Progress towards an ab initio, Standard Model calculation of direct CP-violation in K decay

Christopher Kelly

Columbia University

RBC & UKQCD Collaboration

T.Blum, P.A.Boyle, N.H.Christ, N.Garron, E.Goode, T.Izubuchi, C.Jung, CK, C.Lehner, M.Lightman, Q.Liu, A.T.Lytle, R.D.Mawhinney, C.T.Sachrajda, A.Soni, C.Sturm, D.Zhang.

$\mathbf{K} \to \pi\pi$ *Decays*

- Direct CP-violation first observed in $K \to \pi\pi$ decays.
- Two types of decay:

$$\Delta I = 3/2 \;: K^+ \to (\pi^+\pi^0)_{I=2} \quad \text{with amplitude } A_2$$
$$\Delta I = 1/2 \;: K^0 \to (\pi^+\pi^-)_{I=0}$$
$$K^0 \to (\pi^0\pi^0)_{I=0} \quad \text{with amplitude } A_0$$

- Direct CP-violation: $\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$

  where

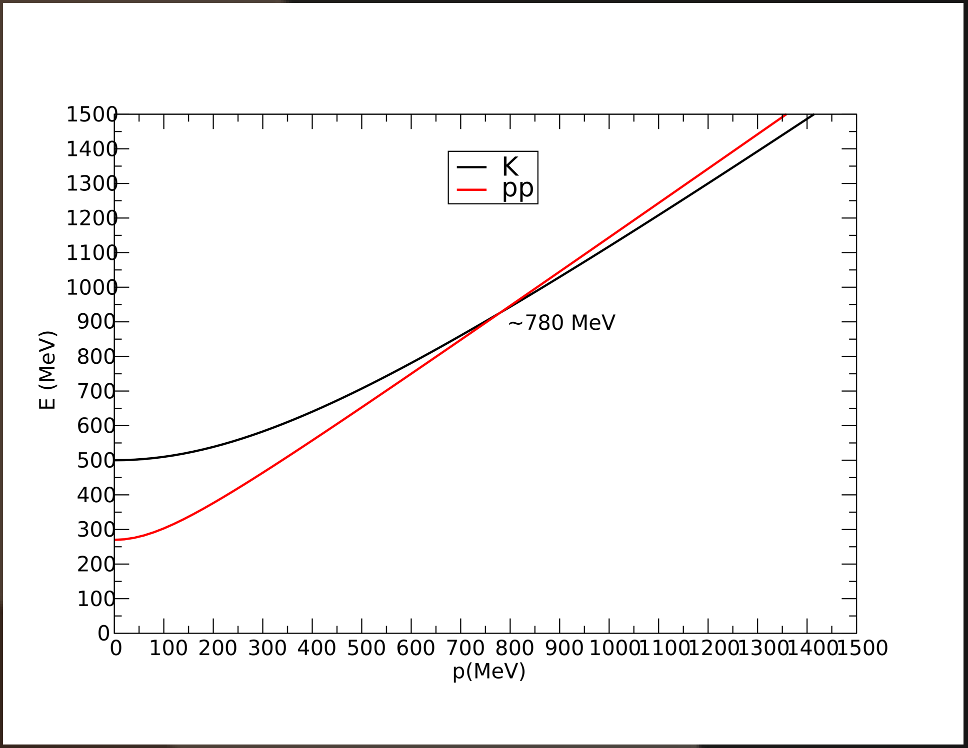
  $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$ and δ_I are strong scattering phase shifts.
- ϵ' is highly sensitive to BSM sources of CPV.
- Strong interactions very important – origin ([arXiv:1212.1474]) of the so-called $\Delta I = 1/2$ rule: preference to decay to $I = 0$ final state.

Physical kinematics and moving pions

- Finite-volume lattice decay amplitudes are related to those in the infinite-volume by the Lellouch-Luscher formula.
- This requires physical kinematics - $E_{\pi\pi} = m_K$
- $m_\pi = 135\ \mathrm{MeV}$ and $m_K = 500\ \mathrm{MeV}$: need moving pions
- However ground state comprises stationary pions.
- Could attempt to tune L such that first excited state energy matches kaon mass.
- This will be extremely noisy, and, especially when there are disconnected diagrams, it is highly unlikely that a decent signal could be extracted.

Moving kaon?

- One possibility is to consider a moving kaon $K(p_K)$ decaying to $\pi(p_\pi)\pi(0)$. Need $$\sqrt{m_K^2 + p_K^2} \approx \sqrt{m_\pi^2 + p_\pi^2} + m_\pi$$
- ~780 MeV energy required.
- SNR decreases exponentially in the energy difference between the state energy and the pion mass: this will be too noisy.



Workaround for $\Delta I = 3/2$ *calculation*

- Circumvent problem by imposing antiperiodic BCs on d-quark propagator. Changes finite-volume momentum discretization:

$$p = \frac{2\pi n}{L} \to \frac{(2n+1)\pi}{L}$$

- Minimum d-quark momentum is π/L : charged pion ground state has momentum! But...
- For neutral pion the momenta can cancel giving stationary ground state. Unfortunately the desired state is $\pi^+\pi^0$, so this does not work. However....
- Wigner-Eckart theorem:

$$\langle(\pi^+\pi^0)_{I=2}|Q^{\Delta I_z=1/2}|K^+\rangle = \frac{\sqrt{3}}{2}\langle(\pi^+\pi^+)_{I=2}|Q^{\Delta I_z=3/2}|K^+\rangle$$

- APBCs on d-quark break isospin symmetry allowing mixing between isospin states: however $\pi^+\pi^+$ is the only charge-2 state hence it cannot mix.

The $\Delta I = 1/2$ *case*

- Must measure $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$
- Wigner-Eckart trick cannot be used for $I = 0$ final state
- If we stay with APBC on d-quarks, isospin-breaking would allow mixing between $I = 0$ and $I = 2$ final states.
- Require moving π^0, but momentum cancels in $d\bar{d}$
- Need to apply BCs that commute with isospin and produce moving π^0 as well as π^+ and π^-.
- G-parity boundary conditions satisfy these criteria.

G-Parity Boundary Conditions

- G-parity is a charge conjugation followed by a 180 degree isospin rotation about the y-axis:

$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \qquad \begin{array}{c} \hat{G}|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle \\ \hat{G}|\pi^{0}\rangle = -|\pi^{0}\rangle \end{array}$$

- Pions are all eigenstates with e-val -1, hence G-parity BCs make pion wavefunctions antiperiodic, with minimum momentum π/L .

Kim, arXiv:hep-lat/0311003 (2003)

- G-parity commutes with isospin.

Wiese, Nucl.Phys.B375, (1992)

- At the quark level:

$$\hat{G} \left(\begin{array}{c} u \\ d \end{array} \right) = \left(\begin{array}{c} -C\bar{d}^T \\ C\bar{u}^T \end{array} \right)$$

where $C = \gamma^2\gamma^4$. in our conventions

- Requires extensive code modifications to treat two flavours that mix at the boundary.

Gauge Field Boundary Conditions

- Dirac operator for $C\bar{u}^T$ field involves conjugate links U^* .
- As this field transitions seamlessly to the d -field at the boundary, the links must also transition from U to U^* , i.e. links obey complex conjugate BCs (equiv to charge conjugation BCs).
- Boundary link gauge transformation is unusual:

$$U_\mu(L-1) \to V^\dagger(L-1)U_\mu(L-1)V^*(0)$$

- Necessitates generation of new ensemble of gauge links satisfying these BCs.

  (Note: for other choices of BC, e.g. APBC, new ensembles would still need to be generated, but due to presence of disconnected diagrams)

Unusual Contractions

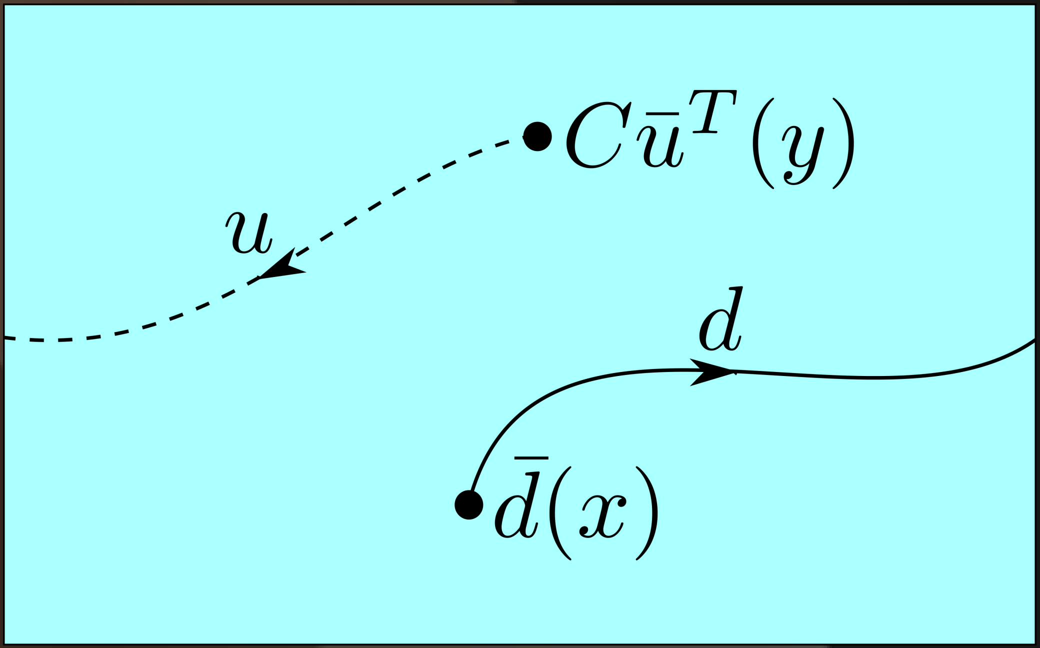
- Flavor mixing at boundary allows contraction of up and down fields:

$$\mathcal{G}^{(2,1)}_{y,x} = C\bar{u}^T_y \bar{d}_x \,,$$
$$\mathcal{G}^{(1,2)}_{x,y} = -d_x u^T_y C^T$$

- Interpret as boundary creating/destroying flavor (violating baryon number)
- More Wick contractions to evaluate.
- Some states mix at the boundary, e.g.

$$uud \leftrightarrow \bar{d}\bar{d}\bar{u}$$

hence the proton is not an eigenstate.



Kaons

- $K \to \pi\pi$ calculation needs stationary K^0.
- Need an eigenstate with e-val +1 for periodic BCs and hence $p_{\min} = 0$.
- $K^0 = \bar{s}d$ is not a G-parity eigenstate: $\bar{s}d \leftrightarrow \bar{s}\bar{u}$
- Introduce 'strange isospin' (I'): s-quark in doublet $\begin{pmatrix} s' \\ s \end{pmatrix}$
- Can now form an eigenstate:
$$K_0^g = \frac{1}{\sqrt{2}}(\bar{s}d + \bar{u}s') = \frac{1}{\sqrt{2}}(K_0 + K_0')$$
with e-val +1.
- Unphysical partner K_0' mixes with physical state K_0. For a physical operator, e.g. $A_\mu = \bar{s}\gamma^5\gamma^\mu d$, K_0' only contributes after propagating through the boundary: suppressed like $e^{-m_K L}$, a sub-% effect.
- Up to these effects, only change is a normalization factor.

Locality

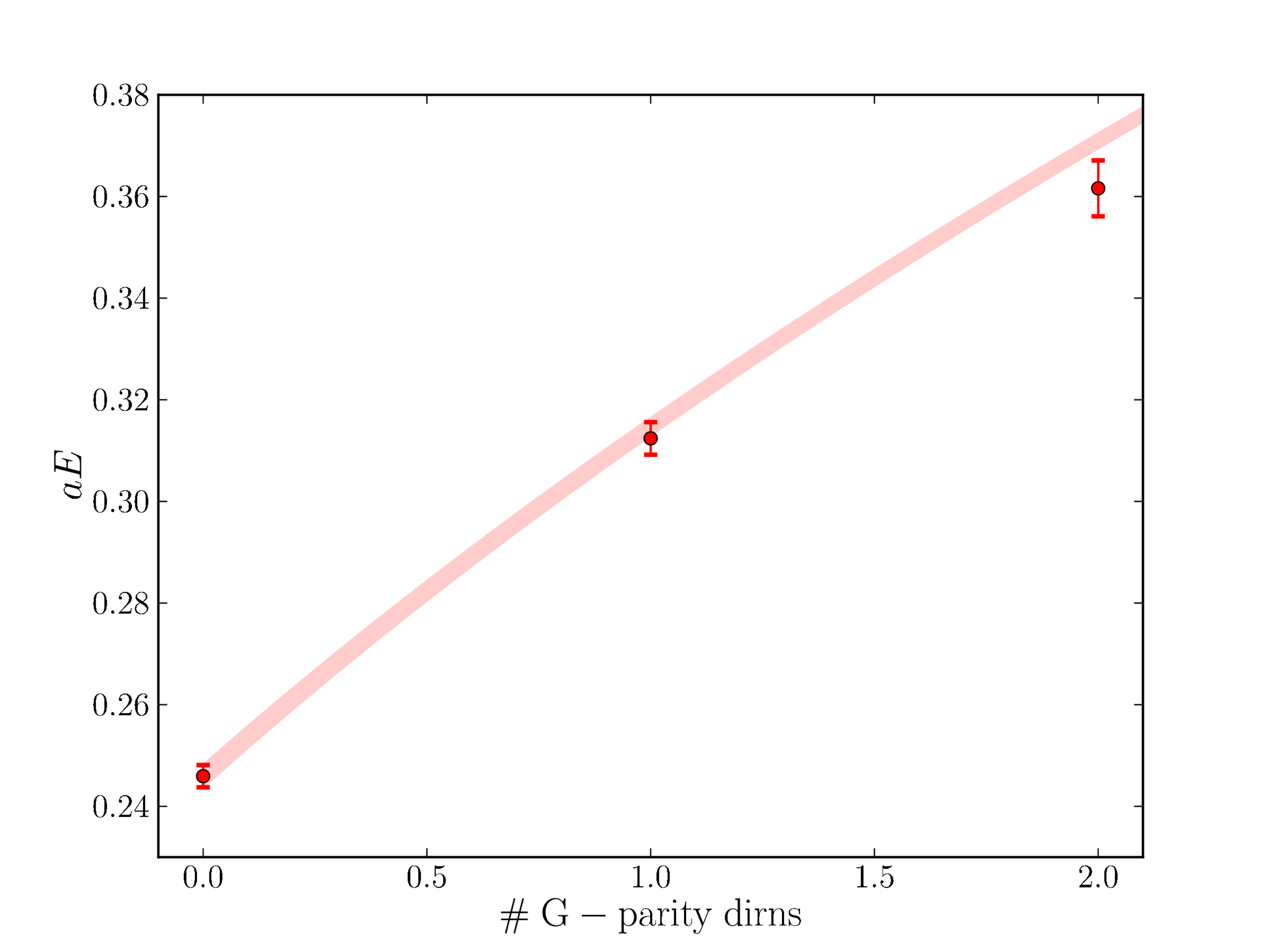
- Theory has one too many flavors. Must take square-root of s'/s determinant in evolution to revert to 3 flavors. Determinant becomes non-local.
- Non-locality is however only a boundary effect that vanishes as $L \to \infty$. With sufficiently large volumes the effect should be benign.
- Estimate size of effect : Staggered ChPT?

Results: Pion Dispersion Relation

- Generated $16^3 \times 32$ fully dynamical test ensembles with G-parity BCs in 0,1,2 directions.

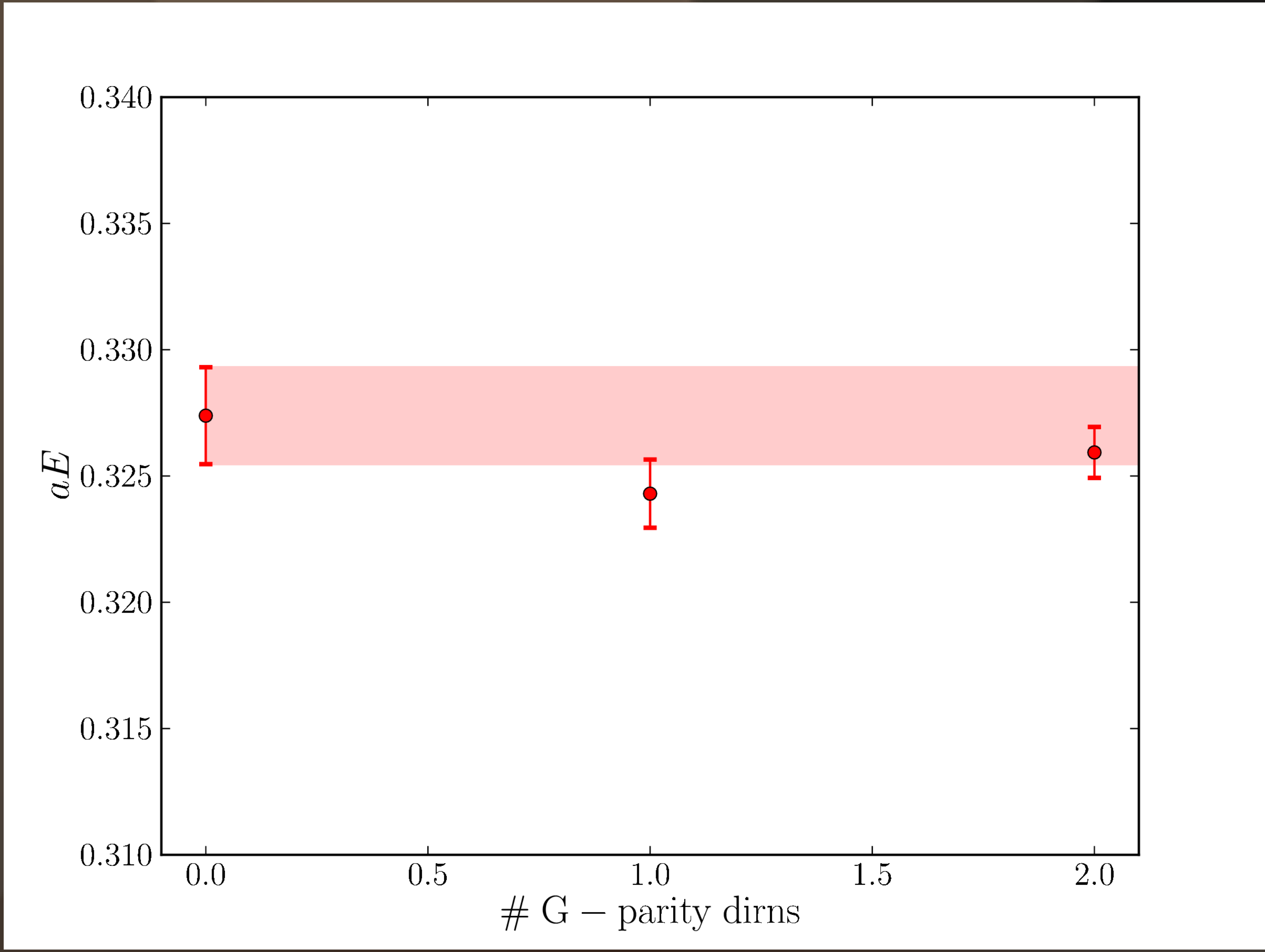
$a^{-1} = 1.73(3)$ GeV

$m_\pi \sim 420$ MeV



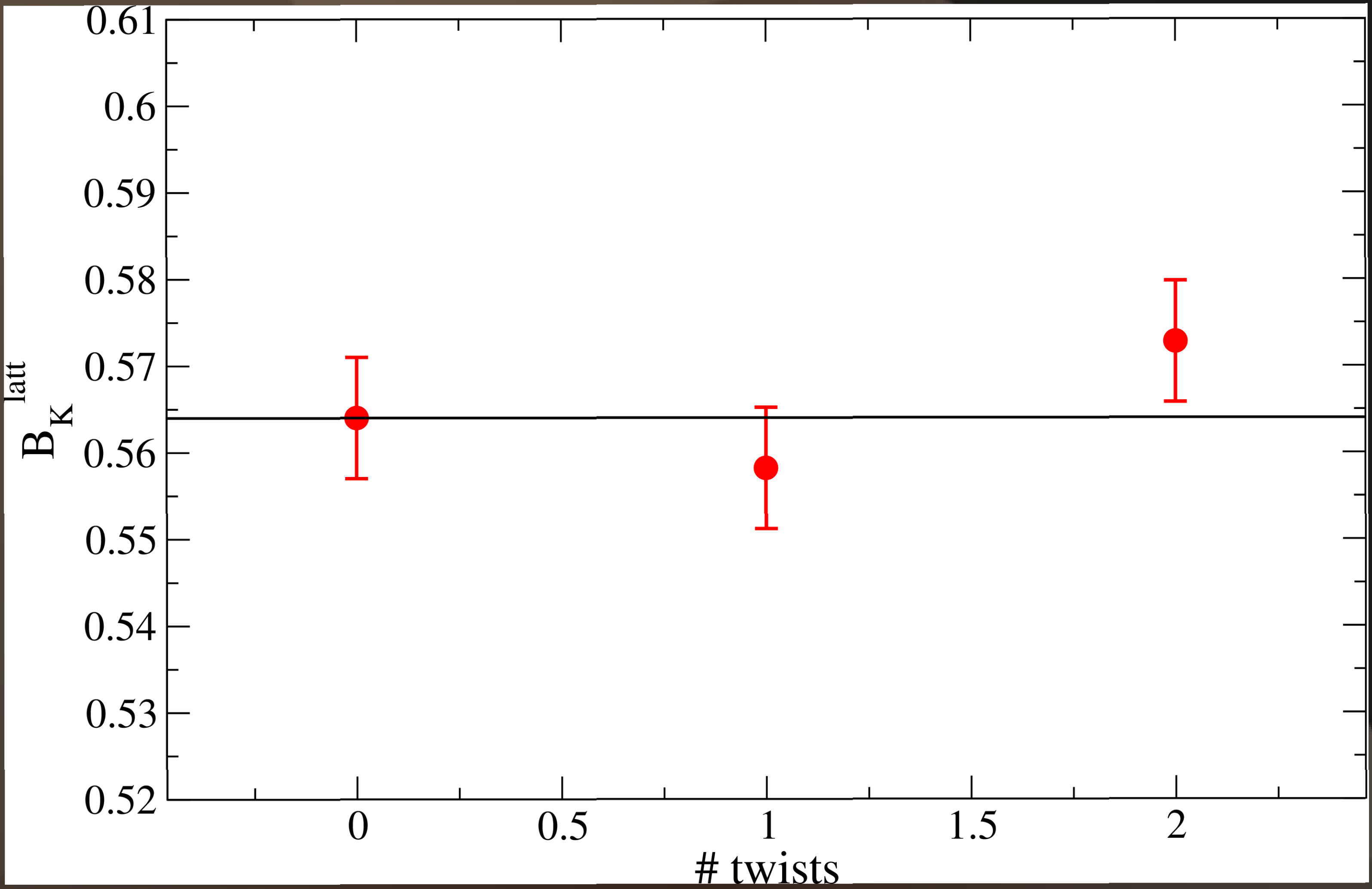
Results: Kaon Dispersion Relation

- Stationary kaon states demonstrated:



Results: B_K

- $\bar{K}^0 \leftrightarrow K^0$ mixing amplitude shown to be independent as expected. These 4-quark effective vertices are similar to those used in $K \rightarrow \pi\pi$ calculation, hence this is a valuable demonstration.

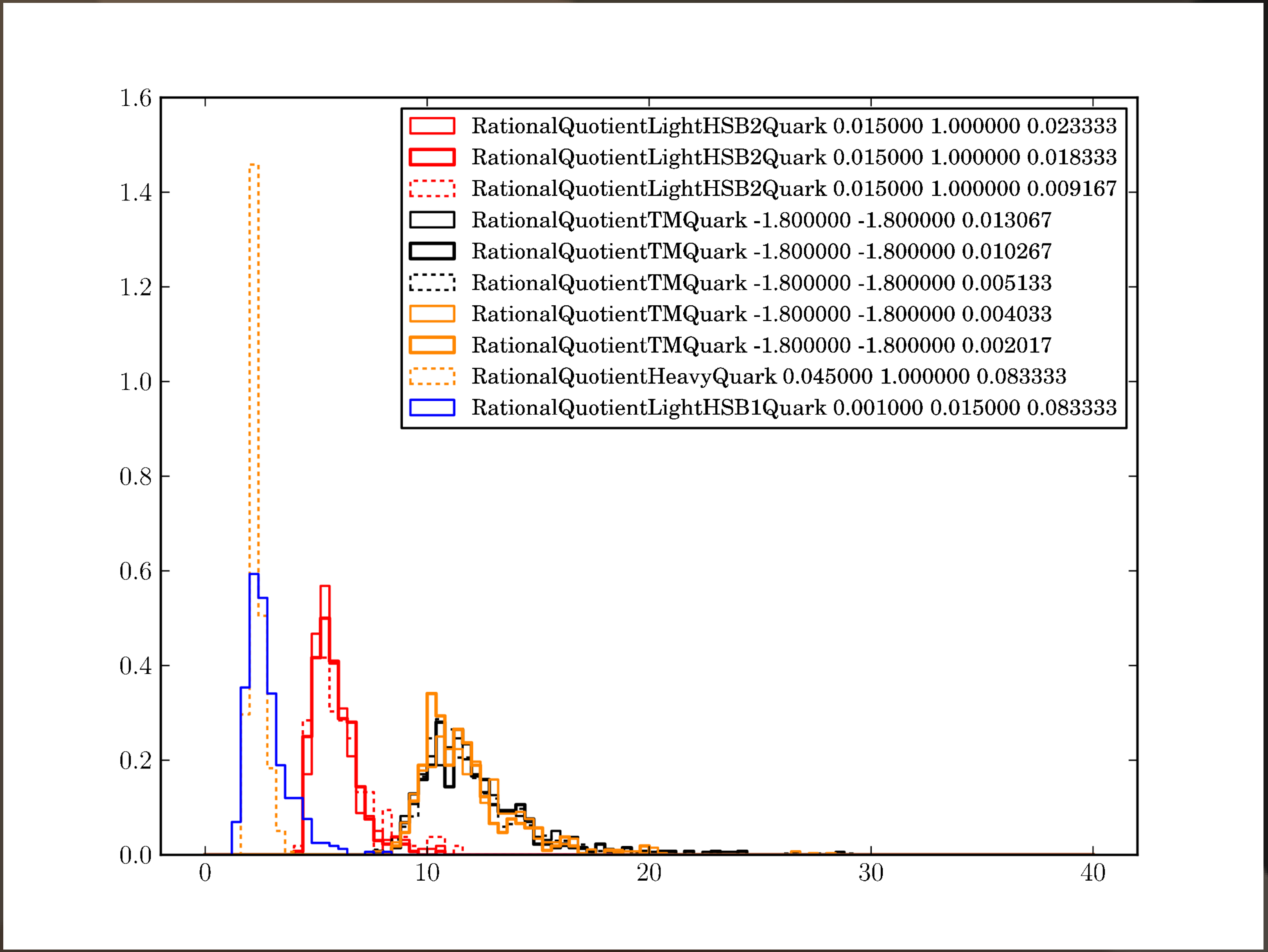


Ensemble for physical $K \to (\pi\pi)_{I=0}$ *calculation*

- Evolution code (CPS+BFM) for Mobius DW and Iwasaki+DSDR gauge action with G-parity BCs is now complete.
- Generation of $32^3 \times 64 \times 16$ configurations has been underway for over a month on the USQCD BGQ half-rack at BNL. Will soon have enough thermalized configurations to begin testing.
- Parameters are the same as the ensemble used for the $\Delta I = 3/2$ calculation: $\beta = 1.75\ (a^{-1} = 1.37(1)\ \mathrm{GeV})$ and $m_\pi = 143(1)\ \mathrm{MeV}$ (PQ) , $171(1)\ \mathrm{MeV}$ (unitary)
- Mobius parameters tuned to match to regular DWF, allows factor of 2 reduction in Ls for same physics.
- Dirac matrix is intrinsically 2-flavor, hence $\det(M^\dagger M)$ contains 4 flavors: must use RHMC even for light quarks.
- RHMC cost overhead (no chronological inverter) makes using multiple Hasenbusch steps more expensive. Ensemble is more difficult to tune.

Force Histogram

- Example force histogram produced during evolution tuning



Status and Outlook

- Substantial progress has been made in the march towards calculating the $\Delta I = 1/2$ amplitude.
- Further investigation of systematic errors associated with G-parity technique is required. However all tests to date have not indicated any sicknesses with the approach.
- Work still to be done in deciding best technique for measuring the amplitudes in the more complex environment of the calculation with G-parity. (cf. Daiqian Zhang's talk in this session)
- G-parity techniques may be useful for controlling errors in other frontier calculations performed by RBC & UKQCD, e.g. $K_L - K_S$ mass difference.
- Please stick around for Tadeusz Janowski's talk (next) in which you will hear about our developments on the $\Delta I = 3/2$ front.

Extra Slides

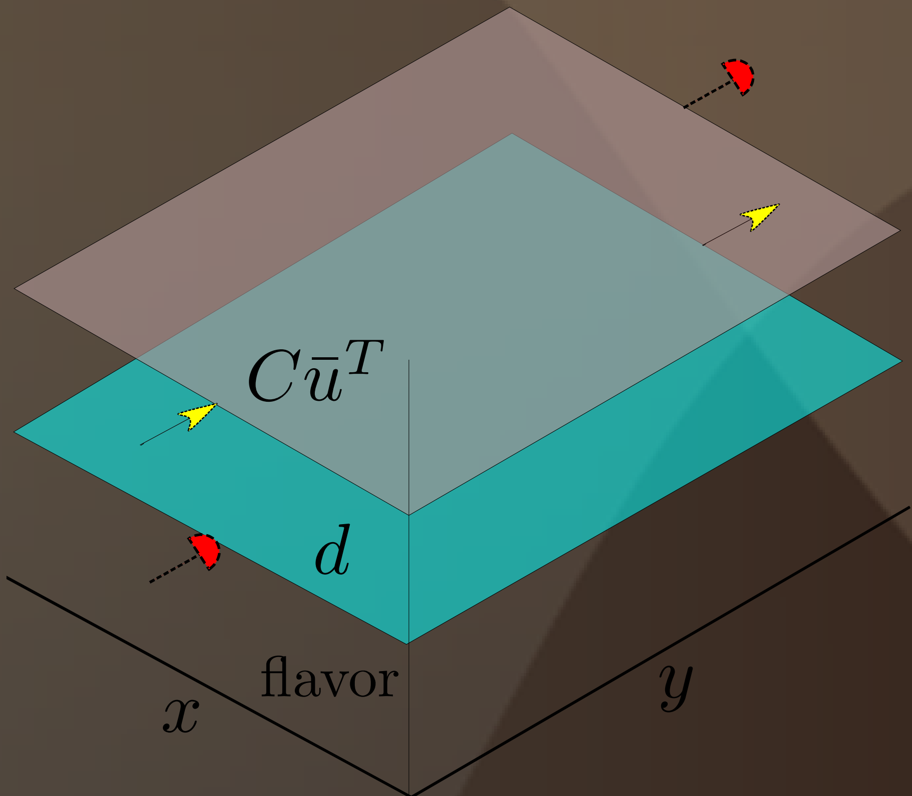
Lattice Determination

- As with B_K, amplitude A_2 is combination of renormalization-scheme dependent perturbative Wilson coeffs $C_i(\mu)$ and non-perturbative matrix elements $M_i(\mu)$:

$$A_2 \propto G_F V_{ud} V_{us} \sum_{i=1}^{10} C_i(\mu) M_i(\mu)$$

- $M_i = \langle (\pi^+ \pi^0)_{I=2} | Q_i | K^0 \rangle$
- Q_i are weak effective four-quark operators.
- Proportionality constant contains Lellouch-Luscher factor.
- Again we use RI-MOM NPR techniques to evaluate the renormalized amplitudes.

Layout of the Problem



- Two fermion fields on each site indexed by flavor index:

$$\psi^{(1)}(x) = d(x),\ \psi^{(2)}(x) = C\bar{u}^T(x)$$

- BCs are:

$$\psi^{(1)}(x + L\hat{y}) = \psi^{(2)}(x),$$
$$\psi^{(2)}(x + L\hat{y}) = -\psi^{(1)}(x),$$

- Periodic BCs in other dirs.
- Single U-field shared by both flavors, with complex conj BCs.
- Dirac op for $\psi^{(2)}$ uses U_μ^*.

Exploiting the Gauge-Field Symmetry

- Quark fields interact with the same gauge fields. Suggests propagators are related in some way.
- In fact, we find that: $$\begin{aligned} \mathcal{G}^{(2,2)}_{x,z} &= -\gamma^5 C \left[\mathcal{G}^{(1,1)}_{x,z}\right]^* C\gamma^5 \\ \mathcal{G}^{(1,2)}_{x,z} &= +\gamma^5 C \left[\mathcal{G}^{(2,1)}_{x,z}\right]^* C\gamma^5 \end{aligned}$$
- Relative sign due to – sign at boundary between u and d.
- Can be rewritten $\mathcal{G}_{x,z} = \gamma^5 C\sigma_1\sigma_3[\mathcal{G}_{x,z}]^*\sigma_3\sigma_1 C^\dagger\gamma^5$ where $\mathcal{G}$ are 2x2 flavour (also spin/colour) matrices.
- Substantially simplifies contractions: form is often identical to standard form up to additional flavour matrices: e.g.

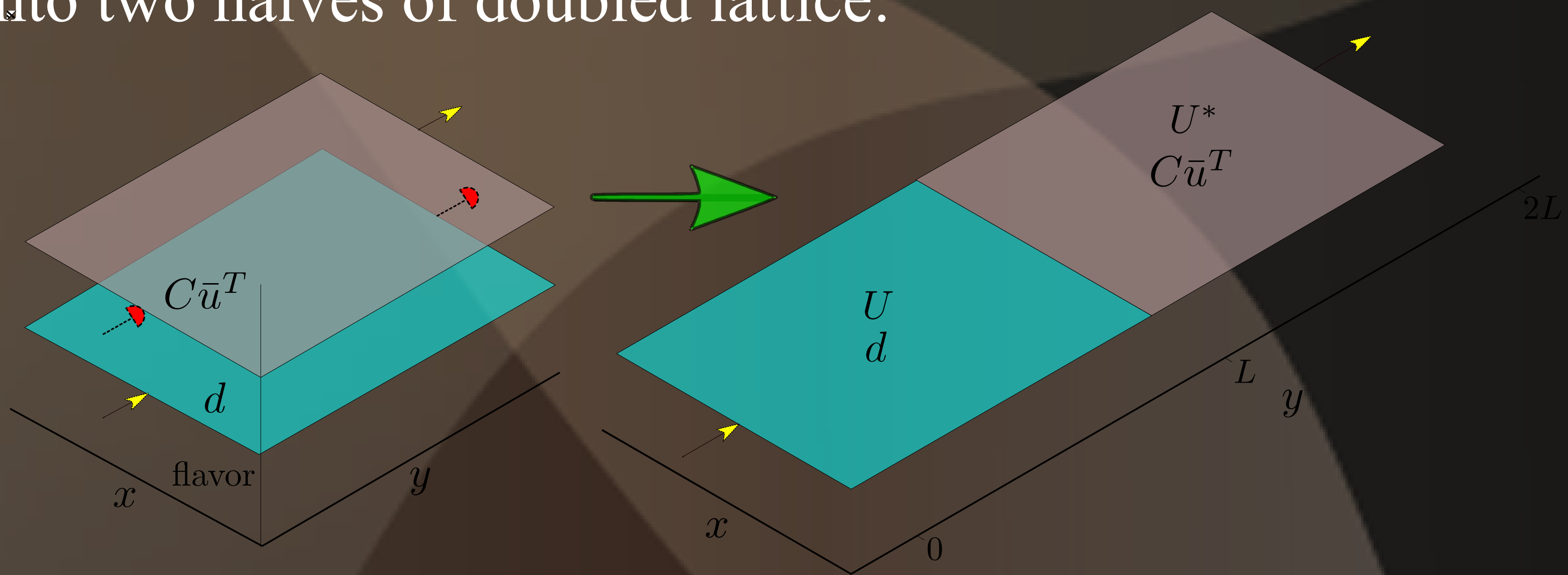
$\langle\pi(x)|\pi(y)\rangle = \mathrm{tr}\ F_1\mathcal{G}_{x,y}\sigma_3\mathcal{G}^\dagger_{x,y}$ (in large time limit)

$F_1 = \frac{1}{2}(1-\sigma_3)$

Usually $\mathrm{tr}\ \mathcal{G}_{x,y}\mathcal{G}^\dagger_{x,y}$

The One-Flavor Method

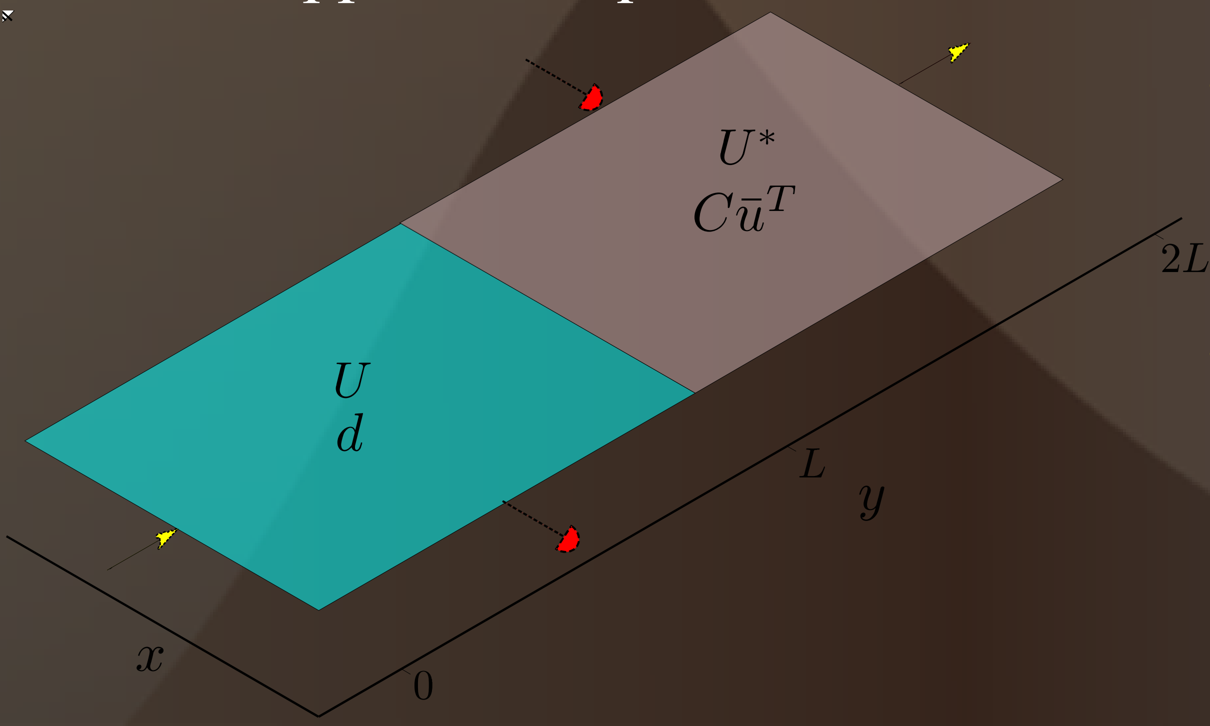
- Obtain equivalent formulation by unwrapping flavor indices onto two halves of doubled lattice:



- Antiperiodic boundary conditions in G-parity direction.
- U -field on first half and U^*-field on second half.

Choosing an Approach

- One flavor setup is much easier to implement.
- However recall that we needed APBC in 2 directions for physical kinematics in $\Delta I = 3/2$ calculation.
- G-parity in >1 dir using one-flavor method requires doubling the lattice again, which is highly inefficient.
- A second approach requires non-nearest neighbour communication:



- Also inefficient depending on machine architecture.
- Choose to implement two-flavor method.

$\mathbf{K} \to \pi\pi$ *Decays on the Lattice*

- Energies are discretized in finite-volume.
- For two non-interacting pions $E_{\pi\pi} = 2\sqrt{m_\pi^2 + k_\pi^2}$, where components of k_π discretized in units of $2\pi/L$ assuming periodic BCs.
- Interactions shift the allowed energies such that k_π are not known a priori and must be measured.
- Allowed k_π are quantized by Luscher condition $\delta(k_\pi) + \phi(k_\pi) = n\pi$, hence we can determine the scattering phase shifts $\delta(k_\pi)$ once k_π is measured.
- Switching on effective weak interaction H_W , allowed energies are further modified:

$$k - k_\pi = \Delta k = \pm \frac{m_K}{4k_\pi} |M| + O(|M|^2)$$

- Note this uses degenerate PT, thus requires $E_{\pi\pi} = m_K$

$\mathbf{K} \to \pi\pi$ *Decays on the Lattice*

- Switching on H_W induces corresponding change in infinite-volume scattering lengths:

$$\Delta\delta_0 = \mp \frac{k_\pi |A|^2}{32\pi m_K^2 |M|} + O(H_W^2)$$

- Imposing Luscher factor allows us to combine equations for $\Delta\delta_0$ and Δk, giving the Lellouch-Luscher formula:

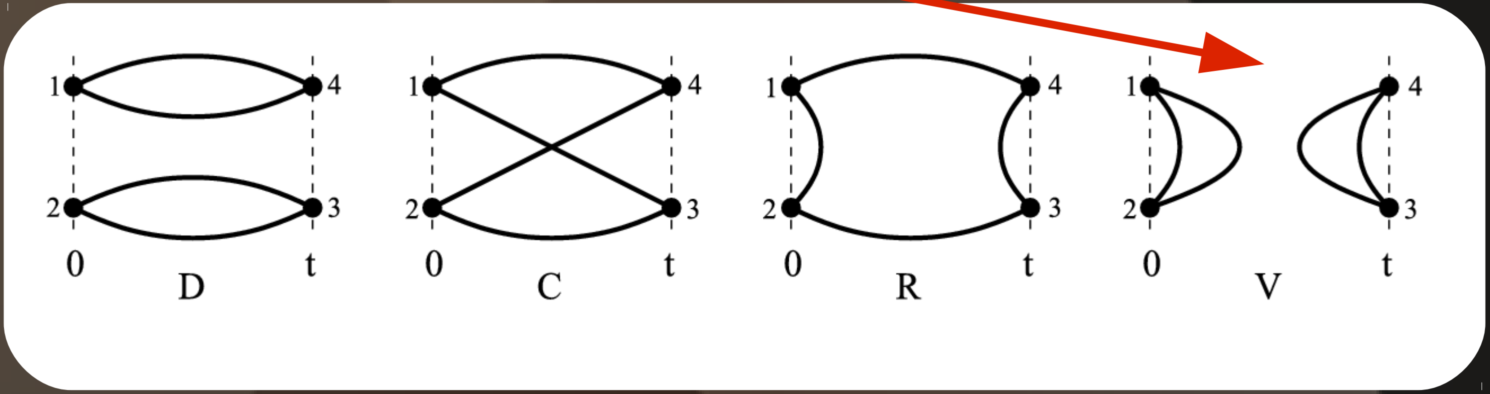
$$|A|^2 = 8\pi \left\{ q\frac{\partial\phi}{\partial q} + k\frac{\partial\delta_0}{\partial k} \right\}_{k=k_\pi} \left(\frac{m_K}{k_\pi}\right)^3 |M|^2$$

  where $q = kL/2\pi$

- As ϕ is analytic, only unknown is $\partial\delta_0/\partial k$. We measure this from the phenomenological curve at the measured k_π .

Challenges: part 1

- Measuring A_0 is considerably more challenging.
- Measure both $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$.
- $\pi\pi$ state has vacuum quantum numbers, hence there are disconnected diagrams:



- Need large statistics and many source positions (or A2A/AMA propagators) to resolve - cf. D. Zhang's talk
- With Blue Gene/Q resources we can now perform such calculations with large enough physical volumes.

$\mathrm{K} \to \pi\pi$ *Decays on the Lattice*

- Infinite-volume S-matrix $\langle \pi\pi | \mathcal{L}_W | K \rangle = A e^{i\delta_0}$ final state scattering induces dependence on s-wave scattering length δ_0 .
- Finite-volume matrix element: $M = \langle \pi\pi | H_W | K \rangle$ where H_W is effective weak vertex.
- Using degenerate PT (requires $E_{\pi\pi} = m_K$) ,weak effective theory and Luscher's quantization condition

$$\delta(k_\pi) + \phi(k_\pi) = n\pi \qquad E_{\pi\pi} = 2\sqrt{m_\pi^2 + k_\pi^2}$$

one obtains the Lellouch-Luscher formula relating them:

$$|A|^2 = 8\pi \left\{ q\frac{\partial \phi}{\partial q} + k\frac{\partial \delta_0}{\partial k} \right\}_{k=k_\pi} \left(\frac{m_K}{k_\pi} \right)^3 |M|^2$$

where $q = kL/2\pi$

- As ϕ is analytic, only unknown is $\partial\delta_0/\partial k$. We measure this from the phenomenological curve at the measured k_π