Determination of $\Delta$ resonance parameters from lattice QCD

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based on arXiv:1305.6081 [hep-lat]

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Motivation

Characterization of baryonic resonance states on the lattice

$\Delta^{++}, \vec{p}\Delta$  

$\pi^+, \vec{p}_\pi$

$\pi^+ - p - \text{bound state}$

- strong decays, transitions from baryonic initial to final states of bound hadrons
- well known example: $\Delta \to \pi + N$
- resonance parameters $E_R, \Gamma$
- volume method: phase shifts via finite volume energy spectrum  
- transfer matrix method: attempt to estimate $\mathcal{M} \sim \langle f | aH | i \rangle$ from lattice QCD  
  $[\text{Phys.Rev. D65, 094505 (2002)}]$  
- first test for $\Delta \to \pi N$  
  $[\text{arXiv:1305.6081}]$
Motivation - transfer matrix method

- no physical transition on a Euclidean lattice in finite volume
- mixing of finite volume lattice states $\leftrightarrow$ if energies of levels are *sufficiently close*
- estimates from lattices finite time extent $\leftrightarrow$ if transition amplitude *sufficiently small*
- final states at non-zero momentum with fine resolution relative momentum $\leftrightarrow$ lattice volume *sufficiently large*
Motivation - transfer matrix method

- start from transfer matrix

\[ T = e^{-a\bar{E}} \begin{pmatrix} e^{-a\delta/2} & ax & \cdots \\ ax & e^{+a\delta/2} & \\ \vdots & \ddots & \ddots \end{pmatrix}, \]

- transition amplitude \( x = \langle \Delta | \pi N \rangle \) parametrized by transfer matrix \( T \)
- \( \bar{E} = (E_\Delta + E_{\pi N})/2 \) and \( \delta = E_{\pi N} - E_\Delta \)
- restrict to 2-state model for the spectrum, \( \text{span} \{ |\Delta\rangle, |\pi N\rangle \} \)
- \( T \) with eigenstates of energy

\[ E_\pm \approx \bar{E} \pm \sqrt{\delta^2/4 + x^2} \]
Motivation - transfer matrix method

- summation over all possibilities of one transition $\Delta \rightarrow \piN$ (leading order)
  \[
  \langle \Delta, t_f | \piN, t_i \rangle = \langle \Delta | e^{-H(t_f-t_i)} | \piN \rangle = \langle \Delta | T^{n_{fi}} | \piN \rangle = ax \frac{\sinh(\delta \Delta t_{fi}/2)}{\sinh(a\delta/2)} e^{-\bar{E}\Delta t_{fi}},
  \]
  where $\Delta t_{fi} = t_f - t_i = an_{fi}$
- approximation of $\delta$-functional in Euclidean time
  \[
  a \frac{\sinh(\delta \Delta t_{fi}/2)}{\sinh(a\delta/2)} \xrightarrow{\Delta t_{fi} \to \infty, a \to 0} 2\pi \delta(p_{\piN}^0 - p_{\Delta}^0)
  \]
  for sufficiently small $\delta$ linear expansion
  \[
  \langle \Delta, t_f | \piN, t_i \rangle = [ax \Delta t_{fi} + O(\delta^2 \Delta t_{fi}^3)] e^{-\bar{E}\Delta t_{fi}} + \ldots
  \]
- ellipsis for higher order contributions, contributions from excited states (no asymptotic $\Delta$ and $\piN$ states), mixing with other states
Lattice calculation

- suitable ratio of 3-point and 2-point functions

\[
R(\Delta t f_i, \bar{Q}, \bar{q}) = \frac{C_{\mu}^{\Delta \to \pi N}(\Delta t f_i, \bar{Q}, \bar{q})}{\sqrt{C_{\mu}^{\Delta}(\Delta t f_i, \bar{Q})}} \frac{C^{\pi N}(\Delta t f_i, \bar{Q})}{C^{\pi}(\Delta t f_i, \bar{Q})}
\]

- isospin channel $3/2$: $\Delta^{++} \to \pi^+ + p$
- standard interpolating operators for $\Delta$, $p$, $\pi$; represent $\pi N$ by

\[
J_\pi(\alpha)(t, \bar{q}, \bar{x}) = \sum_{\bar{y}} J_\pi(t, \bar{y} + \bar{x}) J_N^{\alpha}(t, \bar{x}) e^{-i\bar{q}\bar{y}}
\]

- $\pi - N$ system with relative momentum to generate overlap with $l = 1$ state; dominant contribution from coupling $s_N \oplus l \to J_\Delta = 3/2$
- approximate, $C^{\pi N} \approx C^\pi \times C^N$
Lattice calculation

\[ \gamma_5, \vec{p}_\pi \]

\[ \vec{p}_\Delta \rightarrow \gamma_5, \vec{p}_N \]

\[ C \gamma_\mu \]

\[ C \gamma_5 \]

\[ +\vec{k}_i \rightarrow d \]

\[ -\vec{k}_i \rightarrow d \]

\[ -\vec{k}_i \rightarrow \gamma_5 \]

\[ +\vec{k}_f \rightarrow \gamma_\mu \]

\[ (\gamma_5, +\vec{k}_i) \]

\[ (\gamma_5, -\vec{k}_f) \]

\[ C_1 \]

\[ C_2 \]

\[ C_3 \]

\[ (2) \]

\[ (4) \]

\[ (4) \]
Lattice calculation

- hybrid calculation based on staggered sea with domain wall valence quarks
- MILC ensemble $2864f21b676m010m050$ [Phys.Rev. D64, 054506 (2001)], 210 configurations, 4 measurements per configuration
- unitary set up $m_{PS} \approx 360$ MeV, $L = 3.4$ fm, $a \approx 0.124$ fm [Phys.Rev. D79, 054502 (2009)]
- APE smearing, Jacobi smearing at source and sink
- initial setup: $\vec{q} = 2\pi/L \vec{e}_i$,
- averaging over $i = \pm 1, \pm 2, \pm 3$, forward and backward propagation

![Graph showing lattice data and fit](image)
Lattice calculation

\[ f_1(t) = A + B a \frac{\sinh(\delta t/2)}{\sin(a\delta/2)} \]

\[ f_2(t) = A + B t (+C t^3). \]

<table>
<thead>
<tr>
<th>( t_{\text{min}}/a )</th>
<th>( t_{\text{max}}/a )</th>
<th>( A \cdot 10^2 )</th>
<th>( aB \cdot 10^2 )</th>
<th>( a^3 C \cdot 10^5 )</th>
<th>( \chi^2 / \text{dof} )</th>
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<td>4</td>
<td>9</td>
<td>6.47 (49)</td>
<td>2.62 (15)</td>
<td>0.188 (68)</td>
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<td>0.074 (122)</td>
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<td>2.98 (30)</td>
<td>0.7 (28)</td>
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</table>
Extraction of the coupling

- definition of the matrix element from $B$

$$B_i = \sum_{\sigma_3, \tau_3} \frac{\mathcal{M}(\vec{Q}, \vec{q}, \sigma_3, \tau_3)}{\sqrt{N_\Delta N_{\pi N}}} \sqrt{N_\Delta N_{\pi N}} \delta_{\vec{Q} \vec{Q}} \times \text{spin sum factor}$$

- normalizations of finite volume $\Delta$ and $\pi N$ states

$$N_\Delta = V \frac{E_\Delta}{m_\Delta}$$

$$N_{\pi N} = N_\pi \times N_N = 2V E_\pi \times V \frac{E_N}{m_N}$$.

- decomposition of $\mathcal{M}$ by connecting to LO effective effective field theory with coupling $g_{\pi N}^\Delta$

$$\mathcal{M}(\vec{Q}, \vec{q}, \sigma_3, \tau_3) = \frac{g_{\pi N}^\Delta}{2m_N} \bar{u}_\Delta^{\mu \alpha}(\vec{Q}, \sigma_3) q_\mu u_N^{\alpha}(\vec{Q} + \vec{q}, \tau_3)$$.
Extraction of the coupling

Signal from the 3-point function $C_{i}^{\Delta \rightarrow \pi N}$ for $\vec{q} \propto \vec{e}_{z}$

$q \sim +e_{z}$, $i=1$, real part
imag. part

$q \sim +e_{z}$, $i=2$, real part
imag. part

$q \sim +e_{z}$, $i=3$, real part
imag. part

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combined result

\[ g_{\pi N}^{\Delta}(\text{lat}) = 27.0 \pm 0.6 \pm 1.5 . \]

good agreement with LO effective field theory result (assuming \( \Gamma = 118 (3) \text{ MeV} \))

\[ g_{\pi N}^{\Delta}(\text{lo eft}) = 29.4 (4) \]


\[ g_{\pi N}^{\Delta}(\text{exp}) = 28.6 (3) \]

direct conversion to decay width problematic at this stage

→ significant violation of energy conservation in lattice setup \( \rightarrow \) spatial momentum conservation, but pion-momentum \( k_L \approx 360 \text{ MeV} \) much larger than for the continuum decay \( (k_{\text{exp}} \approx 227 \text{ MeV}) \) \( \Rightarrow \) larger volume

dependence on lattice spacing, pion mass to be checked

full \( \pi - N \) 2-point function

application to a variety of hadronic decays under investigation
Discussion and Outlook

Different kinematical setup: $\Delta$ at rest, same relative momentum as before

- $\vec{p}_\Delta = \vec{p}_\pi$, $\vec{p}_p = 0$
- same number of averaged directions
- (approximated) splitting reduces to $aE_{\pi N} - aE_\Delta = 0.046(21)$
- $g_{\pi N}^\Delta = 26.8 \pm 0.6 \pm 1.4$
Further application - preliminary results $\Sigma^{*+} \to \pi\Lambda$

- full width $\Gamma(\Sigma^{3/2+}) = 36.0 (7)$ MeV
- fraction of decay to $\pi\Lambda$ $\Gamma_i/\Gamma = (87.0 \pm 1.5)\%$
- coupling from LO formula for the width $g_{\Sigma^{*+}_{\pi\Lambda}}(\text{lo eft}) \approx 20.0$
- $\bar{q} \propto +\bar{e}_x$

$[4, 12] \quad aB = 0.0208 (06)$ with $\chi^2$/dof = 1.1 $\Rightarrow g = 21.5 \pm 0.7$

$[6, 12] \quad aB = 0.0228 (16)$ with $\chi^2$/dof = 1.1 $\Rightarrow g = 23.6 \pm 1.6$
Thank you very much for your attention.