Searching for a continuum 4D field theory arising from a 5D non-abelian gauge theory

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Problems in particle physics solved by extra dimensions:

- Hierarchy problem
- Cosmological constant problem

Mechanisms for dimensional reduction to the usual 4D spacetime:

- Compactificaion
  - Based on Kaluza-Klein theory with a mass scale $M_{KK} \sim \frac{1}{R}$
- Localization
  - So called: Brane world scenario
  - All particles are localised on 4D hyperplanes (3-brane), embedded in the bulk
  - Notable examples of models that use localization: Randall-Sundrum(RS), Dvali-Shifman(DS), Fu-Nielsen
  - In most of them, all particles are free to propagate on the branes but are confined in the bulk

Fu-Nielsen idea (1984)

If one imposes an anisotropy between the interactions in the usual 4D space and the extra dimension, a new phase arises in the phase diagram which is called Layer phase. In this phase, the particles can travel freely (exhibit Coulombic behaviour) in the four dimensions, but they are confined along the extra dimension.

So, our observed 4D world can be visualised as a hyperplane embedded in the extra dimension, since neighbouring 4D layers do not interact.

Recipe to look for the existence of a Layer phase

- Write down the action for a 5D model in the continuum
- Discretize it - Define the Wilson Action with an anisotropy
- Investigate the Phase Diagram
- Find the order of the phase transitions
- Second order transition:
  - Can take the continuum limit
  - Layer phase has a physical interpretation
- First order phases transition: lattice artefact.

Literature

What has already been investigated in pure gauge groups?

- Layer phase exists in $U(1)$ [Dimopoulos et al. (2006), Farakos and Vrentzos (2008)]
- Most difficult to show its existence in the Non-Abelian case
- Non-abelian gauge group: $SU(2)$
- Interesting groups for the Electroweak Sector of SM: $U(1)$ and $SU(2)$
- In the Mean Field Approximation the existence of the Layer phase was shown [Irges and Knechtli, arXiv:0905.2757]
- Investigation of $SU(2)$ using Monte Carlo numerical simulations [Farakos and Vrentzos, arXiv:1007.4442]

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- In the Mean Field Approximation the existence of the Layer phase was shown [Irges and Knechtli, arXiv:0905.2757]
- Investigation of $SU(2)$ using Monte Carlo numerical simulations [Farakos and Vrentzos, arXiv:1007.4442]

What are we exploring?

$SU(2)$ model using Monte Carlo simulations, extending the work of Farakos and Vretzos [L.Del Debbio, R.D.Kenway, EL, E.Rinladi, arXiv:1305.0752]

Continuum 5D SU(2) Yang-Mills Euclidean Action:

$$S_E = \int d^4x \int dx_5 \frac{1}{2g_5^2} \mathrm{Tr} F_{MN}^2 \qquad {}_{M,N=1\dots5}$$

Discretized anisotropic Wilson Action:

$$\boxed{S = \beta_4 \sum_x \sum_{1\le\mu<\nu\le4} \left(1 - \frac{1}{2}\ \mathrm{Tr}\ U_{\mu\nu}(x)\right) + \beta_5 \sum_x \sum_{1\le\mu\le4} \left(1 - \frac{1}{2}\ \mathrm{Tr}\ U_{\mu5}(x)\right)_{\mu,\nu=1\dots4}}$$

where:

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}a_4) U_\mu^\dagger(x+\hat{\nu}a_4) U_\nu^\dagger(x)$$
$$U_{\mu5}(x) = U_\mu(x) U_5(x+\hat{\mu}a_4) U_\mu^\dagger(x+\hat{5}a_5) U_5^\dagger(x)$$

with $U_\mu = \exp(ig_5 a_4 A_\mu)$ and $U_5 = \exp(ig_5 a_5 A_5)$.

Discretized anisotropic Wilson Action:

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- β_4 and β_5 set the lattice spacings a_4 and a_5 (in the usual four directions and in the extra one, respectively)

$$\beta_4 = \frac{2N_c a_5}{{g_5}^2} \qquad \beta_5 = \frac{2N_c a_4^2}{a_5 {g_5}^2}$$

- Useful to define anisotropy parameter:

$$\gamma = \sqrt{\frac{\beta_5}{\beta_4}}$$

$\Rightarrow$ At tree level: $\gamma = \frac{a_4}{a_5}$

Observables

- Average Plaquette in the extra dimension

$$\langle \hat{P}_5 \rangle = \left\langle \frac{1}{4VN_c} \sum_x \sum_\mu \mathrm{Tr}(U_{\mu 5}(x)) \right\rangle$$

- Susceptibility of Extra-dimensional Plaquette

$$\chi_{\hat{P}_5} = V\left(\langle \hat{P}_5^2 \rangle - \langle \hat{P}_5 \rangle^2 \right) \tag{1}$$

- Polyakov Loop in temporal direction

$$\mathrm{Poly}_0 = \frac{L_T}{N_c V} \left| \sum_{\vec{x}, x_5} \mathrm{Tr} \prod_{x_1=0}^{(L_T-1)a_4} U_1(x) \right|$$

- Susceptibility of Polyakov Loop in temporal direction

$$\chi_{\mathrm{Poly}_T} = \frac{V}{L_T} \left\langle \left(\mathrm{Poly}_T^2 - \langle \mathrm{Poly}_T \rangle^2 \right) \right\rangle \tag{2}$$

Regimes of anisotropy parameter and dimensional reduction

- $\gamma > 1$ $(a_4 > a_5)$
  Dimensional reduction achieved by compactification

  [Ejiri et al., arXiv:0006.217, P. deForcrand et al. arXiv:1003.4643, L. Del Debbio et al., arXiv: 1203.2116]

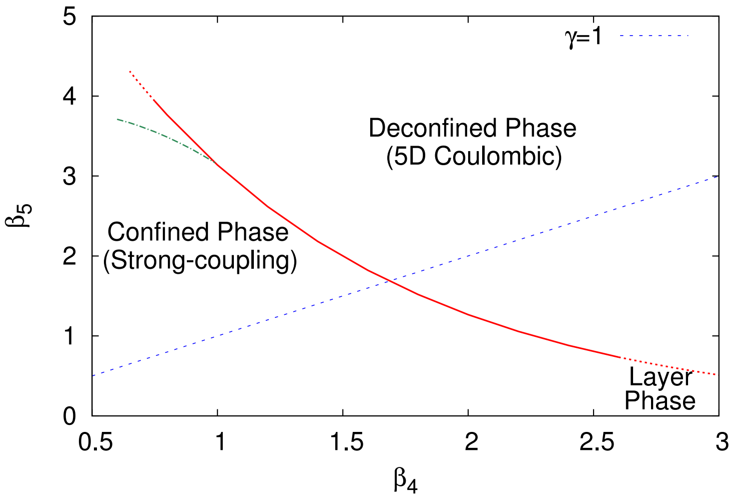
- $\gamma < 1$ $(a_4 < a_5)$
  - Impose one small direction
    $\Rightarrow$ Dimensional reduction via compactification

    [Knechtli et al., arXiv:1110.4210]

  - Keep all directions large enough in size
    $\Rightarrow$ Dimensional reduction via localization

    [Farakos and Vrentzos, arXiv:1007.4442]

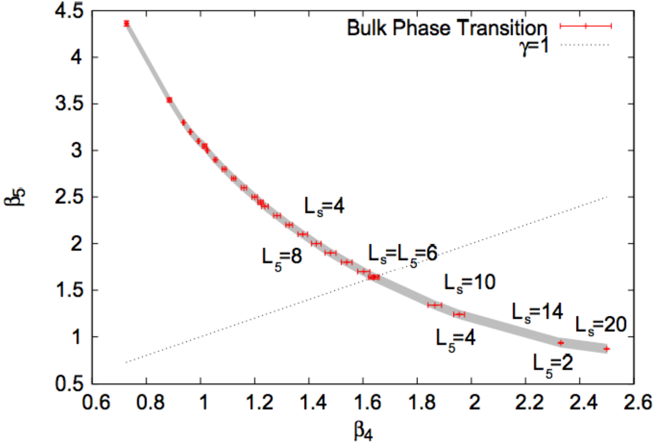
Phase Diagram



Choice of points in the parameter space based on previous works:

- Bulk phase transition exists up to β_4=2.50 [Knechtli et al., 2011]
- For every β_4, there is a minimum lattice size required to see the bulk phase transition
- As β_4 increases, transition gets weaker

Localization



[Knechtli et al., arXiv: 1110.4210]

In the previous Monte Carlo investigation of $SU(2)$ YM model:

[Farakos, Vrentzos, arXiv:1007.4442]

- Points: $\beta_4 = 2.60$ and $\beta_4 = 3.00$
- Reasons for choosing these points:
  - Measurement of the gap of the plaquette

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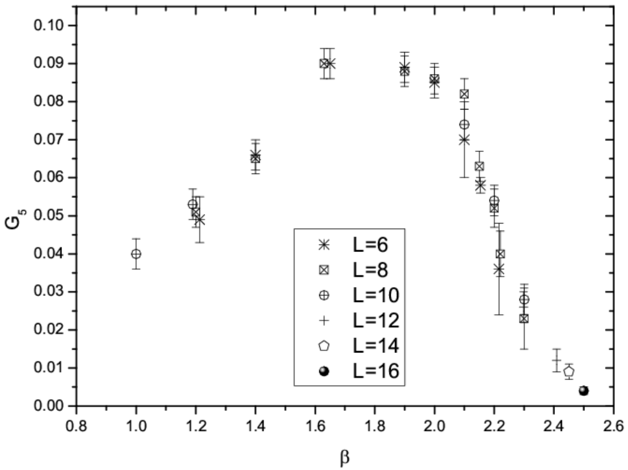

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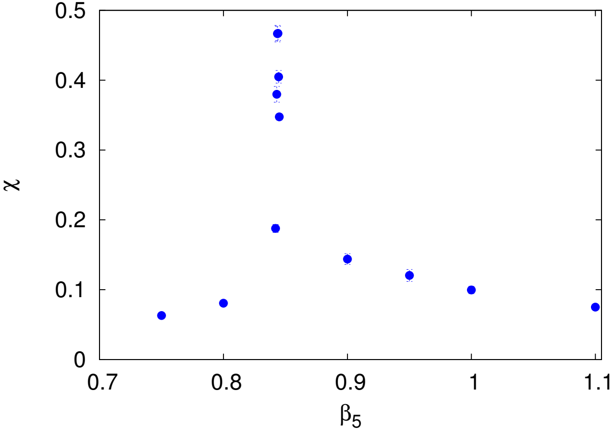
  - $\beta_4 = 2.60$: the critical point at which the order of the transition changes from first to second
  - $\beta_4 = 3.00$: deep in “Layer” phase
- Main problems:
  - Lattice Volumes: up to 16^5 - below the critical size
  - Extrapolation of thermodynamic limit is important!
  - Critical exponents not in a good agreement with the matched 4D Ising Model

Results

- Our choice: $\beta_4 = 2.60$
- Reproduced results for average plaquette in extra dimension for 16^5

Critical point found to be at $\beta_{5_c} = 0.8437(5)$

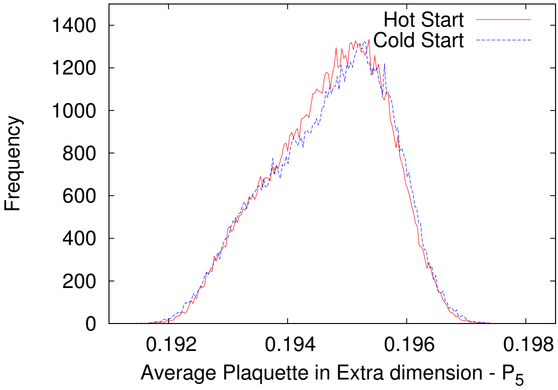


Results

- Bigger Lattice Volumes: $V = 20^4 \times 8$, $24^4 \times 8$ and $32^4 \times 8$
- Measurements of observables taken close to the critical point, starting both from Cold and Hot configurations
- Kennedy-Pendleton Heat Bath Algorithm applied
- 100 000 - 200 000 measurements for each point

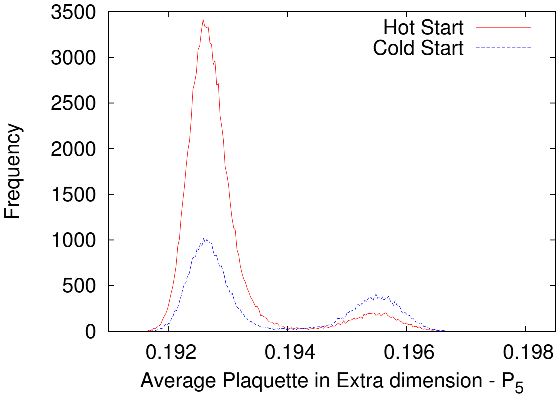
Results

$V = 20^4 \times 8,\ \beta_4 = 2.60,\ \beta_5 = 0.8435$



Fluctuation between two vacua, however there is not a clear gap.

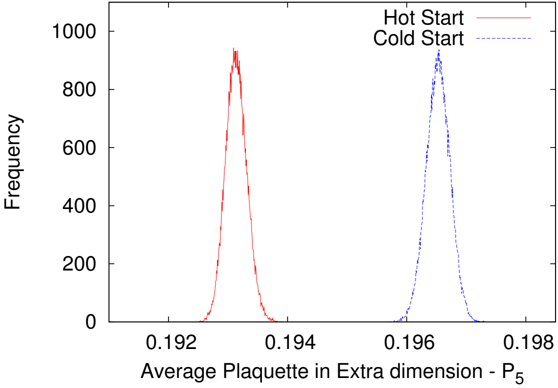
$V = 24^4 \times 8,\ \beta_4 = 2.60,\ \beta_5 = 0.8435$



Clear two-state signal with a bit of overlap

Results

$V = 32^4 \times 8,\ \beta_4 = 2.60,\ \beta_5 = 0.844$



Two-state signal readily apparent $\Rightarrow$ First Order phase transition

What about larger β_4?

- A change in the order of the phase transition for larger values of β_4 cannot be excluded
- However, there is no indication that it will change from first to second
- Further investigation decided not to be done since very big lattice volumes are needed and it does not seem worthwhile

Lattice Volume	Compute time (hours)
$16 \times 16 \times 16 \times 16 \times 16$	190
$20 \times 20 \times 20 \times 20 \times 8$	250
$24 \times 24 \times 24 \times 24 \times 8$	620

Compute time on an NVIDIA Tesla C2070 Computing Processor for 100 000 measurements for a single point

Summary

- The bulk phase transition between confined/deconfined phases continues up to $\beta_4 = 2.60$

- No evidence for a second order transition up to $\beta_4 = 2.60$

- Large volumes are needed to investigate the region where the Layer phase is believed to exist

- The scenario of dimensional reduction of the 5D theory to a continuum 4D theory via the existence of the so-called layer phase seems unpromising.

Thank you!

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Why implement it on the lattice?

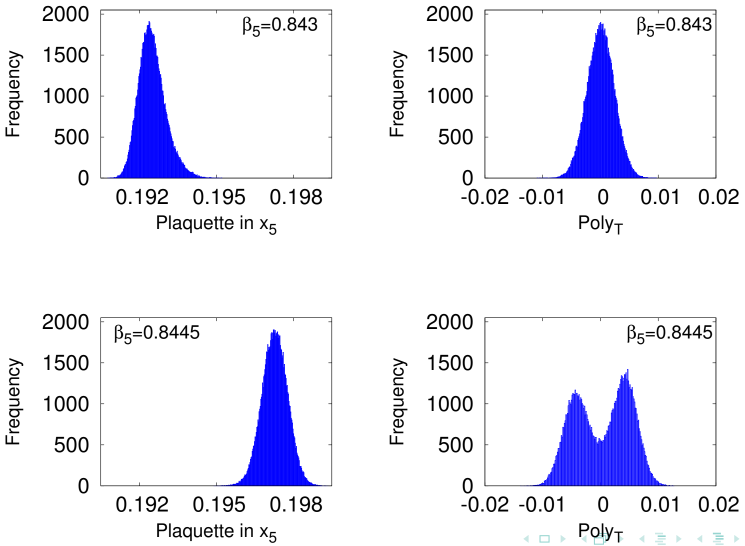
- $SU(2)$ model in extra dimensions is non-renormalizable
- It has a trivial fixed point
- In analytic approaches:
  1. approach fixed point $\Rightarrow$ triviality
  2. go away from fixed point $\Rightarrow$ confinement that has strong dynamics that are hidden in perturbative analysis
- Lattice provides a regulator that preserves gauge invariance and maintains the cutoff ($\Lambda = 1/a$)

Behaviour of Potential ($V(R)$) in different phases

$$\boxed{-lnW_c[U] = V(R)T}$$

Strong	$\sim R$
5D Coulombic	$\sim 1/R^2$
Layer	$\sim 1/R$

$V = 20^4 \times 8,\ \beta_4 = 2.60,\ \beta_5 = 0.8435$



Notable examples of models that use localization

- DS Mechanism

  Based on the idea that a confining theory of a group G can be localized on a topological defect with a symmetry of G', that is: *"A topological defect "eats up" the necessary number of dimensions and breaks the symmetry to a group G' inside the defect".*

  G.R. Dvali and M.A. Shifman, Phys. Lett. B 396 (1997) 64 and B 407 (1997) 452 (E) [arXiv: 9612.128]

- RS model

  All models with non-compact internal spaces suffer from naked singularities. In RS model, we deal with delta-function singularities that are interpreted physically as a 4D domain wall, embedded in the 5D bulk. Even if a 4D graviton is localised on the domain wall, the mechanism of localization of gauge fields is still unknown. L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999) [arXiv: 9906.064] and Phys. Rev. Lett. 83, 3370 (1999) [arXiv: 9905.221]