B, B_s, K and π weak matrix elements with physical light quarks

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Yet another talk on decay constants

UNIVERSITY OF CAMBRIDGE

HPQCD

Motivation

- Errors in decay constants with light quarks dominated by chiral extrapolation (or ambiguities in chiral fit choice)
  - Linear, NLO, NNLO, partially quenched, staggered, $SU(2)$ vs $SU(3)$
  - Pion mass cut?

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- Physical point results will make errors better and more robust
- Flavour physics [A. El-Khadra talk]
  - f_{B_s} used in rate for $B_s \to \mu^+\mu^-$
  - f_{B^+} for $B \to \tau\nu$
  - Ratios used in CKM unitarity tests
- Precision scale determination (w_0, r_1), using f_π

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Contents

- First decay constant results from HPQCD $N_f = 2 + 1 + 1$ program
- f_B, f_{B_s} and f_{B_s}/f_B with NRQCD-HISQ [arXiv:1302.2644]
- f_K/f_π with HISQ [arXiv:1303.1670]

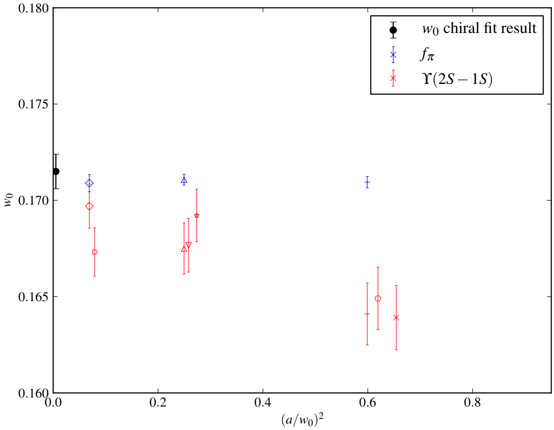
HISQ $N_f = 2+1+1$ ensembles [MILC collaboration]

Set	β	a (fm)	M_π (MeV)	L (fm)	$L/a \times T/a$	$n_{\rm cfg}$
1	5.8	0.15	300	2.5	16×48	1020
2	5.8	0.15	215	3.7	24×48	1000
3	5.8	0.15	130	4.8	32×48	1000
4	6.0	0.12	300	3.0	24×64	1052
5	6.0	0.12	215	3.9	32×64	1000
6	6.0	0.12	130	5.8	48×64	1000
7	6.3	0.09	300	2.9	32×96	1008
8	6.3	0.09	130	5.6	64×96	621

- 3 physical point ensembles
- Large volumes
- Well tuned quark masses
- Bottomonium/B spectra have good agreement with expmt [see backups]

Scale setting

- Compared scale from $\Upsilon(2S-1S)$, w_0, $t_0^{1/2}$ and r_1
- Determined w_0/a with Wilson flow (binned 12 adjacent cfgs) [Lüscher, BMW '12]
- Obtain w_0=0.1715(9) fm (later slides), BMW have $w_0 = 0.1755(18)$ fm
- Good agreement between f_π, $\Upsilon(2S-1S)$ for setting overall scale



We use $\Upsilon(2S-1S)$ for f_B and w_0 (with f_π) for f_K

B-meson decay constants

Analysis

- Improved NRQCD and HISQ actions [arXiv:1110.6887]
- $\sim$32k wall source correlators per ensemble, 3 smearing functions
- $1/m_b$ correction and 1-loop renormalisation for vector current [C.Monahan plenary talk]

Two independent analyses: chiral (8 ensembles), phys pt only (3 ensembles)

- Simultaneous 1-loop $SU(2)$ HMχPT chiral fit to $M_{B_s} - M_B$, f_B, f_{B_s} and ratio

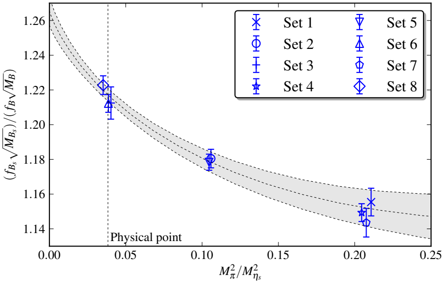
$$\Phi_s = f_{B_s}\sqrt{M_{B_s}} = \Phi_{s0}(1.0 + b_s M_\pi^2/\Lambda_\chi^2) \quad (1)$$

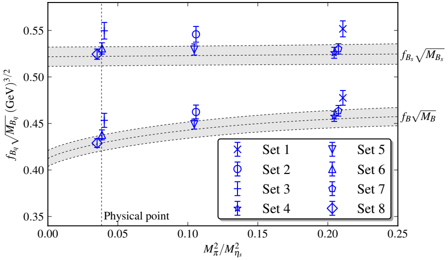
$$\Phi = f_B\sqrt{M_B} = \Phi_0\left(1.0 + b_l\frac{M_\pi^2}{\Lambda_\chi^2} + \frac{1+3g^2}{2\Lambda_\chi^2}\left(-\frac{3}{2}l(M_\pi^2)\right)\right) \quad (2)$$

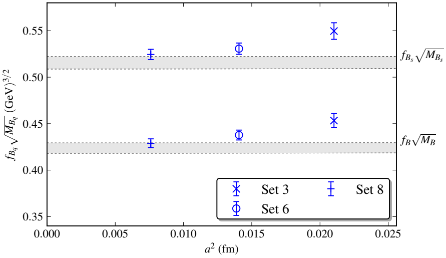
- Finite vol. included via chiral logs
- Discretisation term $(1 + d_1(\Lambda a)^2 + d_2(\Lambda a)^4)$
  - d_i allowed mild am_b dependence
- Prior of 0.5(5) on $g_{B^*B\pi}$
- Stable under changes to priors, adding more terms etc
- Consistent results with SHMχPT fit
- Evaluate fit at fictional $\bar{u}u$ pion mass using $m_u = 0.65(9)m_l$ (2 MeV effect)

f_B results

Consistent results from chiral and phys. pt fits,







$$
\begin{aligned}
f_{B^+} &= 0.184(4)\ \text{GeV} \\
f_{B_s} &= 0.224(4)\ \text{GeV} \\
f_{B_s}/f_{B^+} &= 1.217(8) \\
M_{B_s} - M_B &= 85(2)\ \text{MeV}
\end{aligned}
$$

Splitting agrees with experiment

Error budget

Allowed 2-loop error of 10× 1-loop renormalisation

Error %	Φ_{B_s}/Φ_B	$M_{B_s} - M_B$	Φ_{B_s}	Φ_B
EM:	0.0	1.2	0.0	0.0
a dependence:	0.01	0.9	0.9	0.9
chiral:	0.01	0.2	0.04	0.04
g:	0.01	0.1	0.0	0.01
stat/scale:	0.30	1.2	0.7	0.7
operator:	0.0	0.0	1.3	1.3
relativistic:	0.5	0.5	1.0	1.0
total:	0.6	2.0	2.0	2.0

- Error generally dominated by stats or missing higher order corrections

Light meson decay constants

Analysis

2-point functions:

- 16000 wall source correlators
- Include fictitious $s\bar{s}$ meson η_s
- Additional strange masses on coarsest ensembles
- Bin over 2-4 adjacent cfgs (additional binning has almost no effect)
- Simultaneous multiexponential fit to π, K, η_s 2pt correlators - keep correlations
- No renormalisation required for HISQ

Chiral/continuum fits:

- Remove lattice spacing with w_0/a, i.e. $w_0 f_\pi, w_0 f_K, \ldots$
- w_0 is a parameter in the fit (prior 0.1755(175))
- M_π, M_K used to fix quark masses, f_{π^+} sets the scale

Complementary to MILC analysis with physical point data only

Chiral fit

Physical light quarks $\Longrightarrow$ χPT only used for small quark tunings

- Bayesian fit with $SU(3)$ NLO PQχPT [Sharpe & Shoresh]

$$f_{\mathrm{NLO}}(x_a, x_b, x_\ell^{\mathrm{sea}}, x_s^{\mathrm{sea}}, L) + \delta f_\chi + \delta f_{\mathrm{lat}}. \tag{3}$$

- Express χPT in terms of meson masses
- Subtract 1-loop chiral correction to remove f.vol

$$x_\ell = \frac{M_{0,\pi}^2}{\Lambda_\chi^2}, \quad x_s = \frac{2M_{0,K}^2 - M_{0,\pi}^2}{\Lambda_\chi^2} \tag{4}$$

- Also compared with Staggered χPT, consistent results
- Plus higher orders...

Higher order corrections

- Generic higher order correction terms:

$$\begin{aligned}\delta f_\chi \equiv\; & c_{2a}(x_a+x_b)^2 + c_{2b}(x_a-x_b)^2 + c_{2c}(x_a+x_b)(2x_\ell^{\rm sea}+x_s^{\rm sea}) \\ & + c_{2d}(2x_\ell^{\rm sea}+x_s^{\rm sea})^2 + c_{2e}(2x_\ell^{\rm sea\,2}+x_s^{\rm sea\,2}) + c_{3a}(x_a+x_b)^3 \\ & + c_{3b}(x_a+x_b)(x_a-x_b)^2 + c_{3c}(x_a+x_b)^2(2x_\ell^{\rm sea}+x_s^{\rm sea}) \\ & + c_4(x_a+x_b)^4 + c_5(x_a+x_b)^5 + c_6(x_a+x_b)^6\end{aligned}$$

  Priors of 0(1)

- Discretisation corrections

$$\begin{aligned}\delta f_{\rm lat} &\equiv \sum_{n=1}^{4} d_n \left(\frac{a\Lambda}{\pi}\right)^{2n} \\ d_n &= d_{n,0} + d_{n,1a}(x_a+x_b) + d_{n,1b}(2x_\ell^{\rm sea}+x_s^{\rm sea}) + d_{n,1c}(x_a+x_b)^2, \quad (5)\end{aligned}$$

  Allowed to depend on quark masses (cf. SχPT)

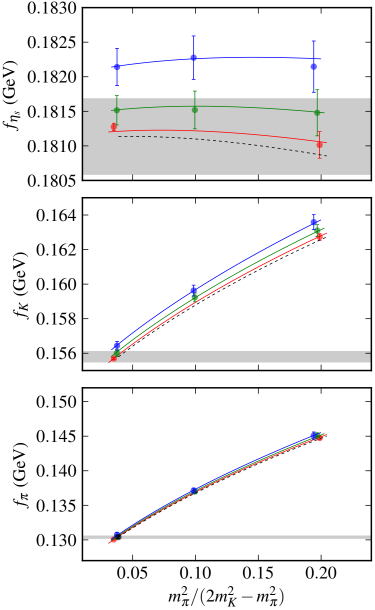
- Priors 0(1). Scale set to $\Lambda = 1.8$ GeV, fit implies much lower scale.

Finite volume

a	m_s/m_ℓ	L	$M_\pi L$	$\Delta_{\mathrm{vol}} f_\pi$	$\Delta_{\mathrm{vol}} f_K$	$\Delta_{\mathrm{vol}} f_{\eta_s}$
0.15 fm	5.3	2.5 fm	3.8	1.24(23)%	0.50(9)%	0.10(0)%
0.15 fm	10.6	3.7 fm	4.0	0.38(7)%	0.12(2)%	0.00(0)%
0.15 fm	26.7	4.8 fm	3.3	0.43(8)%	0.13(2)%	0.00(0)%
0.12 fm	5.0	3.0 fm	4.6	0.37(7)%	0.14(3)%	0.01(0)%
0.12 fm	10.0	3.9 fm	4.3	0.24(5)%	0.08(1)%	0.00(0)%
0.12 fm	27.6	5.8 fm	3.9	0.15(3)%	0.05(1)%	0.00(0)%
0.091 fm	4.9	2.9 fm	4.5	0.41(8)%	0.16(3)%	0.02(0)%
0.088 fm	30.0	5.6 fm	3.7	0.21(4)%	0.07(1)%	0.00(0)%

- Calculated numerically in NLO chiral perturbation theory
- Higher orders allowed with multiplicative prior of 1 ± 0.33
- $\Delta_{\mathrm{vol}} f_\pi \leq 0.5\%$ on all but one ensemble
- Negligible in final error budget

Results



Evaluate fit at π^+, K^+ (see backups)

$$
\begin{aligned}
f_{K^+}/f_{\pi^+} &= 1.1916(21) \\
f_{K^+} &= 155.37(34)\,\mathrm{MeV} \\
M_{\eta_s}^2/(2M_K^2 - M_\pi^2) &= 1.0063(64) \\
f_{\eta_s}/(2f_K - f_\pi) &= 0.9997(17) \\
w_0 &= 0.1715(9)\,\mathrm{fm}
\end{aligned}
$$

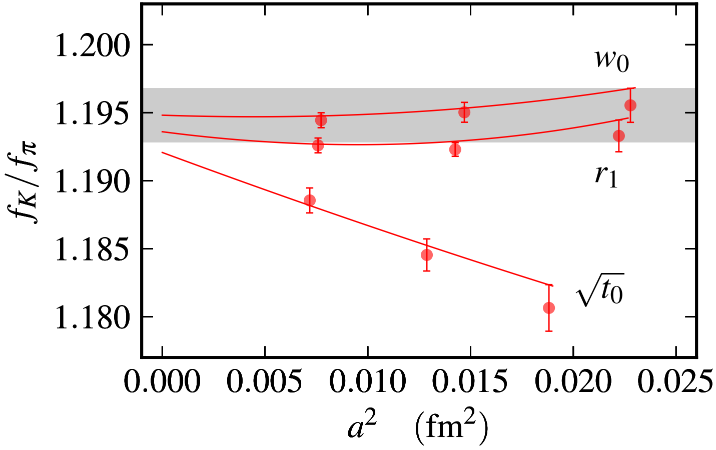
$$|V_{us}| = 0.22564(28)_{\mathrm{Br}(K^+)}(20)_{\mathrm{EM}}(40)_{\mathrm{latt}}(5)_{V_{ud}}$$

Test of first row unitarity, V_{ud} now needs to be improved:

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = -0.00009(51)$$

Results

Consistent results from three scale setting methods



- $\sqrt{t_0}$ has noticably larger disretisation errors
- Wilson flow easier to calculate than r_1

Errors

	f_{K^+}	f_{K^+}/f_{π^+}	m_{η_s}	w_0
statistics + *svd* cut	0.13%	0.13%	0.28%	0.26%
chiral extrapolation	0.03	0.03	0.04	0.15
$a^2 \to 0$ extrapolation	0.10	0.10	0.15	0.27
finite volume correction	0.01	0.01	0.01	0.02
w_0/a uncertainty	0.02	0.02	0.02	0.28
f_{π^+} experiment	0.13	0.03	0.07	0.19
m_u/m_d uncertainty	0.07	0.07	0.00	0.00
Total	0.22%	0.18%	0.33%	0.54%

- "Statistical" error comes largely from SVD cut needed with correlations
- Fit without correlations has much smaller error
- Artificially inflating the errors ($\times 8$) with no SVD cut gives same result

Summary

Heavy and light decay constants at the physical point

$$f_{B^+} = 0.184(4)\ \mathrm{GeV}, \qquad f_{B_s} = 0.224(4)\ \mathrm{GeV}, \qquad f_{B_s}/f_{B^+} = 1.217(8),$$
$$f_{K^+}/f_{\pi^+} = 1.1916(21), \qquad f_{K^+} = 155.37(34)\ \mathrm{MeV}, \qquad w_0 = 0.1715(9)\ \mathrm{fm}$$

- Chiral fit error negligible
- Scale dependence small for both
- NRQCD limited by higher order operators and radiative corrections

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Other HPQCD results on the HISQ ensembles:

- Quark condensates [C.McNeile talk]
- Radiatively improved hyperfine splittings [C.Davies poster]
- $B \to \pi$ at zero recoil at the physical point [C.Davies poster]
- Pion form factor and charge radius [J.Koponen talk]
- Entirely non-perturbative mNPR [B.Chakraborty talk]

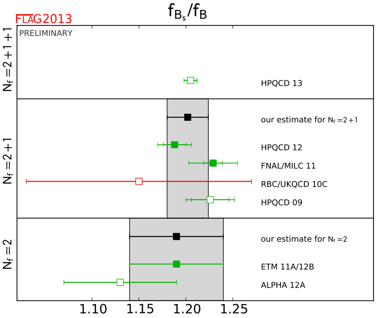
In progress:

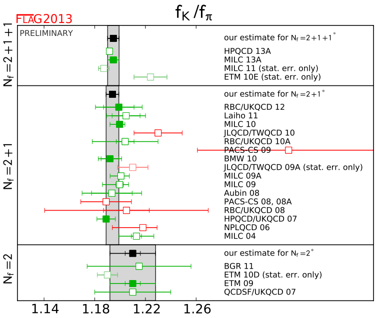
- Radiative decays in bottomonium [C.Hughes poster]
- B-meson bag parameters with NRQCD-HISQ

LATTICE 13

The end

FLAG 2013 comparison





Improved NRQCD action

- v^4 NRQCD action, radiative corrections to Wilson coefficients

$$
\begin{aligned}
aH &= aH_0 + a\delta H; \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b}, \\
a\delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right) \\
&\quad -c_3 \frac{1}{8(am_b)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right) \\
&\quad -c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24am_b} \\
&\quad -c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}.
\end{aligned} \tag{6}
$$

f_B current matching

Lattice currents

$$J_0^{(0)} = \bar{\Psi}_q \gamma_5 \gamma_0 \Psi_Q \quad (7)$$

$$J_0^{(1)} = \frac{-1}{2m_b} \bar{\Psi}_q \gamma_5 \gamma_0 \gamma \cdot \nabla \Psi_Q \quad (8)$$

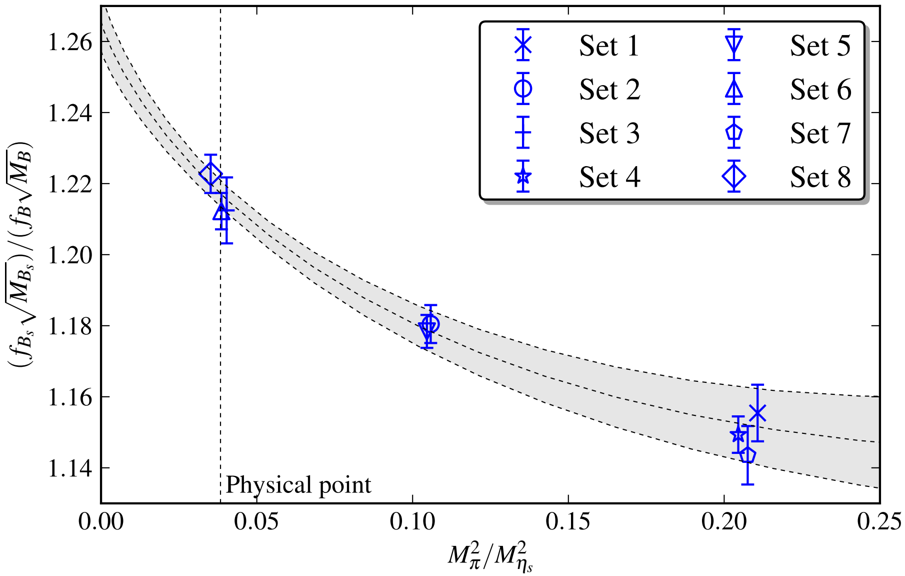
$$J_0^{(2)} = \frac{-1}{2m_b} \bar{\Psi}_q \gamma \cdot \overleftarrow{\nabla} \gamma_5 \gamma_0 \Psi_Q . \quad (9)$$

Related to the full QCD current through $\mathcal{O}\left(\alpha_s, \alpha_s \Lambda_{\mathrm{QCD}}/m_b\right)$ by

$$\langle A_0 \rangle = (1 + \alpha_s z_0)\left(\langle J_0^{(0)} \rangle + (1 + \alpha_s z_1)\langle J_0^{(1)} \rangle + \alpha_s z_2 \langle J_0^{(2)} \rangle\right) \quad (10)$$

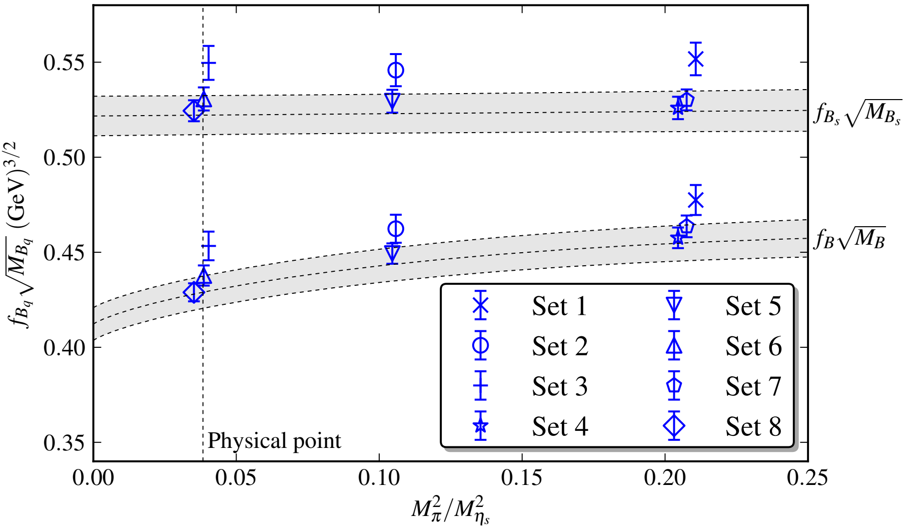
- Coupling $\alpha_V(2/a)$, z_i calculated to 1-loop in pert theory
- Overall renormalisation is small

f_B results



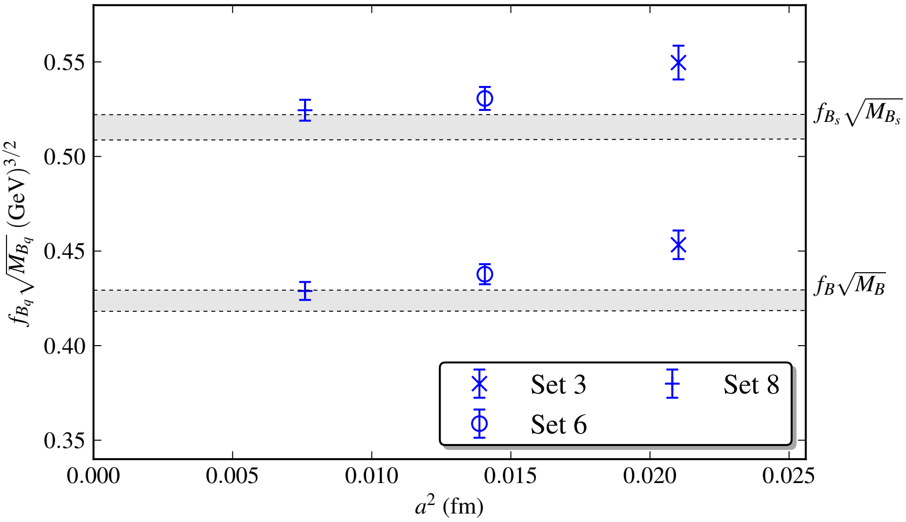
Set 1
Set 2
Set 3
Set 4
Set 5
Set 6
Set 7
Set 8
Physical point
1.26
1.24
1.22
1.20
1.18
1.16
1.14
0.00
0.05
0.10
0.15
0.20
0.25
$(f_{B_s}\sqrt{M_{B_s}})/(f_B\sqrt{M_B})$
$M_\pi^2/M_{\eta_s}^2$

f_B results



0.55
0.50
0.45
0.40
0.35
$f_{B_q}\sqrt{M_{B_q}}$ (GeV)$^{3/2}$
0.00
0.05
0.10
0.15
0.20
0.25
$M_\pi^2/M_{\eta_s}^2$
$f_{B_s}\sqrt{M_{B_s}}$
$f_B\sqrt{M_B}$
Physical point
Set 1
Set 2
Set 3
Set 4
Set 5
Set 6
Set 7
Set 8

f_B results



0.55
0.50
0.45
0.40
0.35
$f_{B_q}\sqrt{M_{B_q}}$ (GeV)$^{3/2}$
0.000
0.005
0.010
0.015
0.020
0.025
a^2 (fm)
$f_{B_s}\sqrt{M_{B_s}}$
$f_B\sqrt{M_B}$
Set 3
Set 8
Set 6

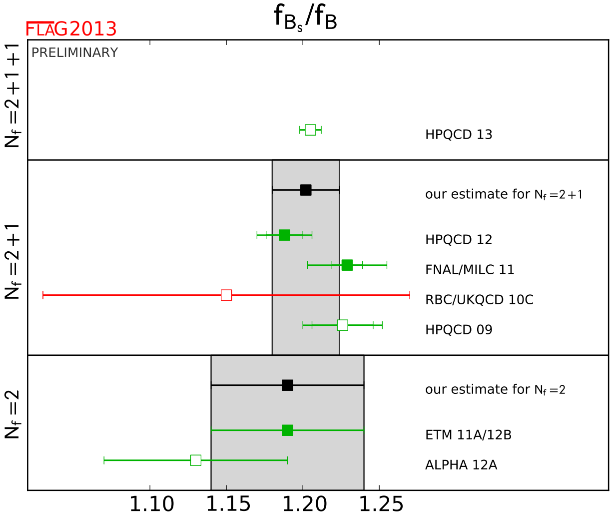
Isospin violation

- Evaluate fit at appropriate point for π^+, K^+
- Noticable shift with small stat errors ($f_K/f_{K^+} = 1.0024(6)$) [N.Tantalo talk]
- For Kaons,

$$(M_K^{\rm phys})^2 \equiv \frac{1}{2}\left[(M_{K^+}^2 + M_{K^0}^2) - (1+\Delta_E)(M_{\pi^+}^2 - M_{\pi^0}^2)\right]. \tag{11}$$

  using $\Delta_E = 0.65(50)$
- These give $m_l = (m_u + m_d)/2$
- For K^+, we need $m_u = 0.65(9) m_l$
- Evaluate fit at $m_\pi = \sqrt{0.65(9)} M_\pi^{\rm phys}$ with $2M_K^2 - m_\pi^2$ fixed

FLAG 2013 comparison



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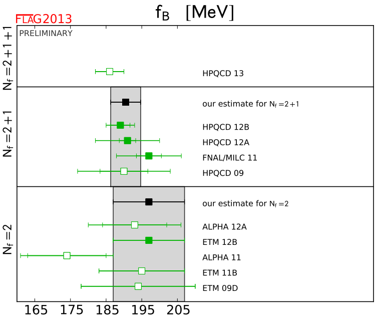

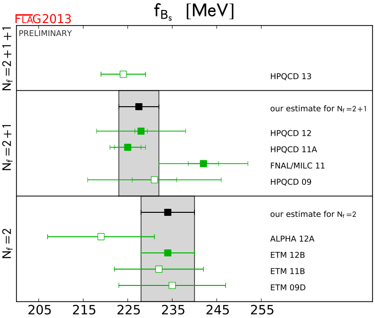

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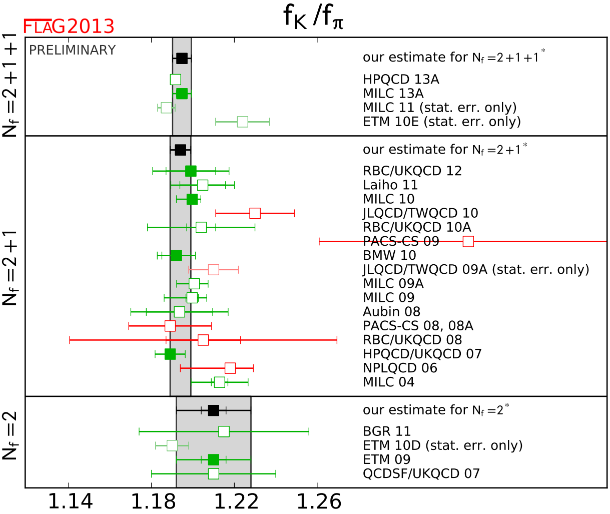

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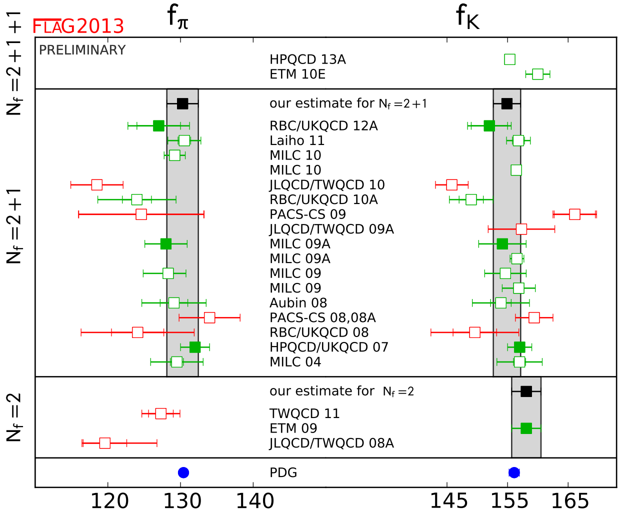
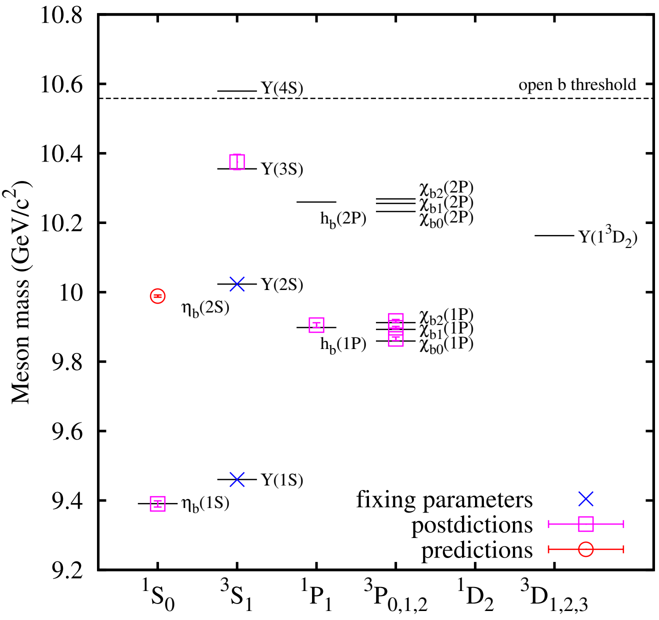
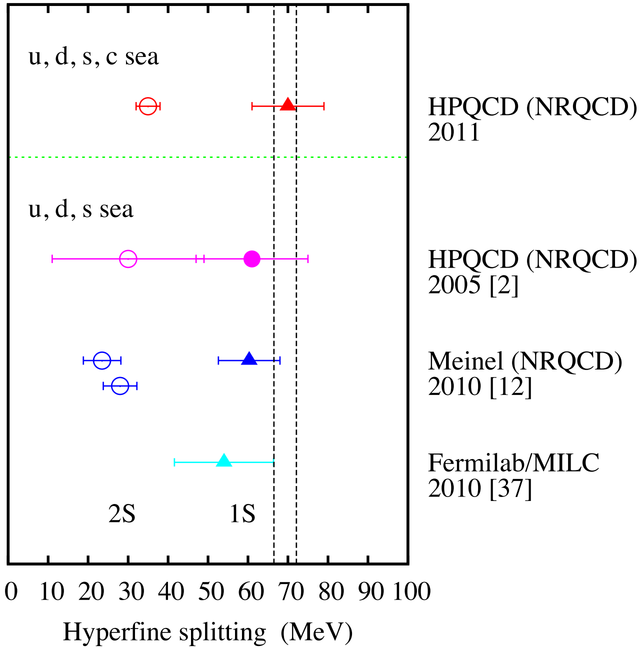
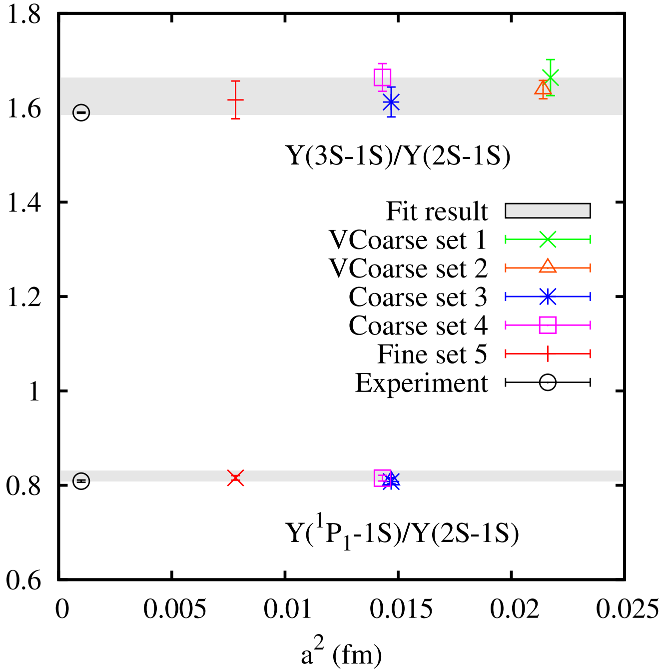




Meson mass (GeV/c^2)
10.8
10.6
10.4
10.2
10
9.8
9.6
9.4
9.2
open b threshold
Υ(4S)
Υ(3S)
Υ(2S)
Υ(1S)
η_b(2S)
η_b(1S)
h_b(2P)
h_b(1P)
χ_{b2}(2P)
χ_{b1}(2P)
χ_{b0}(2P)
χ_{b2}(1P)
χ_{b1}(1P)
χ_{b0}(1P)
Υ(1^3D_2)
fixing parameters
postdictions
predictions
$^1S_0$
$^3S_1$
$^1P_1$
$^3P_{0,1,2}$
$^1D_2$
$^3D_{1,2,3}$


u, d, s, c sea
HPQCD (NRQCD) 2011
u, d, s sea
HPQCD (NRQCD) 2005 [2]
Meinel (NRQCD) 2010 [12]
Fermilab/MILC 2010 [37]
2S
1S
0 10 20 30 40 50 60 70 80 90 100
Hyperfine splitting (MeV)



1.8
1.6
1.4
1.2
1
0.8
0.6
0
0.005
0.01
0.015
0.02
0.025
a^2 (fm)
Υ(3S-1S)/Υ(2S-1S)
Υ(1P1-1S)/Υ(2S-1S)
Fit result
VCoarse set 1
VCoarse set 2
Coarse set 3
Coarse set 4
Fine set 5
Experiment