Weak Decay Measurements from 2+1 flavor DWF Ensembles

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Robert Mawhinney
Columbia University
The RBC and UKQCD Collaborations

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RBC members

Ziyuan Bai
Thomas Blum
Norman Christ
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Taichi Kawanai
Chris Kelly
Hyung-Jin Kim
Christoph Lehner
Jasper Lin
Meifeng Lin
Robert Mawhinney
Greg McGlynn
David Murphy
Shigemi Ohta
Eigo Shintani
Amarjit Soni
Oliver Witzel
Hantao Yin
Jianglei Yu
Daiqian Zhang

UKQCD members

Rudy Arthur
Peter Boyle
Hei-Man Choi
Luigi Del Debbio
Shane Drury
Jonathan Flynn
Julien Frison
Nicolas Garron
Jamie Hudspith
Tadeusz Janowski
Andreas Juettner
Richard Kenway
Andrew Lytle
Marina Marinkovic
Enrico Rinaldi
Brian Pendleton
Antonin Portelli
Chris Sachrajda
Ben Samways
Karthee Sivalingam
Matthew Spraggs
Tobi Tsang

Generic Process	Examples	Experiment	LQCD calculates
$Kl2$	$K^+ \to \mu^+ \nu_\mu$ $K^+ \to e^+ \nu_e$	f_K	f_K (also f_π)
$Kl3$	$K^+ \to \pi^0\, l^+\, \nu_l$ $K^0 \to \pi^-\, l^+\, \nu_l$	$\|V_{us} f^+(0)\|^2$	$f^+(0)$
$Kl4$	$K \to \pi\, \pi\, l\, \bar{\nu}_l$		??
$K \to \pi\pi$ (CP conserving)	$K^0 \to \pi^+\, \pi^-$ $K^+ \to \pi^+\, \pi^0$	$\|A_0\|$ $\|A_2\|$	$\|A_0\|\ \|A_2\|$ (SM_{cpc} inputs)
Δm_K (CP conserving)	$K^0 \leftrightarrow \pi\, \pi \leftrightarrow \overline{K}^0$ (LD) $K^0 \leftrightarrow O_{\Delta S=2} \leftrightarrow \overline{K}^0$ (SD)	Δm_K	Δm_K (SM_{cpc} inputs)
$K^0 \to \pi\ \pi$ (indirect CP violation)	$K_L \to \pi\, \pi$ $\left(K^0 \leftrightarrow \overline{K}^0\right) \to \pi\, \pi$ independent of $\pi\,\pi$ isospin	$\epsilon = \dfrac{\hat{B}_K F_K^2\, \mathrm{SM}}{\Delta m_K}$	$B_K, \dfrac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}$
$K^0 \to \pi\ \pi$ (direct CP violation)	$K_L \to \pi\, \pi$ depends on $\pi\,\pi$ isospin	$\mathrm{Re}(\epsilon'/\epsilon)$ $= f(A_0, A_2, \mathrm{SM})$	$A_0\ \ A_2$ (SM_{cpc} inputs)
$K \to \pi ll$	$K_L \to \pi^0 l^+ l^-$ $K_S \to \pi^0 l^+ l^-$		??

SM_{cpc} = Standard Model CP-conserving parameters

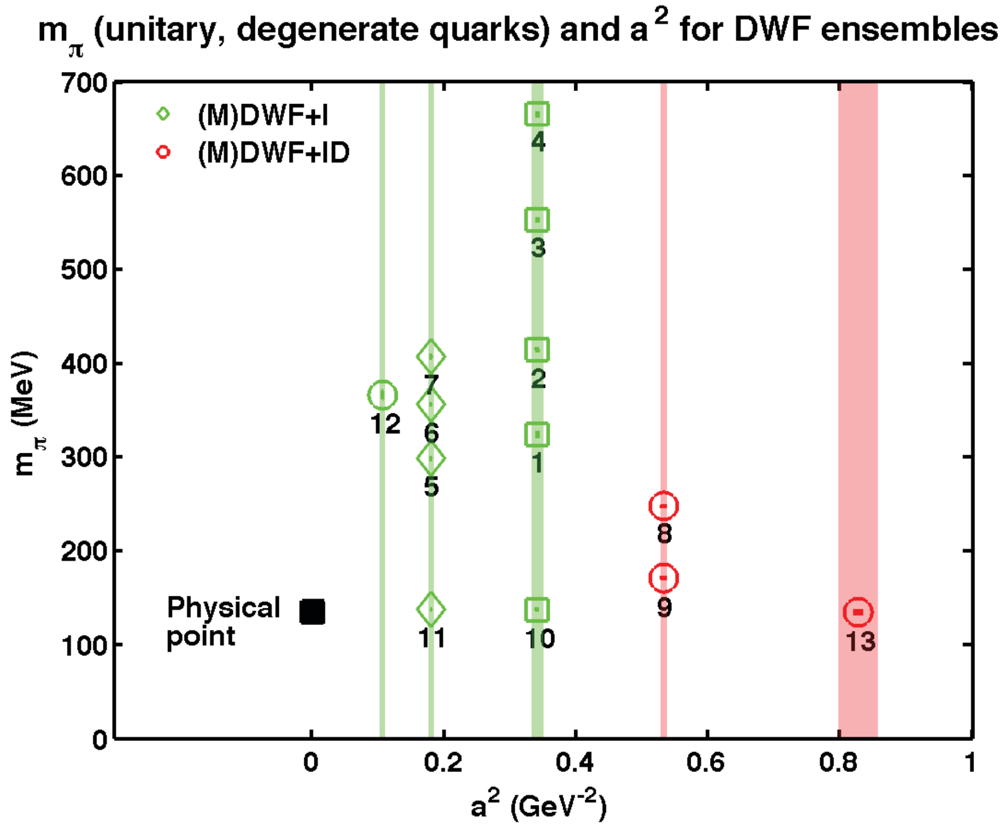
Major Development: Physical Quark Mass DWF Ensembles

RBC, UKQCD and HotQCD

Ens.	Action (G+F)	$1/a$ (GeV)	Lattice volume	m_l	m_s	m_{res}	m_π (MeV)	Size (fm)
				(in lattice units)				
1	DWF+I	1.75(3)	$24^3 \times 64 \times 16$	0.005	0.04	0.00308	330	2.7
2	DWF+I	1.75(3)	$24^3 \times 64 \times 16$	0.01	0.04	0.00308	420	2.7
3	DWF+I	1.75(3)	$24^3 \times 64 \times 16$	0.02	0.04	0.00308	560	2.7
4	DWF+I	1.75(3)	$24^3 \times 64 \times 16$	0.03	0.04	0.00308	670	2.7
5	DWF+I	2.31(4)	$32^3 \times 64 \times 16$	0.004	0.03	0.000664	310	2.6
6	DWF+I	2.31(4)	$32^3 \times 64 \times 16$	0.006	0.03	0.000664	370	2.6
7	DWF+I	2.31(4)	$32^3 \times 64 \times 16$	0.008	0.03	0.000664	420	2.6
8	DWF+ID	1.37(1)	$32^3 \times 64 \times 32$	0.0042	0.046	0.00184	250	4.5
9	DWF+ID	1.37(1)	$32^3 \times 64 \times 32$	0.001	0.046	0.00184	180	4.5
10	MDWF+I	1.75(3)	$48^3 \times 96 \times 24$	0.00078	0.0362	0.000614	138	5.5
11	MDWF+I	2.31(4)	$64^3 \times 128 \times 12$	0.000678	0.02661	0.000314	139	5.5
12	DWF+I	3.06(6)	$32^3 \times 64 \times 12$	0.0047	0.0186	0.00060	380	2.0
13	MDWF+ID	1.12(4)	$32^3 \times 64 \times 24$	0.00022	0.05960	0.0021	135	5.8

Table 1: Dynamical 2+1 flavor domain wall fermion ensembles produced (1-9) and being produced (10-13) by the RBC and UKQCD collaborations (10-12) and the RBC and HotQCD collaborations (13). The gauge and fermion (G+F) action abbreviations are: DWF = domain wall fermions, MDWF = Mobius domain wall fermions, I = Iwasaki gauge action, ID = Iwasaki plus Dislocation Suppressing Determinant Ratio (DSDR) gauge action. The total light quark mass (in lattice units) is $m_l + m_{\mathrm{res}}$ and the total strange quark mass is similarly $m_s + m_{\mathrm{res}}$.

Major Development: Physical Quark Mass DWF Ensembles



Ens.	Action (G+F)	$1/a$ (GeV)	Lattice volume	m_l (in lattice units)	m_s	$m_{\rm res}$	m_π (MeV)	Size (fm)
10	MDWF+I	1.75(3)	$48^3\times 96\times 24$	0.00078	0.0362	0.000614	138	5.5
11	MDWF+I	2.31(4)	$64^3\times 128\times 12$	0.000678	0.02661	0.000314	139	5.5

Using force gradient integrator of Clark and Kennedy, as implemented by Hantao Yin. Gave 2× speed-up over Omelyan at 1/a = 2.31 GeV and m_π = 220 MeV on 48^3. Expect even larger speed-up here, but too expensive to run Omelyan to measure effect.

Generating Physical Quark Mass DWF Ensembles

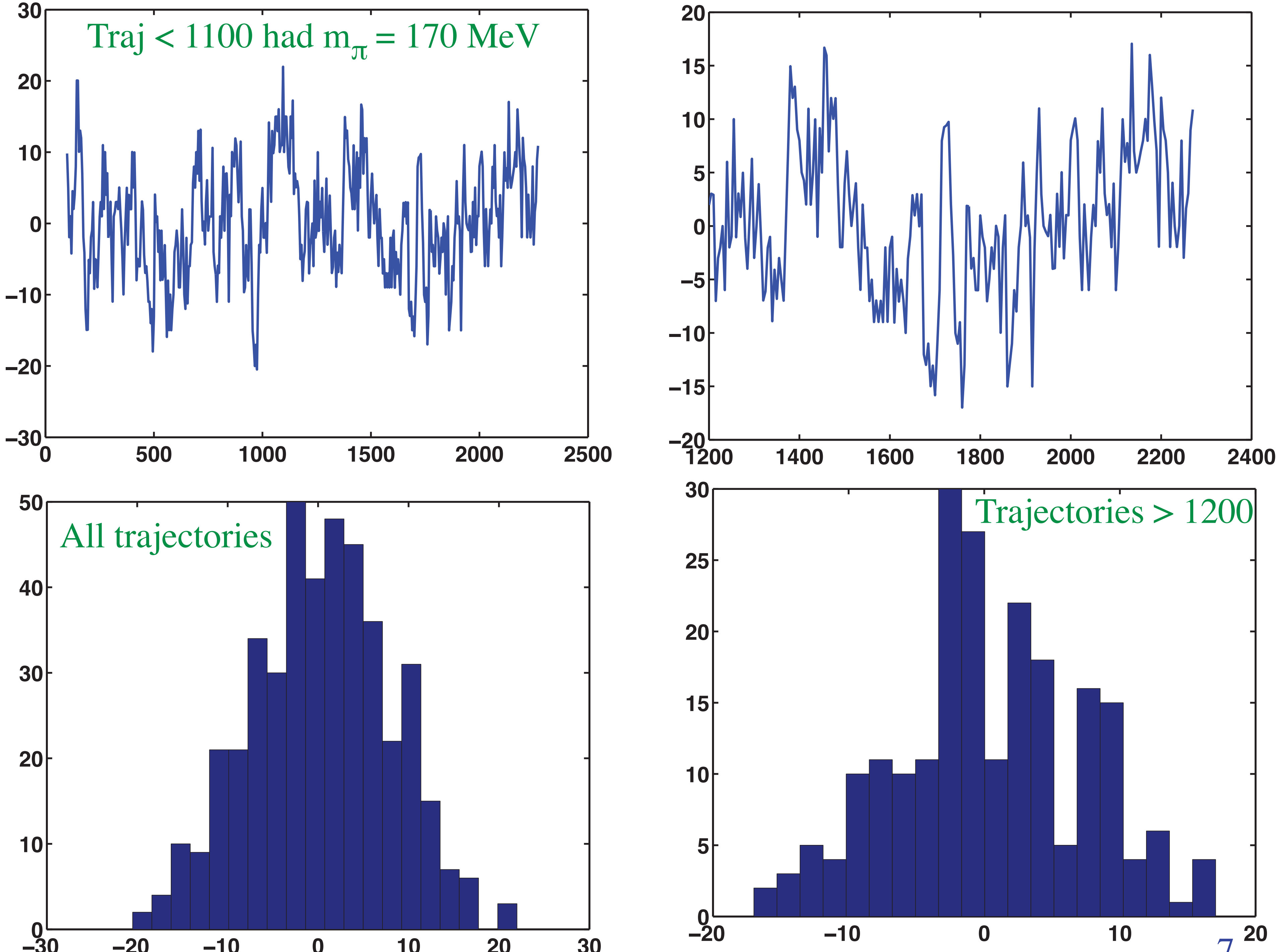
	48^3	64^3
Total # of lattice points	2.55×10^8	4.03×10^8
Hasenbusch masses	$0.00078, 0.005, 0.017, 0.07, 0.18, 0.45$	$0.000678, 0.005, 0.017, 0.07, 0.18, 0.45$
time step, $\Delta\tau$	0.067	0.111
number of steps	15	9
acceptance	83%	87%
Time per trajectory	3.5 hours on 2 BGQ racks 1.5 hours on 4 BGQ racks	0.67 hours on 8 BGQ racks
CG iters/traj	5.9×10^5	6.1×10^5
Single solve max CG iter.	3.5×10^4	4.3×10^4

Multimass solver

$48^3 \times 96 \times 24$ DWF+I			$64^3 \times 128 \times 12$ DWF+I	
$m_s = 0.0362$, 4096 node BG/Q			$m_s = 0.02661$, 8192 node BG/Q	
m_l	0.00078		0.000678	
Local volume	$6^3 \times 12 \times 24$		$16 \times 4 \times 4 \times 12$	
Routines	time(sec)	GFlops/s	time(sec)	GFlops/s
MInv	1538		640	
CG	3366	29	1565	53
GF	56		24	
RF	50		97	
HF	29		20	
Eig	140		140	
Total time(s)	5393		2422	

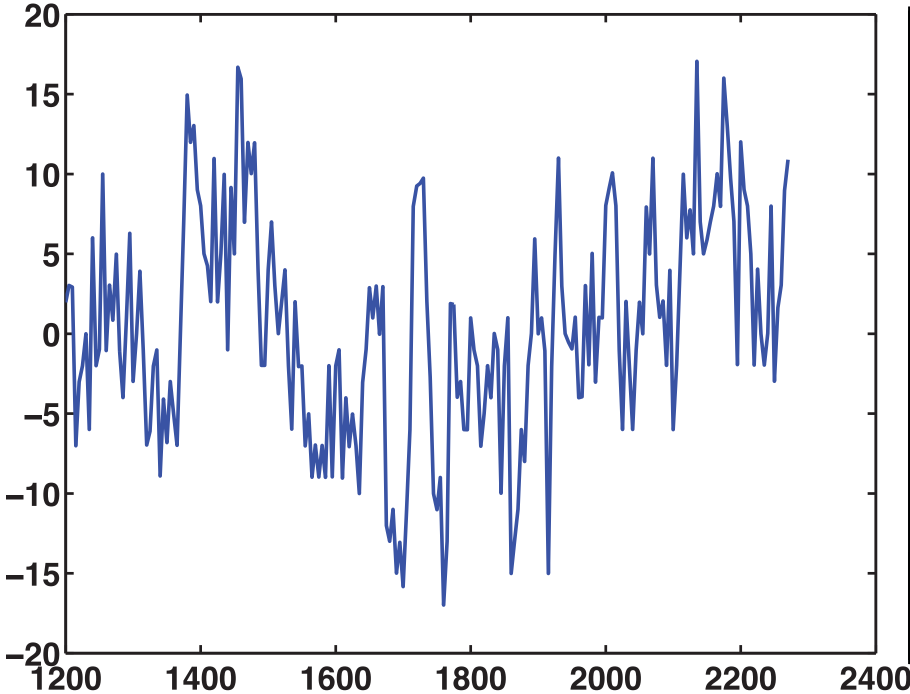
CG+Multimass solver+eigensolver is $\sim$ 90% of total evolution time.

Topology on 64^3 Ensemble

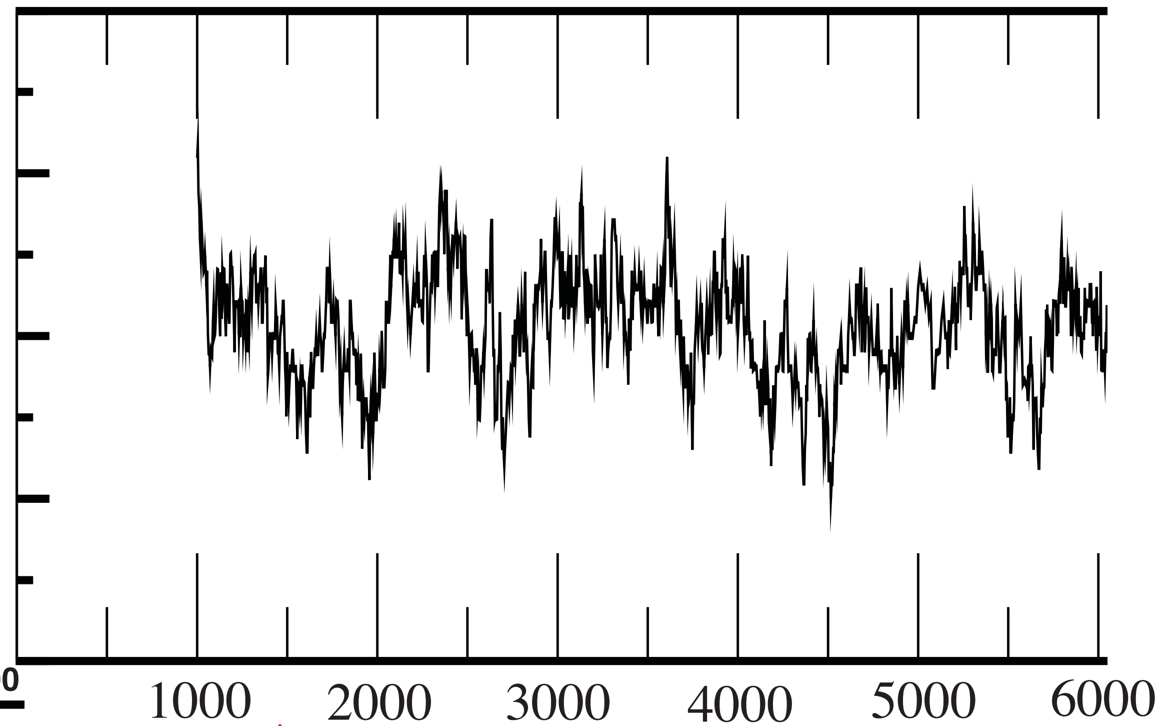


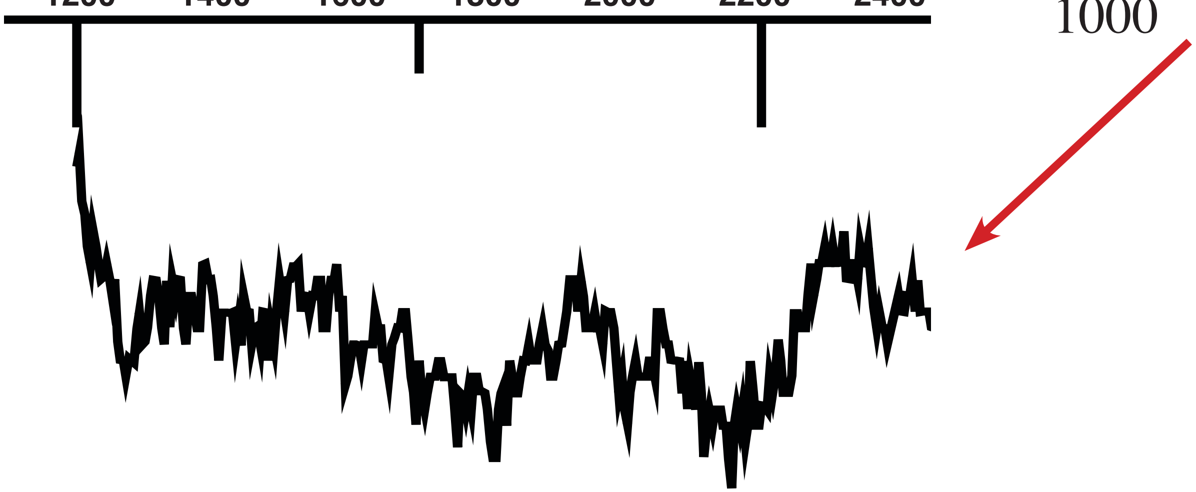
Comparison of Topological Evolution

$64^3 \times 128 \times 12$ MDWF
with $m_\pi = 139$ MeV



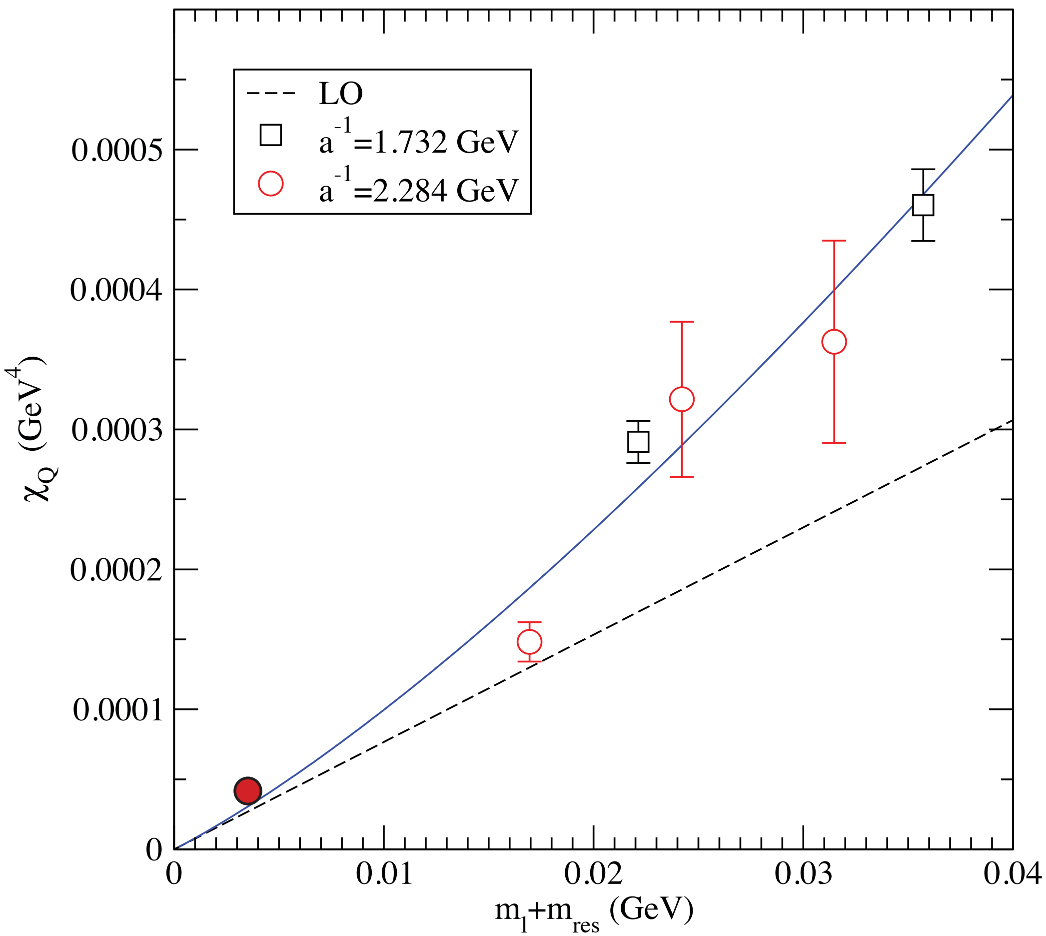
$32^3 \times 64 \times 16$ DWF
with $m_\pi = 295$ MeV





Topological evolution looks better on larger volume, even with smaller quark mass.

Chiral Susceptibility



No attempt at an error yet, but the preliminary result is encouraging.

RBC/UKQCD 2+1 flavor MDWF ensembles

- Using Mobius DWF (MDWF) with the Iwasaki gauge action
- b = 1.5, c = 0.5 for Mobius cuts L_s by 2 w.r.t. DWF, but CG iterations rise by 40-50%
- Very close to physical parameters:

	Physical value	48^3	48^3 deviation	64^3	64^3 deviation
m_π / m_K	0.2723	0.2797(7)	2.7%	0.2739(12)	0.6%
m_π / m_Ω	0.0807	0.0828(5)	2.5%	0.0821(5)	1.7%
m_K / m_Ω	0.2964	0.2959(18)	-0.2%	0.2997(14)	1.1%

- Previous simulations and chiral extrapolations used to choose input parameters
- 48^3 ensemble, 45% of the light quark mass from m_{res}. For the 64^3 ensemble, 30%.
- In a single measurement package, we are measuring basic hadronic masses, along with f_π, f_K, B_K (Frison talk), the K13 form factor $f^+(0)$ (Juettner talk) and the K -> $\pi\pi$, ΔI =2 amplitudes (Janowski talk).
- Also measuring K_L - K_S mass difference on ensembles with 330 MeV pions (J.Yu talk)
- A calculation of the K -> $\pi\pi$, ΔI =0 amplitudes, using G-parity boundary conditions at physical kinematics has begun (Kelly talk), on 1/a = 1.35 GeV lattices.

Measurement Techniques

- For deflation, large volume calculations require substantial storage for eigenvectors. IO can be prohibitive for staging to disk. Large numbers of CPUs also needed.
- BGQ has large memory and long mean time between failure, so measurement jobs can be run with everything in DRAM. ECC protection throughout ensures reliability.
- Earlier $K \to \pi\pi$ calculations by Qi Liu and RBC showed that EigCG (Stathopoulos and Orginos) worked well with DWF. We also had partially optimized EigCG code for BGQ from Qi Liu and further improvements have been made by Hantao Yin.
- Tests by Hantao Yin on $32^3 \times 64$ DWF+ID ensembles showed that error is reduced, particularly for $f^{+}_{K\pi}(0), K \to (\pi\pi)_{I=2}$ from translating calculation to all different times on each configuration.
- Tests showed that the statistical error for a given amount of CPU time from Coulomb gauge fixed wall sources is as small as from box sources, so we could share common sources amongst all of our pion and kaon observables.
- Natural to deflate with EigCG and then use all mode averaging of Blum, Izubuchi and Shintani. This requires a few measurements of observables with small stopping conditions (1e-08 generally) and translated measurements over all times with a larger stopping condition (sloppy solves with 1e-4 or 1e-5 stopping condition).

Measurement Techniques

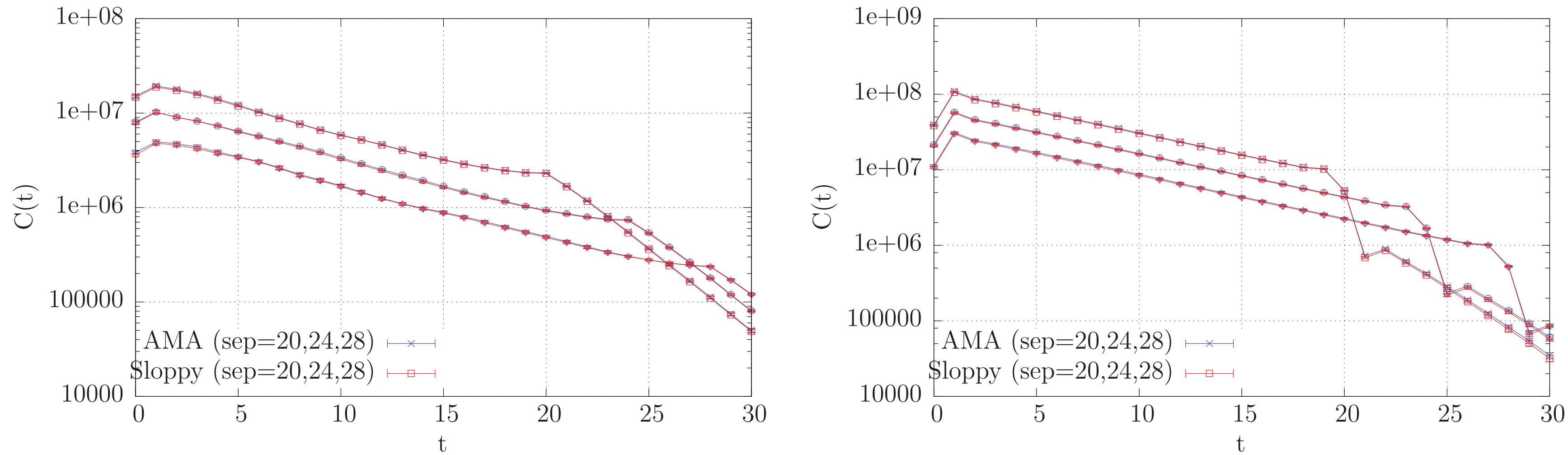
- Run EigCG for light quarks on a volume source - translationally invariant low modes
- On each configuration, 7 high precision wall source propagator solves are done, at definite relative positions. A random number is used for an overall translation.
- Entire procedure is translationally invariant
- CG solves are defect correction solves: solve in single precision, calculate deviation in double precision, recalculate solution for defect in single precision.
- This means deflation is done in single precision - decreases storage for low modes.
- The restart in the defect correction and subsequent reprojection into the subspace compensates for the fact that the EigCG low modes are not all accurate eigenvectors.

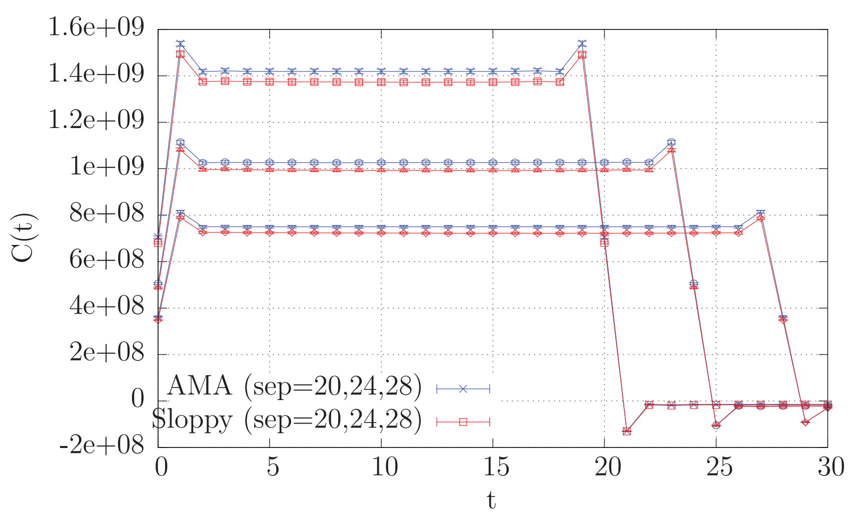
	48^3	64^3
Light modes calculated	600 (7.3 TBytes)	1500 (29 TBytes)
Light quark EigCG setup	29.5	79
Exact light solves (10^{-8})	18.7	12
Inexact light solves (10^{-4})	64	45
Exact strange solves (10^{-8})	8	17
Contractions	3	17
Total BGQ rack-hours	124	170
Time and # of BGQ racks	124 hours on 1 rack	5.3 hours on 32 racks

Times in BGQ rack-hours

Solver sustains ← 1 PFlops

Comparing AMA and Inexact Results for K_{l3}





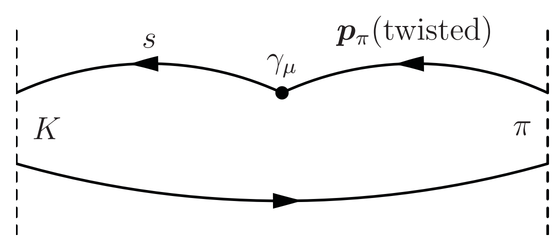


Figure 7.11: Inexact and AMA Kl3 correlators on the $48^3 \times 96$ lattice. Top left: γ_x, top right: γ_t, bottom: $\pi - \pi$ 3 point function for Z_V measurement.

Measurement Speed Up

- Exact light solve, no EigCG takes 1.8 BGQ rack-hours (all spins and colors)
- 7 locations × 3 momenta = 38 BGQ rack-hours
- 112 hours in measurement package for light quark solves (all time slices)
- AMA takes 3× the rack-hours that exact does, but errors are markedly reduced.
  K_{l3} on 48^3 : $\sigma_{exact}/\sigma_{AMA}$ = 4-5

Ensemble	Observable	Exact	AMA	$\sigma_{\mathrm{Exact}}/\sigma_{\mathrm{AMA}}$
$48^3 \times 96$	m_π	0.08006(51)	0.08065(18)	2.81
	m_K	0.28813(55)	0.28840(23)	2.39
	f_π	0.07650(32)	0.07601(13)	2.38
	f_K	0.09099(37)	0.09063(13)	2.94
	f_K/f_π	1.1894(48)	1.1924(18)	2.65
	B_K	0.58132(851)	0.58363(85)	10.0
	Z_A	0.71374(153)	0.71203(20)	7.81
$64^3 \times 128$	m_π	0.05857(48)	0.05891(26)	1.89
	m_K	0.21563(51)	0.21510(21)	2.50
	f_π	0.05555(29)	0.05545(11)	2.71
	f_K	0.06650(32)	0.06643(13)	2.40
	f_K/f_π	1.1972(63)	1.1980(26)	2.44
	B_K	0.5776(118)	0.5623(12)	10.2
	Z_A	0.74302(147)	0.74344(16)	9.28

- Have measurements on 45 configurations for 48^3, separated by 20 trajectories.
  On 64^3 have measurements on 21 configurations, separated by 40 trajectories.

RBC/UKQCD 2+1 flavor Ensembles

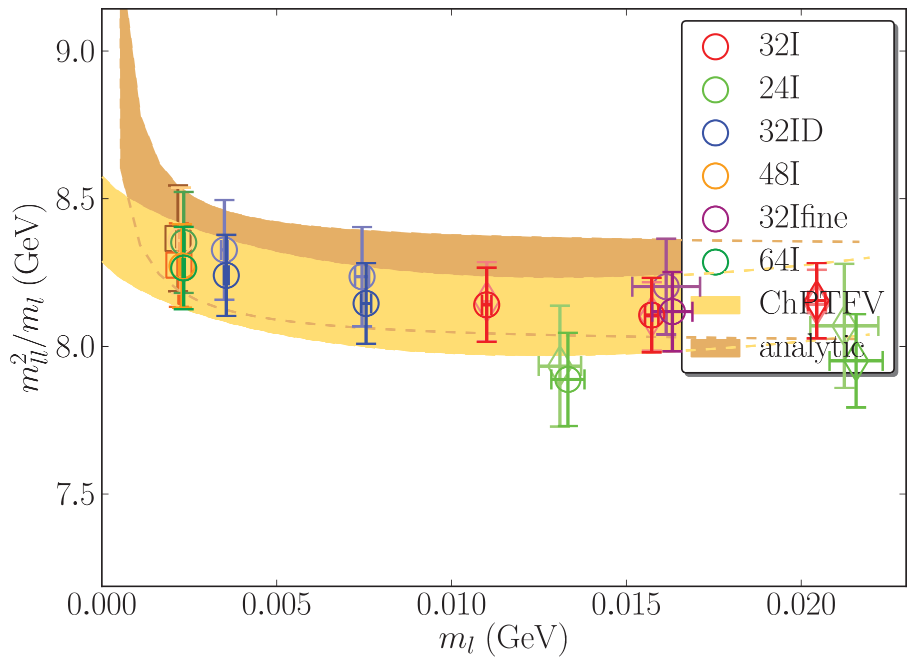
- Ensembles used by RBC and UKQCD Collaborations for kaon and pion physics
- The new ensembles this year are the 140 MeV ensembles, with the large volumes

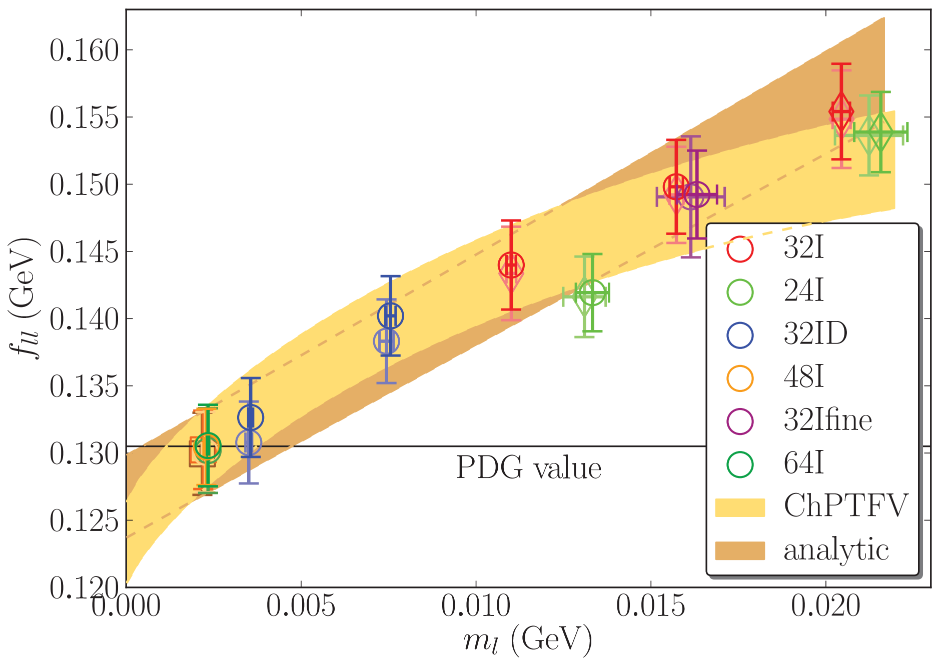
Ensemble Name	a (fm)	Volumes	Unitary m_π (MeV)
1/a= 1.37 GeV	0.146	$(4.7\ \mathrm{fm})^3$	170, 250
1/a = 1.71 GeV	0.117	$(2.8\ \mathrm{fm})^3$ $(5.6\ \mathrm{fm})^3$	140, 320, 410
1/a = 2.36 GeV	0.0847	$(2.7\ \mathrm{fm})^3$ $(5.4\ \mathrm{fm})^3$	140, 295, 350, 400
1/a = 3.07 GeV	0.0651	$(2.1\ \mathrm{fm})^3$	360

- Preliminary global fit to m_π, m_k, m_Ω, f_π, f_K, using m_π, m_k, m_Ω to set (β, m_l, m_s)
- Overweight 48^3 and 64^3 ensembles to make sure NLO ChPT goes through these.
- Heavier quark masses give LEC's for extrapolation and $O(a^2)$ coefficients.
- $f_\pi = 130.2 \pm 2.9_{\mathrm{stat}}$ MeV (Preliminary)
- $f_K = 156.1 \pm 3.2_{\mathrm{stat}}$ MeV (Preliminary)
- $f_\pi / f_K = 1.198 \pm 0.006_{\mathrm{stat}}$ (Preliminary)
- 48^3: $Z_V = 0.7088(15)$, $Z_A = 0.71198(16)$, agree to 0.5%, so good chiral symmetry.

Preliminary Global Fit

- New 48^3 and 64^3 data very close to physical point.
- For preliminary fit, artifically weight these points heavily in global fit.
- Heavier quark mass data to determines chiral extrapolation.
- Other methods in future.
- Can compare NLO ChPT with/without finite volume corrections and analytic fit





$\mathrm{B_K}$

- RBC/UKQCD value from PRD 84 (2011) 014503

$$B_K(\overline{\mathrm{MS}}, 3\,\mathrm{GeV}) = 0.529 \pm 0.005_{\mathrm{stat}} \pm 0.015_{\mathrm{chiral}} \pm 0.002_{\mathrm{finite\,V}} \pm 0.011_{\mathrm{pert}}$$

- RBC/UKQCD value from PRD 87 (2013) 094514

$$B_K(\overline{\mathrm{MS}}, 3\,\mathrm{GeV}) = 0.535 \pm 0.008_{\mathrm{stat}} \pm 0.007_{\mathrm{chiral}} \pm 0.003_{\mathrm{finite\,V}} \pm 0.011_{\mathrm{pert}}$$

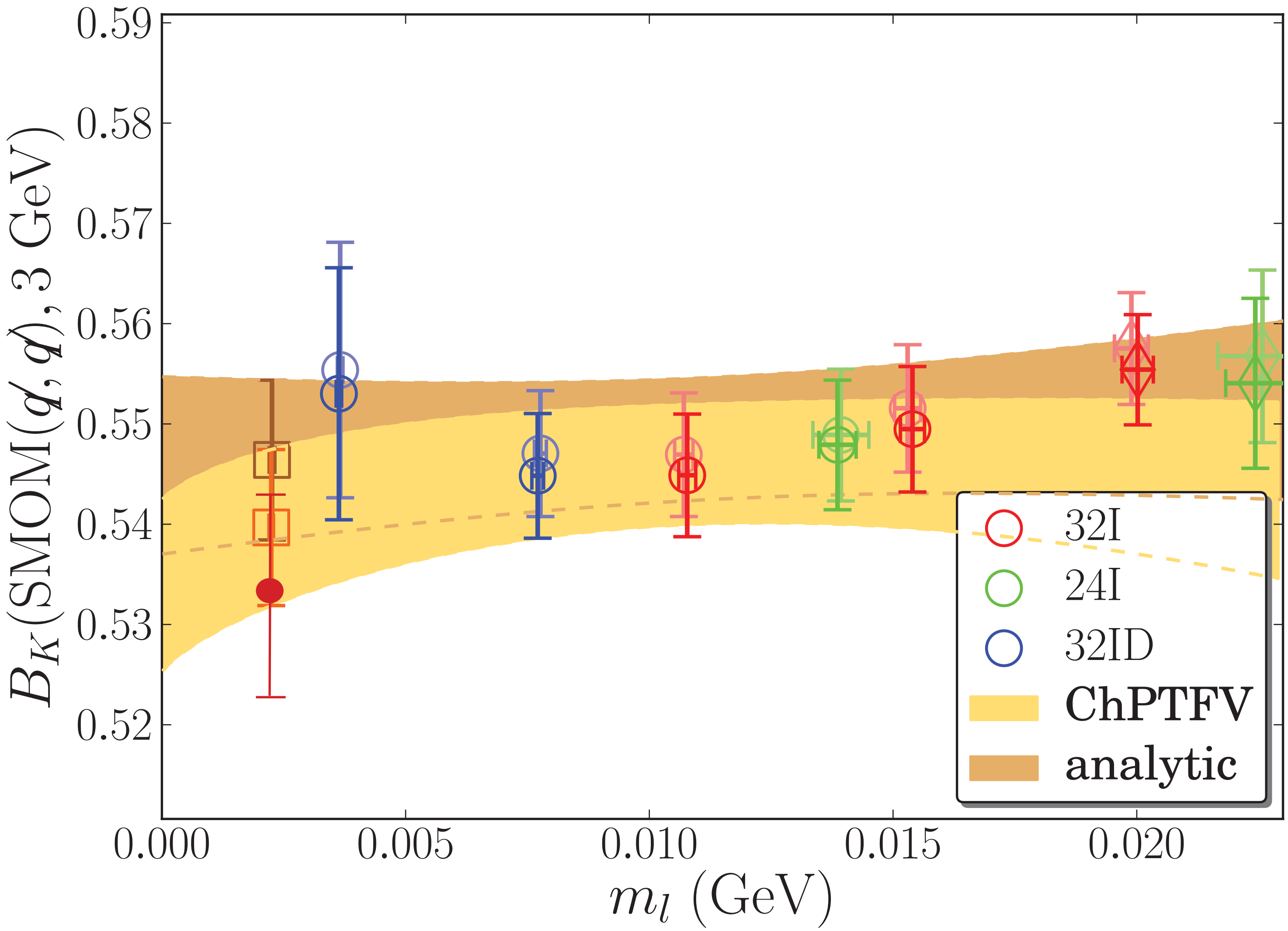
- Using results from our 2 new, physical quark mass lattices gives

$$B_K(\overline{\mathrm{MS}}, 3\,\mathrm{GeV}) = 0.533 \pm 0.003_{\mathrm{stat}} \pm 0.000_{\mathrm{chiral}} \pm 0.002_{\mathrm{finite\,V}} \pm 0.011_{\mathrm{pert}} \text{ (Preliminary)}$$
$$\hat{B}_K = 0.754 \pm 0.004_{\mathrm{stat}} \pm 0.0015_{\mathrm{sys}} \text{ (Preliminary)}$$

- Note the marked reduction in the statistical error and the lack of any chiral extrapolation error.

- Further improvements are possible, if the perturbative error can be reduced through non-perturbative step scaling on the lattice, so that perturbative matching can be done at higher scales.

B_K



New (preliminary) physical point result includes error from matching to $\overline{\mathrm{MS}}$

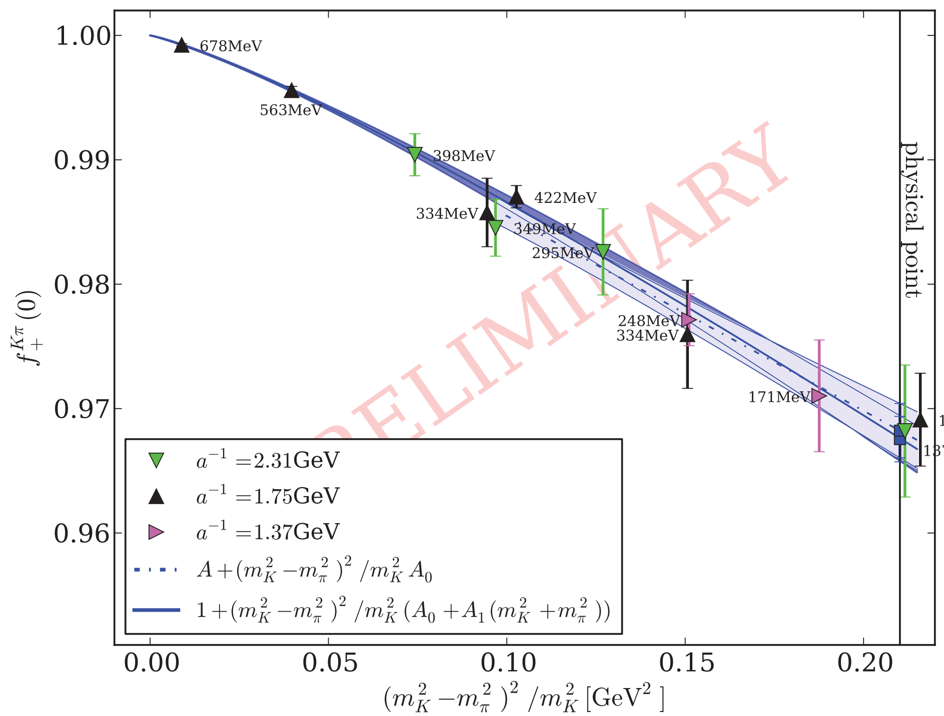
$K \to \pi$ semi-leptonic form factor (RBC+UKQCD Collaborations)

Phys.Rev.Lett. 100 (2008) 141601, Eur.Phys.J. C69 (2010) 159-167, arXiv:1305.7217
Talks by B. Mawhinney (Thursday, 8C, 17:30) and A. Jüttner (Thursday, 7C, 14:20)

- $\langle \pi | V | K \rangle \to f_+^{K\pi}(q^2 = 0)$
- part. twisted boundary conditions
- $N_f = 2 + 1$ domain wall fermions
- a^2-scaling study (0.09fm-0.14fm) $\to$ tiny cut-off effects
- physical point simulation

  m_π: 171–670MeV $\to$ arXiv:1305.7217
  137–670MeV $\to$ PRELIMINARY

  - polynomial ansatz describes data over entire mass range
  - phys. point data eliminates large systematic due to χ extrapolation



$$f_+^{K\pi}(0) = 0.9670(20)\left({}^{+\ 0}_{-42}\right)_{m_q}(7)_{\mathrm{FSE}}(17)_a$$

$$|V_{us}| = 0.2237\,(7)\,\left({}^{+10}_{-\ 0}\right)_{m_q}(2)_{\mathrm{FSE}}(4)_a$$

$\to \approx 0$ with phys. point data

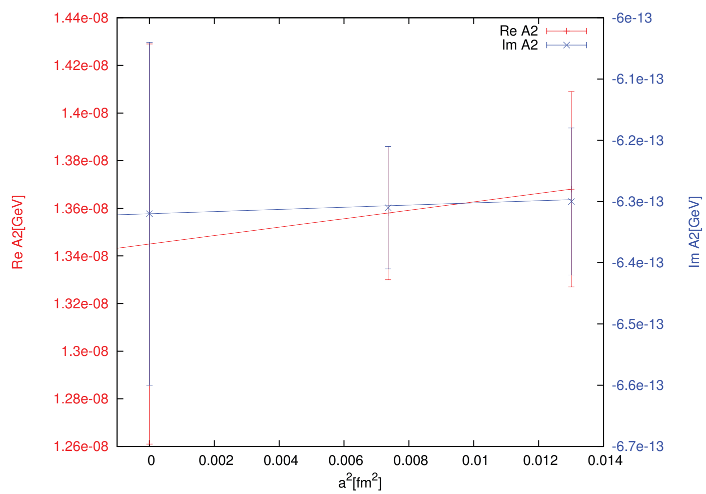
precision $\lesssim 0.3\%$ feasible!

RBC/UKQCD K -> ππ, I = 2 Amplitudes

- Previous published result (PRL 108 (2012) 141601 and PRD 86 (2012) 074513):

$$\mathrm{Re}\,A_2 = (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.135_{\mathrm{sys\,no}\,a^2} \pm 0.207_{a^2}) \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}\,A_2 = -(6.54 \pm 0.46_{\mathrm{stat}} \pm 0.72_{\mathrm{sys\,no}\,a^2} \pm 0.98_{a^2}) \times 10^{-13}\ \mathrm{GeV}$$

- Systematic error dominated by a^2 error estimate for the single DSDR ensemble.
- Now have 2 lattice spacings from new ensembles with the Iwasaki gauge action, allowing a continuum limit extrapolation



$$\mathrm{Re}\,A_2 = (1.345 \pm 0.084_{\mathrm{stat}} \pm 0.135_{\mathrm{sys\,no}\,a^2} \pm 0.0_{a^2}) \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}\,A_2 = -(6.32 \pm 0.28_{\mathrm{stat}} \pm 0.72_{\mathrm{sys\,no}\,a^2} \pm 0.0_{a^2}) \times 10^{-13}\ \mathrm{GeV}$$

(Preliminary)

- Markedly reduced statistical errors for Im A_2
- Other systematic errors will be revisited and likely will be reduced.

Conclusions

- Physical quark mass simulations using MDWF on large $(5.5\ \mathrm{fm})^3$ volumes underway.
- Generation via quotient HMC and RHMC, using force gradient integrator and 5 intermediate Hasenbusch preconditioning masses.
- Mobius DWF helps some here, but helps even more at strong couplings ($1/a \sim 1.3$ GeV) where MDWF thermo is being done.
- Even kaon measurements get noisier when pion is physical
- ~10× speed up in measurement package has been vital to achieving sub-percent statistical errors on many quantities.
- Peter Boyle has HDCG running, offering further speed-up of 2-5× and smaller memory footprint. Being tested in measurement package currently by Chulwoo Jung.

The calculations reported here have been done on BGQ computers of ANL, LLNL, the University of Edinburgh, BNL and the RBRC.