Determination of Δ resonance parameters from lattice QCD

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based on arXiv:1305.6081 [hep-lat]

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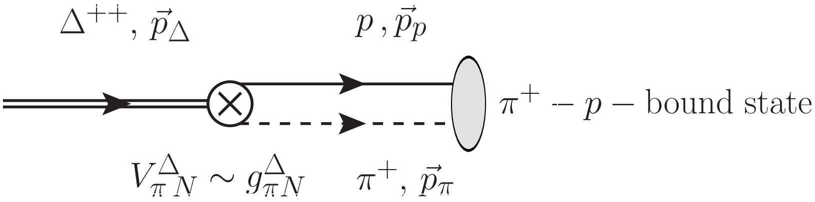
Motivation

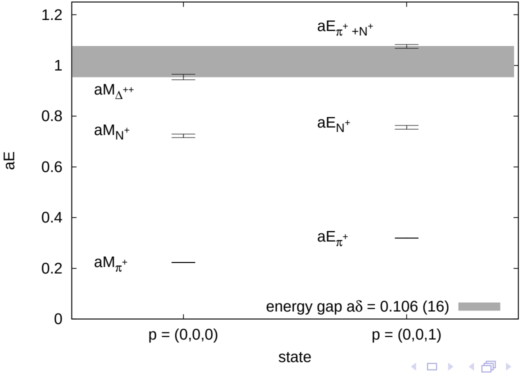
Characterization of baryonic resonance states on the lattice



- strong decays, transitions from baryonic initial to final states of bound hadrons
- well known example: $\Delta \to \pi + N$
- resonance parameters E_R, Γ
- volume method: phase shifts via finite volume energy spectrum *[Commun.Math.Phys. 104, 177 (1986), Commun.Math.Phys. 105, 153 (1986)]*
- transfer matrix method: attempt to estimate $\mathcal{M} \sim \langle f \,|\, aH \,|\, i \rangle$ from lattice QCD *[Phys.Rev. D65, 094505 (2002)]*
- first test for $\Delta \to \pi N$ *[arXiv:1305.6081]*

Motivation - transfer matrix method

- no physical transition on a Euclidean lattice in finite volume
- mixing of finite volume lattice states $\leftrightarrow$ if energies of levels are *sufficiently close*
- estimates from lattices finite time extent $\leftrightarrow$ if transition amplitude *sufficiently small*
- final states at non-zero momentum with fine resolution relative momentum $\leftrightarrow$ lattice volume *sufficiently large*



Motivation - transfer matrix method

- start from transfer matrix

$$\mathrm{T} = \mathrm{e}^{-a\bar{E}} \begin{pmatrix} \mathrm{e}^{-a\delta/2} & ax & \cdots \\ ax & \mathrm{e}^{+a\delta/2} & \\ \vdots & & \ddots \end{pmatrix},$$

- transition amplitude $x = \langle \Delta \,|\, \pi N \rangle$ parametrized by transfer matrix T
- $\bar{E} = (E_\Delta + E_{\pi N})/2$ and $\delta = E_{\pi N} - E_\Delta$
- restrict to 2-state model for the spectrum, $\mathrm{span}\,\{|\Delta\rangle,\, |\pi\, N\rangle\}$
- T with eigenstates of energy

$$E_\pm \approx \bar{E} \pm \sqrt{\delta^2/4 + x^2}$$

Motivation - transfer matrix method

- summation over all possibilities of one transition $\Delta \to \pi N$ (leading order)

$$\begin{aligned}\langle \Delta,\, t_f \,|\, \pi N,\, t_i \rangle &= \langle \Delta \,|\, \mathrm{e}^{-H(t_f - t_i)} \,|\, \pi N \rangle = \langle \Delta \,|\, \mathrm{T}^{n_{fi}} \,|\, \pi N \rangle \\ &= ax \, \frac{\sinh(\delta\, \Delta t_{fi}/2)}{\sinh(a\delta/2)} \, \mathrm{e}^{-\bar{E}\Delta t_{fi}} \,,\end{aligned}$$

where $\Delta t_{fi} = t_f - t_i = a n_{fi}$

- approximation of δ-functional in Euclidean time

$$a \sinh(\delta\, \Delta t_{fi}/2)/\sinh(a\delta/2) \xrightarrow[a\to 0]{\Delta t_{fi}\to\infty} 2\pi\, \delta(p^0_{\pi N} - p^0_{\Delta})$$

- for sufficiently small δ linear expansion

$$\langle \Delta,\, t_f \,|\, \pi N,\, t_i \rangle = \left[ax\, \Delta t_{fi} + \mathcal{O}\left(\delta^2\, \Delta t_{fi}^3\right)\right] \, \mathrm{e}^{-\bar{E}\Delta t_{fi}} + \ldots$$

- ellipsis for higher order contributions, contributions from excited states (no asymptotic Δ and πN states), mixing with other states

Lattice calculation

- suitable ratio of 3-point and 2-point functions

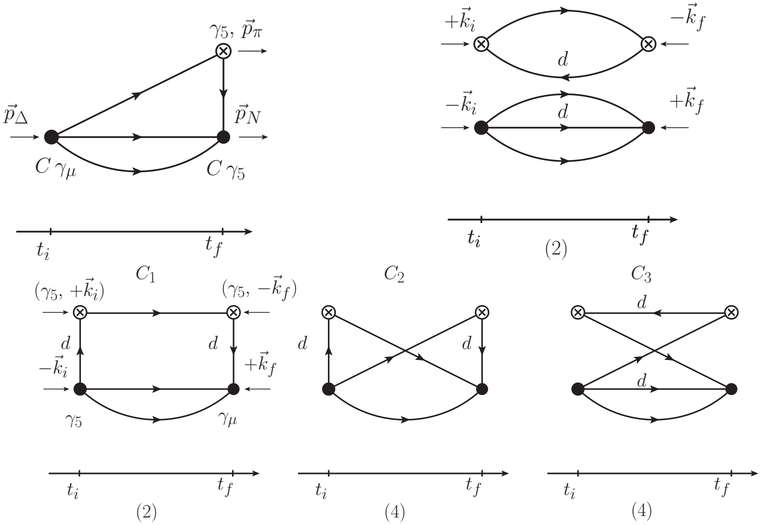
$$R(\Delta t_{fi}, \vec{Q}, \vec{q}) = \frac{C_{\mu}^{\Delta\to\pi N}(\Delta t_{fi},\ \vec{Q}, \vec{q})}{\sqrt{C_{\mu}^{\Delta}(\Delta t_{fi},\ \vec{Q})\ C^{\pi N}(\Delta t_{fi},\ \vec{Q},\ \vec{q})}}$$

- isospin channel 3/2: $\Delta^{++} \to \pi^{+} + p$
- standard interpolating operators for Δ, p, π; represent πN by

$$J_{\pi N}^{\alpha}(t, \vec{q}, \vec{x}) = \sum_{\vec{y}} J_{\pi}(t, \vec{y} + \vec{x})\, J_{N}^{\alpha}(t, \vec{x})\, \mathrm{e}^{-i\vec{q}\vec{y}}$$

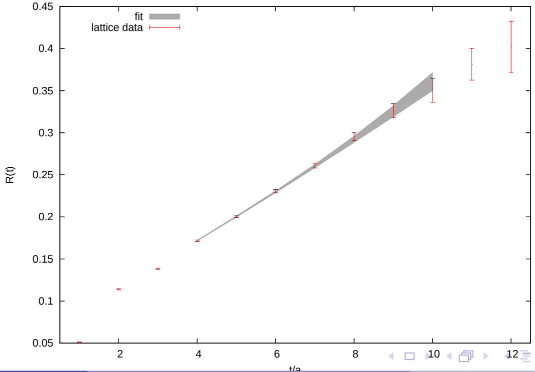
- $\pi - N$ system with relative momentum to generate overlap with $l = 1$ state; dominant contribution from coupling $s_N \oplus l \to J_\Delta = 3/2$
- approximate , $C^{\pi N} \approx C^{\pi} \times C^{N}$

Lattice calculation



Lattice calculation

- hybrid calculation based on staggered sea with domain wall valence quarks
- MILC ensemble *2864f21b676m010m050 [Phys.Rev. D64, 054506 (2001)]*, 210 configurations, 4 measurements per configuration
- unitary set up $m_{PS} \approx 360\,\mathrm{MeV}$, $L = 3.4\,\mathrm{fm}$, $a \approx 0.124\,\mathrm{fm}$ *[Phys.Rev. D79, 054502 (2009)]*
- APE smearing, Jacobi smearing at source and sink
- initial setup: $\vec{q} = 2\pi/L\,\vec{e}_i$,
- averaging over $i = \pm 1, \pm 2, \pm 3$, forward and backward propagation



Lattice calculation

$$f_1(t) = A + B\,a\,\frac{\sinh(\delta\,t/2)}{\sin(a\delta/2)}$$

$$f_2(t) = A + B\,t\,(+C\,t^3)\,.$$

	$t_{\min}/a$	$t_{\max}/a$	$A \cdot 10^2$	$aB \cdot 10^2$	$a^3C \cdot 10^5$	χ^2/dof
f_1	4	9	6.47 (49)	2.62 (15)	0.188 (68)	2.4/3
f_1	4	10	6.24 (47)	2.69 (14)	0.156 (79)	4.3/4
f_1	5	9	5.62 (103)	2.82 (26)	0.140 (104)	1.8/2
f_1	5	10	5.05 (84)	2.98 (21)	0.074 (122)	2.9/3
f_2	4	9	5.62 (25)	2.89 (06)		6.0/4
f_2	4	10	5.63 (25)	2.89 (06)		6.5/5
f_2	5	9	4.75 (51)	3.05 (10)		2.4/3
f_2	5	10	4.78 (52)	3.05 (11)		3.0/4
f_2	4	9	6.51 (53)	2.60 (16)	4.1 (22)	2.4/3
f_2	4	10	6.27 (52)	2.68 (16)	2.9 (21)	4.3/4
f_2	5	9	5.64 (128)	2.82 (33)	2.4 (32)	1.8/2
f_2	5	10	5.05 (117)	2.98 (30)	0.7 (28)	2.9/3

Extraction of the coupling

- definition of the matrix element from B

$$B_i = \sum_{\sigma_3,\,\tau_3} \frac{\mathcal{M}(\vec{Q},\,\vec{q},\,\sigma_3,\,\tau_3)}{\sqrt{N_\Delta\,N_{\pi N}}}\,V\,\delta_{\vec{Q}\vec{Q}} \times \text{spin sum factor}$$

- normalizations of finite volume Δ and πN states

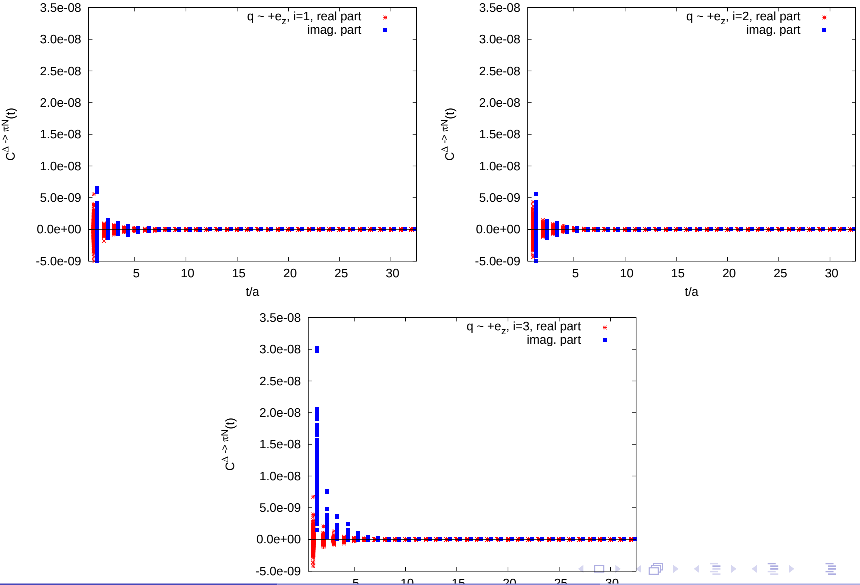
$$\begin{aligned} N_\Delta &= V\,\frac{E_\Delta}{m_\Delta} \\ N_{\pi N} &= N_\pi \times N_N = 2V\,E_\pi \times V\,\frac{E_N}{m_N}\,. \end{aligned}$$

- decomposition of $\mathcal{M}$ by connecting to LO effective effective field theory with coupling $g_{\pi N}^{\Delta}$

$$\mathcal{M}(\vec{Q},\,\vec{q},\,\sigma_3,\,\tau_3) = \frac{g_{\pi N}^{\Delta}}{2m_N}\,\bar{u}_\Delta^{\mu\,\alpha}(\vec{Q},\sigma_3)\,q_\mu\,u_N^\alpha(\vec{Q}+\vec{q},\tau_3)\,,$$

Extraction of the coupling

Signal from the 3-point function $C_i^{\Delta \to \pi N}$ for $\vec{q} \propto \vec{e}_z$



Discussion and Outlook

- combined result

$$g_{\pi N}^{\Delta}(\mathrm{lat}) = 27.0 \pm 0.6 \pm 1.5\,.$$

- good agreement with LO effective field theory result (assuming $\Gamma = 118\,(3)\,\mathrm{MeV}$)

$$g_{\pi N}^{\Delta}(\mathrm{lo\ eft}) = 29.4\,(4)$$

  and result from K-matrix analysis *[Phys.Rev. D51, 158 (1995)]*

$$g_{\pi N}^{\Delta}(\mathrm{exp}) = 28.6\,(3)$$

- direct conversion to decay width problematic at this stage $\rightarrow$ significant violation of energy conservation in lattice setup $\rightarrow$ spatial momentum conservation, but pion-momentum $k_L \approx 360\,\mathrm{MeV}$ much larger than for the continuum decay ($k_{\mathrm{exp}} \approx 227\,\mathrm{MeV}$) $\Rightarrow$ larger volume
- dependence on lattice spacing, pion mass to be checked
- full $\pi - N-$ 2-point function
- application to a variety of hadronic decays under investigation

Discussion and Outlook

Different kinematical setup: Δ at rest, same relative momentum as before

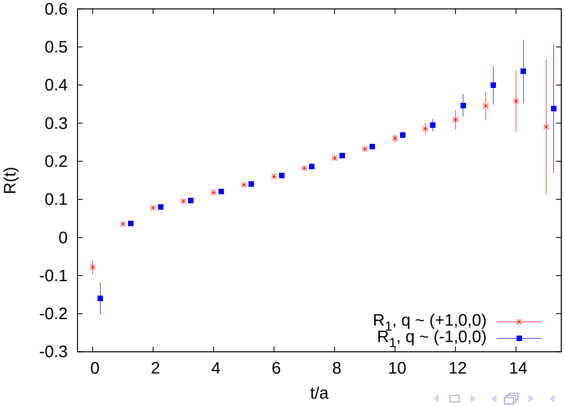
- $\vec{p}_\Delta = \vec{p}_\pi\,,\ \vec{p}_p = 0$
- same number of averaged directions
- (approximated) splitting reduces to $aE_{\pi N} - aE_\Delta = 0.046\,(21)$
- $g^{\Delta}_{\pi N} = 26.8 \pm 0.6 \pm 1.4$

Further application - preliminary results $\Sigma^{*+} \to \pi\Lambda$

- full width $\Gamma(\Sigma^{3/2+}) = 36.0\,(7)\,\mathrm{MeV}$
- fraction of decay to $\pi\Lambda$ $\Gamma_i/\Gamma = (87.0 \pm 1.5)\,\%$
- coupling from LO formula for the width $g_{\pi\Lambda}^{\Sigma^{*+}}(\mathrm{lo\ eft}) \approx 20.0$
- $\vec{q} \propto +\vec{e}_x$

  [4, 12] $aB = 0.0208\,(06)$ with $\chi^2/\mathrm{dof} = 1.1 \Rightarrow g = 21.5 \pm 0.7$

  [6, 12] $aB = 0.0228\,(16)$ with $\chi^2/\mathrm{dof} = 1.1 \Rightarrow g = 23.6 \pm 1.6$



Thank you very much for your attention.