



Electromagnetic Structure of the $\Lambda(1405)$

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Outline

Introduction & Techniques

Electric Form Factors

Magnetic Form Factors

Conclusion



• The $\Lambda(1405)$ is the lowest-lying odd-parity state of the Λ baryon.

The A(1405)

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- It has a mass of $1405.1^{+1.3}_{-1.0}$ MeV.
 - This is lower than the lowest odd-parity nucleon state (N(1535)), even though it has a valence strange quark.

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- It has a mass of $1405.1^{+1.3}_{-1.0}$ MeV.
 - This is lower than the lowest odd-parity nucleon state (N(1535)), even though it has a valence strange quark.
- We now understand this as a consequence of its flavour-singlet structure.





Our recent work has successfully isolated three low-lying states.

BM, W. Kamleh, D. B. Leinweber, M. S. Mahbub, Phys. Rev. Lett. 108, 112001 (2012)

 An extrapolation of the trend of the lowest state reproduces the mass of the Λ(1405).

The $\Lambda(1405)$ and Lattice QCD



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- An extrapolation of the trend of the lowest state reproduces the mass of the Λ(1405).
- Subsequent studies have confirmed these results.

G. P. Engel, C. B. Lang, A. Schäfer, Phys. Rev. D 87, 034502 (2013)

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The $\Lambda(1405)$ and Lattice QCD



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• Our current insertion ($t_{SST} = 21$) is 5 time slices after the source ($t_{src} = 16$).





We are using the PACS-CS (2+1)-flavour ensembles, available through the ILDG.

S. Aoki et al (PACS-CS Collaboration), Phys. Rev. D 79, 034503 (2009)

- Lattice size of $32^3 \times 64$ with $\beta = 1.90$.
- 5 pion masses, ranging from 640 MeV down to 156 MeV.
- Single strange quark mass, with $\kappa_s = 0.13640$.
 - $\circ~$ We partially quench by using $\kappa_s=0.13665$ for the valence strange quarks to reproduce the physical kaon mass.
- We consider both the Sommer and PACS-CS schemes to set the scale.

By using multiple operators, we can isolate and analyse individual energy eigenstates:

Construct the correlation matrix

$$\mathcal{G}_{ij}(\mathbf{p};t) = \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} raket{\Omega|\chi_i(x)\,\overline{\chi}_j(0)|\Omega}$$
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for some set $\{\chi_i\}$ operators that couple to the states of interest.

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• Note: all correlation functions have an implicit trace over the spinor indices with some Dirac matrix:

$$G \equiv \operatorname{tr}(\Gamma G)$$

Solve for the left, v^α(**p**), and right, u^α(**p**), generalised eigenvectors of G(**p**; t + δt) and G(**p**; t):

$$G(\mathbf{p}; t + \delta t) \mathbf{u}^{\alpha}(\mathbf{p}) = e^{-E_{\alpha}(\mathbf{p}) \Delta t} G(\mathbf{p}; t) \mathbf{u}^{\alpha}(\mathbf{p})$$
$$\mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p}) G(\mathbf{p}; t + \delta t) = e^{-E_{\alpha}(\mathbf{p}) \Delta t} \mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p}) G(\mathbf{p}; t)$$

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 These eigenvectors identify "ideal" combinations of the original operators that perfectly isolate individual energy eigenstates at momentum p:

$$\phi^{\alpha} = \mathbf{v}_{i}^{\alpha}(\mathbf{p}) \chi_{i} \qquad \overline{\phi}^{\alpha} = u_{i}^{\alpha}(\mathbf{p}) \overline{\chi}_{i}$$

Eigenstate-Projected Correlation Functions

• Using these "perfect" operators, we can extract correlation functions for these energy eigenstates using

$$\begin{split} \mathcal{G}_{\alpha}(\mathbf{p};t) &= \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} \left\langle \Omega | \phi^{\alpha}(x) \,\overline{\phi}^{\alpha}(0) | \Omega \right\rangle \\ &= \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} \left\langle \Omega | v_{i}^{\alpha}(\mathbf{p}) \,\chi_{i}(x) \,\overline{\chi}_{j}(0) \,u_{j}^{\alpha}(\mathbf{p}) | \Omega \right\rangle \\ &= \mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p}) \, \mathcal{G}(\mathbf{p};t) \,\mathbf{u}^{\alpha}(\mathbf{p}) \end{split}$$

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- We have a lot of flexibility in the operators we choose.
 - $\circ~$ Only requirement is that they couple to the states of interest.
- However:
 - $\circ\;$ too few operators and the states won't be sufficiently isolated, and
 - insufficiently independent operators and the matrix will be too ill-conditioned to solve for the eigenvectors.

There are a number of operators that have the correct quantum numbers to couple to the Λ channel. We use

the flavour-octet operators

$$\chi_{1}^{8} = \frac{1}{\sqrt{6}} \varepsilon^{abc} (2(u^{a}C\gamma_{5}d^{b})s^{c} + (u^{a}C\gamma_{5}s^{b})d^{c} - (d^{a}C\gamma_{5}s^{b})u^{c})$$

$$\chi_{2}^{8} = \frac{1}{\sqrt{6}} \varepsilon^{abc} (2(u^{a}Cd^{b})\gamma_{5}s^{c} + (u^{a}Cs^{b})\gamma_{5}d^{c} - (d^{a}Cs^{b})\gamma_{5}u^{c})$$

• the flavour-singlet operator

$$\chi^1 = 2\varepsilon^{abc}((u^a C\gamma_5 d^b)s^c - (u^a C\gamma_5 s^b)d^c + (d^a C\gamma_5 s^b)u^c)$$

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- These results use 16 and 100 sweeps.
 - $\circ~$ Gives a 6 \times 6 matrix.
- Also considered 35 and 100 sweeps.
 - Results are consistent, however the statistical noise increases due to the increased smearing.

Extracting Form Factors from Lattice QCD

• To extract the form factors for a state α , we need to calculate the three-point correlation function

$$G^{\mu}_{\alpha}(\mathbf{p}',\mathbf{p};t_{2},t_{1}) = \sum_{\mathbf{x}_{1},\mathbf{x}_{2}} e^{-i\mathbf{p}'\cdot\mathbf{x}_{2}} e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_{1}} \langle \Omega | \phi^{\alpha}(x_{2}) j^{\mu}(x_{1}) \,\overline{\phi}^{\alpha}(0) | \Omega \rangle$$

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This takes the form

$$e^{-E_{\alpha}(\mathbf{p}')(t_{2}-t_{1})}e^{-E_{\alpha}(\mathbf{p})t_{1}}\sum_{s,s'}\left\langle \Omega|\phi^{\alpha}|p',s'\right\rangle \left\langle p',s'|j^{\mu}|p,s\right\rangle \left\langle p,s|\overline{\phi}^{\alpha}|\Omega\right\rangle$$

where $\langle p', s' | j^{\mu} | p, s \rangle$ encodes the form factors of the interaction.

Excited State Form Factors

• Using the nature of these "perfect" operators, the eigenstate-projected correlation function is

$$\begin{aligned} G^{\mu}_{\alpha}(\mathbf{p}',\mathbf{p};t_{2},t_{1}) &= \sum_{\mathbf{x}_{1},\mathbf{x}_{2}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}'\cdot\mathbf{x}_{2}} \mathrm{e}^{\mathrm{i}(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_{1}} \times \\ &\times \langle \Omega | v_{i}^{\alpha}(\mathbf{p}') \, \chi_{i}(x_{2}) \, j^{\mu}(x_{1}) \, \overline{\chi}_{j}(0) \, u_{i}^{\alpha}(\mathbf{p}) | \Omega \rangle \\ &= \mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p}') \, G^{\mu}_{ij}(\mathbf{p}',\mathbf{p};t_{2},t_{1}) \, \mathbf{u}^{\alpha}(\mathbf{p}) \end{aligned}$$

where

$$G_{ij}^{\mu}(\mathbf{p}',\mathbf{p};t_2,t_1) = \sum_{\mathbf{x}_1,\mathbf{x}_2} e^{-i\mathbf{p}'\cdot\mathbf{x}_2} e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_1} \langle \Omega | \chi_i(x_2) j^{\mu}(x_1) \overline{\chi}_j(0) | \Omega \rangle$$

is the matrix constructed from the three-point correlation functions of the original operators { χ_i }.

Extracting Form Factors from Lattice QCD

• To eliminate the time dependence of the three-point correlation function, we construct the ratio

$$R^{\mu}_{\alpha}(\mathbf{p}',\mathbf{p};t_2,t_1) = \left(\frac{G^{\mu}_{\alpha}(\mathbf{p}',\mathbf{p};t_2,t_1) G^{\mu}_{\alpha}(\mathbf{p},\mathbf{p}';t_2,t_1)}{G_{\alpha}(\mathbf{p}';t_2) G_{\alpha}(\mathbf{p};t_2)}\right)^{1/2}$$

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To further simply things, we define the reduced ratio

$$\overline{R}^{\mu}_{\alpha} = \left(\frac{2E_{\alpha}(\mathbf{p})}{E_{\alpha}(\mathbf{p}) + m_{\alpha}}\right)^{1/2} \left(\frac{2E_{\alpha}(\mathbf{p}')}{E_{\alpha}(\mathbf{p}') + m_{\alpha}}\right)^{1/2} R^{\mu}_{\alpha}$$

Current Matrix Element for Spin-1/2 Baryons

The current matrix element for spin-1/2 baryons can be written in the form

$$\langle p', s' | j^{\mu} | p, s \rangle = \left(\frac{m_{\alpha}^{2}}{E_{\alpha}(\mathbf{p}) E_{\alpha}(\mathbf{p}')} \right)^{1/2} \times \\ \times \overline{u} \left(F_{1}(q^{2}) \gamma^{\mu} + \mathrm{i} F_{2}(q^{2}) \sigma^{\mu\nu} \frac{q^{\nu}}{2m_{\alpha}} \right) u$$

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• The Dirac and Pauli form factors are related to the Sachs form factors through

$$egin{split} \mathcal{G}_{\mathsf{E}}(q^2) &= F_1(q^2) - rac{q^2}{(2m^{lpha})^2}F_2(q^2) \ \mathcal{G}_{\mathsf{M}}(q^2) &= F_1(q^2) + F_2(q^2) \end{split}$$

Sachs Form Factors for Spin-1/2 Baryons

 A suitable choice of momentum (q = (q, 0, 0)) and the (implicit) Dirac matrices allows us to directly access the Sachs form factors:
 o for G_E: using Γ[±]₄ for both two- and three-point,

$$\mathcal{G}^{\alpha}_{\mathsf{E}}(q^2) = \overline{R}^4_{\alpha}(\mathbf{q},\mathbf{0};t_2,t_1)$$

 $\,\circ\,$ for $\mathcal{G}_{\mathsf{M}}:$ using $\mathsf{\Gamma}_4^\pm$ for two-point and $\mathsf{\Gamma}_j^\pm$ for three-point,

$$|\varepsilon_{ijk} q^i| \mathcal{G}^{lpha}_{\mathsf{M}}(q^2) = (E_{lpha}(\mathbf{q}) + m_{lpha}) \overline{R}^k_{lpha}(\mathbf{q}, \mathbf{0}; t_2, t_1)$$

where for positive parity states,

$$\Gamma_j^+ = \frac{1}{2} \begin{bmatrix} \sigma_j & 0\\ 0 & 0 \end{bmatrix} \qquad \Gamma_4^+ = \frac{1}{2} \begin{bmatrix} \mathbb{I} & 0\\ 0 & 0 \end{bmatrix}$$

and for negative parity states,

$$\Gamma_j^- = -\gamma_5 \Gamma_j^+ \gamma_5 = -\frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \sigma_j \end{bmatrix} \qquad \Gamma_4^- = -\gamma_5 \Gamma_4^+ \gamma_5 = -\frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \mathbb{I} \end{bmatrix}$$

${\cal G}_{\sf E}$ for the A(1405) at $Q^2 \sim 0.15\,{ m GeV}^2$



• assume a dipole dependence on Q^2 :

$$\mathcal{G}_{\mathsf{E}}(Q^2) = \left(\frac{\Lambda}{\Lambda + Q^2}\right)^2 \mathcal{G}_{\mathsf{E}}(0)$$

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- solve for Λ using $\mathcal{G}_{\mathsf{E}}(0)=1$ (unit charge quarks) for each m_π^2
- evaluate \mathcal{G}_{E} at a common Q^2 (we used 0.16 GeV²)

${\cal G}_{\sf E}$ for the $\Lambda(1405)$ at $Q^2=0.16\,{ m GeV}^2$



Structure of the $\Lambda(1405)$

These results are consistent with the development of a non-trivial \overline{KN} component at light quark masses.

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- Noting that the centre of mass of the KN is nearer the heavier N, compared to the ground state,
 - $\circ~$ the anti–light-quark contribution is distributed further out by the \overline{K} and thus leaves an enhanced light-quark form factor.

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- Noting that the centre of mass of the KN is nearer the heavier N, compared to the ground state,
 - $\circ\,$ the anti–light-quark contribution is distributed further out by the $\overline{K}\,$ and thus leaves an enhanced light-quark form factor.
 - $\circ\,$ the strange quark is distributed further out by the \overline{K} and thus has a smaller form factor.

Structure of the $\Lambda(1405)$



\mathcal{G}_{M} for the A at $\mathit{Q}^2 \sim 0.15\,\mathsf{GeV}^2$



${\cal G}_{\sf M}$ for the Λ at ${\it Q}^2\sim 0.15\,{ m GeV}^2$

The ground state Λ is flavour-octet:

- the light quarks form a scalar diquark, which has no spin
- the strange quark is the dominant contribution to the magnetic moment

\mathcal{G}_{M} for the A at $\mathit{Q}^2 \sim 0.15\,\mathsf{GeV}^2$



${\cal G}_{\sf M}$ for the A(1405) at ${\it Q}^2\sim 0.15\,{ m GeV}^2$

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• Expect $\mu = 0$ in the SU(3) limit, so that $\mathcal{G}_{M}^{\text{light}} = \mathcal{G}_{M}^{\text{strange}}$.

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- Expect $\mu = 0$ in the SU(3) limit, so that $\mathcal{G}_{M}^{\text{light}} = \mathcal{G}_{M}^{\text{strange}}$.
- The magnetic moment is proportional to 1/m, so as the light quark mass decreases, the form factor should increase.

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- The light quark sector will be governed by p + n.
 - This is a positive, non-analytic contribution to the dressing and will enhance the light form factor.

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 - This will suppress the strange quark form factor of the $\Lambda(1405)$.

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- There is no contribution to the strange quark magnetic moment
 - $\,\circ\,$ This will suppress the strange quark form factor of the $\Lambda(1405).$
- $\mathcal{G}_{M}^{\text{strange}}$ is consistent with zero!
- The structure of the $\Lambda(1405)$ is dominated by a bound $\overline{K}N$.

Fitted $\mathcal{G}_{ m M}(t)$ at $m_\pi^2=0.151\,{ m GeV}^2$, $Q^2=0.160\,{ m GeV}^2$



Fitted $\mathcal{G}_{ m M}(t)$ at $m_\pi^2=0.030\,{ m GeV}^2$, $Q^2=0.169\,{ m GeV}^2$



Conclusions

- After decade of speculation, the nature of $\Lambda(1405)$ is finally revealed from the first principles of QCD.
- Our results are consistent with the development of a non-trivial $\overline{K}N$ component as the quark masses become light.
- At the physical point the structure of the $\Lambda(1405)$ is dominated by a bound $\overline{K}N.$