
- g_A^0, axial charge
- c_A, $\pi N \Delta$ coupling
- c_V, magnetic $\gamma N \Delta$ coupling

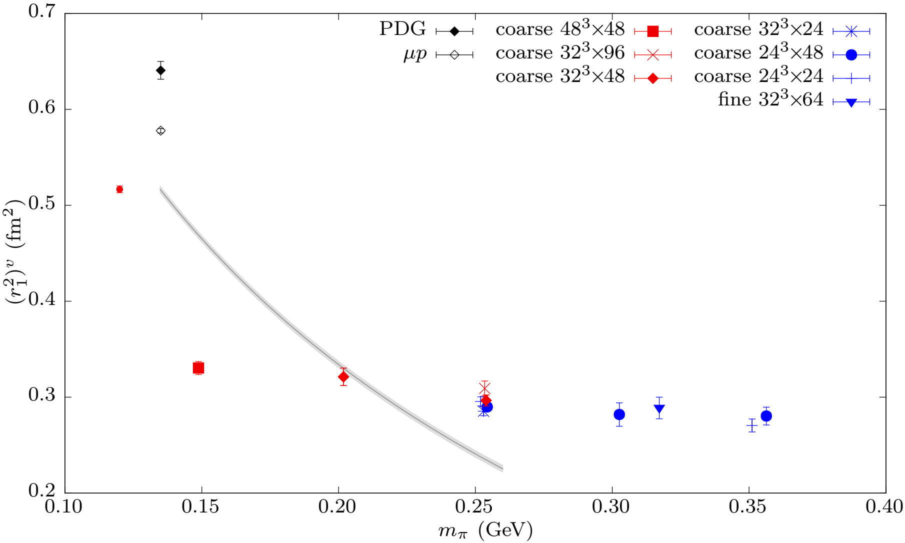
Fit parameters

- $(r_1^2)^v$: 1
- κ^v: 2
- $\kappa^v (r_2^2)^v$: 1

Dipole fitting to $F_1^v(Q^2)$

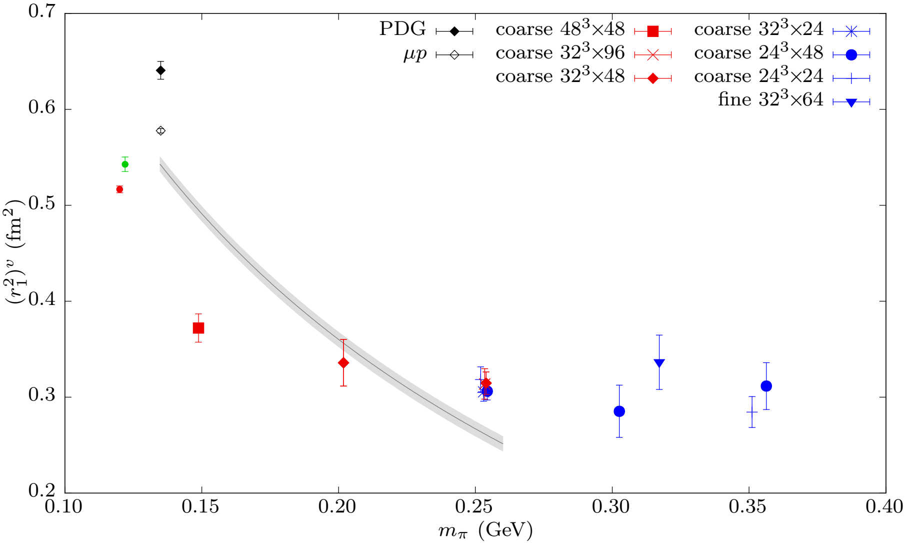


Isovector Dirac radius $(r_1^2)^v$



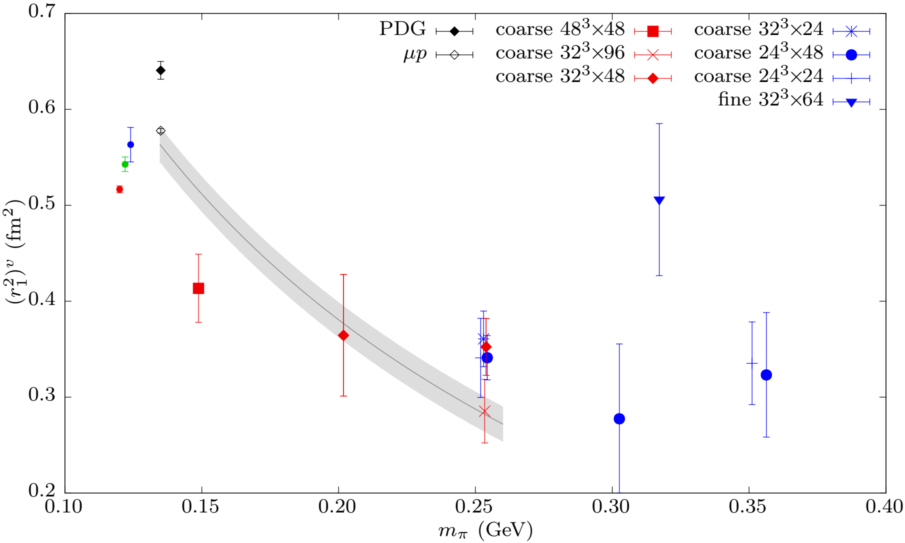
ratio method: shortest T

Isovector Dirac radius $(r_1^2)^v$



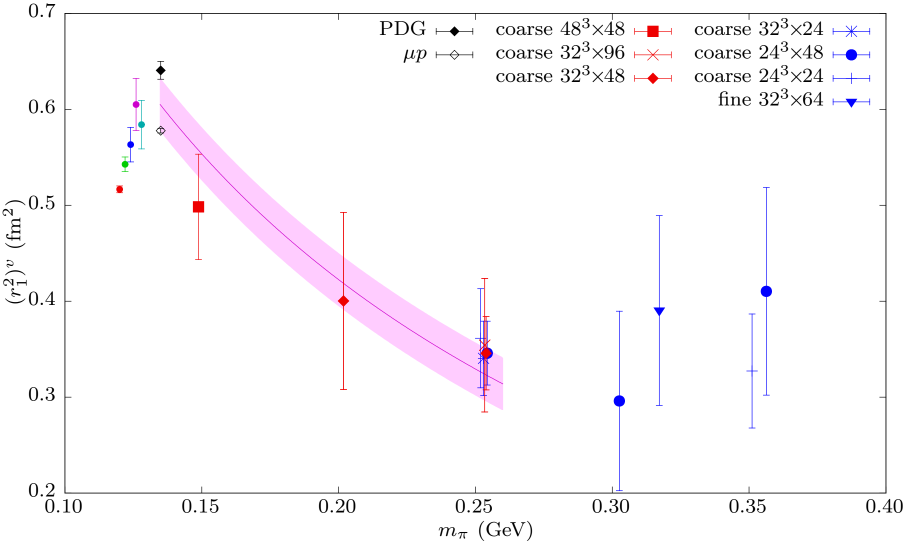
ratio method: middle T

Isovector Dirac radius $(r_1^2)^v$



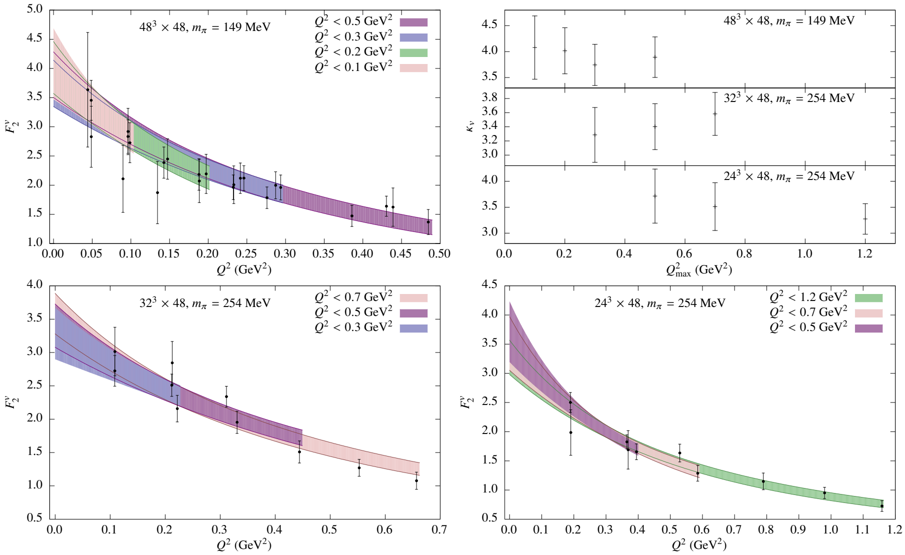
ratio method: largest T

Isovector Dirac radius $(r_1^2)^v$

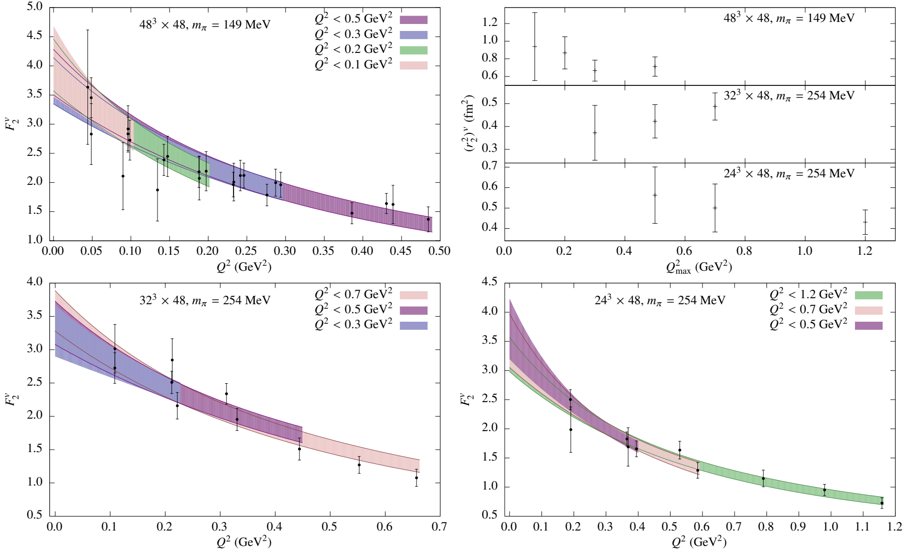


summation method

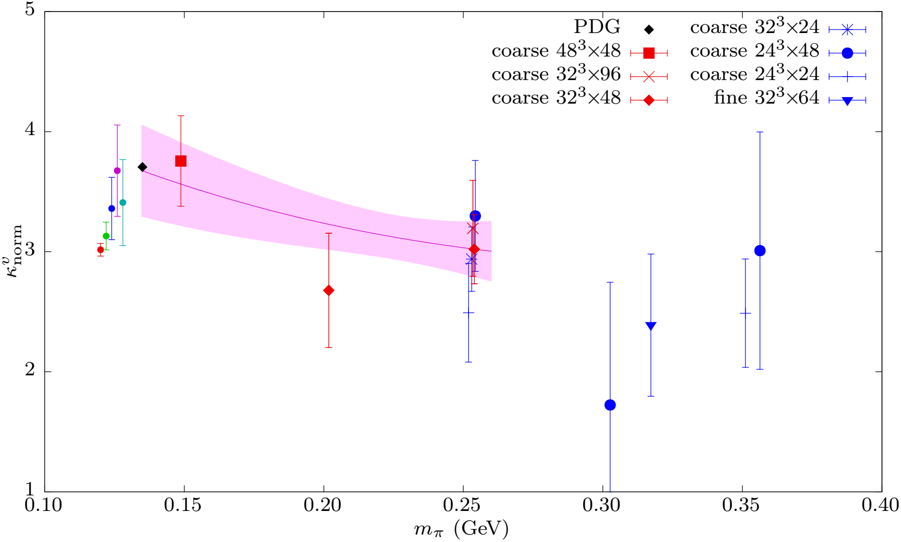
Dipole fitting to $F_2^v(Q^2)$



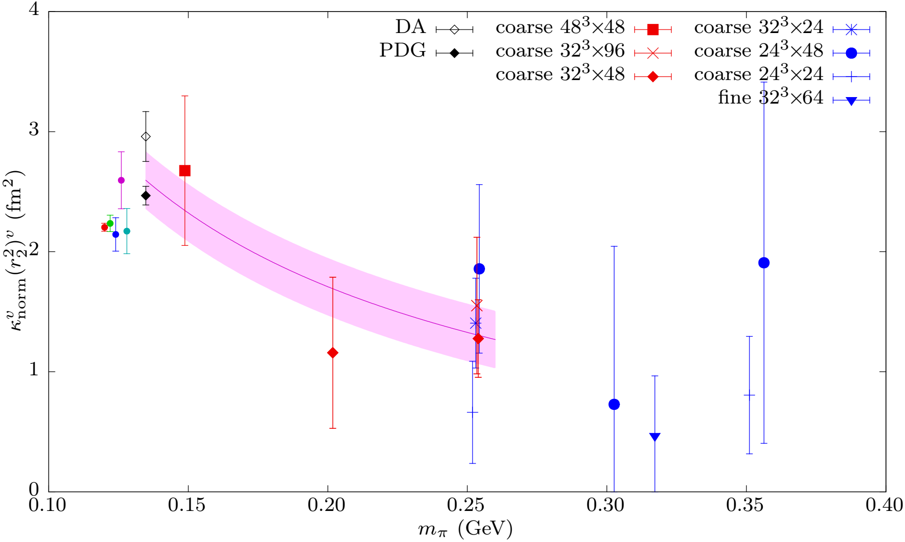
Dipole fitting to $F_2^v(Q^2)$



Isovector anomalous magnetic moment κ^v



Isovector Pauli radius $(r_2^2)^v$

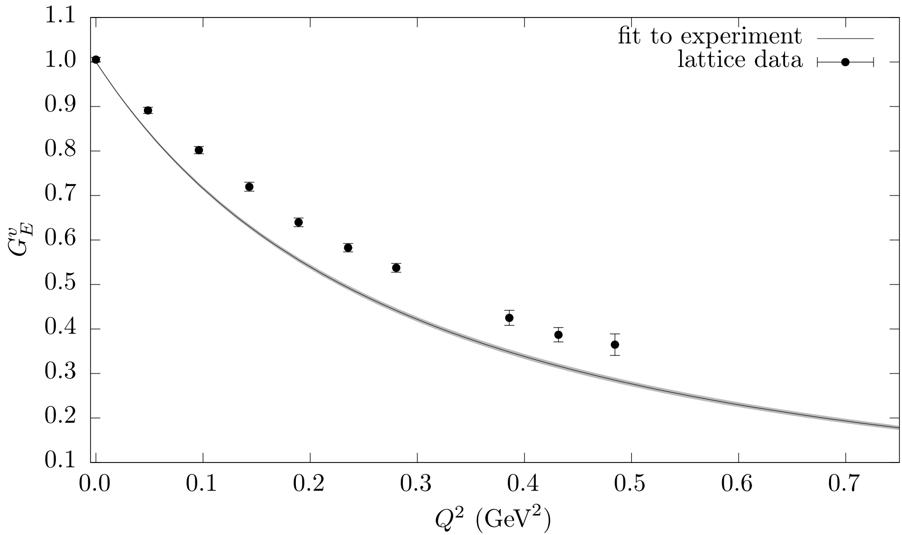


Sachs form factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{2m_N} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

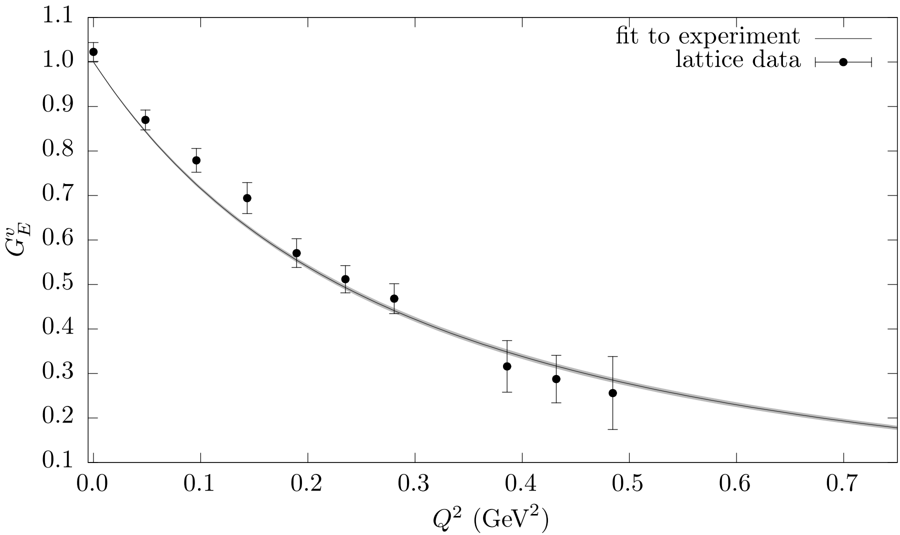
- Slopes at $Q^2 = 0$ give rms charge and magnetic radii.
- Compare:
  1. Isovector $G_E(Q^2)$, $G_M(Q^2)$ from lattice calculation.
  2. Parameterization of experimental data:
     W. M. Alberico, S. M. Bilenky, C. Giunti, K. M. Graczyk, "Electromagnetic form factors of the nucleon: new fit and analysis of uncertainties," Phys. Rev. C **79**, 065204 (2009).
     4 parameters for each of G_{Ep}, G_{Mp}, G_{Mn}; 2 parameters for G_{En}: determined from fit to experiment.

Electric form factor $G_E^v(Q^2)$



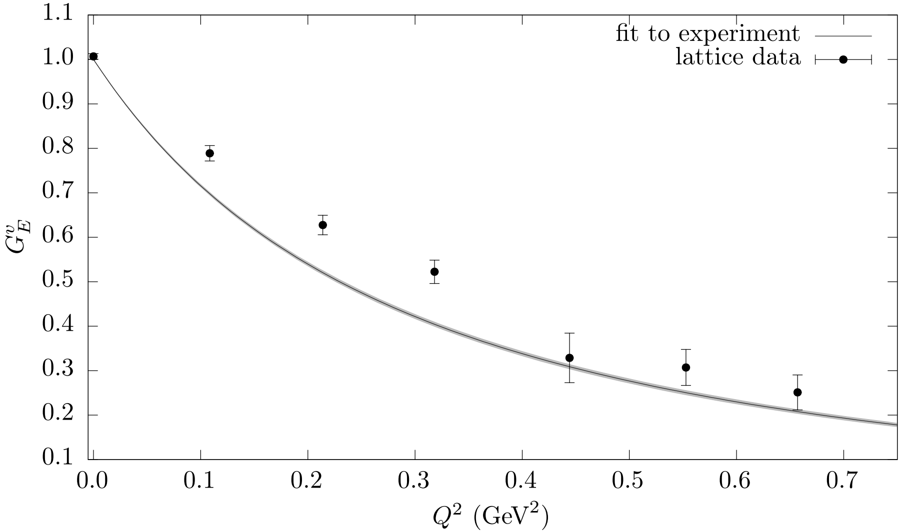
$m_\pi = 149$ MeV, ratio method, $T = 10a$

Electric form factor $G_E^v(Q^2)$



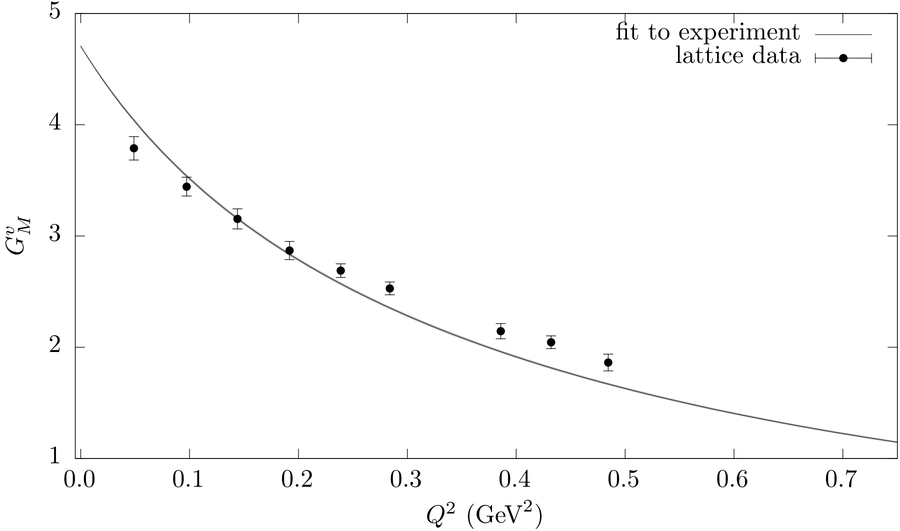
$m_\pi = 149$ MeV, summation method; $p = 0.64$

Electric form factor $G_E^v(Q^2)$



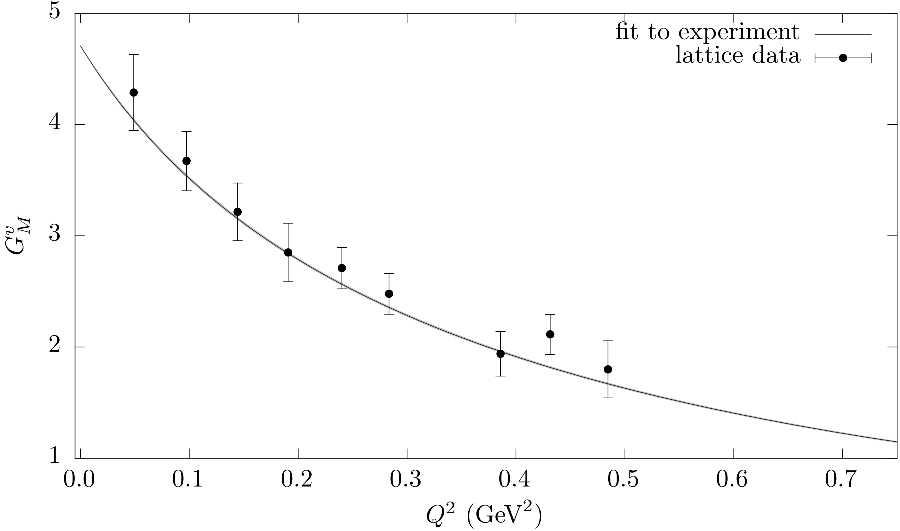
$m_\pi = 254$ MeV, summation method; $p = 3 \times 10^{-5}$

Magnetic form factor $G_M^v(Q^2)$



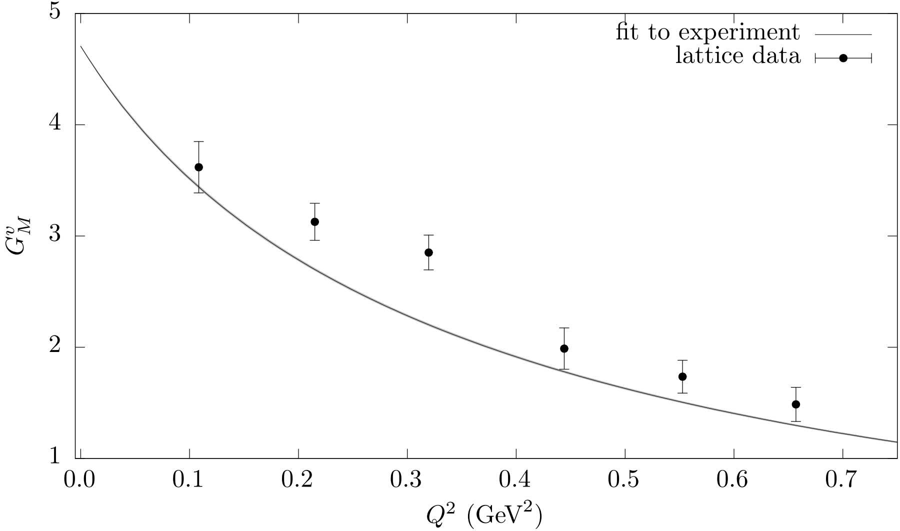
$m_\pi = 149$ MeV, ratio method, $T = 10a$; $p = 0.0006$

Magnetic form factor $G_M^v(Q^2)$



$m_\pi = 149$ MeV, summation method; $p = 0.81$

Magnetic form factor $G_M^v(Q^2)$



$m_\pi = 254$ MeV, summation method; $p = 0.0007$

Controlled study of finite L_s and L_t effects

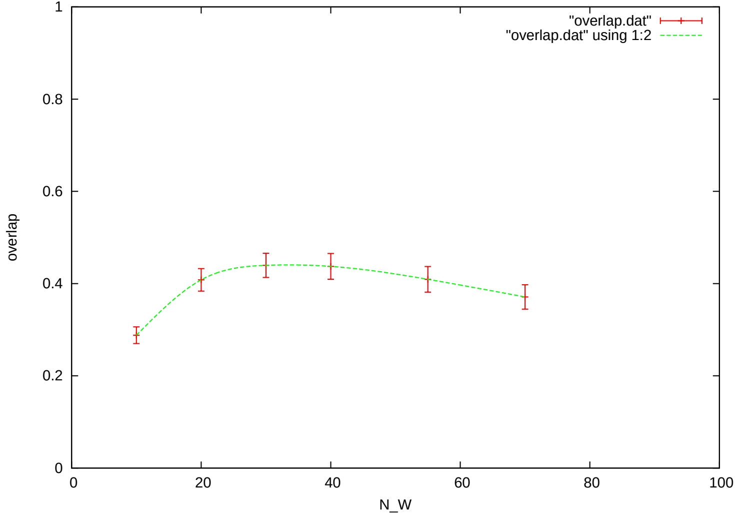
- Four lattice ensembles at $m_\pi \approx 250$ MeV with $a = 0.116$ fm that differ only in their volume: $32^3 \times 48$, $24^3 \times 48$, $32^3 \times 24$, and $24^3 \times 24$.
- $m_\pi L_s = 3.6$ and 4.8
- $m_\pi L_t = 3.6$ and 7.2
- Test "$m_\pi L = 4$" rule of thumb.
- Results:
    - Effects consistent with zero when comparing noisy summation data.
    - Shortest source-sink separation data suggest at $m_\pi L_s = 4$:
        - possible $\sim -5\%$ effect on κ^v and $(r_2^2)^v$
        - effect on $(r_1^2)^v$ is consistent with zero and less than 2%.
    - No effect seen for g_A.

Summary

- With both
  1. near-physical m_π
  2. reduced excited-state contributions

  good agreement is achieved with experiment for isovector vector form factors.
- High-precision calculations at $m_\pi \approx 317$ MeV corroborate the excited-state behavior seen in the noisier calculations at near-physical pion masses.
- Dedicated finite-L_s and L_t study at $m_\pi \approx 250$ MeV finds that $m_\pi L = 4$, effects are generally consistent with zero.

Source tuning, high-precision ensemble



Wuppertal smearing with $\alpha = 3$, $N = 35$ using APE-smeared ($A = 2.85$, $N = 25$) gauge links.

Other form factors

Axial and induced pseudoscalar form factors:

$$\langle p',\lambda'|\bar{q}\gamma^\mu\gamma_5 q|p,\lambda\rangle = \bar{u}(p',\lambda')\left(\gamma^\mu G_A^q(Q^2) + \frac{\Delta^\mu}{2m} G_P^q(Q^2)\right)\gamma_5 u(p,\lambda).$$

Generalized form factors of the quark energy-momentum operator

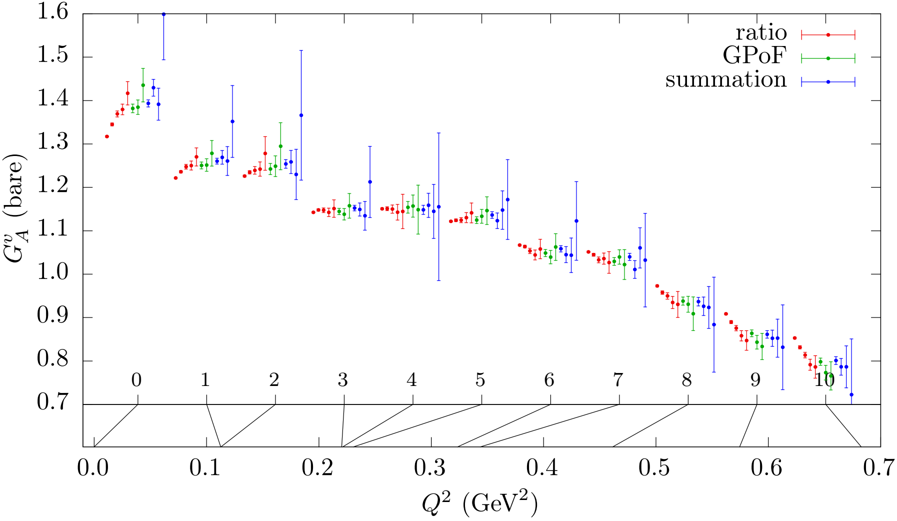
$$\langle p',\lambda'|\bar{q}\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}} q|p,\lambda\rangle = \bar{u}(p',\lambda')\left(\bar{p}^{\{\mu}\gamma^{\nu\}} A_{20}^q(Q^2) + \frac{i\bar{p}^{\{\mu}\sigma^{\mu\}\alpha}\Delta_\alpha}{2m} B_{20}^q(Q^2) + \frac{\Delta^{\{\mu}\Delta^{\nu\}}}{m} C_2^q(Q^2)\right) u(p,\lambda)$$

Source and sink momenta

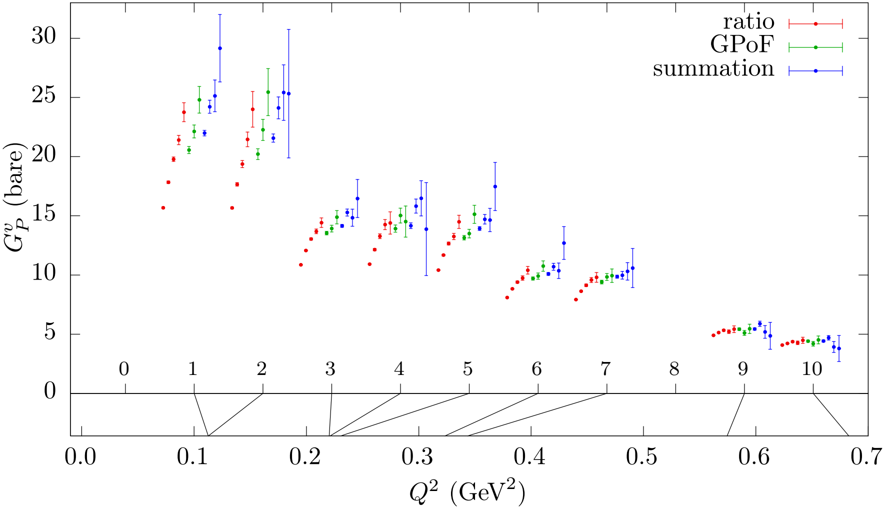
#	$\langle \vec{n}' \vert \vec{n} \rangle$
0	$\langle 0,0,0 \vert 0,0,0 \rangle$, $\langle -1,0,0 \vert -1,0,0 \rangle$
1	$\langle 0,0,0 \vert 1,0,0 \rangle$
2	$\langle -1,0,0 \vert -1,1,0 \rangle$
3	$\langle 0,0,0 \vert 1,1,0 \rangle$
4	$\langle -1,0,0 \vert -1,1,1 \rangle$
5	$\langle -1,0,0 \vert 0,1,0 \rangle$
6	$\langle 0,0,0 \vert 1,1,1 \rangle$
7	$\langle -1,0,0 \vert 0,1,1 \rangle$
8	$\langle -1,0,0 \vert 1,0,0 \rangle$
9	$\langle -1,0,0 \vert 1,1,0 \rangle$
10	$\langle -1,0,0 \vert 1,1,1 \rangle$

$\vec{p} = \frac{2\pi}{L_s}\vec{n}$

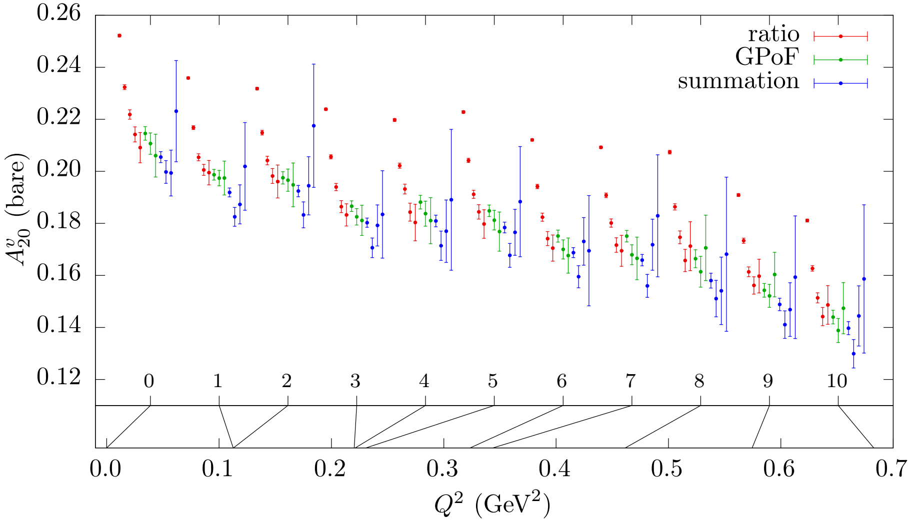
Isovector Axial form factor $G_A^v(Q^2)$



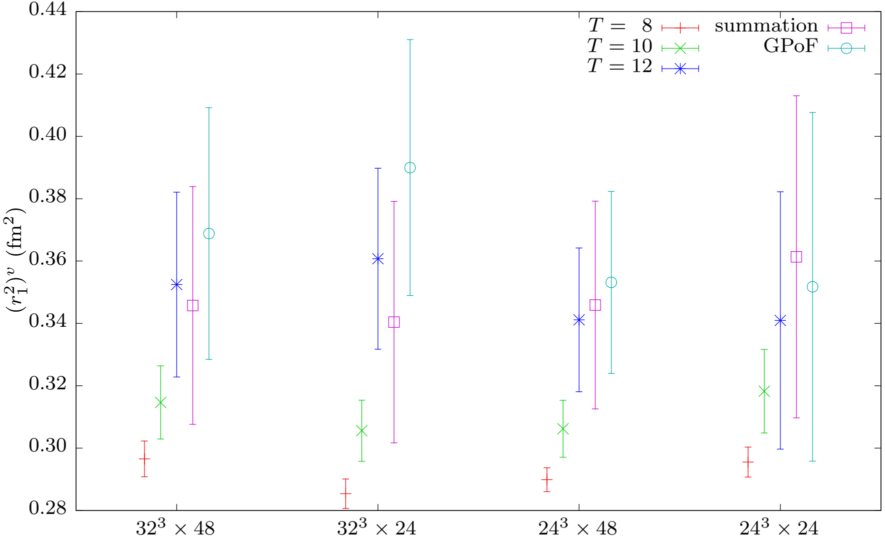
Isovector induced pseudoscalar form factor $G_P^v(Q^2)$



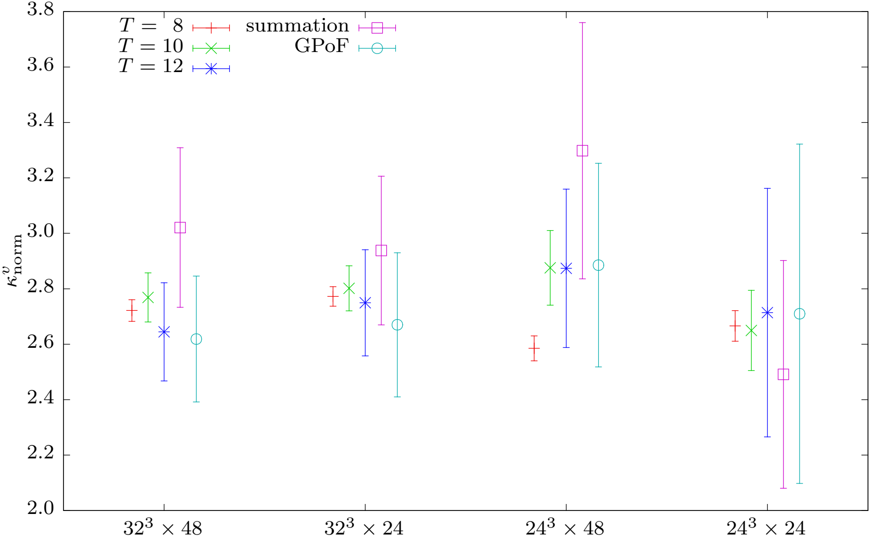
Isovector generalized form factor $A_{20}^v(Q^2)$



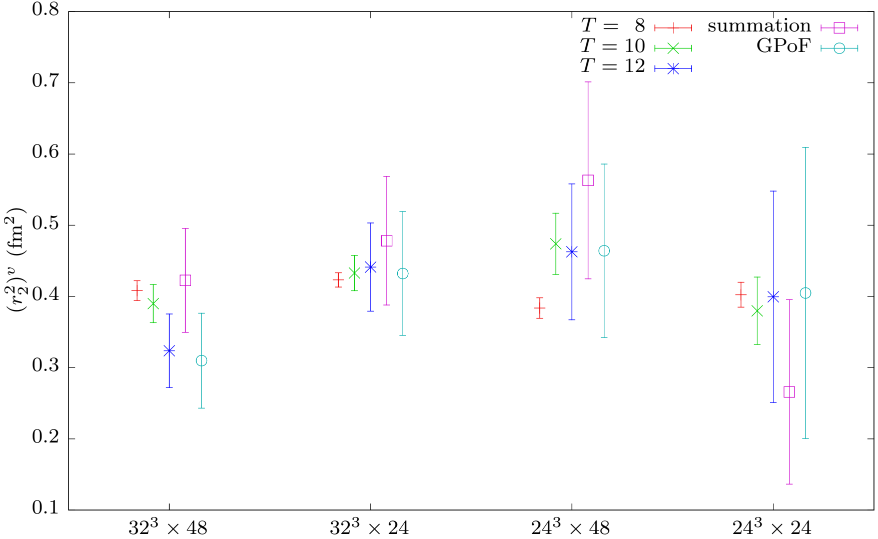
Isovector Dirac radius $(r_1^2)^v$



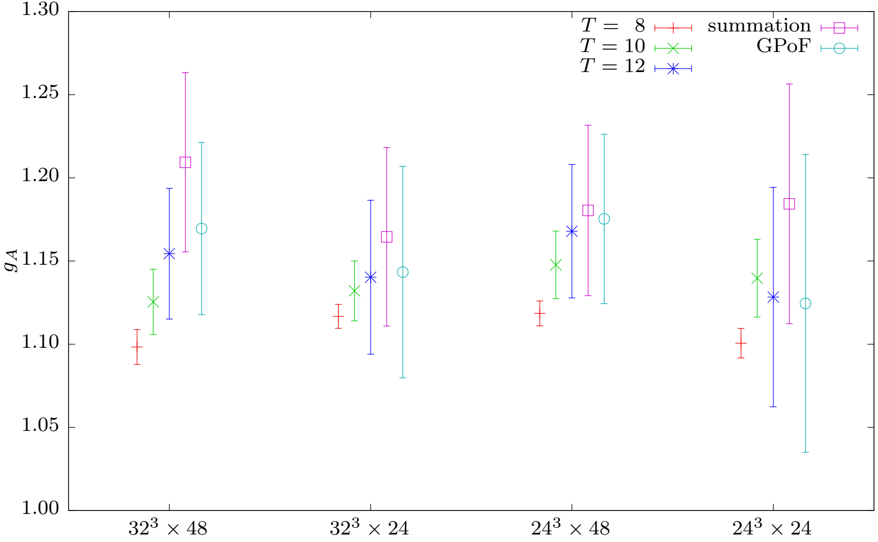
Isovector anomalous magnetic moment κ^v



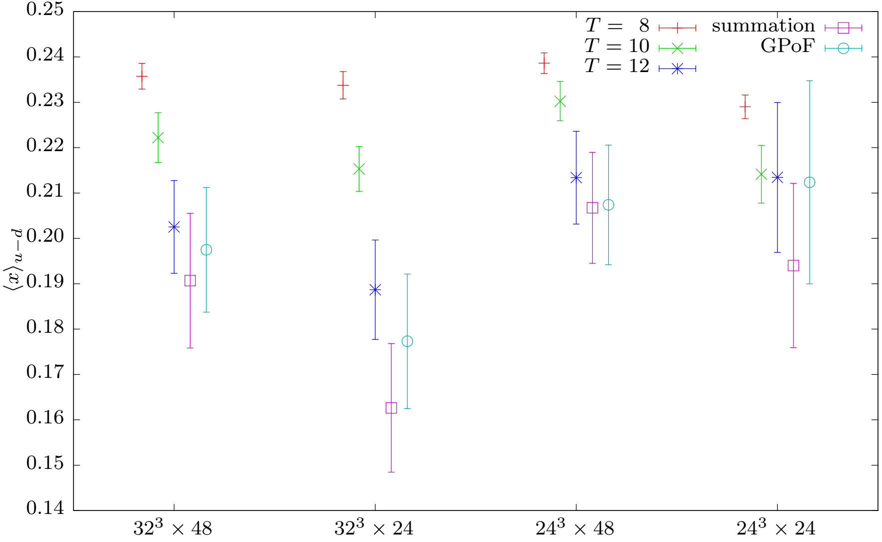
Isovector Pauli radius $(r_2^2)^v$



Axial charge g_A



Isovector quark momentum fraction $\langle x \rangle_{u-d}$

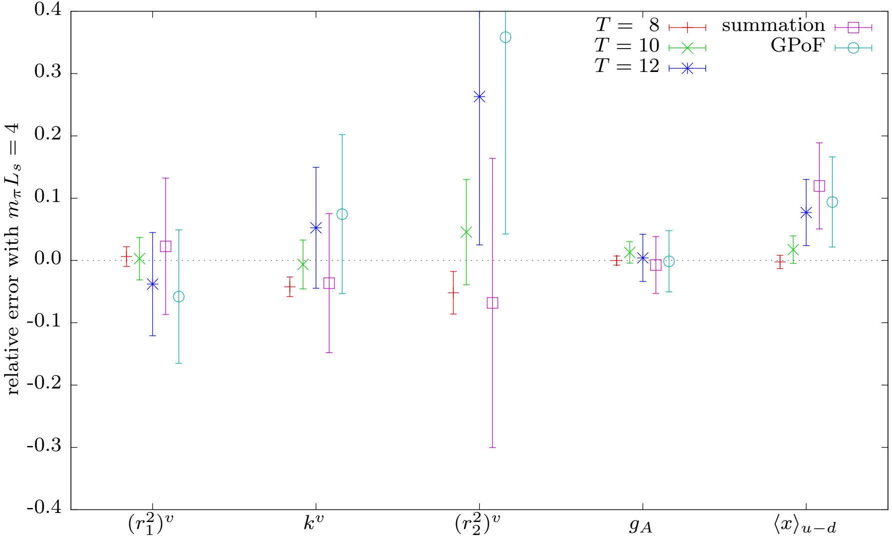


$L_s \to \infty$, $L_t \to \infty$ extrapolation

Fit $A + Be^{-m_\pi L_s} + Ce^{-m_\pi L_t}$ to summation data:

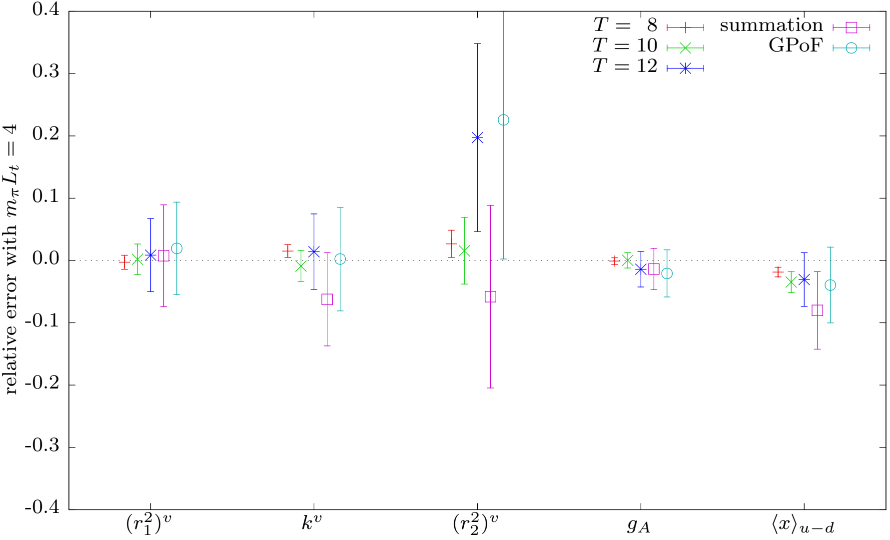
	A	Be^{-4}	Ce^{-4}
$(r_1^2)^v$	0.338(48)	0.008(38)	0.003(28)
κ^v	3.19(36)	−0.12(35)	−0.20(23)
$(r_2^2)^v$	0.476(98)	−0.032(106)	−0.028(68)
g_A	1.204(67)	−0.009(54)	−0.016(39)
$\langle x \rangle_{u-d}$	0.178(18)	0.021(14)	−0.014(10)

Relative error, $m_\pi L_s = 4$



Plotted: $e^{-4} B/A$

Relative error, $m_\pi L_t = 4$



Plotted: $e^{-4}C/A$