Electromagnetic Structure of the $\Lambda(1405)$

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Outline

Introduction & Techniques

Electric Form Factors

Magnetic Form Factors

Conclusion
The $\Lambda(1405)$

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- It has a mass of $1405.1^{+1.3}_{-1.0}$ MeV.
  - This is lower than the lowest odd-parity nucleon state ($N(1535)$), even though it has a valence strange quark.
The $\Lambda(1405)$

![Graph showing the mass distribution of $\Lambda$ and $N$ particles with mass values ranging from 500 to 2000 MeV.]
The Λ(1405)

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- It has a mass of $1405.1^{+1.3}_{-1.0}$ MeV.
  - This is lower than the lowest odd-parity nucleon state ($N(1535)$), even though it has a valence strange quark.
- We now understand this as a consequence of its flavour-singlet structure.
The $\Lambda(1405)$

![Graph showing unit eigenvector component vs. $m_{\pi}^2$ [GeV$^2$] with data points for 16 and 100 sweeps.](image)
The $\Lambda(1405)$ and Lattice QCD

Our recent work has successfully isolated three low-lying states.


- An extrapolation of the trend of the lowest state reproduces the mass of the $\Lambda(1405)$.
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• An extrapolation of the trend of the lowest state reproduces the mass of the $\Lambda(1405)$.
• Subsequent studies have confirmed these results.

The $\Lambda(1405)$ and Lattice QCD

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- Our current insertion ($t_{SS-T} = 21$) is 5 time slices after the source ($t_{src} = 16$).
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\[ \chi^2/dof = 0.55 \]
Simulation Details

We are using the PACS-CS $(2 + 1)$-flavour ensembles, available through the ILDG.


- Lattice size of $32^3 \times 64$ with $\beta = 1.90$.
- 5 pion masses, ranging from 640 MeV down to 156 MeV.
- Single strange quark mass, with $\kappa_s = 0.13640$.
  - We partially quench by using $\kappa_s = 0.13665$ for the valence strange quarks to reproduce the physical kaon mass.
- We consider both the Sommer and PACS-CS schemes to set the scale.
Variational Analysis

By using multiple operators, we can isolate and analyse individual energy eigenstates:

- Construct the correlation matrix

\[ G_{ij}(p; t) = \sum_x e^{-ip \cdot x} \langle \Omega | \chi_i(x) \chi_j(0) | \Omega \rangle, \]

for some set \( \{ \chi_i \} \) operators that couple to the states of interest.
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• Note: all correlation functions have an implicit trace over the spinor indices with some Dirac matrix:

\[ G \equiv \text{tr}(\Gamma G) \]
Variational Analysis

• Solve for the left, $v^\alpha(p)$, and right, $u^\alpha(p)$, generalised eigenvectors of $G(p; t + \delta t)$ and $G(p; t)$:

$$\begin{align*}
G(p; t + \delta t) u^\alpha(p) &= e^{-E^\alpha(p) \Delta t} G(p; t) u^\alpha(p) \\
v^{\alpha T}(p) G(p; t + \delta t) &= e^{-E^\alpha(p) \Delta t} v^{\alpha T}(p) G(p; t)
\end{align*}$$
Variational Analysis

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  $$ G(p; t + \delta t) u^\alpha(p) = e^{-E^\alpha(p) \Delta t} G(p; t) u^\alpha(p) $$

  $$ v^{\alpha_T}(p) G(p; t + \delta t) = e^{-E^\alpha(p) \Delta t} v^{\alpha_T}(p) G(p; t) $$

- These eigenvectors identify “ideal” combinations of the original operators that perfectly isolate individual energy eigenstates at momentum $p$:

  $$ \phi^\alpha = v^\alpha_i(p) \chi_i $$

  $$ \phi^\alpha = u^\alpha_i(p) \bar{\chi}_i $$
Using these “perfect” operators, we can extract correlation functions for these energy eigenstates using

\[ G_\alpha(p; t) = \sum_x e^{-ip \cdot x} \langle \Omega | \phi_\alpha(x) \bar{\phi}_\alpha(0) | \Omega \rangle = \sum_x e^{-ip \cdot x} \langle \Omega | v_\alpha^T(p) \chi_i(x) \bar{\chi}_j(0) u_\alpha^T(p) | \Omega \rangle = v_\alpha^T(p) G(p; t) u_\alpha^T(p) \]
Choice of Operators

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  ◦ Only requirement is that they couple to the states of interest.

• However:
  ◦ too few operators and the states won’t be sufficiently isolated, and
  ◦ insufficiently independent operators and the matrix will be too ill-conditioned to solve for the eigenvectors.
Operators Used in $\Lambda(1405)$ Analysis

There are a number of operators that have the correct quantum numbers to couple to the $\Lambda$ channel. We use

- the flavour-octet operators

\[
\chi_1^8 = \frac{1}{\sqrt{6}} \varepsilon^{abc} (2(u^a C \gamma_5 d^b)s^c + (u^a C \gamma_5 s^b)d^c - (d^a C \gamma_5 s^b)u^c)
\]

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- the flavour-singlet operator

\[
\chi_1^1 = 2\varepsilon^{abc} ((u^a C \gamma_5 d^b)s^c - (u^a C \gamma_5 s^b)d^c + (d^a C \gamma_5 s^b)u^c)
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Operators Used in $\Lambda(1405)$ Analysis

We also use gauge-invariant Gaussian smearing to increase our operator basis.

- These results use 16 and 100 sweeps.
  - Gives a $6 \times 6$ matrix.

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- Results are consistent, however the statistical noise increases due to the increased smearing.
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To extract the form factors for a state $\alpha$, we need to calculate the three-point correlation function

$$G^\mu_\alpha(p', p; t_2, t_1) = \sum_{x_1, x_2} e^{-i p' \cdot x_2} e^{i (p' - p) \cdot x_1} \langle \Omega | \phi^\alpha (x_2) j^\mu (x_1) \overline{\phi}^\alpha (0) | \Omega \rangle$$
Extracting Form Factors from Lattice QCD

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• This takes the form

$$e^{-E_\alpha(p')(t_2 - t_1)} e^{-E_\alpha(p)t_1} \sum_{s, s'} \langle \Omega | \phi^\alpha | p', s' \rangle \langle p', s' | j^\mu | p, s \rangle \langle p, s | \overline{\phi}^\alpha | \Omega \rangle$$

where $\langle p', s' | j^\mu | p, s \rangle$ encodes the form factors of the interaction.
Excited State Form Factors

- Using the nature of these “perfect” operators, the eigenstate-projected correlation function is

\[ G_{\alpha}^{\mu}(p', p; t_2, t_1) = \sum_{x_1, x_2} e^{-i p' \cdot x_2} e^{i (p' - p) \cdot x_1} \times \]

\[ \langle \Omega | v_\alpha^{\chi}(p') \chi_i(x_2) j^\mu(x_1) \bar{\chi}_j(0) u^\alpha_i(p) | \Omega \rangle \]

\[ = v^{\alpha T}(p') \cdot G_{ij}^{\mu}(p', p; t_2, t_1) \cdot u^\alpha(p) \]

where

\[ G_{ij}^{\mu}(p', p; t_2, t_1) = \sum_{x_1, x_2} e^{-i p' \cdot x_2} e^{i (p' - p) \cdot x_1} \langle \Omega | \chi_i(x_2) j^\mu(x_1) \bar{\chi}_j(0) | \Omega \rangle \]

is the matrix constructed from the three-point correlation functions of the original operators \( \{ \chi_i \} \).
To eliminate the time dependence of the three-point correlation function, we construct the ratio

\[ R^\mu(\mathbf{p}', \mathbf{p}; t_2, t_1) = \left( \frac{G^\mu_\alpha(\mathbf{p}', \mathbf{p}; t_2, t_1) G^\mu_\alpha(\mathbf{p}, \mathbf{p}'; t_2, t_1)}{G_\alpha(\mathbf{p}'; t_2) G_\alpha(\mathbf{p}; t_2)} \right)^{1/2} \]
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- To further simplify things, we define the reduced ratio

\[ \overline{R}_{\alpha}^\mu = \left( \frac{2E_{\alpha}(\mathbf{p})}{E_{\alpha}(\mathbf{p}) + m_{\alpha}} \right)^{1/2} \left( \frac{2E_{\alpha}(\mathbf{p}')}{E_{\alpha}(\mathbf{p}') + m_{\alpha}} \right)^{1/2} R_{\alpha}^\mu \]
Current Matrix Element for Spin-1/2 Baryons

The current matrix element for spin-1/2 baryons can be written in the form

\[ \langle p', s' | j_\mu | p, s \rangle = \left( \frac{m^2_\alpha}{E_\alpha(p)E_\alpha(p')} \right)^{1/2} \times \]

\[ \times \bar{u} \left( F_1(q^2) \gamma^\mu + i F_2(q^2) \sigma^{\mu\nu} \frac{q^\nu}{2m_\alpha} \right) u \]
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\]

- The Dirac and Pauli form factors are related to the Sachs form factors through

\[
G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m_\alpha)^2} F_2(q^2)
\]

\[
G_M(q^2) = F_1(q^2) + F_2(q^2)
\]
Sachs Form Factors for Spin-1/2 Baryons

- A suitable choice of momentum \( \mathbf{q} = (q, 0, 0) \) and the (implicit) Dirac matrices allows us to directly access the Sachs form factors:
  - for \( G_E \): using \( \Gamma_4^\pm \) for both two- and three-point,
    \[
    G_E^\alpha(q^2) = \bar{R}_\alpha^4(q, 0; t_2, t_1)
    \]
  - for \( G_M \): using \( \Gamma_4^\pm \) for two-point and \( \Gamma_j^\pm \) for three-point,
    \[
    |\varepsilon_{ijk} \, q^i| \, G_M^\alpha(q^2) = (E_\alpha(q) + m_\alpha) \bar{R}_\alpha^k(q, 0; t_2, t_1)
    \]
  - where for positive parity states,
    \[
    \Gamma_j^+ = \frac{1}{2} \begin{bmatrix} \sigma_j & 0 \\ 0 & 0 \end{bmatrix} \quad \Gamma_4^+ = \frac{1}{2} \begin{bmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{bmatrix}
    \]
    and for negative parity states,
    \[
    \Gamma_j^- = -\gamma_5 \Gamma_j^+ \gamma_5 = -\frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \sigma_j \end{bmatrix} \quad \Gamma_4^- = -\gamma_5 \Gamma_4^+ \gamma_5 = -\frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \mathbb{I} \end{bmatrix}
    \]
$G_E$ for the $\Lambda(1405)$ at $Q^2 \sim 0.15 \text{ GeV}^2$
Correcting for Varying $Q^2$

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- solve for $\Lambda$ using $G_E(0) = 1$ (unit charge quarks) for each $m^2_\pi$
- evaluate $G_E$ at a common $Q^2$ (we used 0.16 GeV$^2$)
$G_E$ for the $\Lambda(1405)$ at $Q^2 = 0.16$ GeV$^2$
Structure of the $\Lambda(1405)$

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- Noting that the centre of mass of the $\bar{K}N$ is nearer the heavier $N$, compared to the ground state,
  - the anti–light-quark contribution is distributed further out by the $\bar{K}$ and thus leaves an enhanced light-quark form factor.
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  - the strange quark is distributed further out by the $\bar{K}$ and thus has a smaller form factor.
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The ground state $\Lambda$ is flavour-octet:

- the light quarks form a scalar diquark, which has no spin
- the strange quark is the dominant contribution to the magnetic moment
$G_M$ for the $\Lambda$ at $Q^2 \sim 0.15 \text{GeV}^2$
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• Expect $\mu = 0$ in the SU(3) limit, so that $G_M^{\text{light}} = G_M^{\text{strange}}$. 
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- Expect $\mu = 0$ in the SU(3) limit, so that $G_M^{\text{light}} = G_M^{\text{strange}}$.
- The magnetic moment is proportional to $1/m$, so as the light quark mass decreases, the form factor should increase.
$G_M$ for the $\Lambda(1405)$ at $Q^2 \sim 0.15 \text{ GeV}^2$
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If we consider a $\bar{K}N$ dressing,

\[ \Lambda^* \rightarrow n, p \rightarrow \Lambda^* \]

\[ \bar{K}^0, K^- \]
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- There is no contribution to the strange quark magnetic moment.
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- $G_M^{\text{strange}}$ is consistent with zero!
- The structure of the $\Lambda(1405)$ is dominated by a bound $\bar{K}N$. 

\[ G_M \text{ for the } \Lambda(1405) \text{ at } Q^2 \sim 0.15 \text{ GeV}^2 \]
Fitted $G_M(t)$ at $m_\pi^2 = 0.151$ GeV$^2$, $Q^2 = 0.160$ GeV$^2$
Fitted $G_M(t)$ at $m^2_\pi = 0.030$ GeV$^2$, $Q^2 = 0.169$ GeV$^2$
Conclusions

- After decade of speculation, the nature of $\Lambda(1405)$ is finally revealed from the first principles of QCD.
- Our results are consistent with the development of a non-trivial $\bar{K}N$ component as the quark masses become light.
- At the physical point the structure of the $\Lambda(1405)$ is dominated by a bound $\bar{K}N$. 