A possible new phase in gauge-fixed Yang-Mills theory

Maarten Golterman (IFAE, Barcelona and San Francisco State University)

with Yigal Shamir, Phys.Rev. D87 (2013) 054501

Outline:

- non-perturbative gauge fixing: equivariant BRST symmetry (review)
- spontaneous symmetry breaking in a topological field theory?
- phase diagram of equivariantly gauge-fixed SU(2) Yang-Mills (speculation)

Equivariant gauge fixing

- Standard BRST gauge fixing: insert into $Z = \int [dU] \exp[-S(U)]$

$$\text{constant} = Z_{\text{gf}}(U,\xi) = \int [d\phi][dc][d\bar{c}] \exp[-S_{\text{gf}}(U^\phi, c, \bar{c})]$$

$Z_{\text{gf}}(U,\xi)$ independent of U and gf parameter ξ, but $\text{constant} = 0$
U^ϕ is gauge transform of U (Neuberger '87)

- equivariant gauge fixing: gauge fix only the coset $SU(2)/U(1)$

- $C = C_1\tau_1/2 + C_2\tau_2/2$, etc. coset valued: $sC = 0$ instead of $sC = -iC^2$

- $s^2(\text{field}) = \delta_{U(1)}(\text{field})$: "equivariant" nilpotency

- $S_{\text{gf}} = \frac{1}{\xi g^2} \operatorname{tr} \left(F(U^\phi)\right)^2 + 2 \operatorname{tr} \left(\overline{C} M(U^\phi) C\right) -2\xi g^2 \operatorname{tr} \left(C^2 \overline{C}^2\right)$

- choose $F(U) \sim D_\mu(A) W_\mu$, $V_\mu = \frac{1}{2}\left(W_\mu^1 \tau_1 + W_\mu^2 \tau_2 + A_\mu \tau_3\right)$

- Invariance theorem: $\langle \mathcal{O}(\mathcal{U}) \rangle_{\text{unfixed}} = \langle \mathcal{O}(\mathcal{U}) \rangle_{\text{eBRST}}$ (Schaden'98, G&S '04)

  $Z_{\text{gf}}(U, \xi g^2) \neq 0$ does not depend on U, $\tilde{g}^2 = \xi g^2$: topological field theory

- Reduced model: take U pure gauge (on trivial orbit, i.e., $g \to 0$)

$$Z_{\text{gf}}(1, \tilde{g}^2) = \int [d\phi][dC][d\overline{C}] \exp\left[-S_{\text{gf}}(1^\phi, C, \overline{C})\right]$$

$$S_{\text{gf}} = \frac{1}{\xi g^2} \operatorname{tr}\left(F(1^\phi)\right)^2 + 2 \operatorname{tr}\left(\overline{C} M(1^\phi) C\right) - 2\xi g^2 \operatorname{tr}\left(C^2 \overline{C}^2\right)$$

  with $1^\phi = \phi_x \phi^\dagger_{x+\mu}$, keep $\tilde{g}^2 = \xi g^2$ fixed

  This is a strongly interacting theory with asymptotically free coupling $\tilde{g}$

- Symmetries: $\phi_x \to h_x \phi_x g^\dagger$, $s\phi = -iC\phi$, etc.; still TFT $\longrightarrow$?

  h_x: $U(1)_L$ (local) $g^\dagger$: $SU(2)_R$ (global) ($U(1)$: ϕ coset valued)

- Order parameter: $\phi^\dagger \tau_3 \phi$ (invariant under unfixed $U(1)_L$)
  $\langle \phi^\dagger \tau_3 \phi \rangle \neq 0$ breaks $SU(2)_R \to U(1)_R$

- Effective potential for order parameter:

$$\exp(-V_{\text{eff}}(\tilde{A})) = \int [d\phi][dC][d\overline{C}]\, \delta\left(\tilde{A} - \frac{1}{V}\sum_x \phi_x^\dagger \tau_3 \phi_x\right)\, \exp(-S_{\text{gf}})$$

  But, in view of $dZ_{\text{gf}}/d\tilde{g} = 0$, can $V_{\text{eff}}(\tilde{A})$ be non-trivial?
  In other words, can SSB ever take place in a TFT?

- First, consider a toy model

A toy model: zero-dimensional TFT (aka integral)

$$Z = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} d\phi \int dc d\bar{c} \exp[-f^2(\phi)/4 + \bar{c} f'(\phi) c]$$

b(aby)BRST: $s\phi = c \ , \quad sc = 0 \ , \quad s\bar{c} = f(\phi)/2$ (onshell form)

Of course, $Z = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} d\phi\, f'(\phi) \exp[-f^2(\phi)/4] = 1$ $(f \to \pm\infty \text{ for } x \to \pm\infty)$

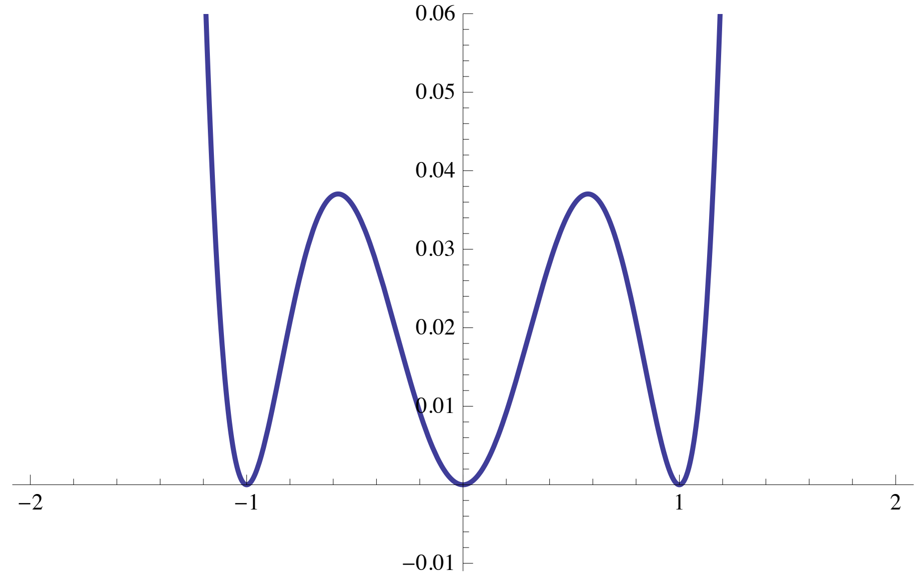
TFT because Z does not depend on $f(\phi)$

Choose $f(\phi) = \frac{1}{\lambda}\left(\phi^3 - v^2\phi\right)$ then model has bBRST and Z_2 symm. $\phi \to -\phi$

classical minima: $\phi = 0$ (Z_2 unbroken) , $\phi = \pm v$ (Z_2 broken)

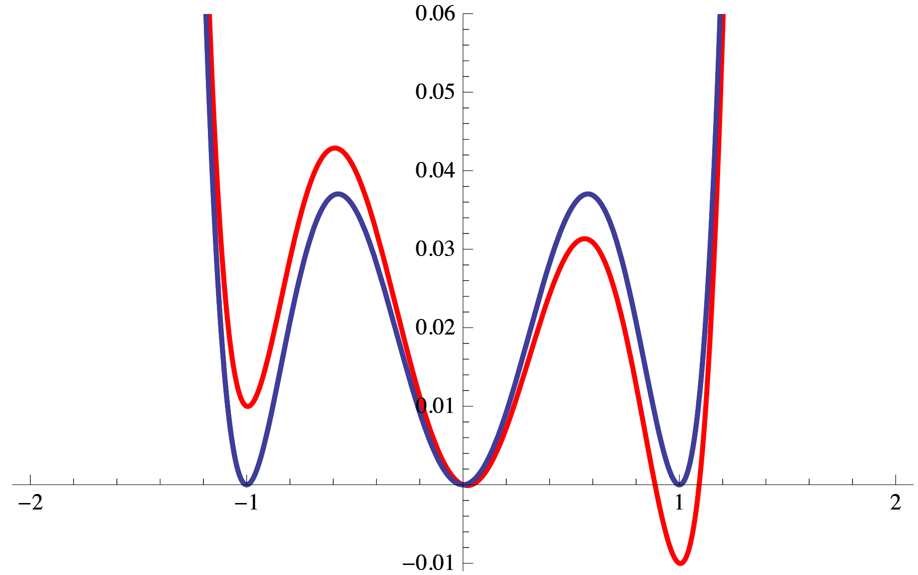
However: always $\langle \phi \rangle = 0$ because of the invariance theorem!

Now recall how to study SSB:



Now recall how to study SSB:

- turn on seed $S_{\text{seed}} = -\epsilon\phi$
- then take $V \to \infty$
- only then take $\epsilon \to 0$



seed breaks bBRST and Z_2
and selects minimum $\phi = v$, dominates saddlepoint approx.

Find: $Z_v = 1$ to all orders in λ

$$\langle\phi\rangle_v = v\left(1 - \frac{3}{4}\frac{\lambda^2}{v^6} + \ldots\right)$$ breaks Z_2 but not bBRST

For bBRST to be broken, need $\langle sX\rangle_v \neq 0$ for some X, does not happen

But $\langle\phi\rangle \neq 0$ and non-trivial: Z_2 can be, and is broken!

Back to reduced model: $S_{\text{seed}} = -\text{tr}\left(h\tau_3\,\phi^\dagger\tau_3\phi\right)$

breaks $SU(2)_R \to U(1)_R$ and eBRST symmetries, invariant under $U(1)_{\text{gauge}}$

- invariance theorem does not apply as long as $h \neq 0$
- whether it applies after $V \to \infty$ and $h \to 0$ is a dynamical question!

Integrate out ghosts in $1/\tilde{g}^2$ expansion, apply mean field to resulting $S_{\text{eff}}(\phi)$:

- 1st order phase transition at $\tilde{g} = \tilde{g}_c$ with $\langle \phi^\dagger\tau_3\phi\rangle \begin{cases} = 0\,, & \tilde{g} > \tilde{g}_c \\ \neq 0\,, & \tilde{g} < \tilde{g}_c \end{cases}$
- breaks $SU(2)_R \to U(1)_R$, implies massive W, massless photon!
- fate of eBRST: don't know! But (1) $\phi^\dagger\tau_3\phi \neq s(\text{anything})$

  (2) effective mass term $S_m = m^2 \int d^4x\,\text{tr}\left(\frac{1}{2}W_\mu^2 + \overline{C}C\right)$ is eBRST invariant

Phase diagram

A confining phase, mass gap

B Higgs phase, no mass gap, massless photon

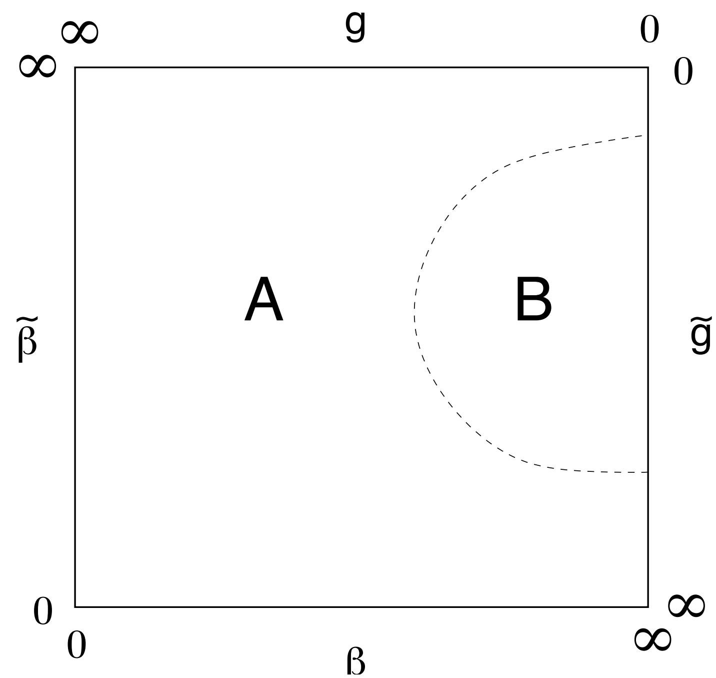
clear phase separation also in full theory

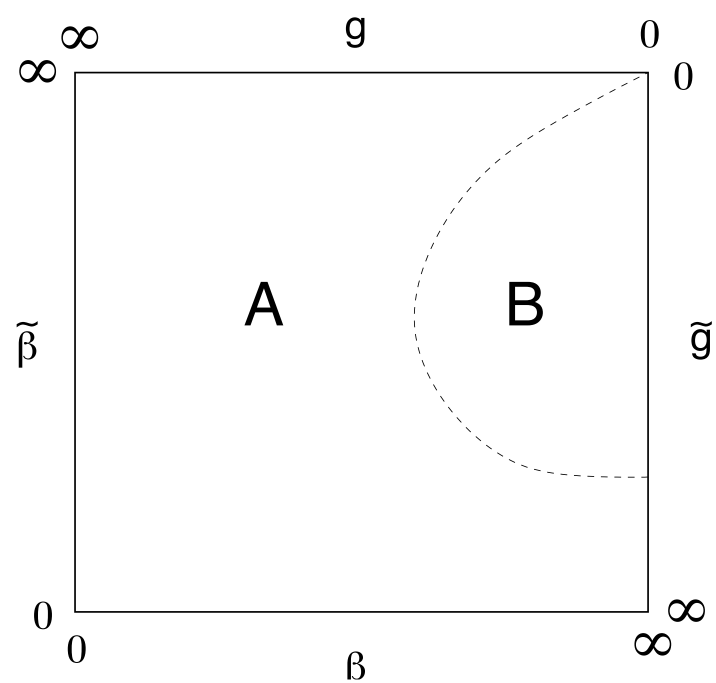
boundaries:

$\tilde{g} \to \infty$: gf sector decouples ⇒ confinement

$\tilde{g} \to 0$: 4-ghost term unimportant ⇒ conf.

$g \to \infty$: like analysis gives confinement





Phase diagram

upper diagram: B is lattice artifact

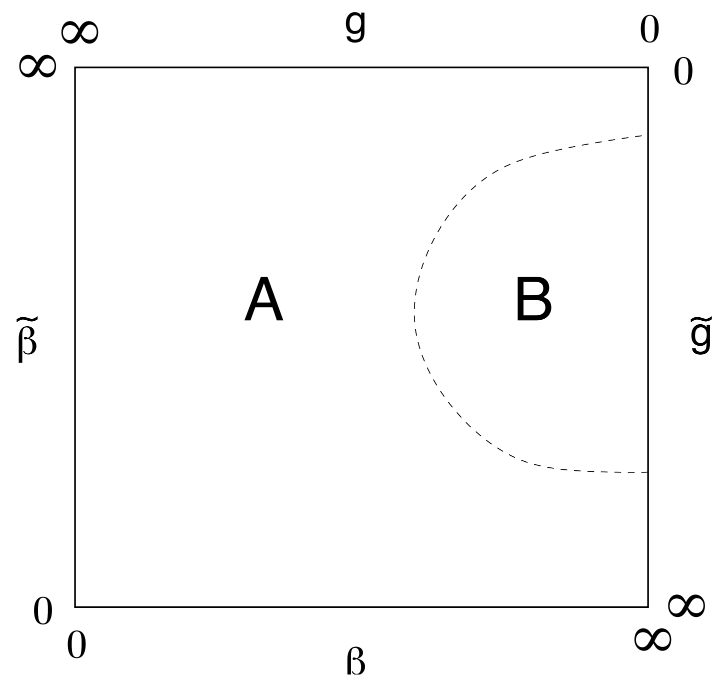
lower diagram: A and B contain continuum limit (at point $g = \tilde{g} = 0$)

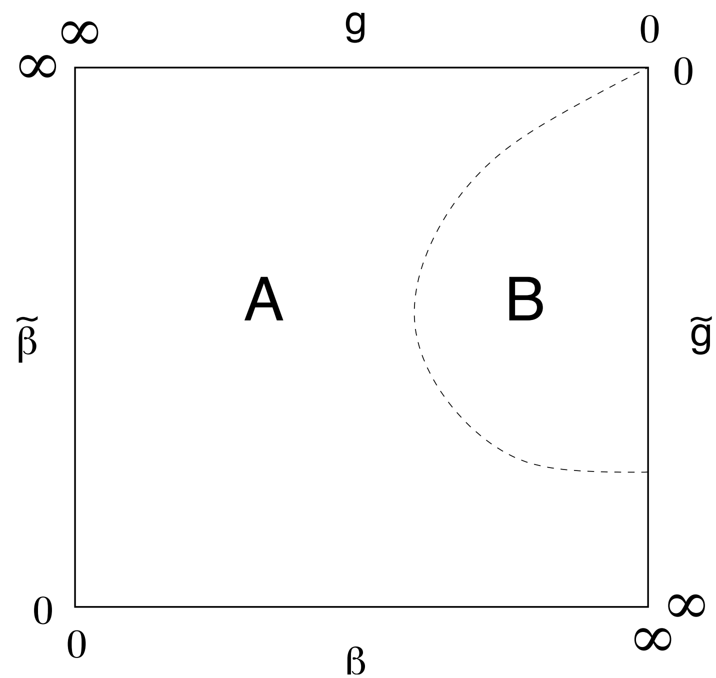
one-loop RG analysis: (MG & Shamir, 06)

associate a scale Λ with g, $\tilde{\Lambda}$ with $\tilde{g}$

$\tilde{\Lambda} \approx \Lambda$, only one scale, confining?

$\tilde{\Lambda} \gg \Lambda$, $\tilde{\Lambda}$ dominates, $m_W \sim g\tilde{\Lambda}$?





Questions and conclusions

1) Does the new phase exist? Numerical

2) Does it extend to $g = \tilde{g} = 0$? Small volume, large N

3) Is the continuum limit unitary? Does eBRST remain unbroken?

4) What distinguishes two phases microscopically in full theory?
   In full theory seed should break eBRST (not $SU(2)_R$), biases sum over Gribov copies

- Scenario consistent, SSB can occur in a topological field theory

- If SSB occurs in reduced model, Higgs mechanism unavoidable