SU(3) flavour breaking and baryon structure

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QCDSF Collaboration
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Lattice 2013, July 29 - August 3, 2013, Mainz, Germany

See also poster by Ashley Cooke

Outline

- Introduction
- Tuning and simulation parameters [arXiv:1003.1114 (PLB), 1102.5300 (PRD)]
- Hyperon Results
    - Electromagnetic Form Factors
    - Axial Charges
- SU(3) Flavour Breaking Expansions

See also poster by Ashley Cooke

- Semileptonic decays

Motivation for Investigation of Hadron Structure

- We know the nucleon is not a point-like particle but in fact is composed of quarks and gluons

- But how are these constituents distributed inside the nucleon?

- How do they combine to produce its experimentally observed properties?

- For example

  - “Spin crisis”: quarks carry on ~30% of the proton’s spin

  - gluons? orbital angular momentum?

- Understanding how the nucleon is built from its quark and gluon constituents remains one the most important and challenging questions in modern nuclear physics.

- Lattice has a big role to play in tackling these questions.

Motivation for Investigation of Hadron Structure

- A lot of progress in computing nucleon and pion observables on the Lattice

  see plenary by S. Syritsyn and other parallel talks, e.g. Alexandrou, Collins, Jäger, Najjar, ...

- Little has been done for the Hyperons

  [hep-lat/0604022 (CSSM, quenched)]
  [arXiv:0712.1214, 0812.4456 (HWL,KO)]
  [arXiv:0910.4190 (Lang et al.)]
  [arXiv:1209.6115 (Sasaki)]

- Only a few Hyperon properties have been measured experimentally

  - Magnetic moments
  - Charge radii Σ^-
  - Axial charges (not directly, experimental decay rates + $SU(3)_f$/ChPT)
  - Semileptonic decay rates

    $\Xi^0 \to \Sigma^+ \ell \nu_\ell$ $\quad$ $\Sigma^- \to n \ell \nu_\ell$

    $\Xi^- \to \Lambda^0 \ell \nu_\ell$ $\quad$ $\Lambda^0 \to p \ell \nu_\ell$

- While on the Lattice, they should (in principle) be easier than nucleon observables

Motivation for Investigation of Hadron Structure

- Charge and magnetic distribution of hyperons

- Examine the role of SU(3) flavour symmetry breaking in these distributions

- Insights into the role of hidden flavour (e.g. strangeness in the proton)

- Lattice simulations currently in isospin-symmetric limit $m_u = m_d$

Recent progress in isospin-breaking effects in masses, etc (see plenary by Nazario)

- QCD is flavour-blind, so could think of the s quark as a very heavy d quark and compare lattice results for $\Sigma^+(uus)$ with $p(uud)$ to gain an idea of what to expect when we move to 1+1+1-flavour simulations

Idea behind the charge-symmetry violation determinations in [arXiv:1012.0215] and [arXiv:1204.3492]

Lattice Set-Up

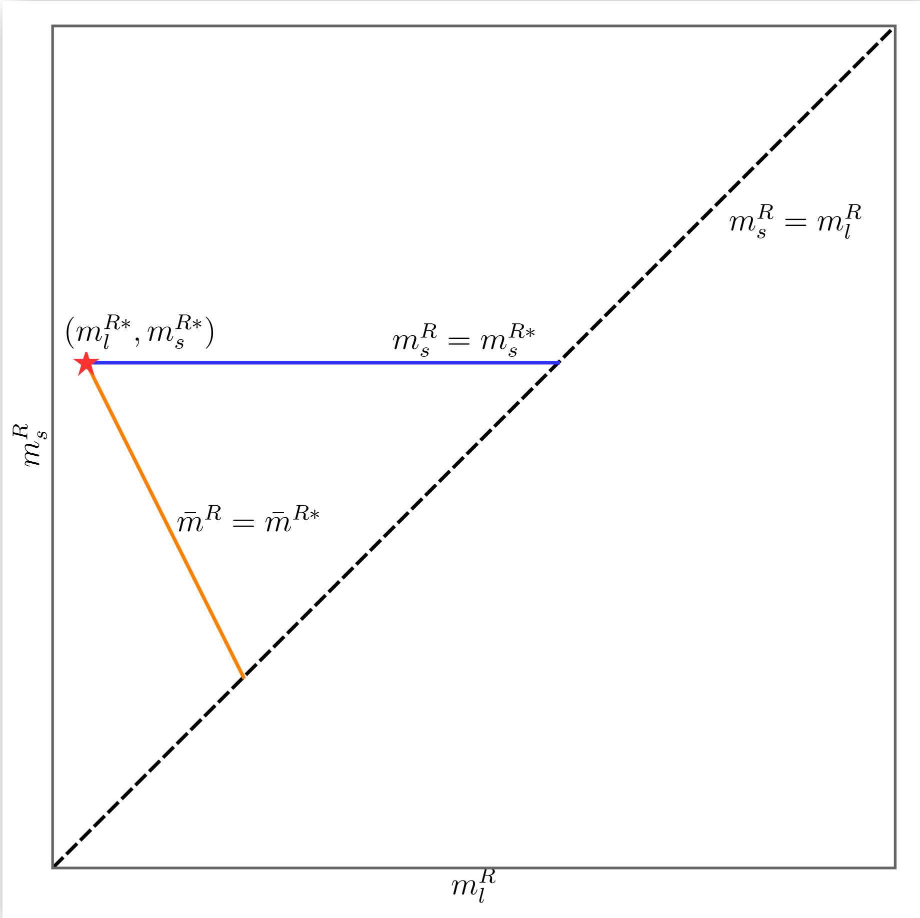
- N_f =2+1 O(a)-improved Clover fermions ("SLiNC" action)
- Tree-level Symanzik gluon action (plaq + rect)
- Most results from a single lattice spacing (a~0.08fm), but simulations and preliminary results becoming available at a~0.06fm
- Novel method for tuning the quark masses [arXiv:1003.1114 (PLB), 1102.5300 (PRD)]

  See also parallel talk by R.Horsley

- No study of systematics (disconnected, excited states, ...)
    - Hopefully similar for the octet

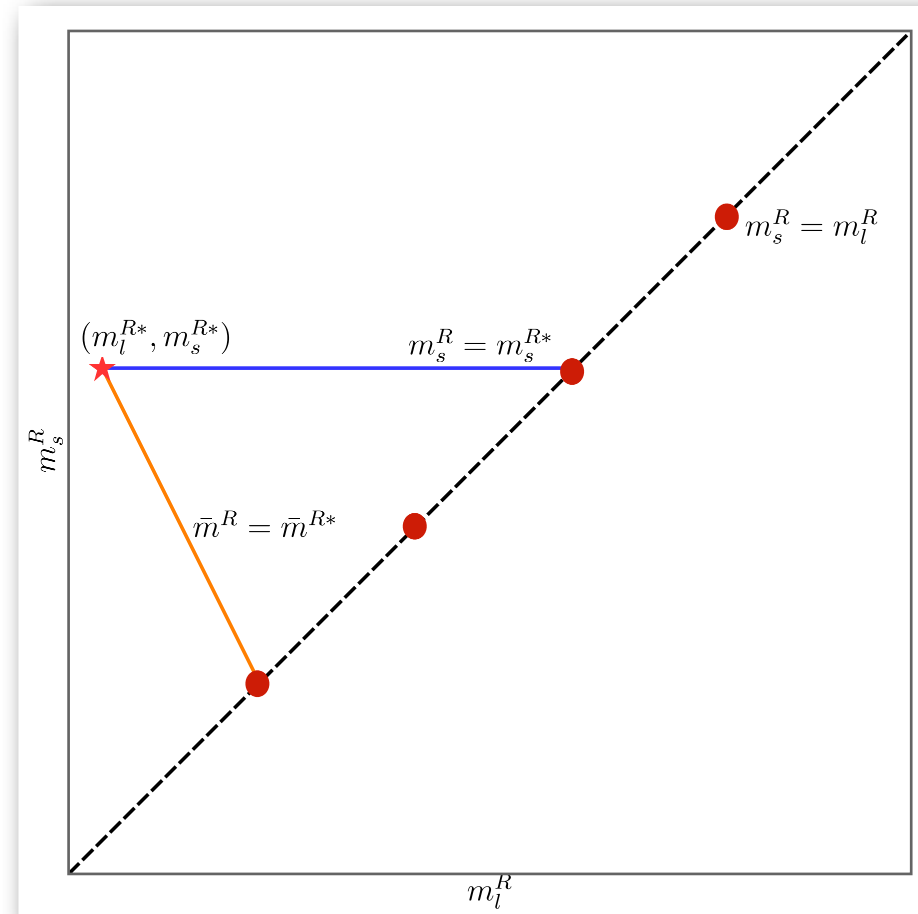
Tuning 2+1

- Need to choose a path to physical point



Tuning 2+1

- Need to choose a path to physical point
- Start from a point on the SU(3)-symmetric line



Tuning 2+1

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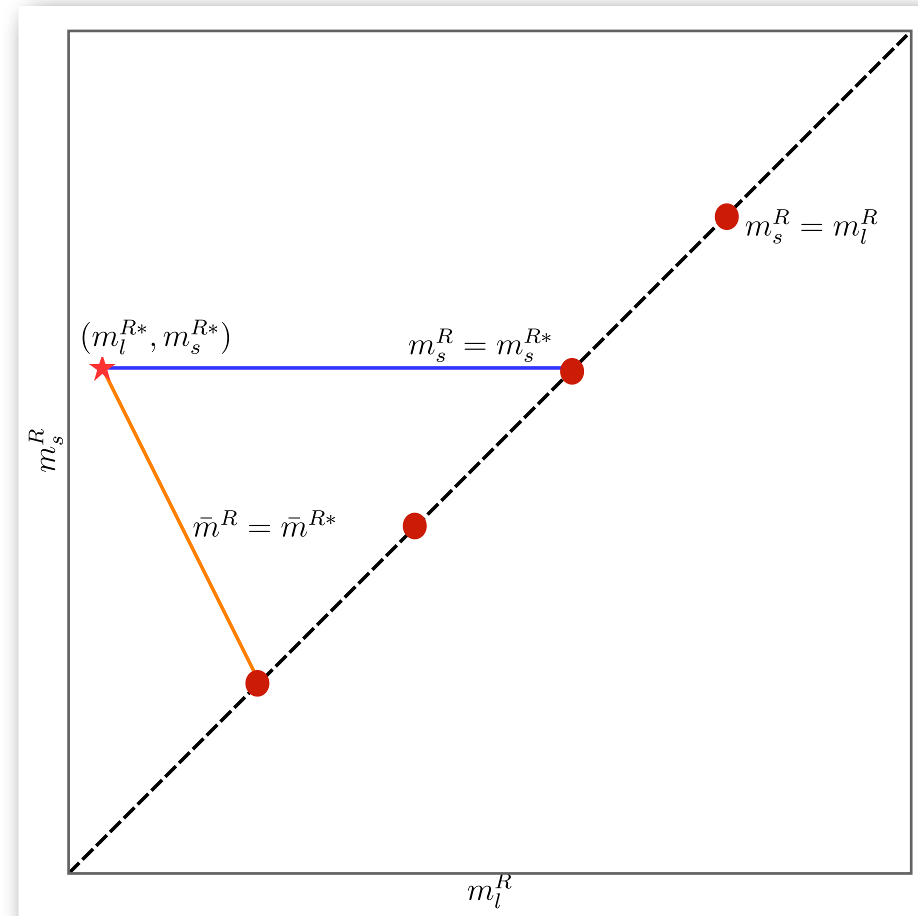
- Extrapolate to physical point by
    - e.g. keeping m_s fixed



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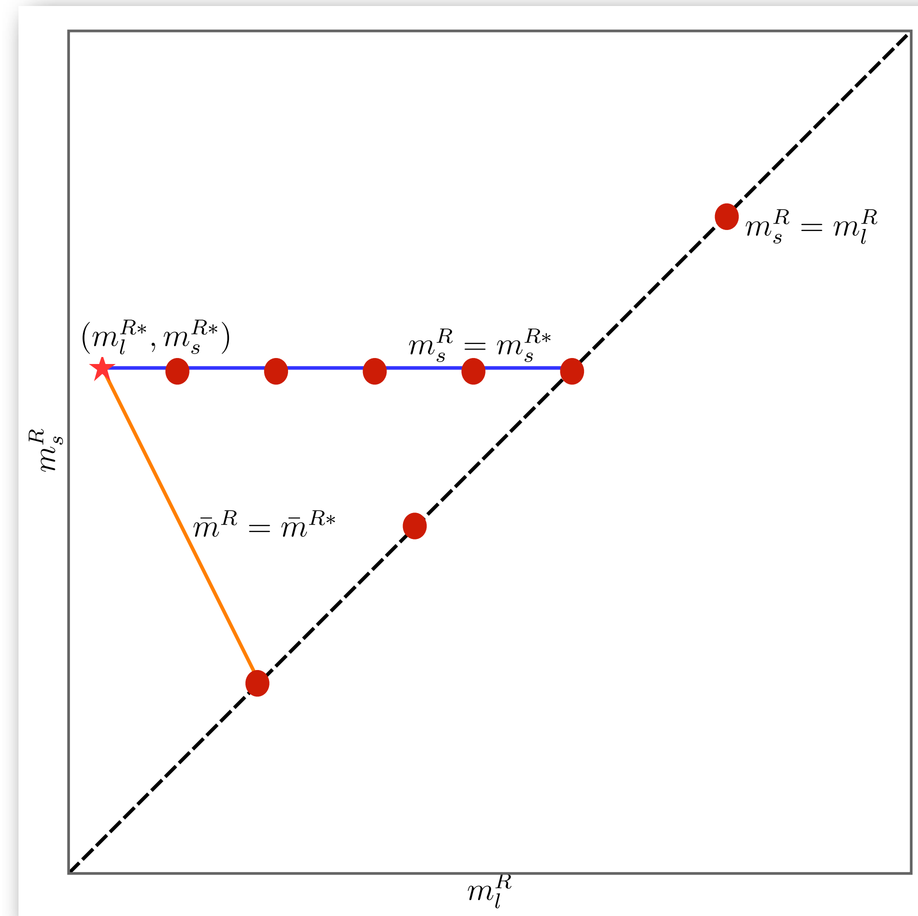
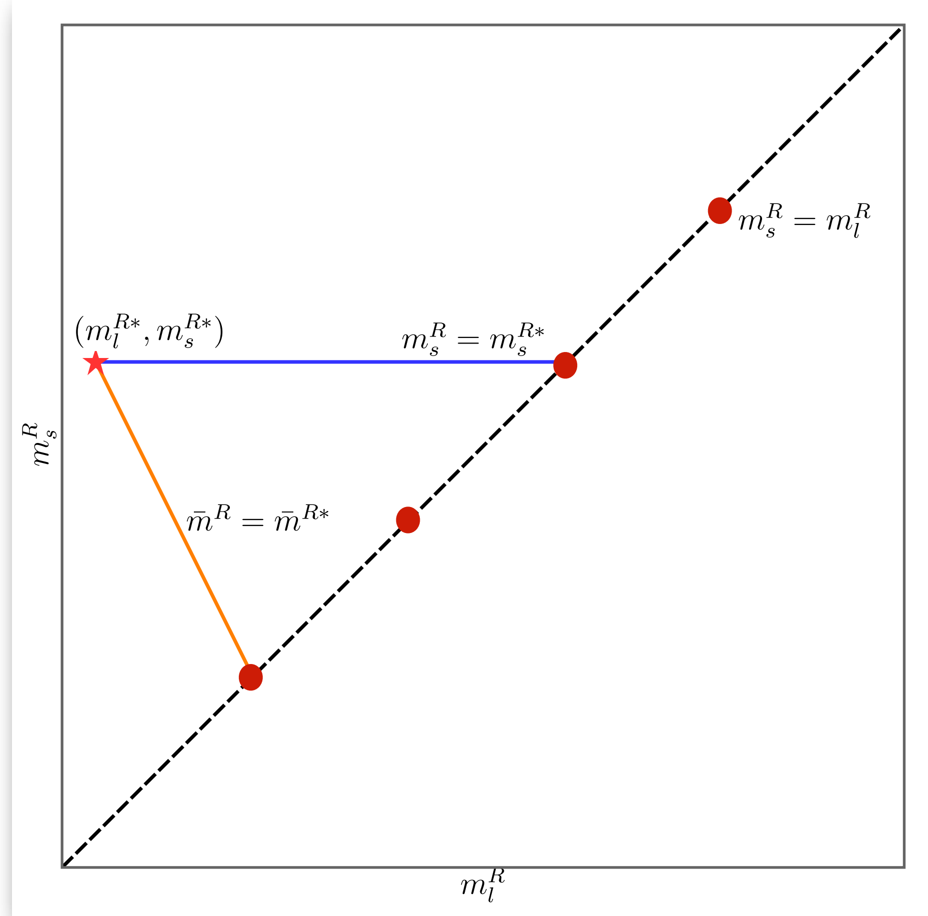


Tuning 2+1

- Need to choose a path to physical point
- Start from a point on the SU(3)-symmetric line
- Our choice is
  - to keep the singlet quark mass fixed

$$\overline{m}^R = \frac{1}{3}(2m_l^R + m_s^R)$$



- Several benefits:
  - Any flavour singlet quantity can be used to set the scale $(r_0, X_\pi, X_N, t_0, w_0, ...)$
  - Simplified SU(3)-flavour expansions [arXiv:1102.5300 (PRD)]
  - Kaon mass approached its physical value from below (better convergence of SU(3) ChPT?

A. Schäfer (Wed, 5D, 08:50)

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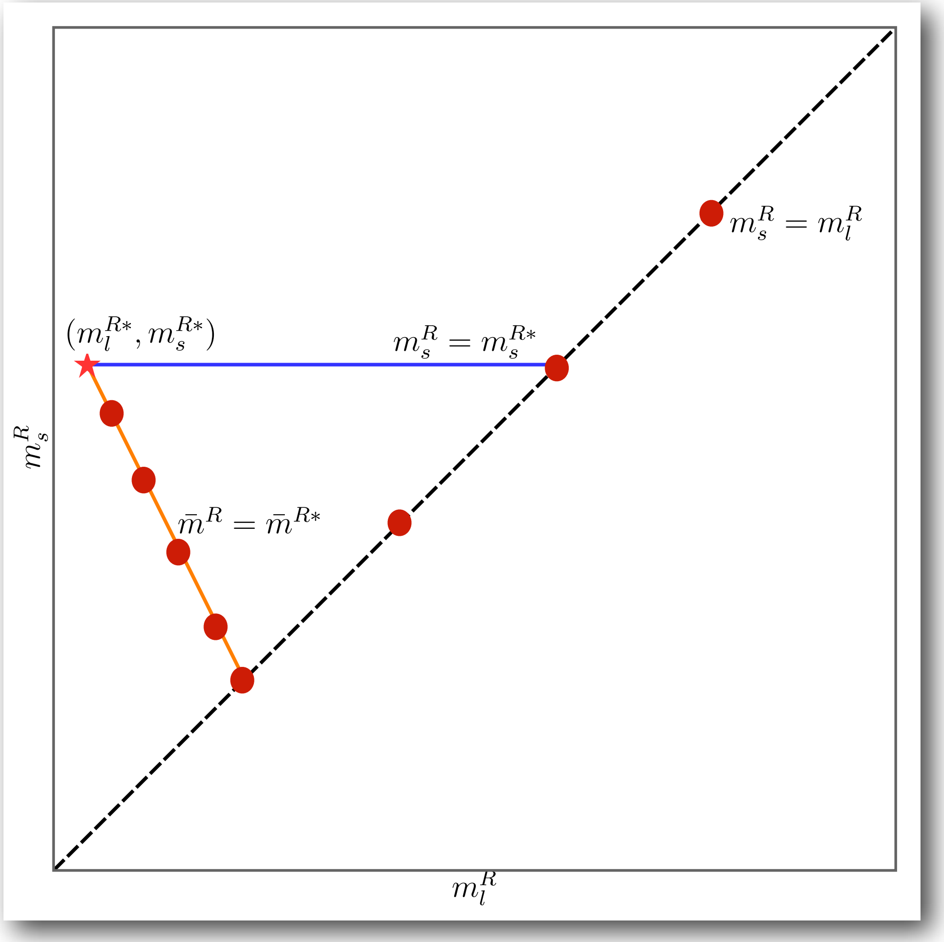

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Simulation parameters

$$\beta = 5.50, \kappa_0 = 0.12090$$

κ_l	κ_s	$N_S^3 \times N_T$	(X_π) a [fm]	m_π [MeV]	m_K [MeV]	$m_\pi L$	
0.120830	0.121040	$24^3 \times 48$		448	391	4.64	#
0.120900	0.120900	$24^3 \times 48$		411	411	4.27	#
0.120950	0.120800	$24^3 \times 48$	0.083	382	424	3.99	#
0.121000	0.120700	$24^3 \times 48$		350	438	3.65	#
0.121040	0.120620	$24^3 \times 48$		324	448	3.38	#
0.120900	0.120900	$32^3 \times 64$		411	411	5.59	
0.121040	0.120620	$32^3 \times 64$	0.083	319	450	4.32	
0.121095	0.120512	$32^3 \times 64$		276	464	3.72	
0.121145	0.120413	$32^3 \times 64$		231	480	3.10	
0.121166	0.120371	$48^3 \times 96$	0.083	205	485	4.11	*
0.122810	0.122810	$32^3 \times 64$	0.060	411	411	3.85	
0.122810	0.122810	$48^3 \times 96$	0.060	411	411	5.77	*

Poster by A. Cooke: Results for more extensive set of observables + $SU(3)_f$ breaking expansions

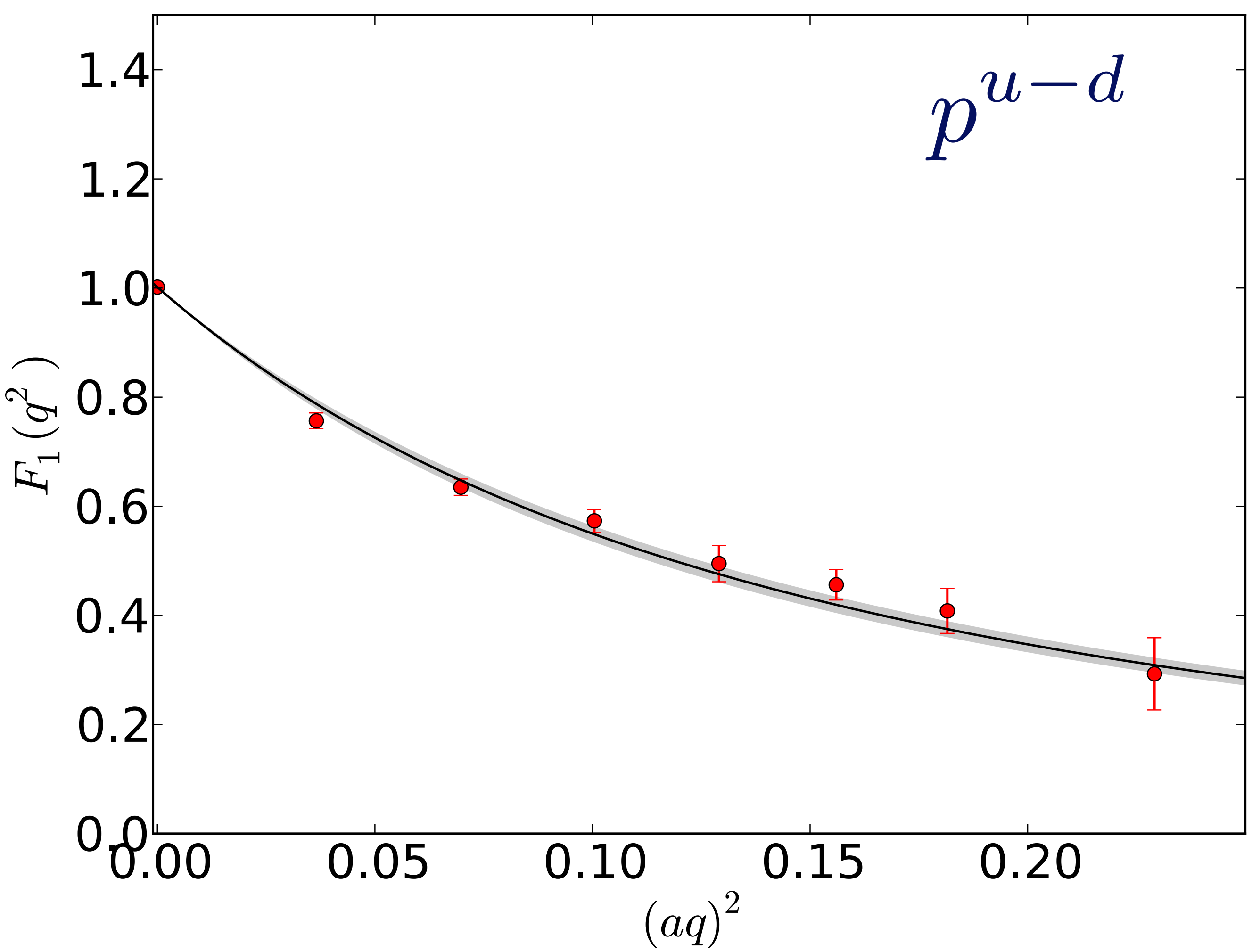
* Currently running

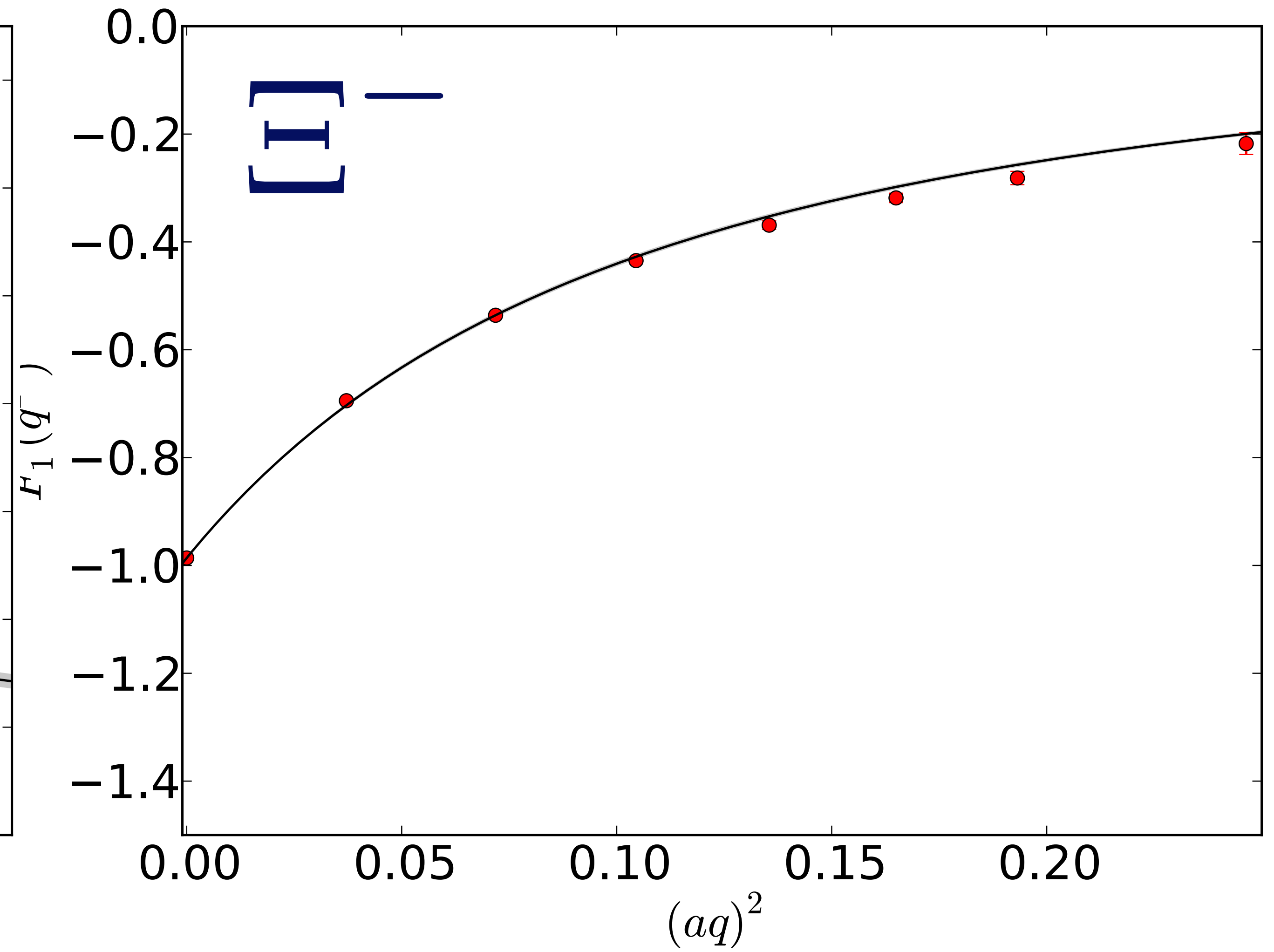
Hyperon Form Factors

Hyperon Form Factors

$m_\pi = 280$ [MeV], $32^3 \times 64$

Dipole: $\boxed{F_1(Q^2) = \frac{F(0)}{(1+Q^2/M^2)^2}}$





$F_1(Q^2)$

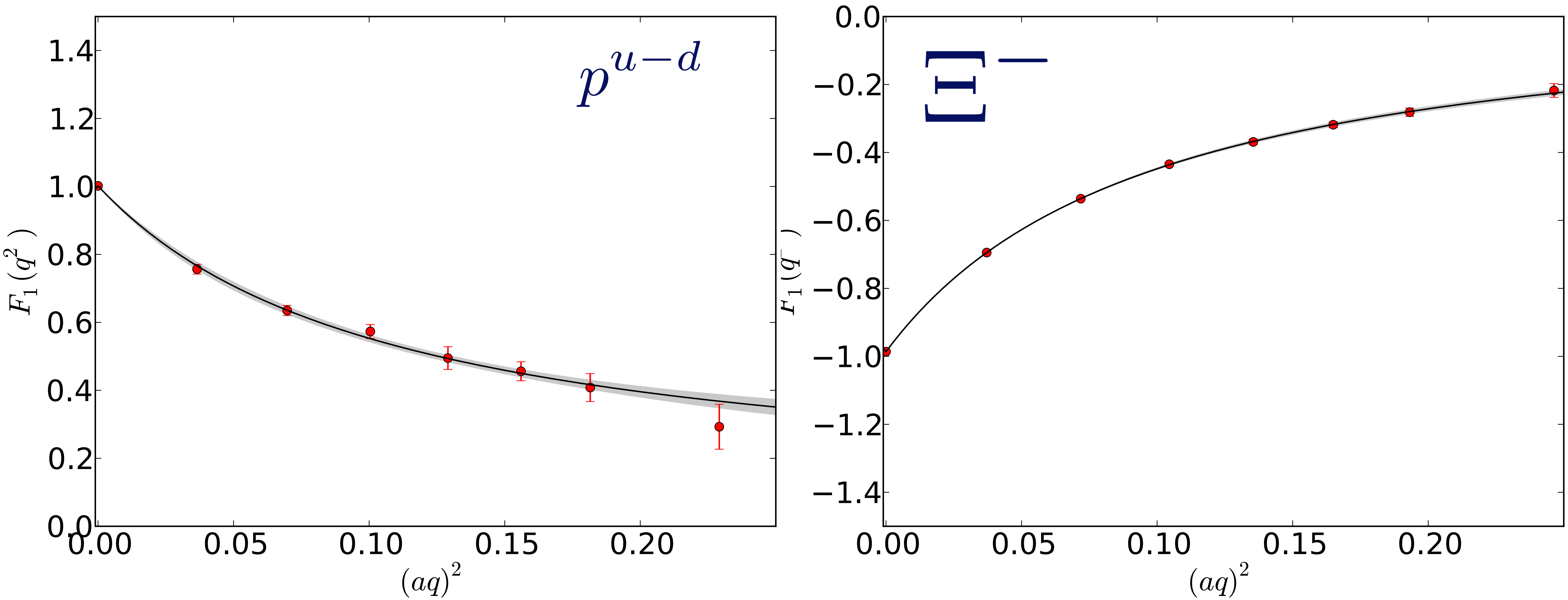
Hyperon Form Factors

$m_\pi = 280$ [MeV], $32^3 \times 64$

PRD 84, 074507 (2011) [arXiv:1106.3580]

"Kelly"

$$\boxed{F_1(Q^2) = \frac{F_1(0)}{(1 + c_{12}Q^2 + c_{14}Q^4)}}$$

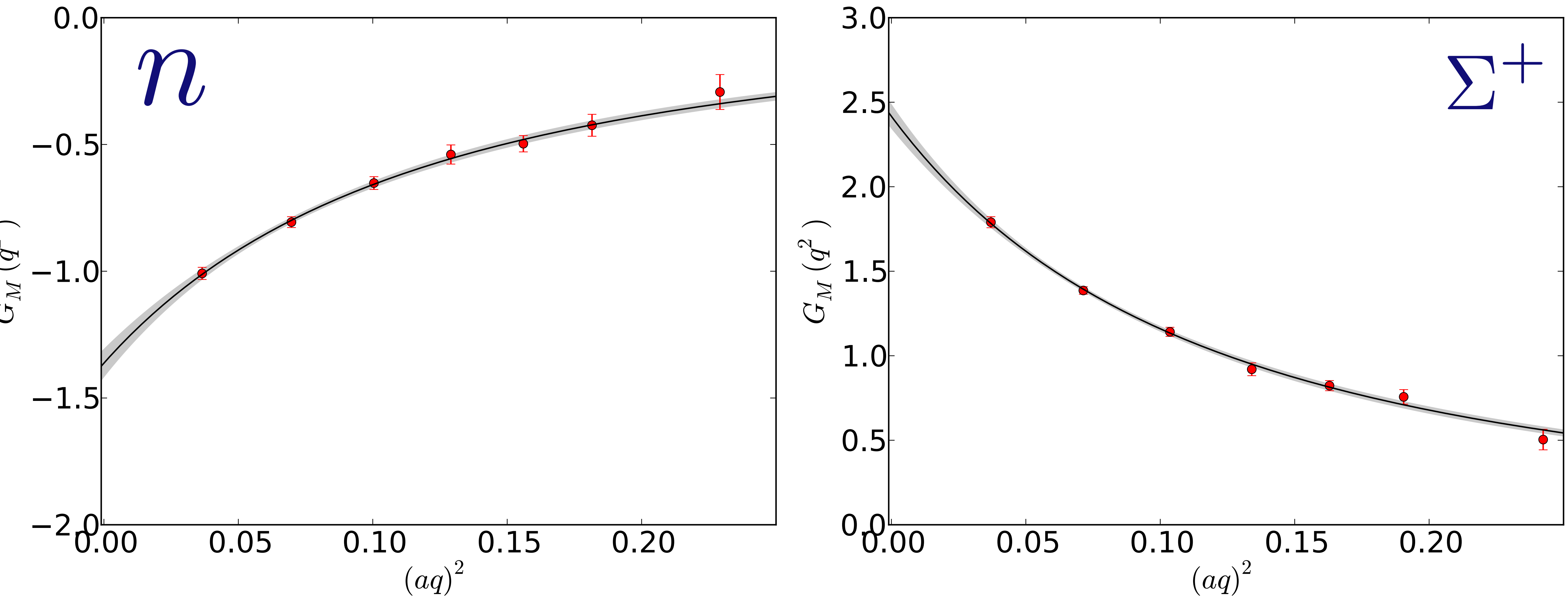


$$F_1(Q^2)$$

Hyperon Form Factors

$m_\pi = 280$ [MeV], $32^3 \times 64$

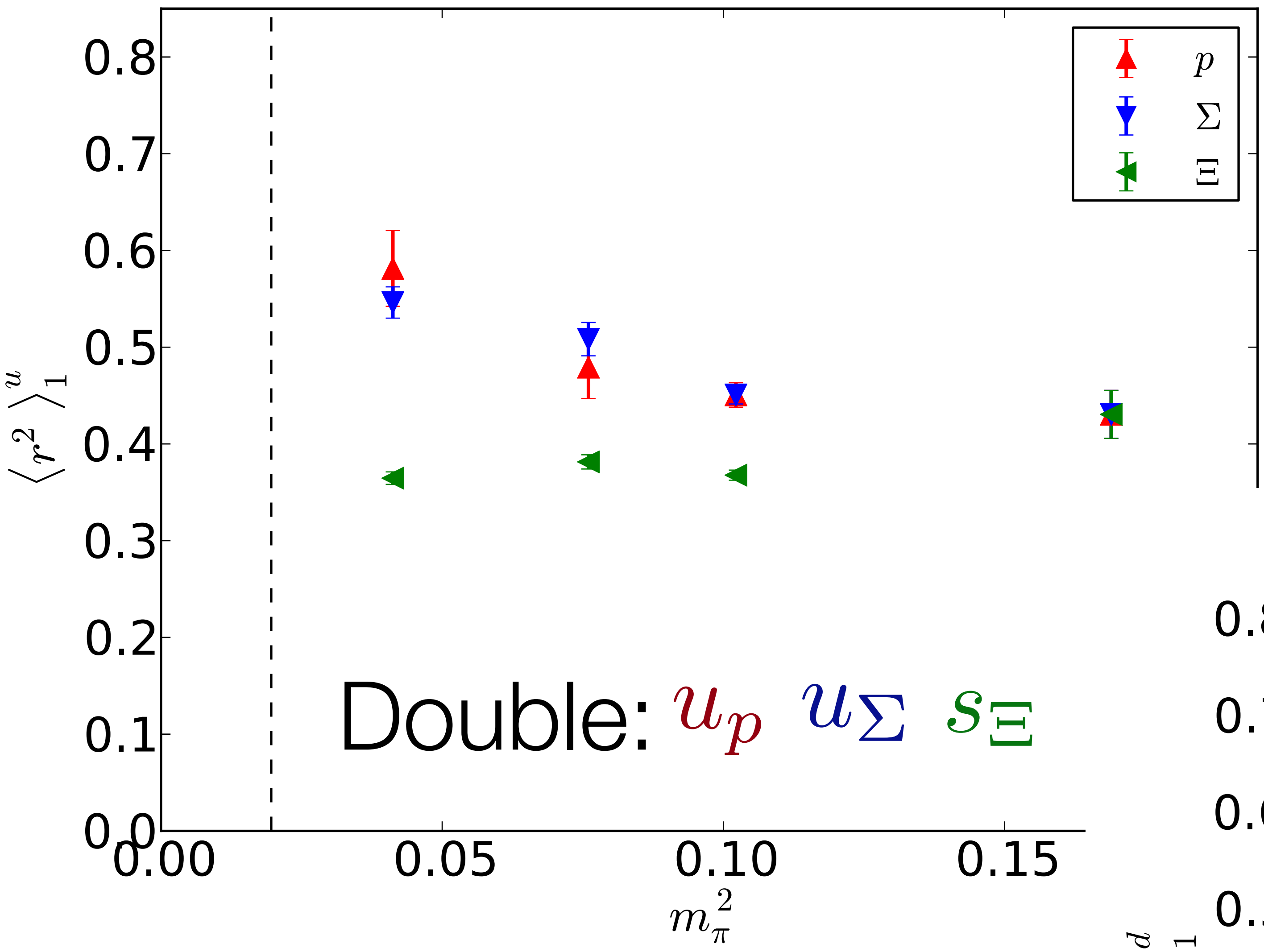
Dipole: $$\frac{\mu}{(1 + Q^2/M^2)^2}$$

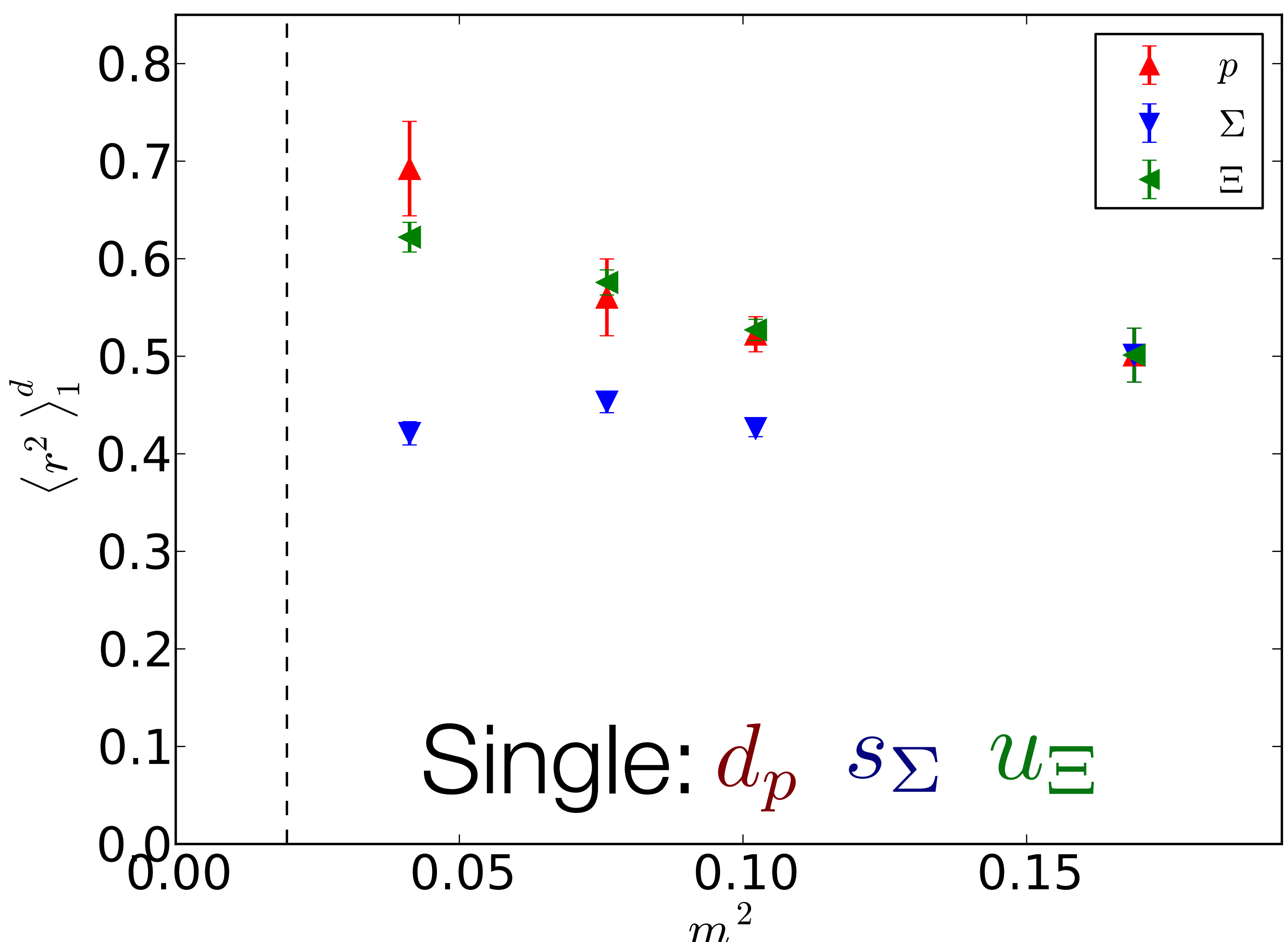


$$G_M(Q^2)$$

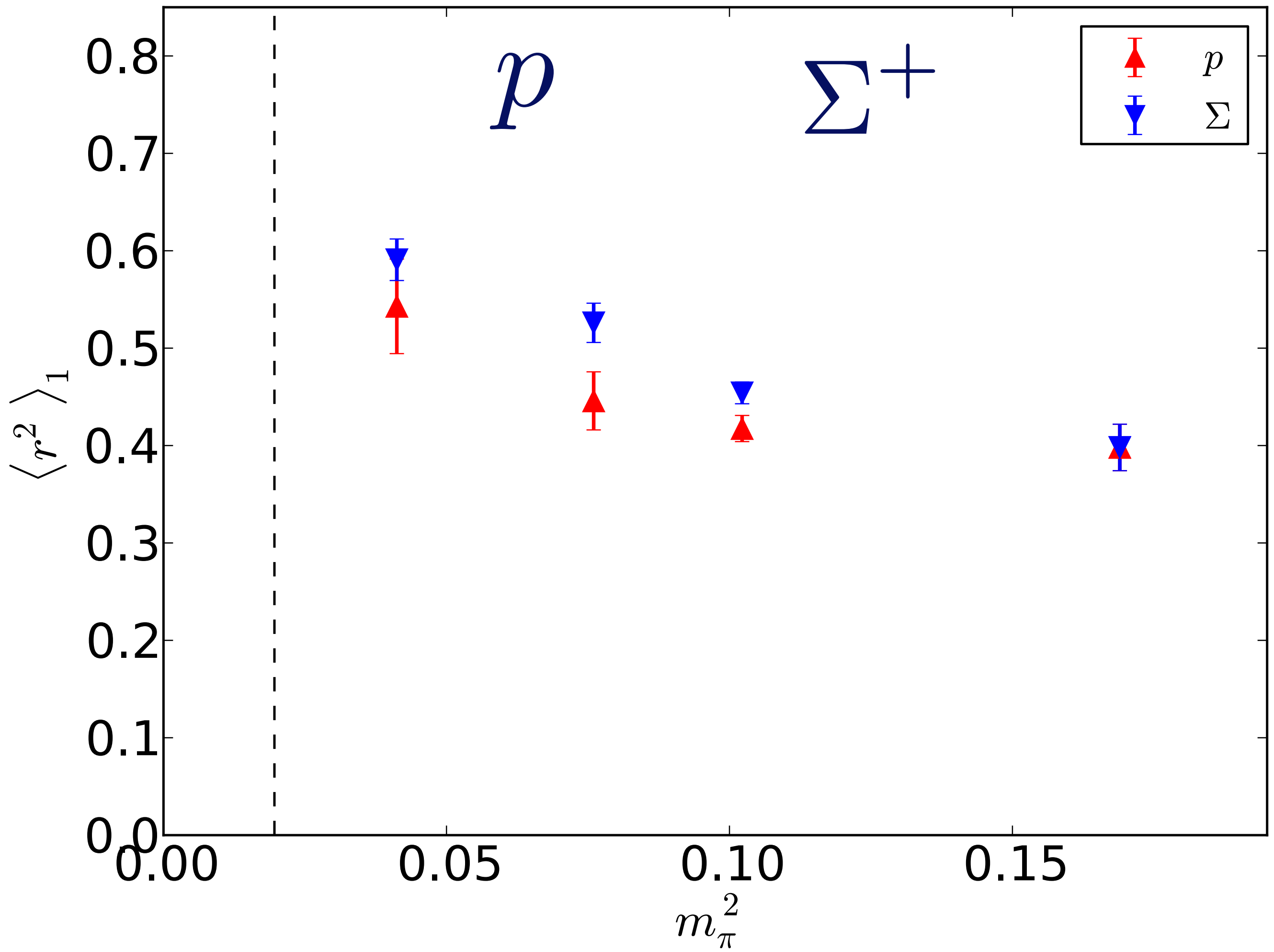
Quark Sectors

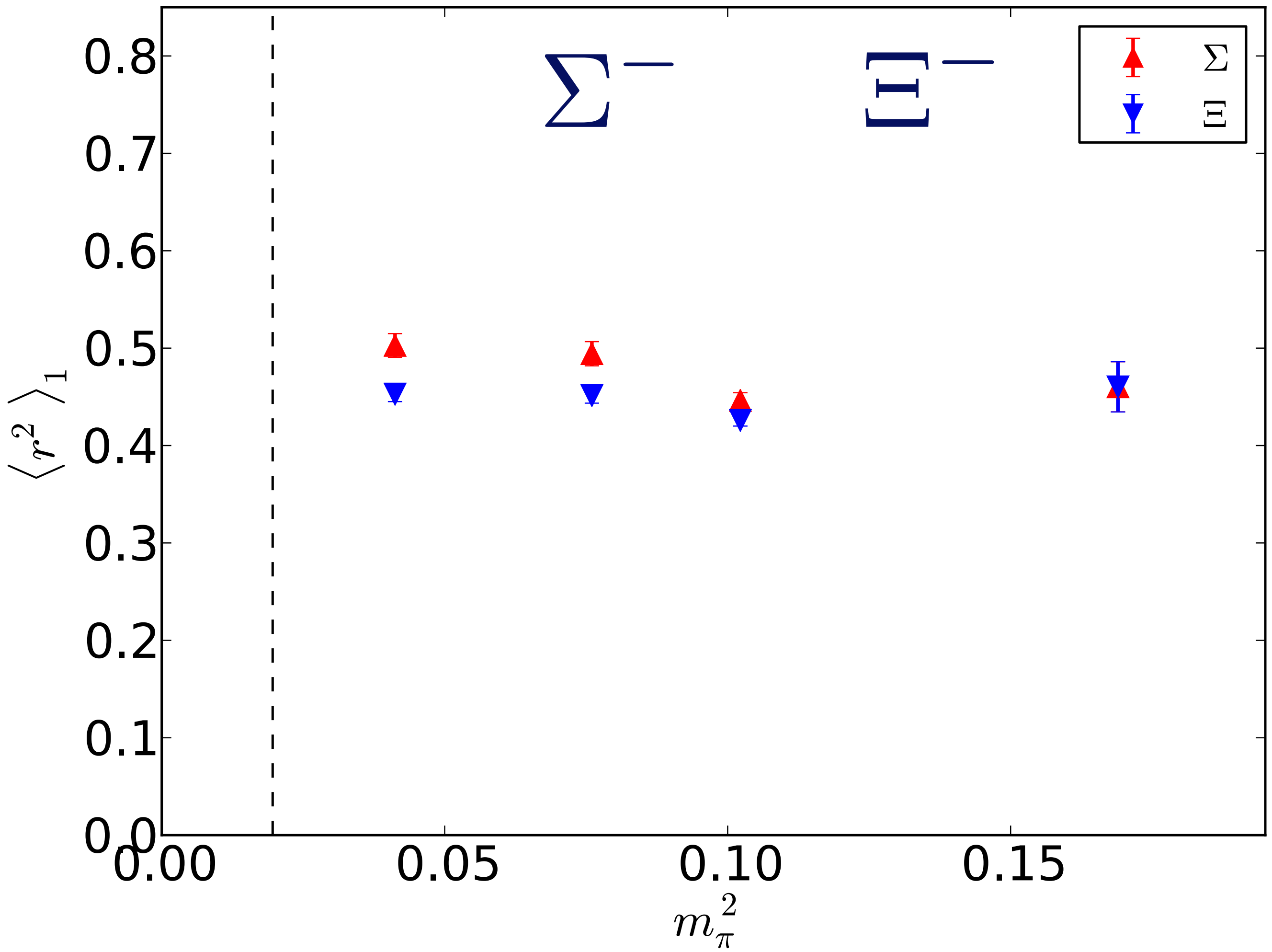
$\langle r^2 \rangle_1^q$





Hyperon Charge Radii





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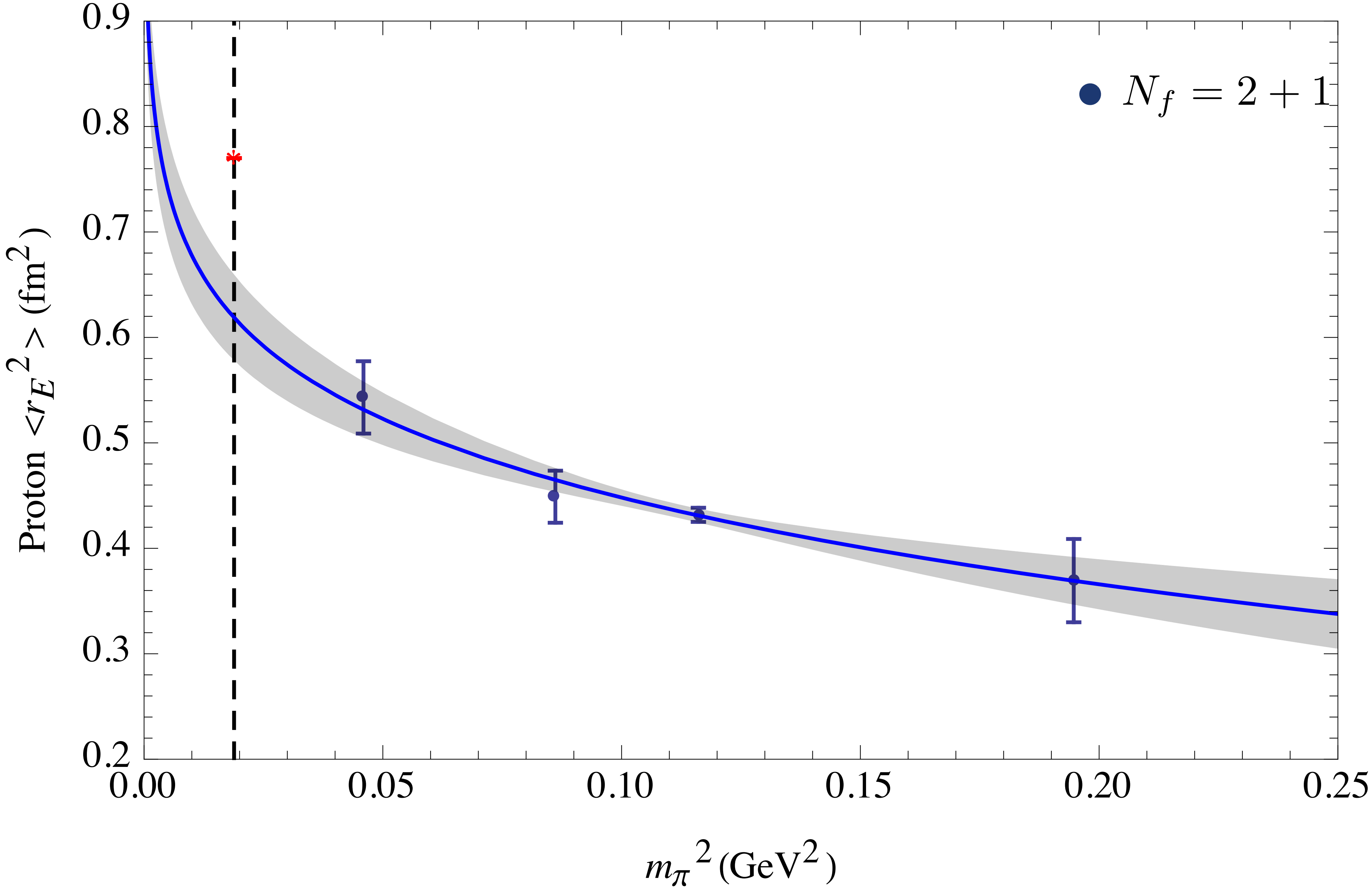
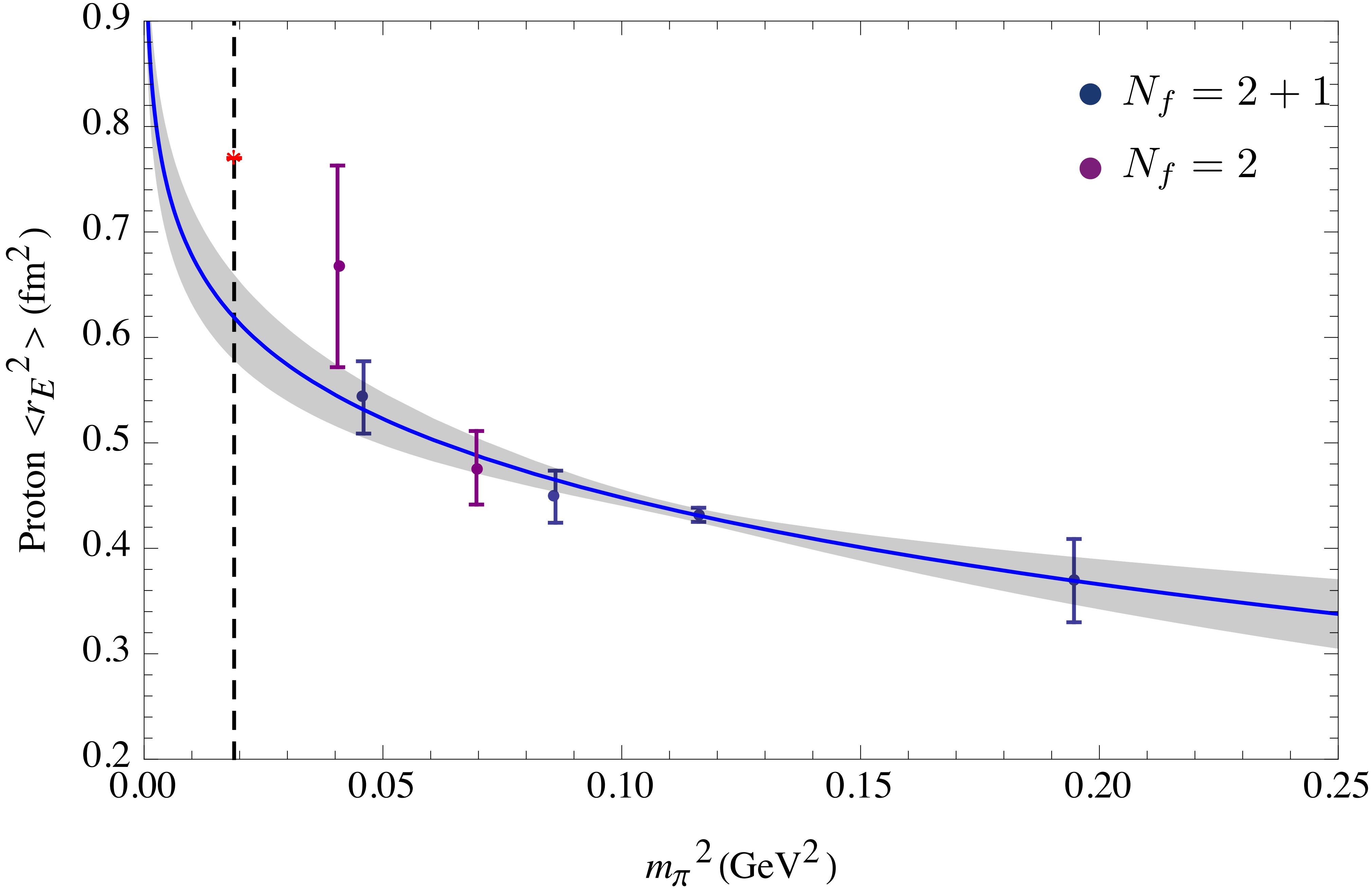


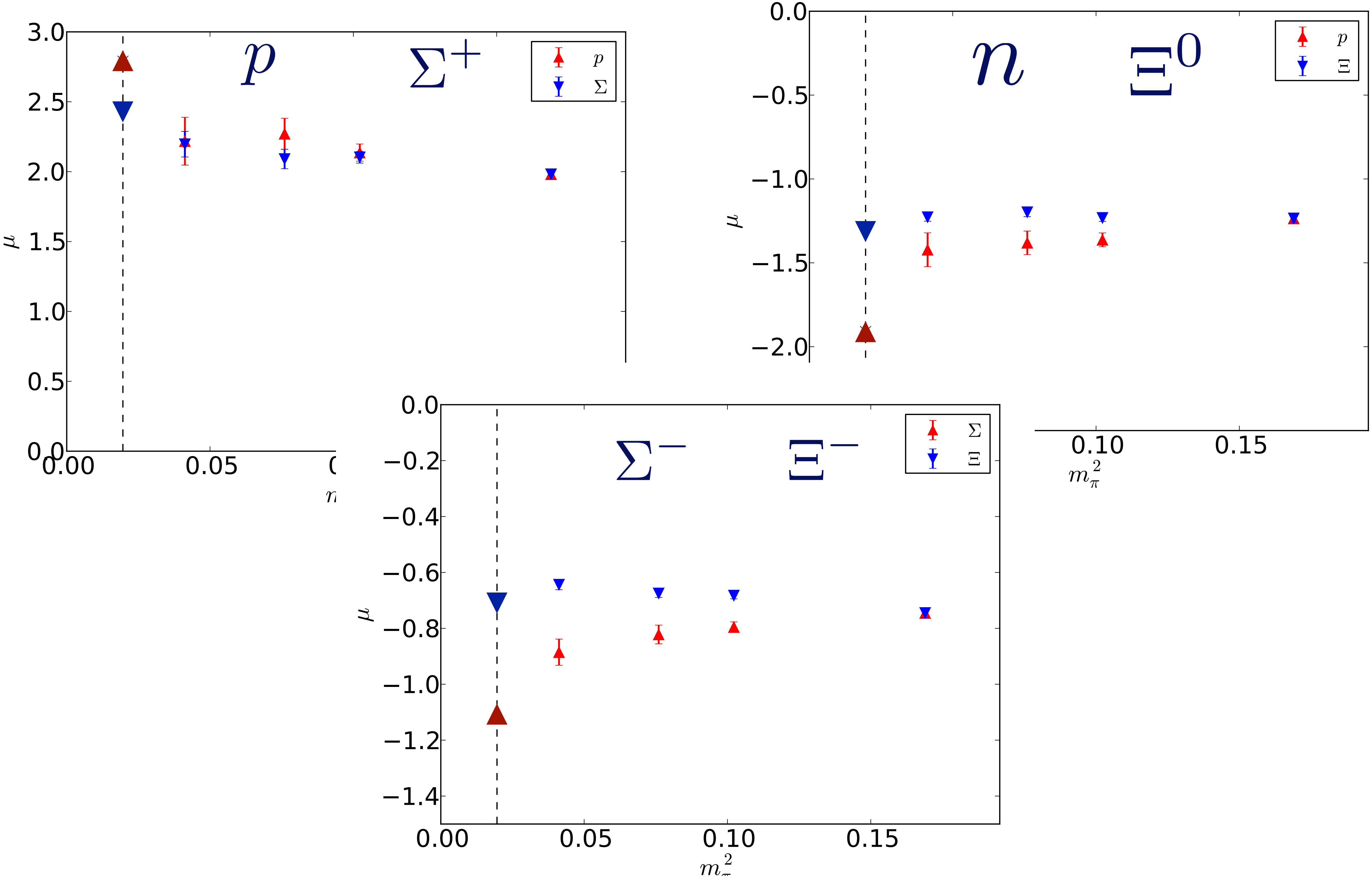
Fit using Finite Range Regularised ChPT by Phiala Shanahan

Hyperon Charge Radii



Fit using Finite Range Regularised ChPT by Phiala Shanahan

Hyperon Magnetic Moments



Hyperon Axial Charges

Hyperon Axial Charges

- Important for low-energy effective field theory description of octet baryons

- SU(3)$_f$

$$g_{A\,NN} = F + D, \qquad g_{A\,\Xi\Xi} = F - D, \qquad g_{A\,\Sigma\Sigma} = F,$$
$$g_{A\,\Lambda\Xi} = F - \frac{1}{3}D, \qquad g_{A\,\Sigma\Xi} = F + D,$$
$$g_{A\,\Lambda N} = F + \frac{1}{3}D, \qquad g_{A\,\Sigma N} = F - D, \qquad g_{A\,\Lambda\Sigma} = D.$$

- *D* and *F* enter chiral expansion of every baryonic quantity (e.g. masses, hyperon semi-leptonic decays, B-B' scattering phase shifts, ...)

- Poorly (or not at all) determined experimentally

- Quark Model F=0.46 , D=0.68 [K.-S.Choi, 1005.0337]

- Fits to Hyperon beta decay F=0.46 , D=0.8 [Close & Roberts, PLB316, 165 (1993)]

- ChPT, Large Nc predicts $0.3 \leq g_{\Sigma\Sigma} \leq 0.55$ $\quad 0.18 \leq -g_{\Xi\Xi} \leq 0.36$

Hyperon Spin Content

- Proton "Spin Crisis": only 33(3)(5)% of the proton spin carried by quarks
- Is this suppression a property of the nucleon, or a universal feature?
- Do we observe $SU(3)_f$ breaking effects

$$
\begin{aligned}
F_1 \equiv \frac{1}{\sqrt{3}}(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}) &= 2f - \frac{2}{\sqrt{3}} s_2 \delta m_l \\
F_2 \equiv (A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}) &= 2f + 4 s_1 \delta m_l \\
F_3 \equiv A_{\bar{\Sigma}\pi\Sigma} &= 2f + (-2s_1 + \sqrt{3} s_2)\delta m_l \\
F_4 \equiv \frac{1}{\sqrt{2}} \Re (A_{\bar{\Sigma}K\Xi} - A_{\bar{N}K\Sigma}) &= 2f - 2 s_1 \delta m_l \\
F_5 \equiv \frac{1}{\sqrt{3}} \Re (A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}) &= 2f + \frac{2}{\sqrt{3}} (\sqrt{3} s_1 - s_2) \delta m_l \,.
\end{aligned}
$$

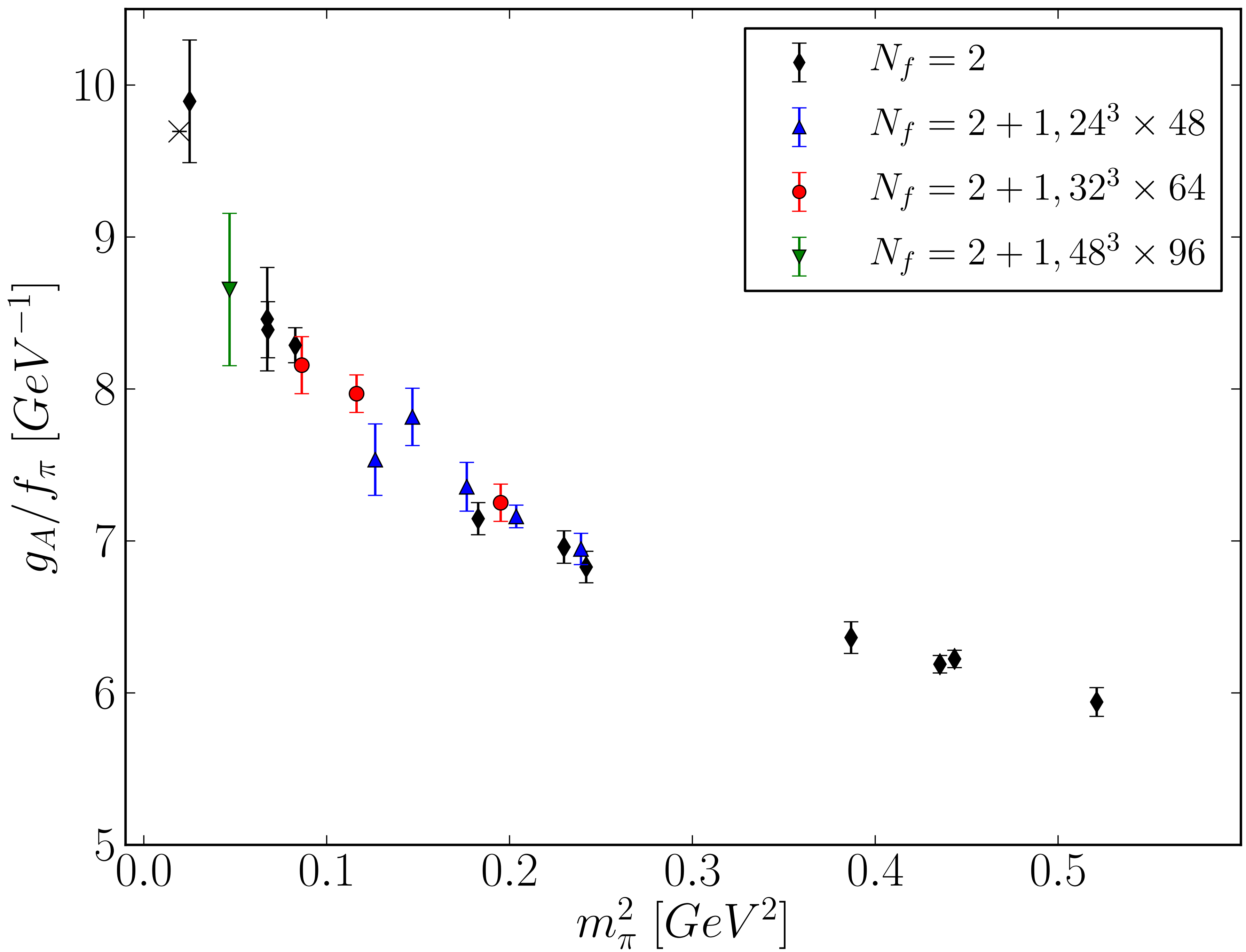
A. Cooke '13 poster,
Latt'12, 1212.2564

$$f = \frac{1}{\sqrt{2}} F$$

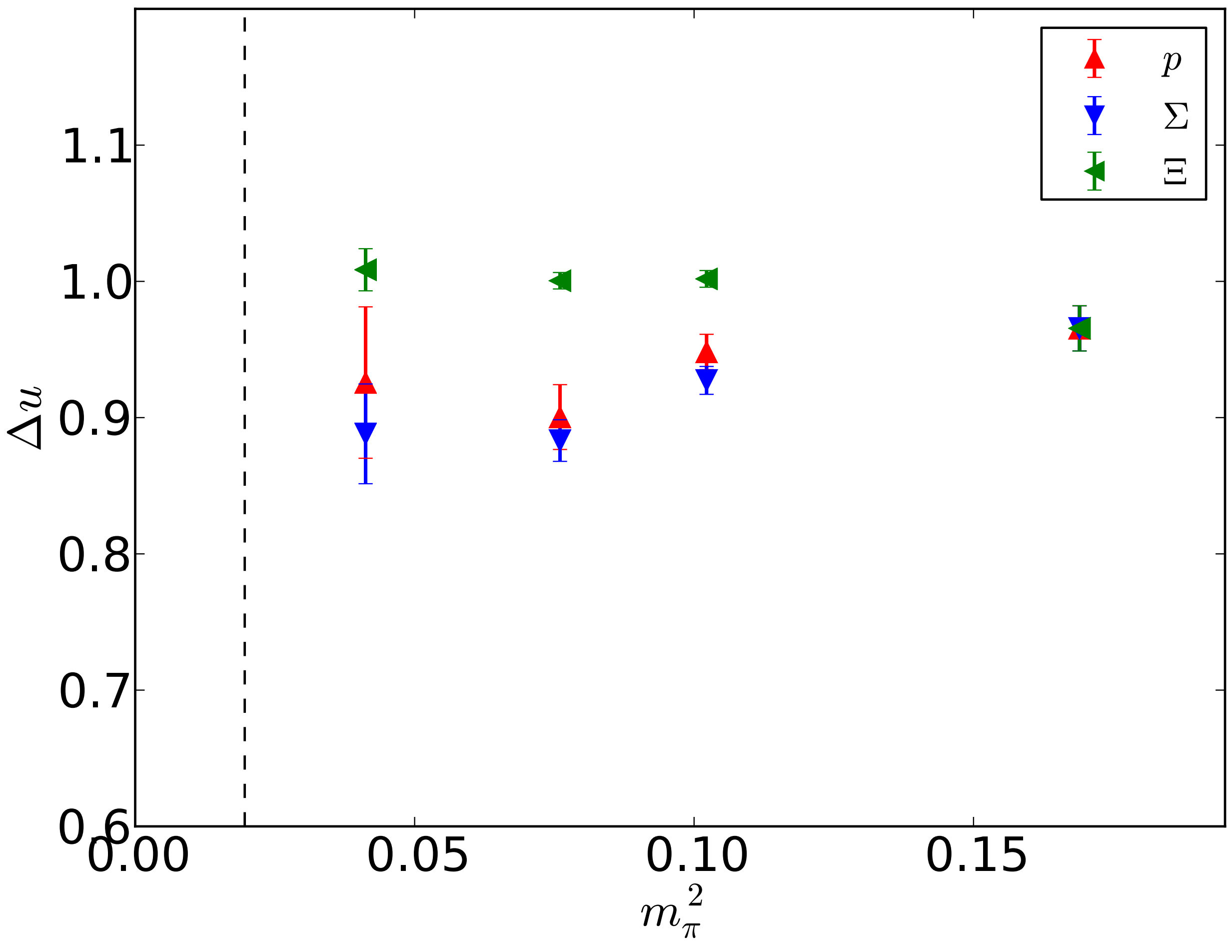
Nucleon Axial Charge, g_A

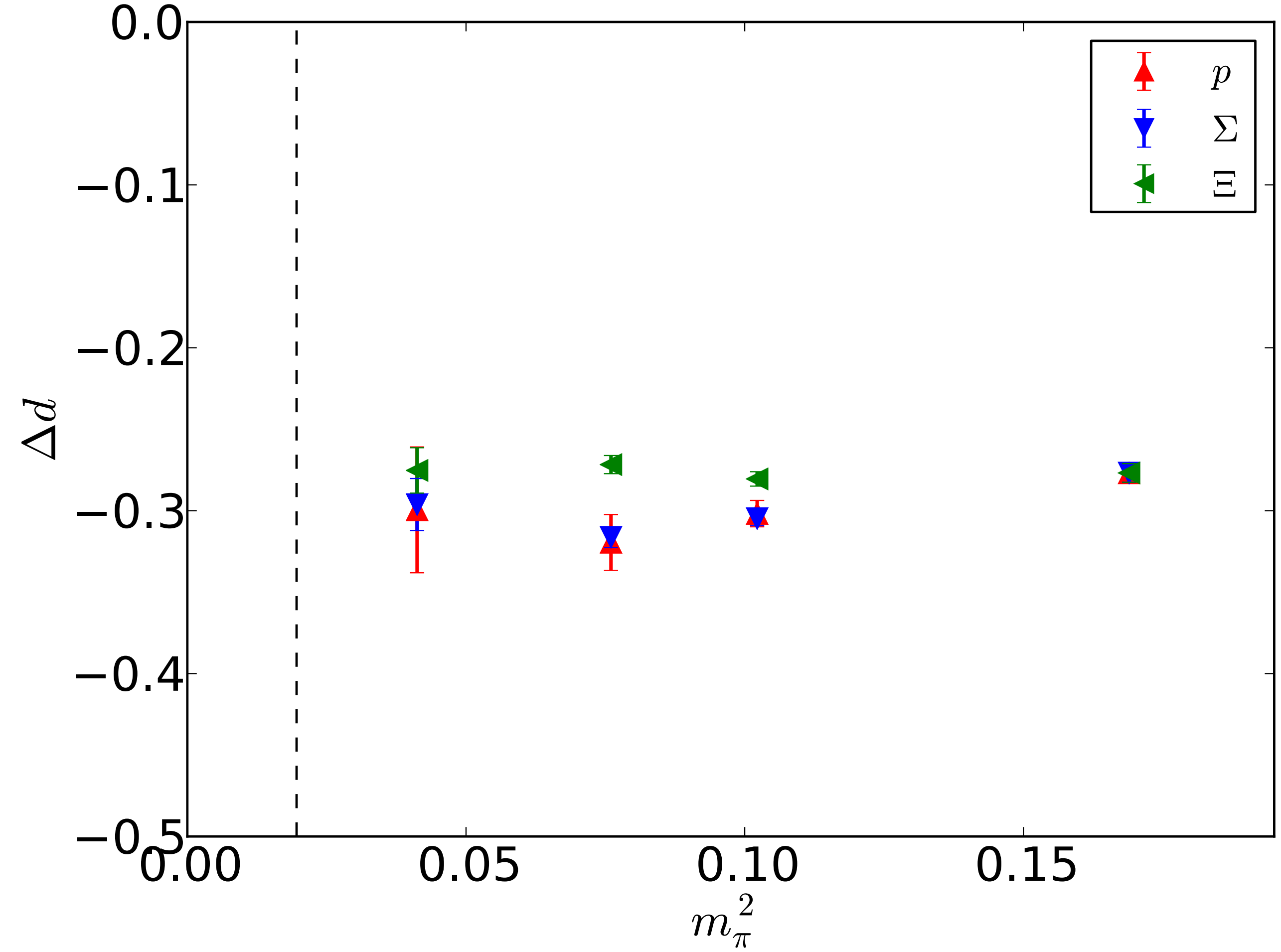
- Z_A almost complete (0.85-0.9)
- Cancels in ratio g_A/f_π
- Compare with N_f=2

[QCDSF: 1302.2233]



Quark Spin Contributions (Unrenormalised)

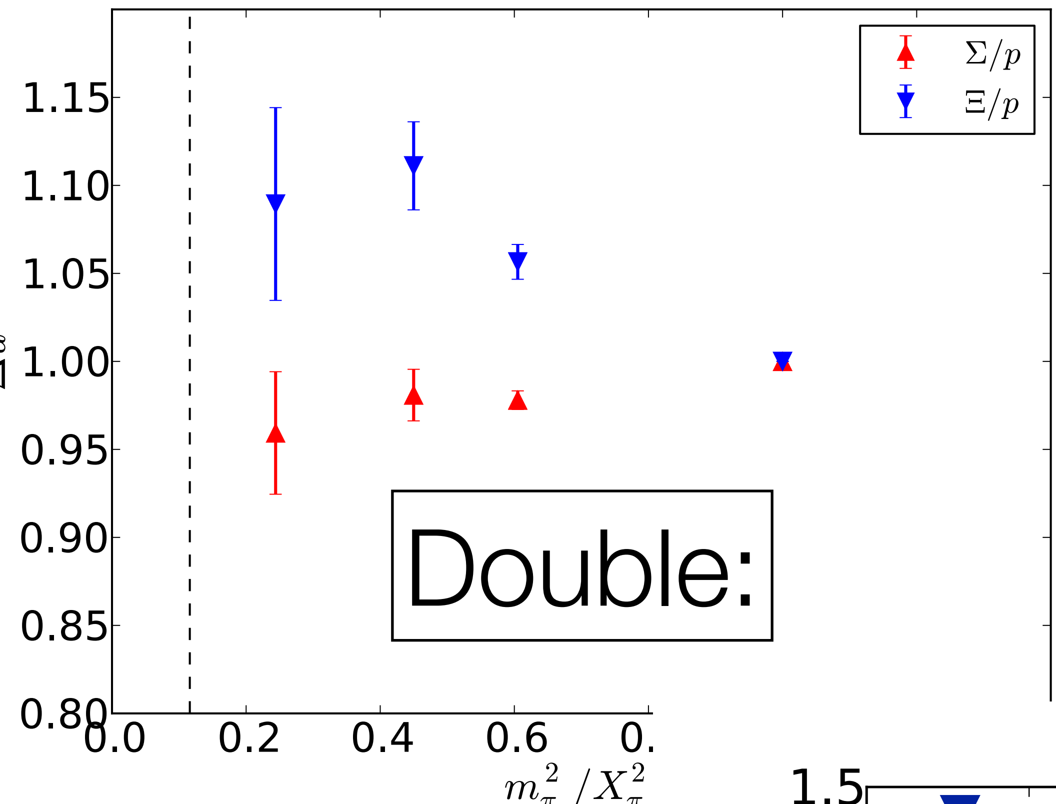


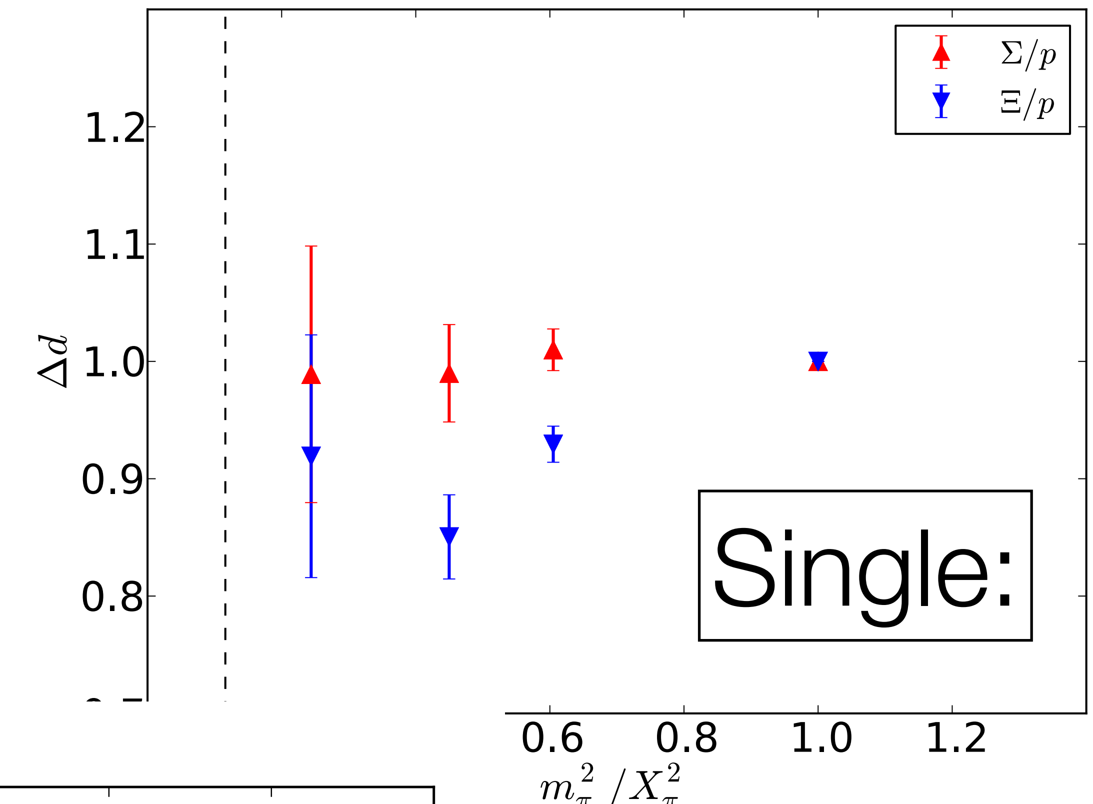


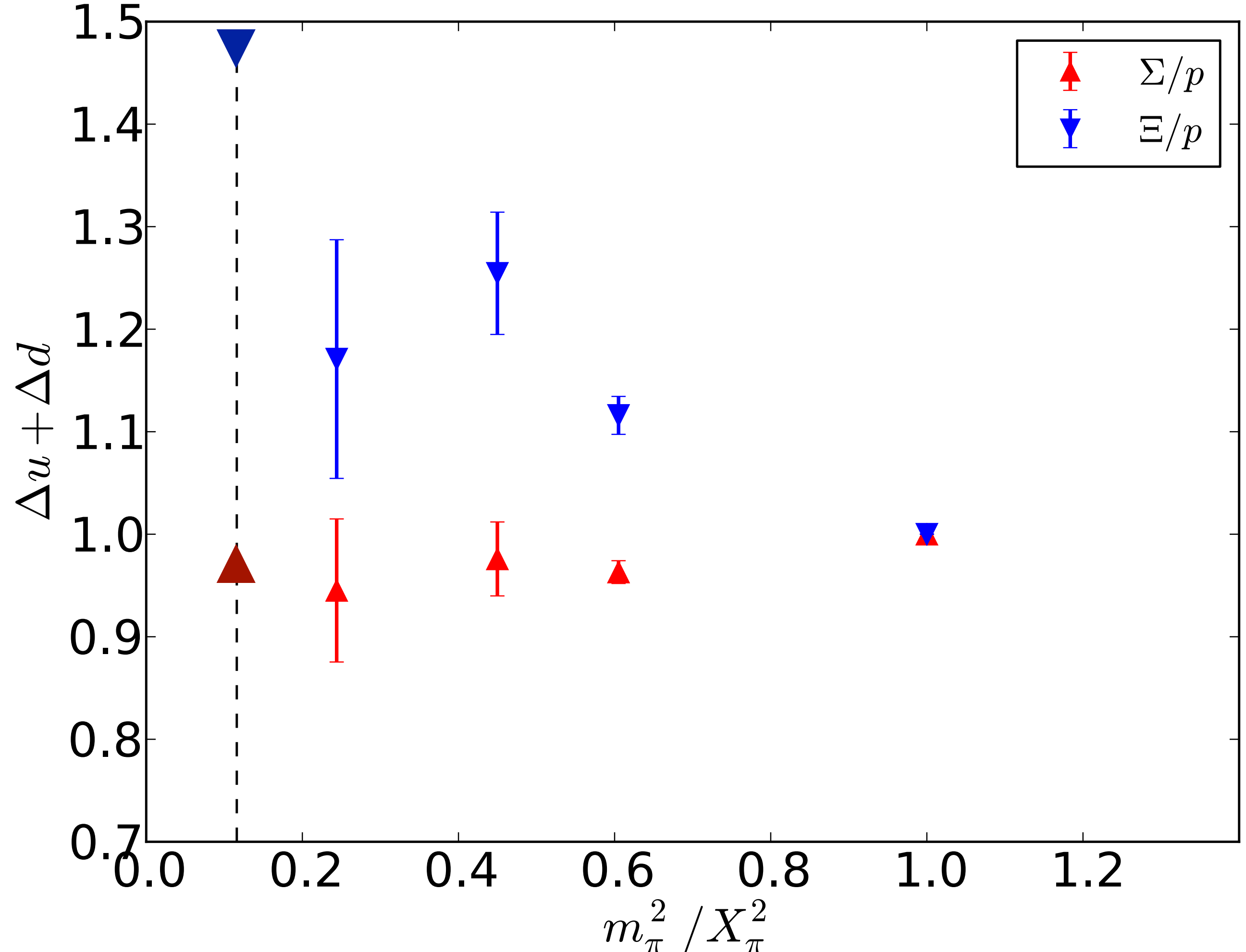
Double: u_p u_Σ s_Ξ

Single: d_p s_Σ u_Ξ

Quark Spin Contributions







[c.f. P.Shanahan et al., arXiv:1302.6300]

Spin suppression perhaps not universal

SU(3)$_f$ Expansions

A. Cooke '13 poster,
Latt'12, 1212.2564

$$
\begin{aligned}
F_1 &\equiv \frac{1}{\sqrt{3}}\left(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}\right) = 2f - \frac{2}{\sqrt{3}} s_2 \delta m_l \\
F_2 &\equiv \left(A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}\right) = 2f + 4 s_1 \delta m_l \\
F_3 &\equiv A_{\bar{\Sigma}\pi\Sigma} = 2f + \left(-2s_1 + \sqrt{3} s_2\right)\delta m_l \\
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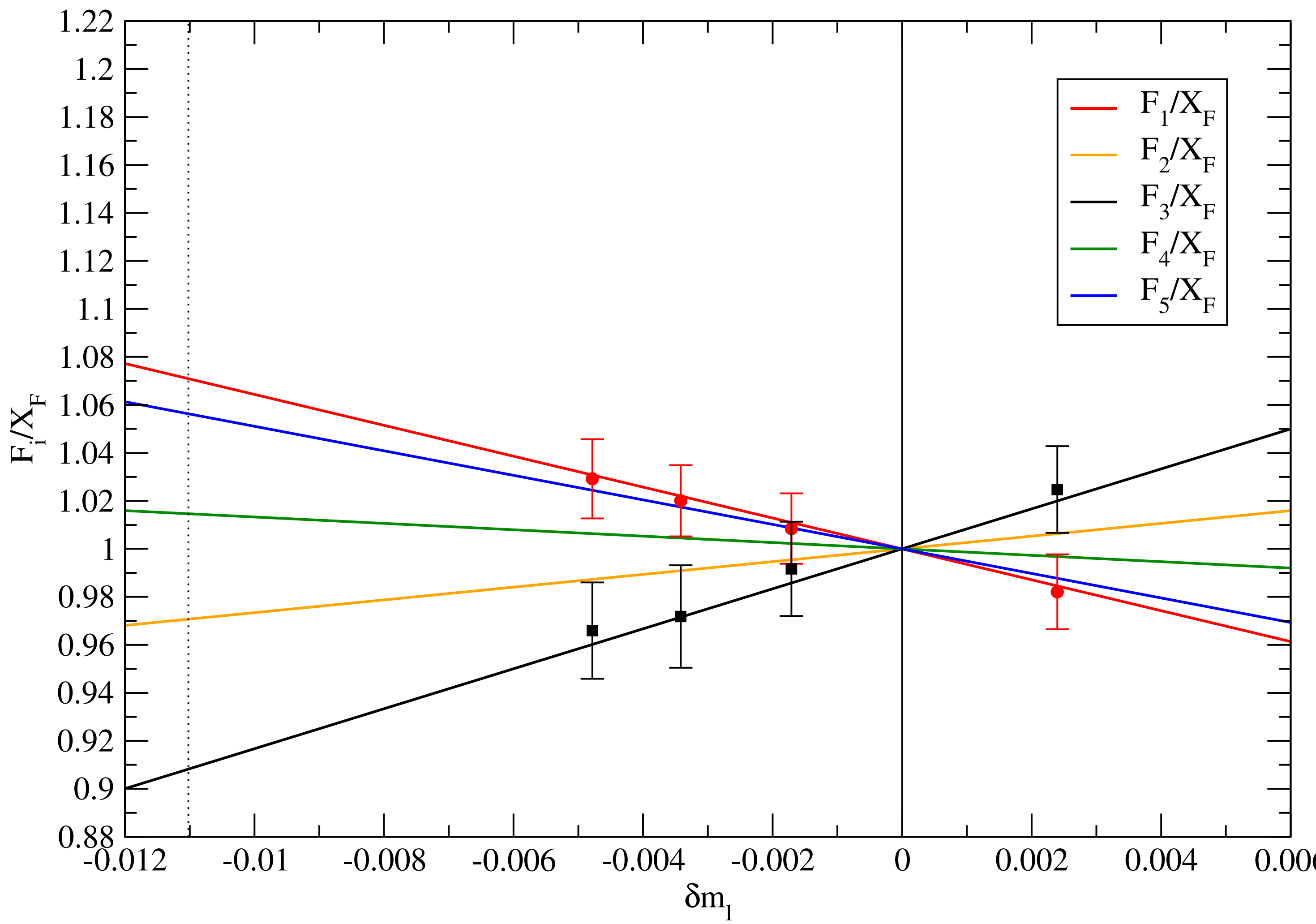
F_5 &\equiv \frac{1}{\sqrt{3}}\Re\left(A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}\right) = 2f + \frac{2}{\sqrt{3}}\left(\sqrt{3} s_1 - s_2\right)\delta m_l \,.
\end{aligned}
$$

$$\pi = \frac{1}{\sqrt{2}}(\bar{u}\gamma u - \bar{d}\gamma d)$$

$$\eta = \frac{1}{\sqrt{6}}(\bar{u}\gamma u + \bar{d}\gamma d - 2\bar{s}\gamma s)$$



Semileptonic Form Factors

Hyperon Semileptonic Form Factors

$$B \to b\ell\nu$$

- Provides an alternative method to K_{l3} for determining $|V_{us}|$
  - Using the experimental value for

$$|V_{us} f_1(0)|^2 \left(1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2\right)$$

  - Obtain f_1 and g_1/f_1 from lattice matrix elements

$$\langle b(p')|V_\mu(x) + A_\mu(x)|B(p)\rangle = \overline{u}_b(p')(\mathcal{O}_\mu^V(q) + \mathcal{O}_\mu^A(q))u_B(p)e^{iq\cdot x}$$

$$\mathcal{O}_\mu^V(q) = f_1(q^2)\gamma_\mu + f_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_b + M_B} + f_3(q^2)i\frac{q_\mu}{M_b + M_B}$$

$$\mathcal{O}_\mu^A(q) = g_1(q^2)\gamma_\mu\gamma_5 + g_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_b + M_B}\gamma_5 + g_3(q^2)i\frac{q_\mu}{M_b + M_B}\gamma_5$$

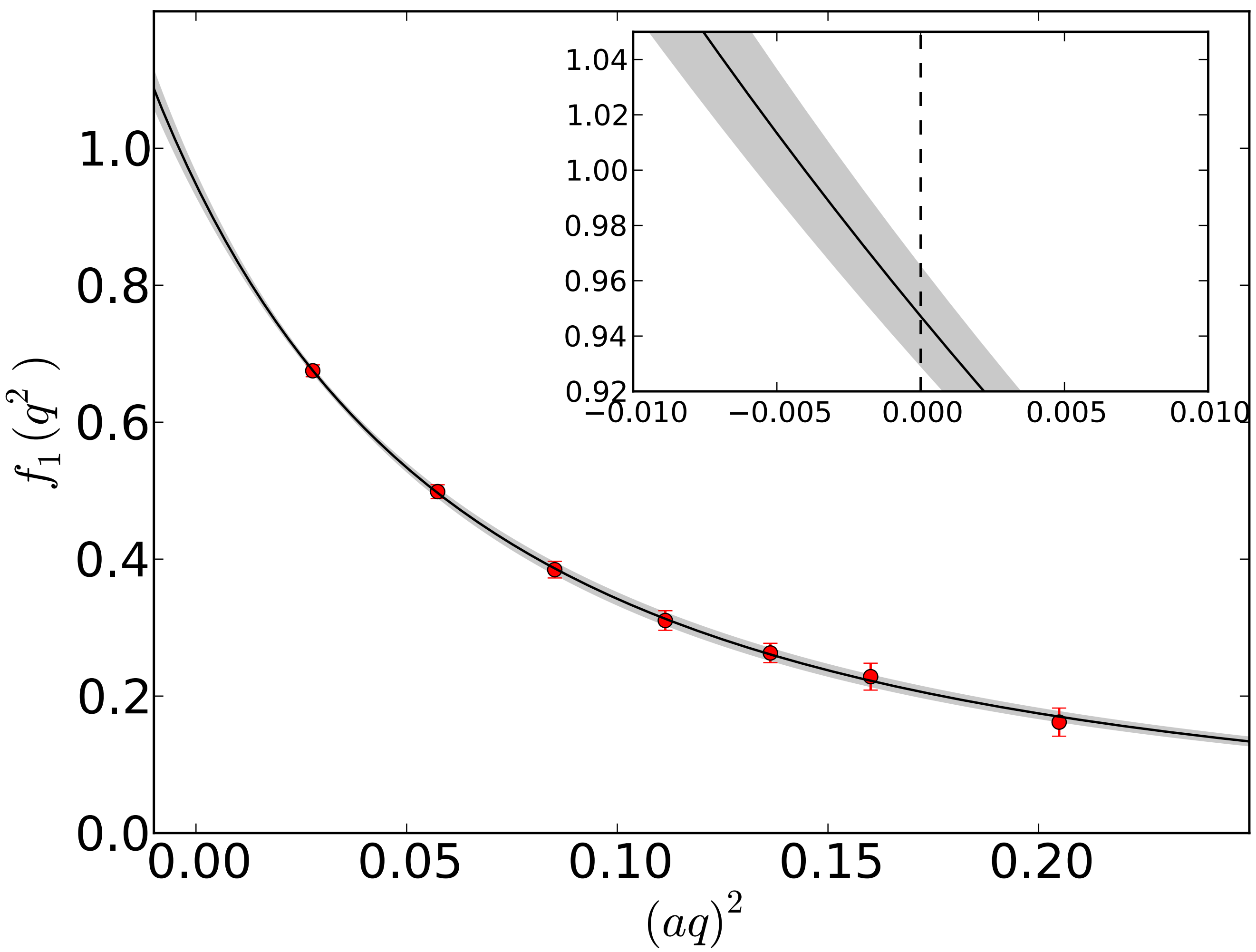
$$\boxed{f_0(q^2) = f_1(q^2) + \frac{q^2}{m_\Sigma^2 - m_n^2}f_3(q^2)}$$

[See also S. Sasaki, Fri, 10D, 18:10]

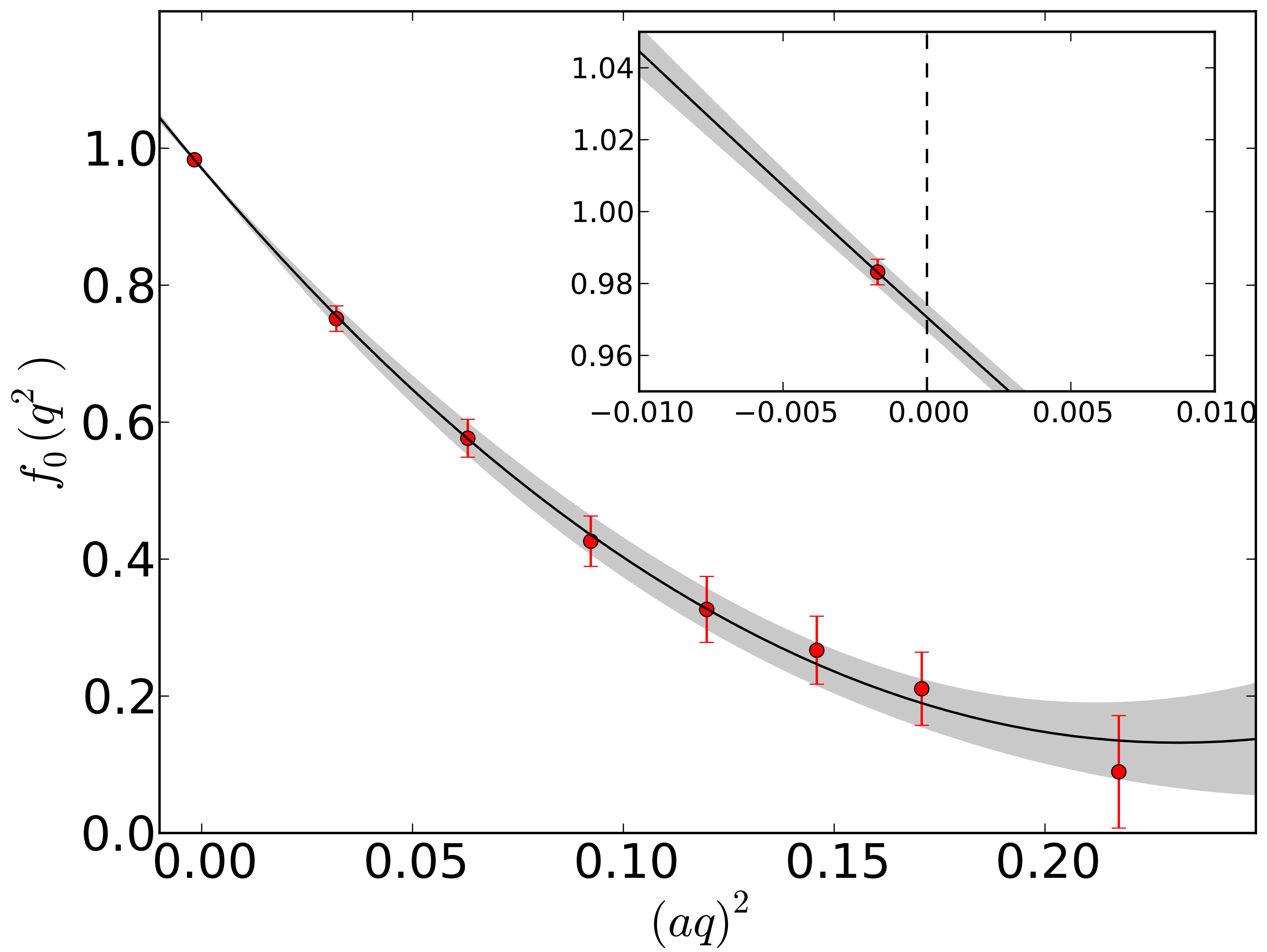
Hyperon Semileptonic Form Factors

$m_\pi = 319$ [MeV], $32^3 \times 64$

$$\Sigma^- \to n \ell \nu_\ell$$



$$f_1(q^2) = \frac{a}{1 - q^2/M^2}$$



$$f_0(q^2) = a + bq^2 + cq^4$$

Hyperon Semileptonic Form Factors

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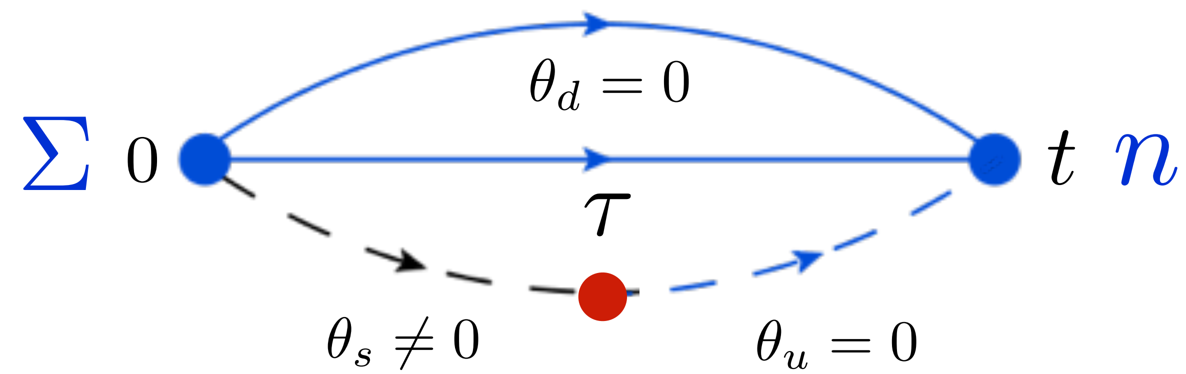
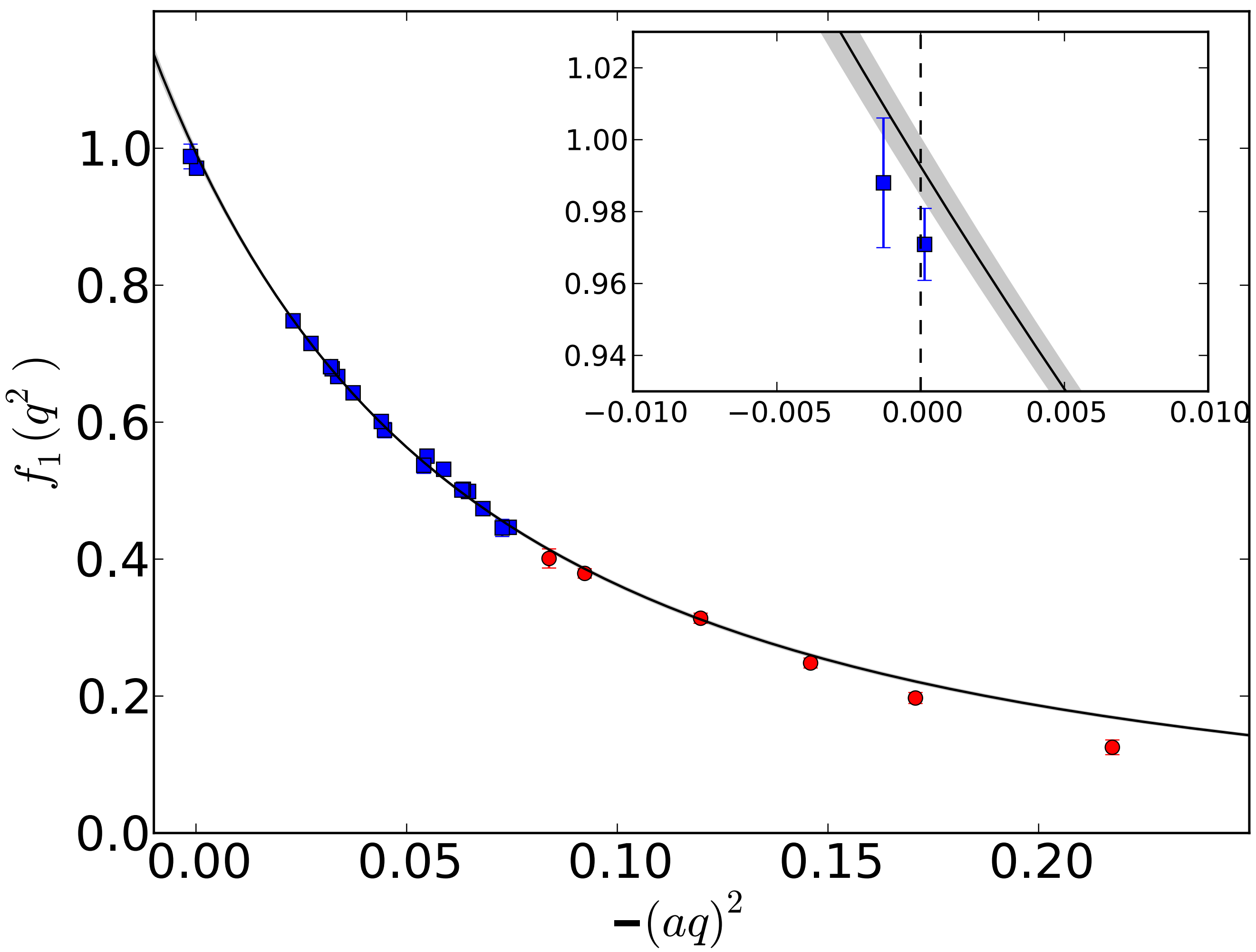
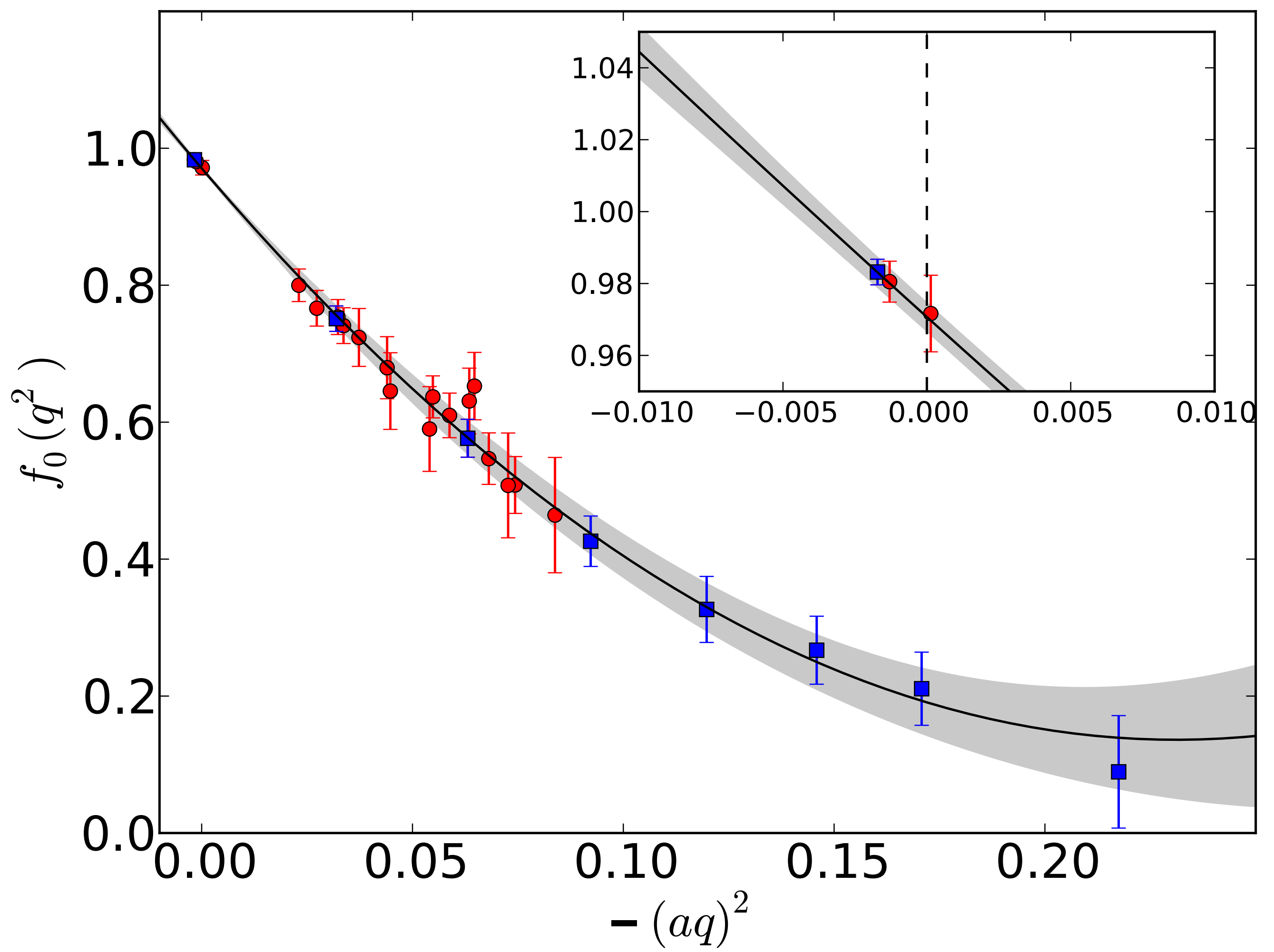
Twisted Boundary Conditions







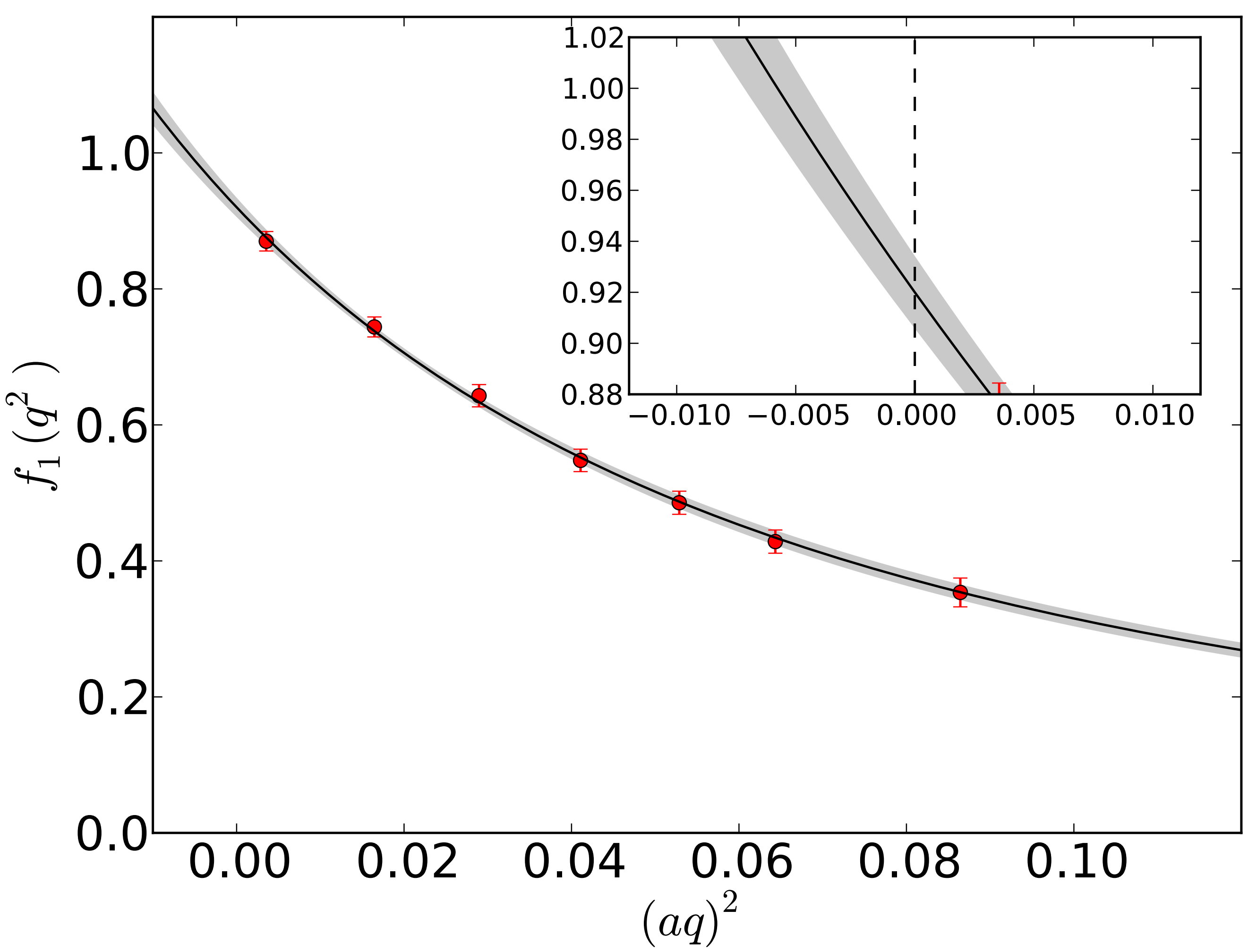
$$f_1(q^2) = \frac{a}{1 - q^2/M^2}$$

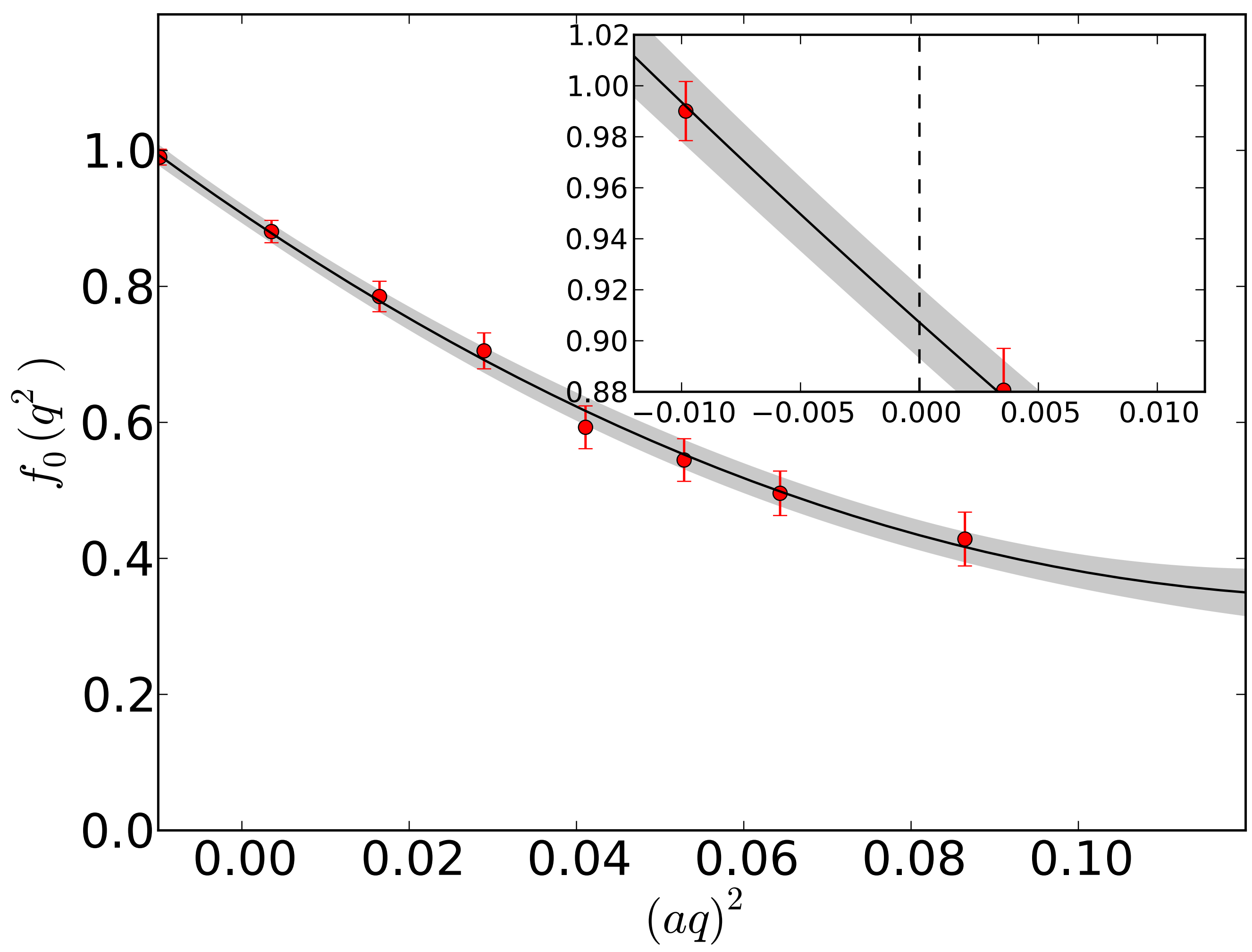
$$f_0(q^2) = a + bq^2 + cq^4$$

Hyperon Semileptonic Form Factors

$m_\pi = 205$ [MeV], $48^3 \times 96$

$$\Sigma^- \to n \ell \nu_\ell$$

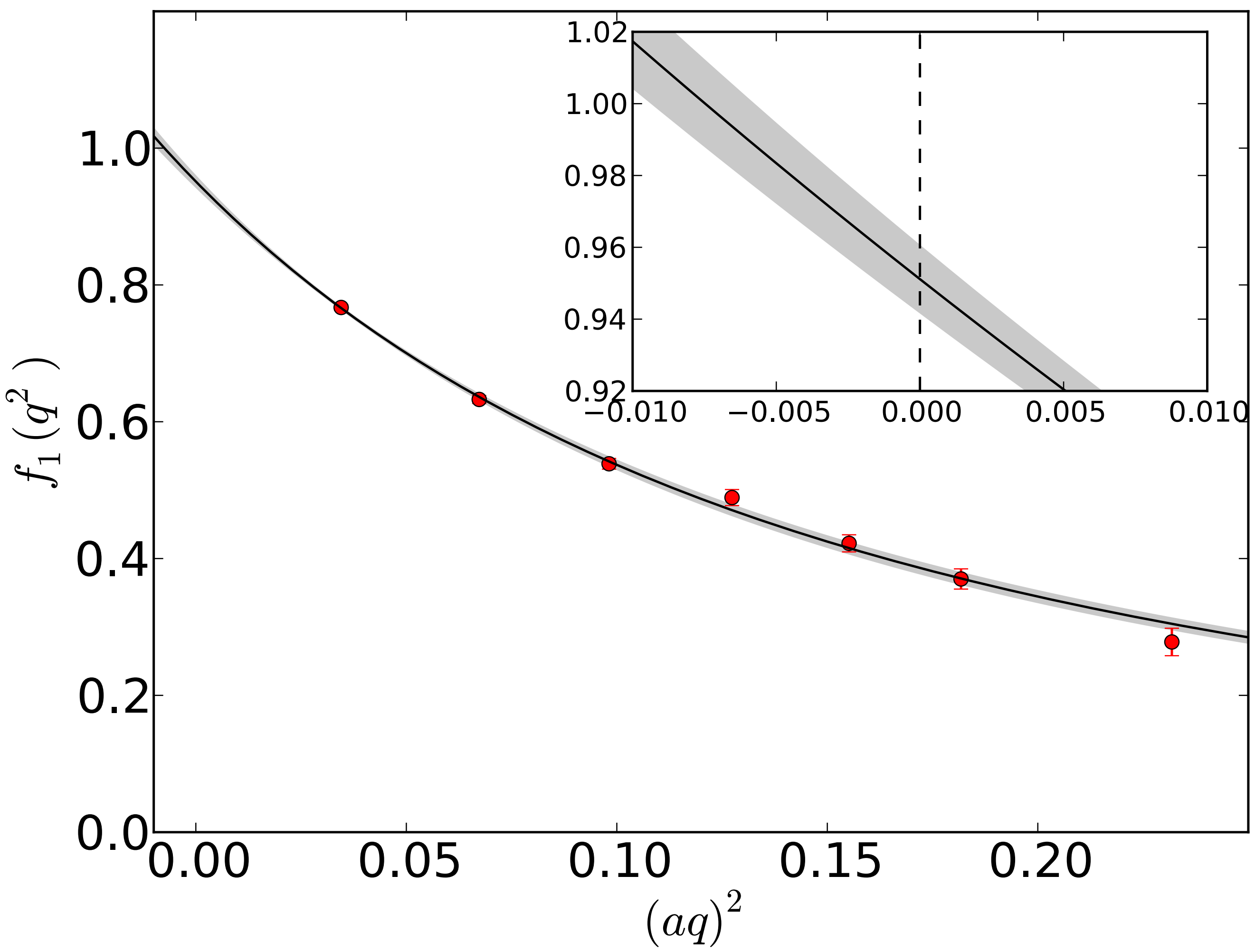


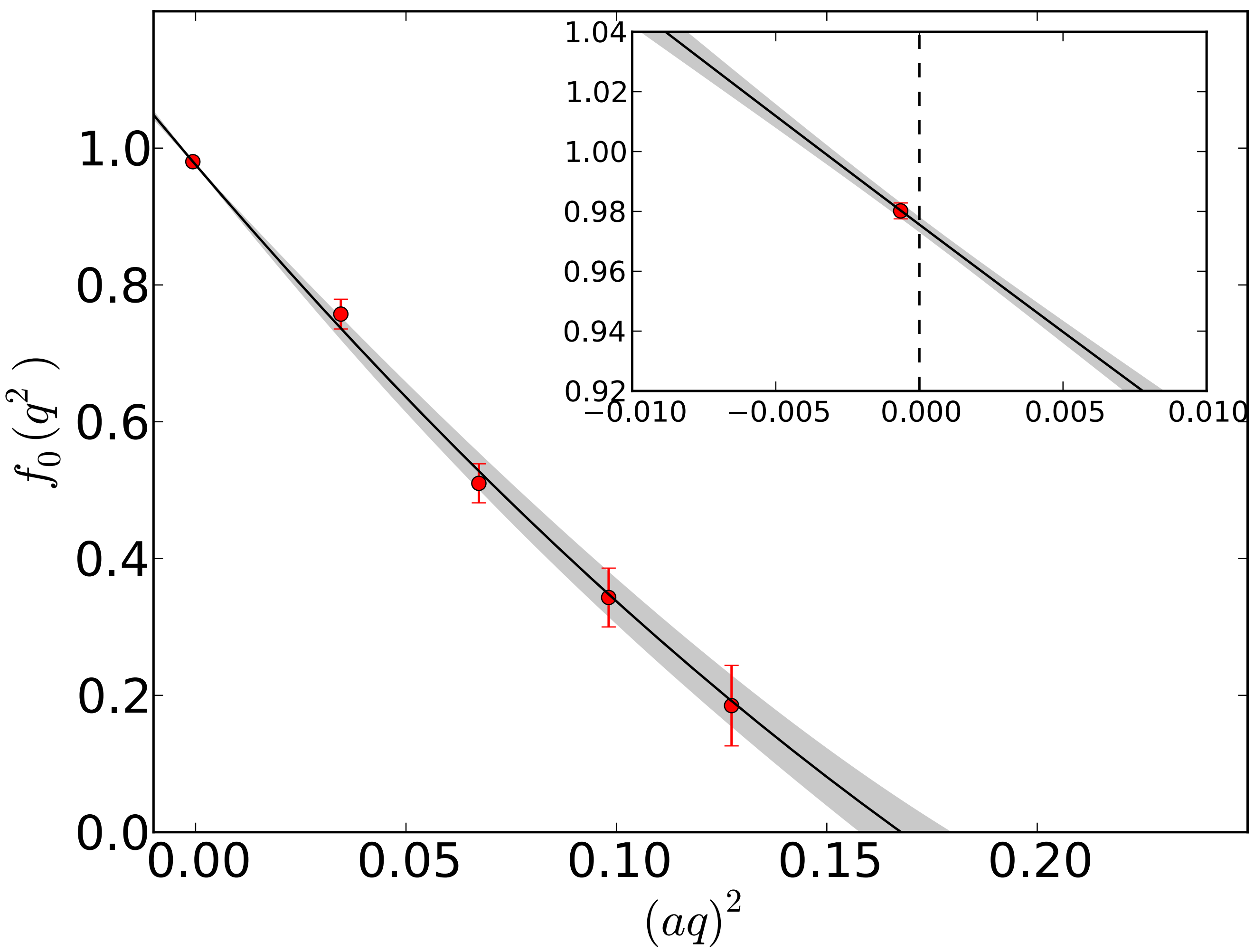


Hyperon Semileptonic Form Factors

$m_\pi = 320$ [MeV], $32^3 \times 64$

$$\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e$$





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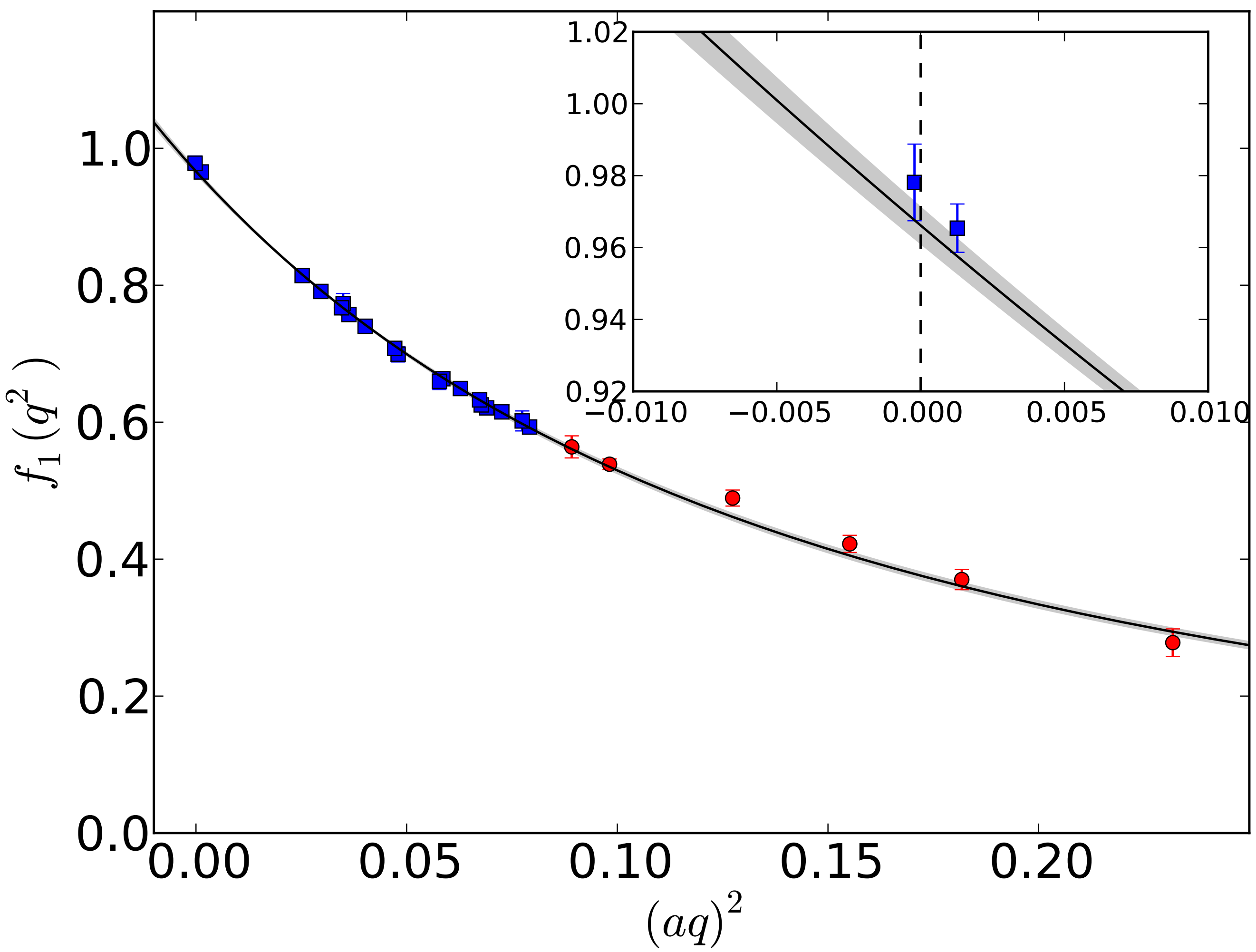
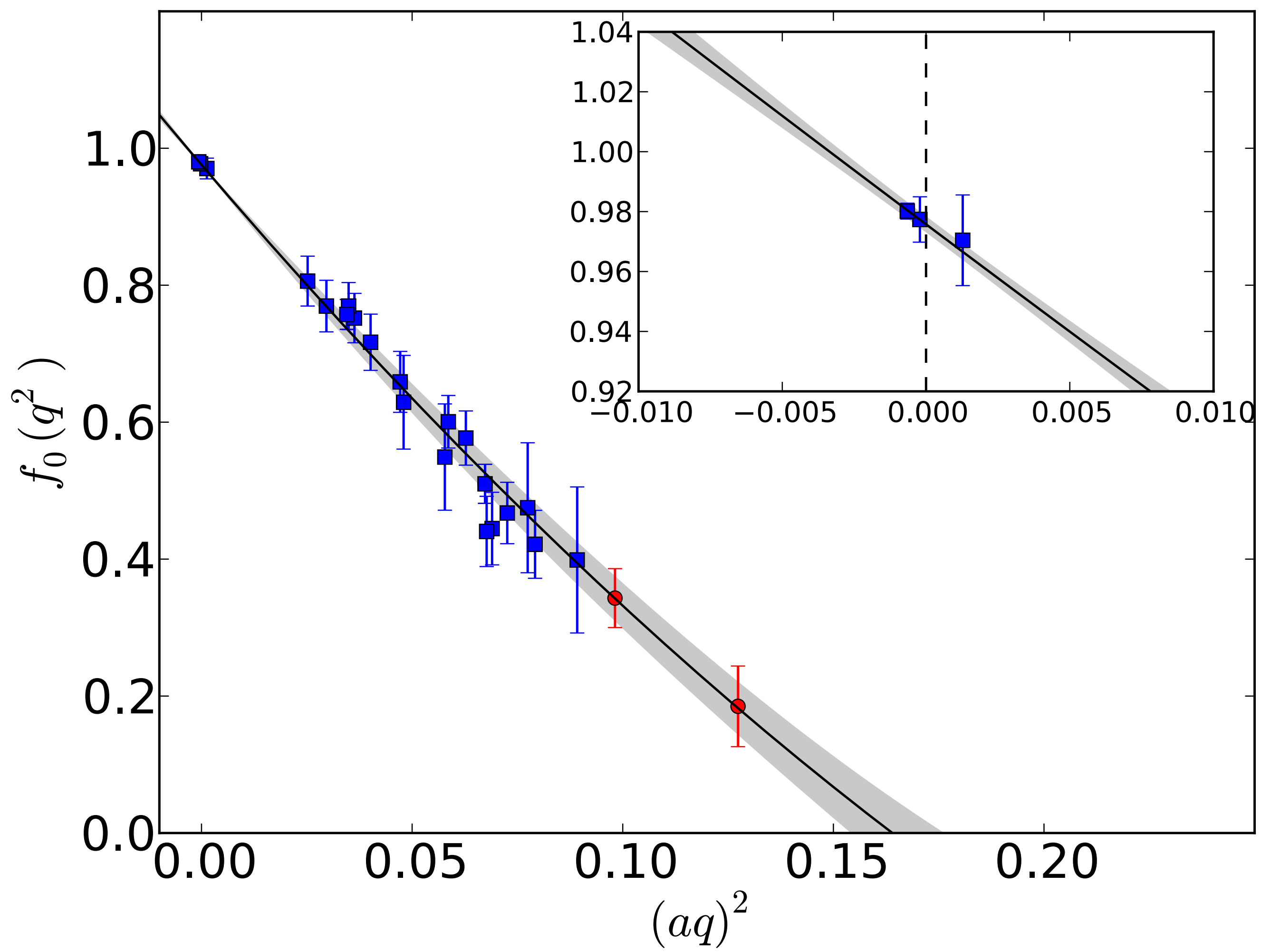
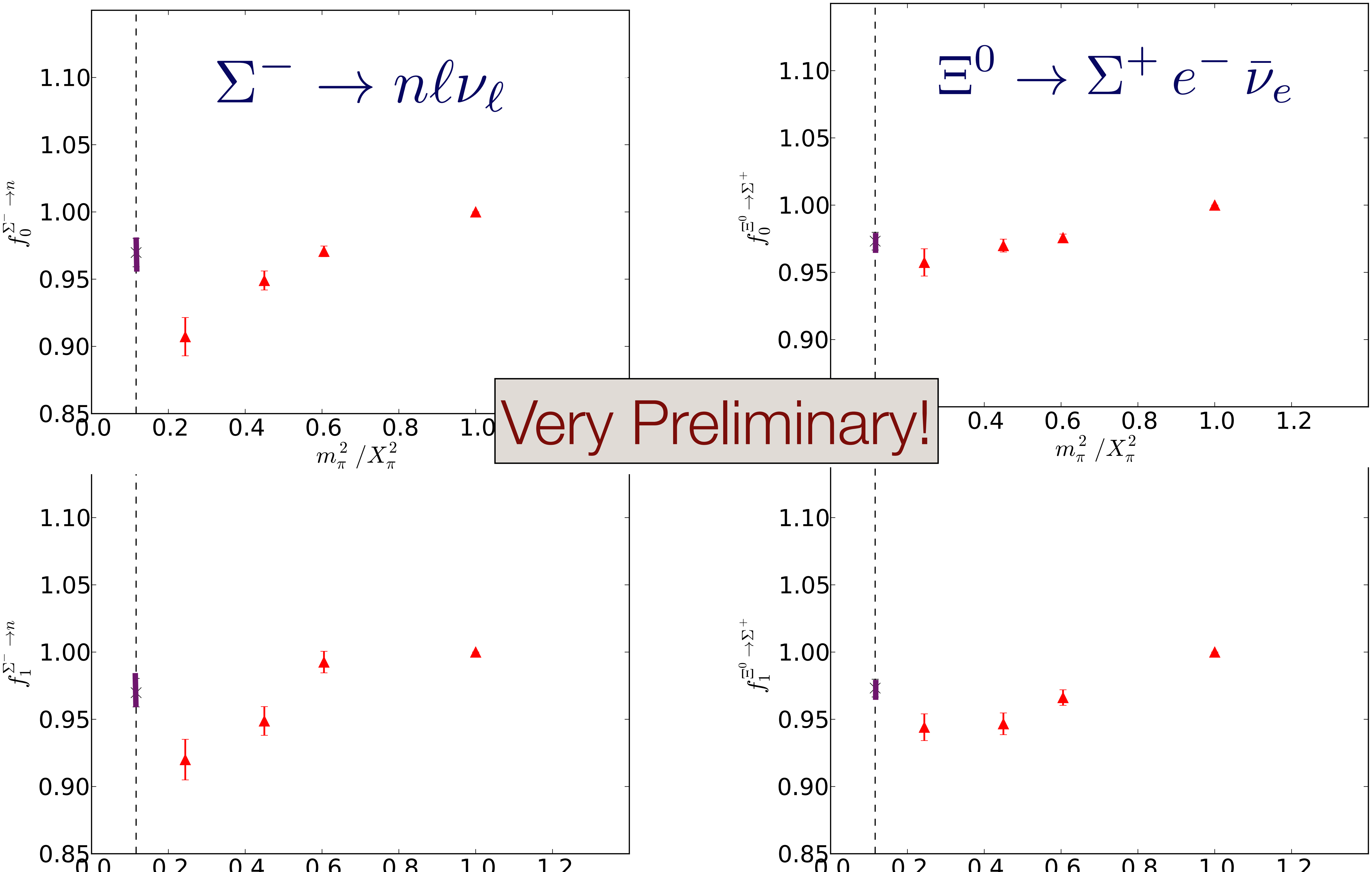




Hyperon Semileptonic Form Factors

S.Sasaki 1209.6115



Summary

- Hyperon electromagnetic form factors
    - Environmental dependence in radii & magnetic moments
- Hyperon axial charges and spin content
    - g_A/f_π agrees with N_f=2
    - $SU(3)_f$ breaking effects in quark spin contributions
    - "Spin crisis" not so severe for, e.g. Ξ
- Hyperon semileptonic decays
    - Results for $f_0(0) = f_1(0)$ can now be combined with exp.$|V_{us} f_1(0)|^2$ to determine $|V_{us}|$
    - also need to determine $$|V_{us} f_1(0)|^2 \left(1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2\right)$$
- $SU(3)_f$ breaking expansions