A classification of 2-dim Lattice Theory

Mario Kieburg

Fakultät für Physik
Universität Bielefeld (Germany)

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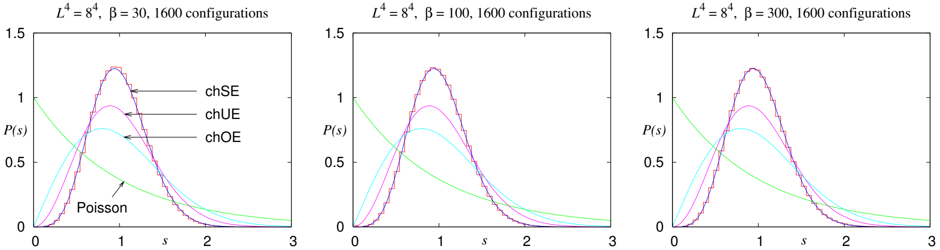
in collaboration with Jacobus Verbaarschot and Savvas Zafeiropoulos

Global Symmetries of Staggered Fermions

Example 1: 4-D, **SU**(2), fermions in fundamental representation



- Bruckmann, Keppeler, Panero, Wettig (2008)
- $\beta \to \infty$: splitting of spectrum into three scales (plateaux, clusters, level spacing)

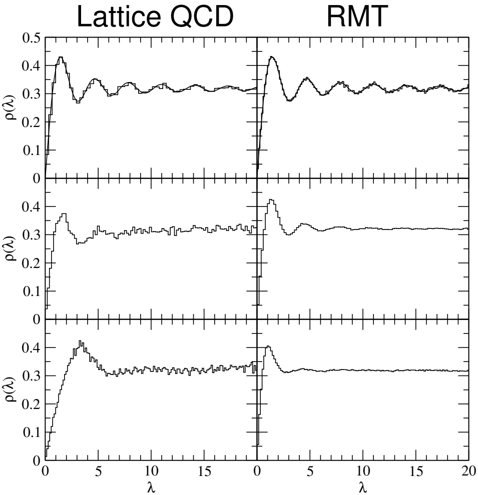
$\Rightarrow$ statistics on scale of plateaux and clusters: Poisson

$\Rightarrow$ statistics on scale of level spacing: χGSE

- continuum statistics: χGOE!

Global Symmetries of Staggered Fermions

Example 2: 3-D, **SU**(3), fermions in fundamental representation



$\beta \ll 1$ $\begin{pmatrix} 0 & W \\ -W^\dagger & 0 \end{pmatrix}$

$\beta \gg 1$ $\begin{pmatrix} 0 & H \\ -H & 0 \end{pmatrix}, H = H^\dagger$

- Bialas, Burda, Petersson (2010)

Do we have a transition of global symmetries?

The mechanism of such a transition?

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The mechanism of such a transition?

RMT may help to solve the puzzle!

RMT-applicable Regime: The ϵ-regime of QCD

- infrared limit of QCD
- large Compton wavelength of Mesons $\gg$ box size $V^{1/4} = L$
- lattice volume (space-time volume) $V \to \infty$

Saddlepoint approximation:

- spontaneous breaking of chiral symmetry

+

Integration over all kinetic modes
(important for applicability of RMT)

$$\Downarrow$$

$$\chi\text{Lagrangian: } \mathcal{L}(U) = \frac{\Sigma V}{2} \mathrm{tr}\, M(U + U^\dagger) + \mathcal{L}_{\mathrm{correction}}(V, a, U)$$

Why 2-D?

- exhibits the same effect in a simpler setting

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Coleman–Mermin–Wagner theorem?

- well-known (but still puzzling) effect of spontaneous breaking of chiral symmetry for quenched 2-D QCD (eg. Damgaard, Heller, Narayana, Svetitsky (2005, Schwinger-model))

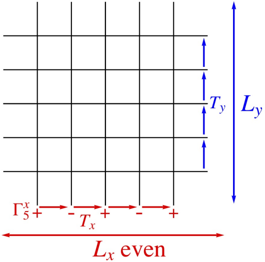
- our guess: integration over all kinetic modes + still small lattice volume V ($\approx 10^2$)
(maybe logarithmic divergent in V)

General RMT model: $D = \begin{bmatrix} 0 & W \\ -W^{\dagger} & 0 \end{bmatrix}$

Original Classification (Verbaarschot, 90's):

$$W \text{ is } \begin{cases} \text{real}, \\ \text{complex}, \\ \text{quaternion}, \end{cases}$$

Artificial chiral structure



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Yesterday

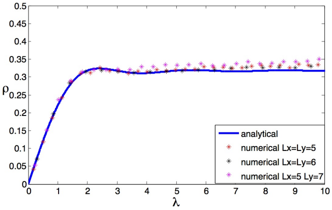
Reasons:
other dimensions = other global symmetries
(DeJonghe, Frey, Imbo, 2012)

+

Artificial symmetry: $\Gamma_5^x T_x \Gamma_5^x = -T_x, \;\; \Gamma_5^x T_y \Gamma_5^x = T_y$
$\Rightarrow$ change of global symmetries

Let us do some simulations!

Comparison: Lattice Data $\leftrightarrow$ RMT
2-D & Two colors ($\mathrm{SU}(2)$) & fundamental representation ($\psi \to U_\mu \psi$)



odd-odd

=

level rep.: $|\lambda_i - \lambda_j|$

+

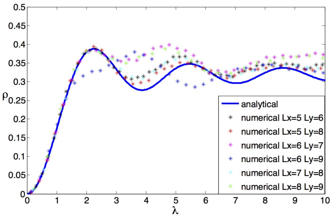
$\mathrm{USp}(2N_\mathrm{f}) \times \mathrm{USp}(2N_\mathrm{f})$

$\downarrow$

$\mathrm{USp}(2N_\mathrm{f})$

=

2-D continuum QCD



odd-even

=?

level rep.: $|\lambda_i - \lambda_j|^2$

+

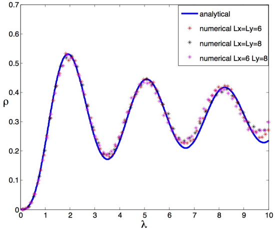
$\mathrm{USp}(4N_\mathrm{f})$

$\downarrow$

$\mathrm{USp}(2N_\mathrm{f}) \times \mathrm{USp}(2N_\mathrm{f})$

=

3-D continuum QCD



even-even

=

level rep.: $|\lambda_i - \lambda_j|^4$

+

$\mathrm{SU}(4N_\mathrm{f})$

$\downarrow$

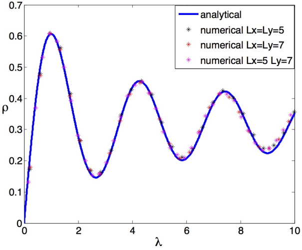
$\mathrm{SO}(4N_\mathrm{f})$

=

2-D staggered fermions

Comparison: Lattice Data $\leftrightarrow$ RMT

2-D & Three colors ($\mathrm{SU}(3)$) & adjoint representation ($\psi \to U_\mu \psi U_\mu^{-1}$)



odd-odd

=

level rep.: $|\lambda_i - \lambda_j|^4$

+

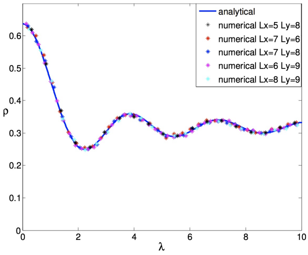
$\mathrm{SO}(2N_\mathrm{f}) \times \mathrm{SO}(2N_\mathrm{f})$

$\downarrow$

$\mathrm{SO}(2N_\mathrm{f})$

=

2-D continuum QCD



odd-even

=

level rep.: $|\lambda_i - \lambda_j|^2$

+

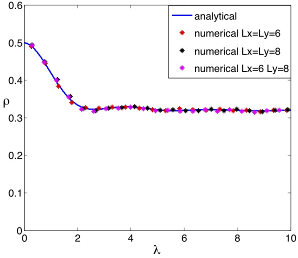
$\mathrm{SO}(4N_\mathrm{f})$

$\downarrow$

$\mathrm{SO}(2N_\mathrm{f}) \times \mathrm{SO}(2N_\mathrm{f})$

=

3-D continuum QCD



even-even

=

level rep.: $|\lambda_i - \lambda_j|$

+

$\mathrm{SU}(4N_\mathrm{f})$

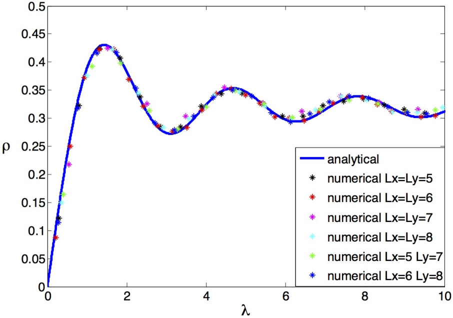
$\downarrow$

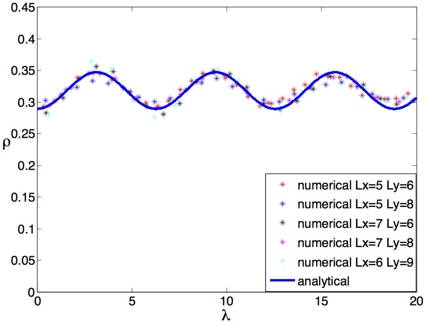
$\mathrm{Sp}(4N_\mathrm{f})$

=

2-D staggered fermions

Comparison: Lattice Data $\leftrightarrow$ RMT
2-D & Three colors ($\mathrm{SU}(3)$) & fundamental representation ($\psi \to U_\mu \psi$)





odd-odd & even-even

=

level rep.: $|\lambda_i - \lambda_j|^2$

+

$\mathrm{SU}(2N_{\mathrm{f}}) \times \mathrm{SU}(2N_{\mathrm{f}}) \to \mathrm{SU}(2N_{\mathrm{f}})$

=

2-D continuum QCD

odd-even

=

level rep.: $|\lambda_i - \lambda_j|^2$

+

$\mathrm{SU}(2N_{\mathrm{f}}) \to \mathrm{SU}(N_{\mathrm{f}}) \times \mathrm{SU}(N_{\mathrm{f}})$

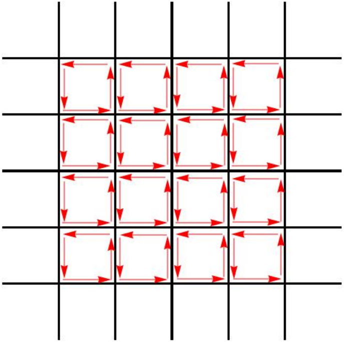
=

3-D continuum QCD

What is with the action?

$$S_{\mathrm{YM}} \longrightarrow \underline{\text{Wilson action:}}\ S = \beta(a) \sum_{\mu \neq \nu} \mathrm{tr}\, T_\mu T_\nu T_\mu^{-1} T_\nu^{-1}$$

= Wilson loop:



Competition between

- Haar measure of $U_\mu(x)$
- strong coupling limit: $\beta \to 0$ $\longleftrightarrow$
- independent T_μ

- Wilson action: e^S
- weak coupling limit: $\beta \to \infty$

$\Rightarrow$ path independence: $T_\mu T_\nu = T_\nu T_\mu$

What is happening?

strong coupling		weak coupling
independent T_μ	$\longleftrightarrow$	$T_\mu T_\nu = T_\nu T_\mu$

How does the symmetry of D_{stag} change?

What is happening?

strong coupling		weak coupling
independent T_μ	$\longleftrightarrow$	$T_\mu T_\nu = T_\nu T_\mu$

How does the symmetry of $D_{\rm stag}$ change?

RMT-Model by Osborn (2004/2011), fundamental 4-D staggered

$\mathcal{T}$	$S_\mathcal{T}$	$-V\mathcal{V}$
$\begin{pmatrix} 0 & iX \\ iX^\dagger & 0 \end{pmatrix} \otimes \Gamma$	$\beta N \langle X^\dagger X \rangle$	$\frac{\alpha N}{\beta} \langle \Gamma U \Gamma U^\dagger \rangle$
$\begin{pmatrix} iA & 0 \\ 0 & iB \end{pmatrix} \otimes \Gamma$	$\beta N [\langle A^2 \rangle + \langle B^2 \rangle]$	$\frac{\alpha N}{4\beta} \langle \Gamma U \Gamma U + \Gamma U^\dagger \Gamma U^\dagger \rangle$
$\begin{pmatrix} ib \otimes \mathbb{I}_{N+\nu} & 0 \\ 0 & ib \otimes \mathbb{I}_N \end{pmatrix} \otimes \Gamma$	$\beta N b^2$	$\frac{\alpha N}{4\beta} \langle \Gamma U + \Gamma U^\dagger \rangle^2$
$\begin{pmatrix} ic \otimes \mathbb{I}_{N+\nu} & 0 \\ 0 & -ic \otimes \mathbb{I}_N \end{pmatrix} \otimes \Gamma$	$\beta N c^2$	$\frac{\alpha N}{4\beta} \langle \Gamma U - \Gamma U^\dagger \rangle^2$

Is there a simpler model?

Achieved:

- identification of important symmetries and matrix blocks

$\Rightarrow$ classification of the naive Dirac-operator

$\Rightarrow$ mechanism of getting the wrong global symmetries as continuum QCD

Our goal!

- construction of tractable RMT models (recall the model by Bialas, Burda, Petersson (2010))
- restriction of low energy constants
- understanding of mechanism when changing global symmetries
  (interplay of Haar measure of gauge group and Wilson action)

Stay tuned for upcoming battles!

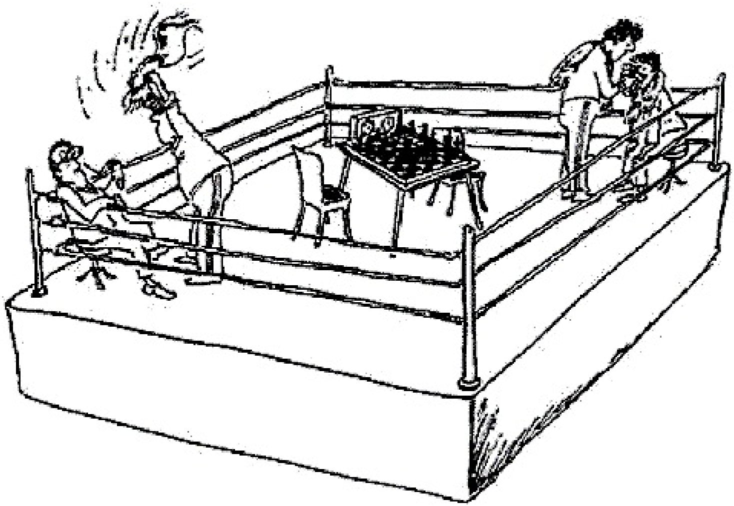


image from chessbase.de