Sigma-terms and axial charge for hyperons and charmed baryons

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1 August, 2013

with

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The XXXI International Symposium on Lattice Field Theory, Mainz, 29 Jul- 3 Aug. 2013
Outline

- Hyperons and charmed baryons
- Axial charge for various baryons
- Sigma terms for various baryons
- Stochastic Method for computing connected three point functions
We use $N_f = 2 + 1 + 1$ twisted mass fermions with dynamical strange and charm quark masses fixed to their physical values.

$SU(4)_{flavour}$ representations

$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \bar{4}$

$\square \otimes \square \otimes \square = \square \oplus \square \oplus \square \oplus \bar{4}$

$\Rightarrow$ A 20-plet with $SU(3)$ octet and a 20-plet with $SU(3)$ decuplet.
Tune quark masses

Tuning of strange and charm quarks to their physical values  

B. Blossier et al arXiv:0709.4574

We use mixed action setup to simulate valance quarks

- For strange quark we match the kaon to its unitary mass
- For charm quark we use D-meson

\[
a^2 M^2_{PS}(a\mu_l, a\mu_h) = a_1(\mu_l+\mu_h) + a_2(\mu_l+\mu_h)^2 + a_3(\mu_l+\mu_h)^3 + a_4(\mu_l+\mu_h)(\mu_l-\mu_h)^2
\]

using linear fit

\[
a^2 M^2_{PS}(a\mu_s) = c_1 + c_2 a\mu_s, \ aM_{PS}(a\mu_c) = d_1 + d_2 a\mu_s
\]
Masses of Hyperons and Charmed baryons

Two-point Correlators

\[ C(t) = \sum_{\vec{x}} \langle \Omega | \Gamma_0 \chi(\vec{x}, t) \bar{\chi}(\vec{0}, 0) | \Omega \rangle \]

Effective mass

\[ m_{\text{eff}}(t) = \log \left( \frac{C(t)}{C(t+1)} \right) \approx m + \log \left( \frac{1+c_1 e^{-\Delta_1 t}}{1+c_1 e^{-\Delta_1 (t+1)}} \right) \]

\[ \xrightarrow{t \to \infty} m \]

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Content</th>
<th>Intepolating field</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Xi^0)</td>
<td>uss</td>
<td>(\epsilon_{abc} (s^T_a C \gamma_5 u_b) s_c)</td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td>uds</td>
<td>(\frac{1}{\sqrt{6}} \epsilon_{abc} ((u^T_a C \gamma_5 s_b) d_c + (d^T_a C \gamma_5 s_b) u_c))</td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>uds</td>
<td>(\frac{1}{\sqrt{2}} \epsilon_{abc} ((u^T_a C \gamma_5 s_b) d_c + (d^T_a C \gamma_5 s_b) u_c))</td>
</tr>
<tr>
<td>(\Omega_{cc}^+)</td>
<td>scc</td>
<td>(\epsilon_{abc} (c^T_a C \gamma_5 s_b) c_c)</td>
</tr>
<tr>
<td>(\Xi_{cc}^{++})</td>
<td>ucc</td>
<td>(\epsilon_{abc} (c^T_a C \gamma_5 u_b) c_c)</td>
</tr>
<tr>
<td>(\Xi_c^{*+})</td>
<td>usc</td>
<td>(\epsilon_{abc} (s^T_a C \gamma_5 u_b) c_c)</td>
</tr>
</tbody>
</table>

\(I\) and \(I_z\) values:

- \(\Xi^0\) (uss): \(I = 1/2, I_z = +1/2\)
- \(\Sigma^0\) (uds): \(I = 1, I_z = 0\)
- \(\Lambda\) (uds): \(I = 0, I_z = 0\)
- \(\Omega_{cc}^+\) (scc): \(I = 0, I_z = 0\)
- \(\Xi_{cc}^{++}\) (ucc): \(I = 1/2, I_z = +1/2\)
- \(\Xi_c^{*+}\) (usc): \(I = 1/2, I_z = +1/2\)
Lattice computation of matrix elements

Evaluation of three-point functions

\[ G^\mu(t_2, t_1; \vec{p}', \vec{p}; \Gamma) = \sum_{\vec{x}_2, \vec{x}_1} \langle \Omega | \Gamma^\beta_\alpha \chi_\alpha(\vec{x}_2, t_2) \mathcal{O}^\mu(\vec{x}_1, t_1) \bar{\chi}_\beta(\vec{0}, 0) | \Omega \rangle e^{-i\vec{x}_2 \cdot \vec{p}'} e^{+ix_1 \cdot (\vec{p}' - \vec{p})} \]

In the non-relativistic basis the projectors are

\[ \Gamma_0 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \Gamma_i = \frac{1}{2} \begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix} \]

We will consider the axial charge and sigma-terms
Consider the following ratio

\[ R^\mu(\Gamma, \vec{q}, t_2, t_1) = \frac{G^\mu(\Gamma, \vec{q}, t_1)}{G(\vec{0}, t_2)} \sqrt{\frac{G(\vec{p}, t_2 - t_1)G(\vec{0}, t_1)G(\vec{0}, t_2)}{G(\vec{0}, t_2 - t_1)G(\vec{p}, t_1)G(\vec{p}, t_2)}} \]

For sufficiently large separations \( t_2 - t_1 \) and \( t_1 \) this ratio becomes time-independent (plateau)

\[ \lim_{t_2-t_1 \gg 0} \lim_{t_1 \gg 0} R^\mu(\Gamma, \vec{q}, t_2, t_1) \longrightarrow \Pi^\mu(\Gamma, \vec{q}) \]
Axial charge for various hadrons

Axial-Vector matrix element decomposition

\[ A^3_\mu \equiv \bar{\psi}(x)\gamma_\mu\gamma_5 \frac{\tau_3}{2}\psi(x) \implies \]

\[ \langle N(p')|A^3_\mu|N(p)\rangle = \bar{u}_N(p') \frac{1}{2} \left[ G_A(Q^2)\gamma_\mu\gamma_5 + \frac{q_\mu\gamma_5}{2m_N}G_p(Q^2) \right] u_N(p) \implies G_A(Q^2 = 0) = g_A \]

Evaluation of connected diagrams using the sequential propagator

**Two approaches**

**Fixed sink method**

- Advantage: Calculate all Operators for a specific particle.
- Disadvantage: Fix the particle state and the projector.

**Fixed insertion method** (extract axial charge for all baryons)

- Advantage: Calculate any particle state for any projector.
- Disadvantage: Fix the insertion operator.
Axial charge for hyperons

If exact SU(3) flavor symmetry: \( W. Lin \) and \( K. Orginos, \) PRD 79, 034507 (2009)

\[
\begin{align*}
g_N^A &= F + D, \quad g_\Sigma^A = 2F, \quad g_\Xi^A = -D + F \quad \implies \quad g_N^A - g_\Sigma^A + g_\Xi^A = 0 \\
\text{Probe deviation:} \quad \delta_{SU(3)} &= g_N^A - g_\Sigma^A + g_\Xi^A \quad \text{versus} \quad x = \frac{m_K^2 - m_\pi^2}{4\pi^2 f_\pi^2}
\end{align*}
\]

Breaking \( \sim x^2 \) leads to about 15\% at the physical point \( x_{phy} = 0.33 \)

Study of SU(3) breaking for the decuplet
Axial charge for various hadrons

Axial charge of \((\Xi_{c}^{*+}(usc))\) and \((\Omega_{cc}^{*+}(scc))\)

Results and connection to phenomenology

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Another example using the fixed insertion are the sigma-terms (this requires a new set of inversions)

- From light sector we can extract \( \sigma_{\pi N} = m_l \langle N | \bar{u}u + \bar{d}d | N \rangle \)
- Similarly for the strange quark \( \sigma_s = m_s \langle N | \bar{s}s | N \rangle \)
- Using Feynman-Hellman theorem \( \sigma_{\pi N} = m_l \frac{\partial m_N}{\partial m_l} \)

\( N_f = 2 + 1 + 1 \) twisted mass fermions, \( a = 0.082 \text{ fm} \) and \( m_\pi \sim 372 \text{ MeV} \)
Sigma terms for hyperons and charmed baryons

\[ \Omega^- \]
\[ \Omega^{*0}_{cc} \]
\[ \Omega^{*+}_{cc} \]
\[ \Omega^{++}_{ccc} \]
\[ \Omega^{*0}_c \]

\( t_1 = 5 \)
\( t_1 = 7 \)
\( t_1 = 9 \)
In order **NOT** to require new set of inversions for each operator we investigate a stochastic method.

**Fixed sink method**
- Advantage: Calculate all Operators for a specific particle.
- Disadvantage: Fix the particle state and the projector.

**Fixed insertion method**
- Advantage: Calculate any particle state for any projector.
- Disadvantage: Fix the insertion operator.

**Stochastic method**
- Advantage: Calculate any operator for any particle state and any projector.
- Disadvantage: Introduces stochastic error.
Formulation of stochastic Method

- Noise vector using $Z(4)$ random numbers
  
  \[
  \frac{1}{N_r} \sum_{r=1}^{N_r \to \infty} \xi_r(x)^a_\mu \to 0 , \quad \frac{1}{N_r} \sum_{r=1}^{N_r \to \infty} \xi_r(x)^a_\mu \xi_r^*(y)^b_\nu \to \delta(x - y)\delta_{\mu\nu}\delta^{ab}
  \]

- Reconstruction of all-to-all propagator
  
  \[
  \phi_r(x)^a_\mu = G(x; y)^{ab}_{\mu\nu} \xi_r(y)^b_\nu , \quad G(x; y)^{ab}_{\mu\nu} = \frac{1}{N_r} \sum_{r=1}^{N_r \to \infty} \phi_r(x)^a_\mu \xi_r^*(y)^b_\nu
  \]

- Decomposition of double sum to two single sums
  
  \[
  \sum_{\vec{y}} \sum_{\vec{x}} e^{-i\vec{p}' \cdot \vec{x}} e^{-i\vec{p} \cdot \vec{y}} G(x; y) \Gamma G(y; 0) \longrightarrow
  \]
  
  \[
  \frac{1}{N_r} \sum_{r=1}^{N_r} \left( \sum_{\vec{x}} e^{-i\vec{p}' \cdot \vec{x}} \phi_r(x) \right) \left( \sum_{\vec{y}} e^{-i\vec{p} \cdot \vec{y}} \xi_r^*(y) \Gamma G(y; 0) \right)
  \]
Electromagnetic Form Factor

\[ \langle N(\vec{p})|j_\mu^3|N(\vec{p})\rangle = \bar{u}_N(\vec{p}) \gamma_\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N} F_2(Q^2) \] \[ u_N(\vec{p}) \]

Axial Form Factor

\[ \langle N(\vec{p})|A_\mu^3|N(\vec{p})\rangle = \bar{u}_N(\vec{p}) \frac{1}{2} \left[ G_A(Q^2) \gamma_\mu \gamma_5 + \frac{q_\mu q_5}{2m_N} G_P(Q^2) \right] u_N(\vec{p}) \]

- Fully diluted (spin, color) noise vectors \( \Rightarrow \) for \( N_r = 1 \) we need 12 inversions
- For the fixed sink method we need also 12 inversions for each sequential source
- As you increase the statistics you need less noise vectors
One Derivative Form Factors

\[ O_{V}^{\mu\nu} \equiv \bar{\psi} i \gamma^{\left\{ \mu \right\} \bar{D}^{\nu} \rho} \frac{\tau^{3}}{2} \psi \rightarrow \langle N(p') | O_{V}^{\mu\nu} | N(p) \rangle = \bar{u}_{N}(p') \frac{1}{2} \left[ A_{20}(Q^{2}) \gamma^{\left\{ \mu P \right\} \rho} + B_{20}(Q^{2}) \frac{\sigma^{\left\{ \mu Q_{\alpha} P^{\nu} \right\} \rho}}{2m} + C_{20}(Q^{2}) \frac{1}{m} q^{\left\{ \mu q \right\} \rho} \right] u_{N}(p) \]

Stochastic method shows the same behavior for many operators.
Exploit full power of stochastic method for Axial form factor

\[ G_A^{u-d} = 2 \]
\[ N_r = 4 \]
\[ N_r = 6 \]
\[ \text{Fixed Sink} \]

\[ G_P^{u-d} \]

\[ Q^2(\text{GeV}) \]

<table>
<thead>
<tr>
<th>Projectors</th>
<th>States</th>
<th># ( N_r )</th>
<th># Inversions</th>
<th>( \text{Err(Stoch)}/\text{Err(Fixed Sink)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Sink</td>
<td>( \sum_k \Gamma_k )</td>
<td>( p )</td>
<td>-</td>
<td>( 24 \times 24 )</td>
</tr>
<tr>
<td>Stochastic</td>
<td>( \sum_k \Gamma_k )</td>
<td>( p )</td>
<td>2</td>
<td>( 2 \times 24 )</td>
</tr>
<tr>
<td>Stochastic</td>
<td>( \Gamma_1, \Gamma_2, \Gamma_3 )</td>
<td>( (p + n)/2 )</td>
<td>2</td>
<td>( 2 \times 24 )</td>
</tr>
</tbody>
</table>
The combined error (gauge and stochastic) drops as $V^{-1/2}$ for large volumes. C. Alexandrou et al arXiv:1302.2608

→ For larger volumes you may need less stochastic vectors.
Summary

- Predictions for axial charge and sigma-terms for Hyperons and charmed baryons
- Stochastic Method can be an alternative method to compute connected three point functions
- Stochastic Method utilizes the advantages of fixed sink and insertion method
- We will use this method to evaluate many hadronic elements.