Gauge Symmetry Breaking in Strongly Coupled Theories

Simon Catterall and Aarti Veernala

Upshot

Strongly coupled vector lattice theory with reduced staggered fermions and gauge fields

↓ exhibits

Higgs Mechanism

↓ via

Gauge invariant 4-fermion condensate

Upshot

Strongly coupled twisted theory with reduced staggered fermions and gauge fields

↓ exhibits

Higgs Mechanism

↓ via

Gauge invariant 4-fermion condensate

Broken lattice Symmetries

↓ continuum

Chiral Symmetries

However, the lattice theory is NOT a lattice chiral gauge theory

Contents

Reduced Staggered Fermions

- Free staggered theory : $\mathrm{S} = -\sum_{x,\mu} \frac{1}{2} \eta_\mu(x) \overline{\chi}(x) \left[\chi(x+\mu) - \chi(x-\mu)\right]$

- Restricting the staggered fields to even and odd sites :

$$\lambda_+(x) = \frac{1}{2}\left[1+\epsilon(x)\right]\chi(x)$$
$$\psi_-(x) = \frac{1}{2}\left[1-\epsilon(x)\right]\chi(x)$$

- Staggered action :

$$\mathrm{S} = \frac{1}{2}\sum_{x,\mu} \eta_\mu(x)\overline{\psi}_+(x)\left[\psi_-(x+\mu) - \psi_-(x-\mu)\right]$$
$$+\frac{1}{2}\sum_{x,\mu} \eta_\mu(x)\overline{\lambda}_-(x)\left[\lambda_+(x+\mu) - \lambda_+(x-\mu)\right]$$

- Can gauge the ψ and the λ fields independently.

Staggered and Continuum Fermions

- Staggered fermions arise from discretization of matrix theory

$$\Psi(x) = \frac{1}{8}\sum_b (\gamma^{x+b})\chi(x+b)$$

$$\overline{\Psi}(x) = \frac{1}{8}\sum_{b'} (\gamma^{x+b'})^\dagger \overline{\chi}(x+b')$$

- Original free staggered action results from the continuum action :

$$S = \int d^4x \ \mathrm{Tr}\left(\overline{\Psi}\gamma_\mu\partial_\mu\Psi\right)$$

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- Continuum Projection : $\Psi_\pm = \frac{1}{2}\left(\Psi \pm \gamma_5 \Psi \gamma_5\right)$
- Projected Action : $S = \int d^4x \; \mathrm{Tr}(\overline{\Psi}_+ \gamma_\mu \partial_\mu \Psi_-) + \mathrm{Tr}(\overline{\Psi}_- \gamma_\mu \partial_\mu \Psi_+)$
- Can be gauged independently

Matrix Fermions and Twisting

- Matrix fermions arise by twisting the theory:

$$SO'(4) = \mathrm{diag}\left(SO_{\mathrm{Lorentz}} \times SO_{\mathrm{flavor}}(4)\right)$$

- Fermion transformation: $\psi_{\alpha\, i} \to L_{\alpha\beta}\psi^{\beta\, j} F^T_{ji}$

- Twisting corresponds to setting L = F

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- Continuum Projection : $\Psi_\pm = \frac{1}{2}\left(\Psi \pm \gamma_5 \Psi \gamma_5\right)$

- Chiral basis :

$$\Psi_+ = \begin{pmatrix} \lambda_R & 0 \\ 0 & \lambda_L \end{pmatrix} \quad \Psi_- = \begin{pmatrix} 0 & \psi_R \\ \psi_L & 0 \end{pmatrix}$$

$$\bar{\Psi}_+ = \begin{pmatrix} \bar{\psi}_L & 0 \\ 0 & \bar{\psi}_R \end{pmatrix} \quad \bar{\Psi}_- = \begin{pmatrix} 0 & \bar{\lambda}_R \\ \bar{\lambda}_L & 0 \end{pmatrix}$$

Symmetries of the Continuum Twisted Theory

Continuum gauged action : $S = \int \mathrm{Tr}(\overline{\Psi}_+ \gamma_\mu D_\mu \Psi_-) + \mathrm{Tr}(\overline{\Psi}_- \gamma_\mu D'_\mu \Psi_+)$

$$S = \int d^4x \ \mathrm{tr} \left(\overline{\psi}_L \sigma_\mu D_\mu \psi_L + \overline{\psi}_R \overline{\sigma}_\mu D_\mu \psi_R + \overline{\lambda}_L \sigma_\mu D'_\mu \lambda_L + \overline{\lambda}_R \overline{\sigma}_\mu D'_\mu \lambda_R \right)$$

$$\Psi'_+ = \begin{pmatrix} H\lambda_R & 0 \\ 0 & H\lambda_L \end{pmatrix}$$

$$\Psi'_- = \begin{pmatrix} 0 & G\psi_R \\ G\psi_L & 0 \end{pmatrix}$$

$\Psi' = \Psi'_+ + \Psi'_-$	$\overline{\Psi}' = \overline{\Psi}'_+ + \overline{\Psi}'_-$
$\begin{pmatrix} H\lambda_R & G\psi_R \\ G\psi_L & H\lambda_L \end{pmatrix}$	$\begin{pmatrix} \overline{\psi}_L G^\dagger & \overline{\lambda}_R H^\dagger \\ \overline{\lambda}_L H^\dagger & \overline{\psi}_R G^\dagger \end{pmatrix}$

$$\overline{\Psi}'_+ = \begin{pmatrix} \overline{\psi}_L G^\dagger & 0 \\ 0 & \overline{\psi}_R G^\dagger \end{pmatrix}$$

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$\Psi' = \Psi'_+ + \Psi'_-$	$\overline{\Psi}' = \overline{\Psi}'_+ + \overline{\Psi}'_-$
$\begin{pmatrix} H\lambda_R & G\psi_R \\ G\psi_L & H\lambda_L \end{pmatrix}$	$\begin{pmatrix} \overline{\psi}_L G^\dagger & \overline{\lambda}_R H^\dagger \\ \overline{\lambda}_L H^\dagger & \overline{\psi}_R G^\dagger \end{pmatrix}$

Chiral

Vector

$$\Psi'_+ = \begin{pmatrix} H\lambda_R & 0 \\ 0 & H\lambda_L \end{pmatrix}$$

$$\Psi'_- = \begin{pmatrix} 0 & G\psi_R \\ G\psi_L & 0 \end{pmatrix}$$

Lattice **Vector** Theory

$$\overline{\Psi}'_+ = \begin{pmatrix} \overline{\psi}_L G^\dagger & 0 \\ 0 & \overline{\psi}_R G^\dagger \end{pmatrix}$$

$$\overline{\Psi}'_- = \begin{pmatrix} 0 & \overline{\lambda}_R H^\dagger \\ \overline{\lambda}_L H^\dagger & 0 \end{pmatrix}$$

Mass Terms

Continuum	Lattice
$\overline{\Psi}_+ \rightarrow \overline{\Psi}_+ G^\dagger$	$\overline{\psi}_+ \rightarrow \overline{\psi}_+ G^\dagger$
$\Psi_- \rightarrow G\Psi_-$	$\psi_- \rightarrow G\psi_-$
$\overline{\Psi}_- \rightarrow \overline{\Psi}_- H^\dagger$	$\overline{\lambda}_- \rightarrow \overline{\lambda}_- H^\dagger$
$\Psi_+ \rightarrow H\Psi_+$	$\lambda_+ \rightarrow H\lambda_+$

- Twisted Lorentz invariant mass terms : $\mathrm{Tr}\left(\overline{\Psi}\Psi\right) = \mathrm{Tr}\left(\overline{\Psi}_+\Psi_+ + \overline{\Psi}_-\Psi_-\right)$
- Single site mass parameter : $\left(\overline{\psi}_+(x)\lambda_+(x) + \overline{\lambda}_-(x)\psi_-(x)\right)$

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$\overline{\Psi}_+ \to \overline{\Psi}_+ G^\dagger$	$\overline{\psi}_+ \to \overline{\psi}_+ G^\dagger$
$\Psi_- \to G\Psi_-$	$\psi_- \to G\psi_-$
$\overline{\Psi}_- \to \overline{\Psi}_- H^\dagger$	$\overline{\lambda}_- \to \overline{\lambda}_- H^\dagger$
$\Psi_+ \to H\Psi_+$	$\lambda_+ \to H\lambda_+$

- Twisted Lorentz invariant mass terms : $\mathrm{Tr}\left(\overline{\Psi}\Psi\right) = \mathrm{Tr}\left(\overline{\Psi}_+\Psi_+ + \overline{\Psi}_-\Psi_-\right)$
- Single site mass parameter : $\left(\overline{\psi}_+(x)\lambda_+(x) + \overline{\lambda}_-(x)\psi_-(x)\right)$
- When gauged independently : $\overline{\Psi}_+ G^\dagger H \Psi_+ + \overline{\Psi}_- H^\dagger G \Psi_-$
- Mass term (twist invariant) breaks Gauge Invariance

Symmetry Breaking via Higgs Mechanism

- VEV $\left(\overline{\psi}_+(x)\lambda_+(x) + \overline{\lambda}_-(x)\psi_-(x)\right)$ $\xrightarrow[\text{Elitzur}]{\text{NOT Gauge Invariant}}$ Vanishes

- Gauge Invariant four fermion term that can condense and Higgs the system:

$$\sum_{\mu} u_+(x) S_\mu(x) T_\mu^\dagger(x) u_-^\dagger(x+\mu)$$

- Effective Higgs field- a composite

$$u_+(x) = \overline{\psi}_+(x)\lambda_+(x)$$

$$u_-^\dagger(x) = \overline{\lambda}_-(x)\psi_-(x)$$

Toy Technicolour Model

- Technicolor like model - Strong and Weak groups

Strong	Weak
SU(N)	SU(M)
β_s	β_w
V_μ	U_μ

- Kinetic Term - Sterile Case

$$\sum_{x,\mu} \overline{\psi}_+(x) \left[U_\mu(x)V_\mu(x)\psi_-(x+\mu) - U_\mu^\dagger(x-\mu)V_\mu^\dagger(x-\mu)\psi_-(x-\mu)\right]$$

$$\sum_{x,\mu} \overline{\lambda}_-(x) \left[V_\mu(x)\lambda_+(x+\mu) - V_\mu^\dagger(x-\mu)\lambda_+(x-\mu)\right]$$

- Yukawa interaction term and the scalar quadratic term

$$\sum_x g\left[\phi(x)\overline{\psi}_+(x)\lambda_+(x) + \phi^*(x)\psi_-(x)\overline{\lambda}_-(x)\right] ; \sum_x \phi^\dagger(x)\phi(x)$$

- Take g $\longrightarrow$ 0 in the thermodynamic limit

Measured Observables

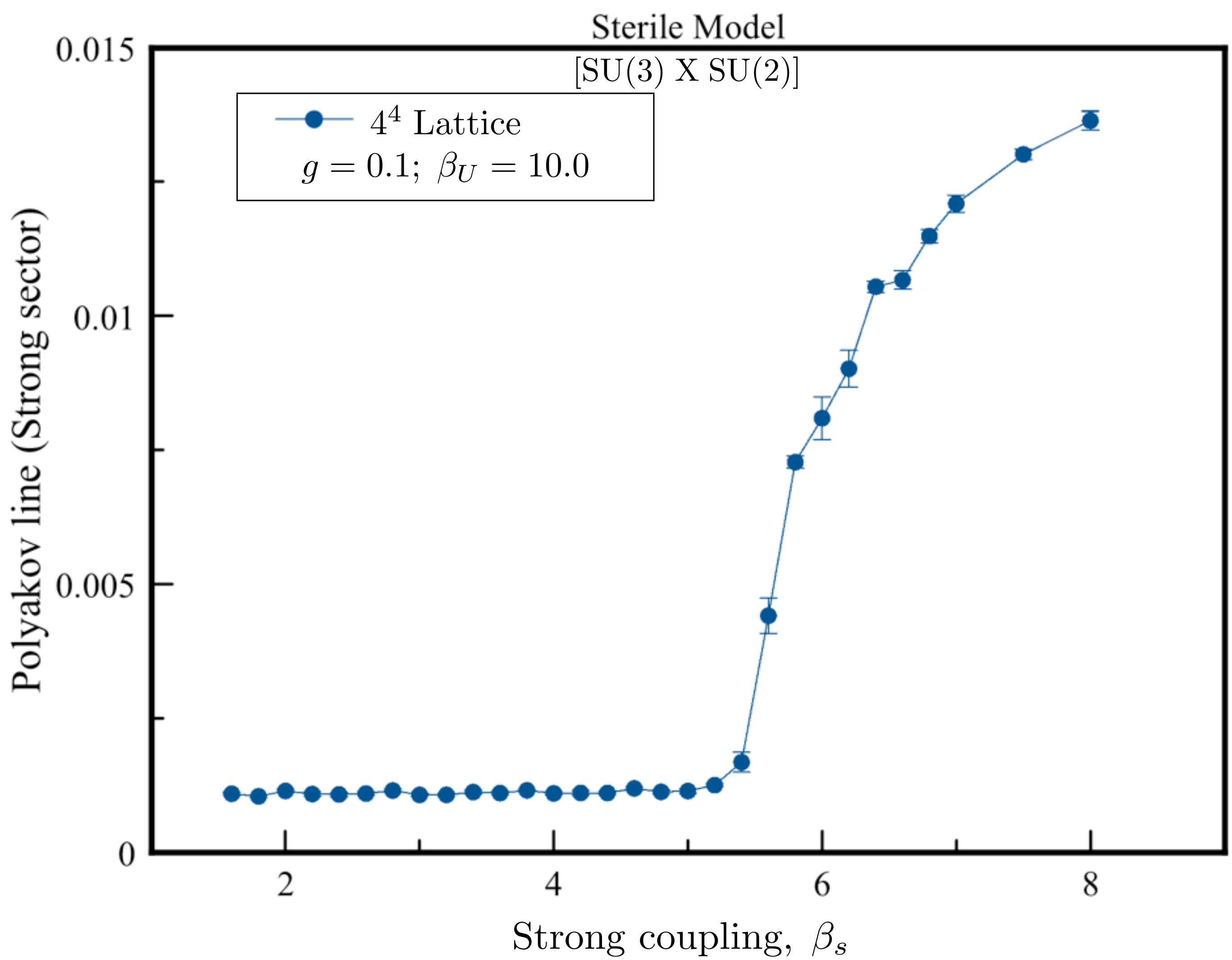
- Four fermion condensate

$$\overline{\psi}_+(x)\lambda_+(x)U_\mu(x)\overline{\lambda}_-(x+\mu)\psi_-(x+\mu)$$

- The weak Polyakov line (corresponding to the weak SU(2) group
- The strong Polyakov line (corresponding to the strong SU(3) group

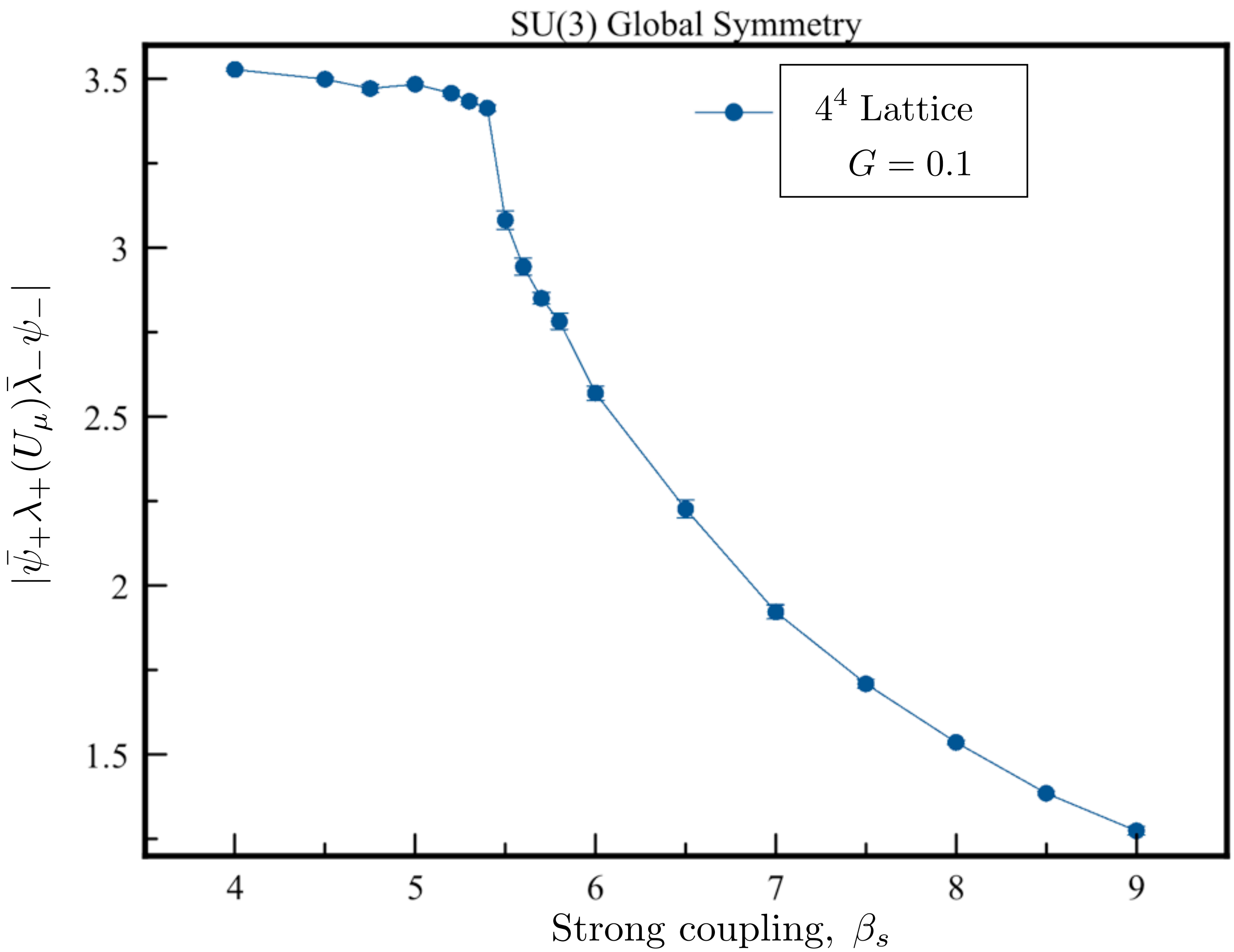
Global SU(2) Symmetry

Strong Sector Polyakov Line- SU(3)

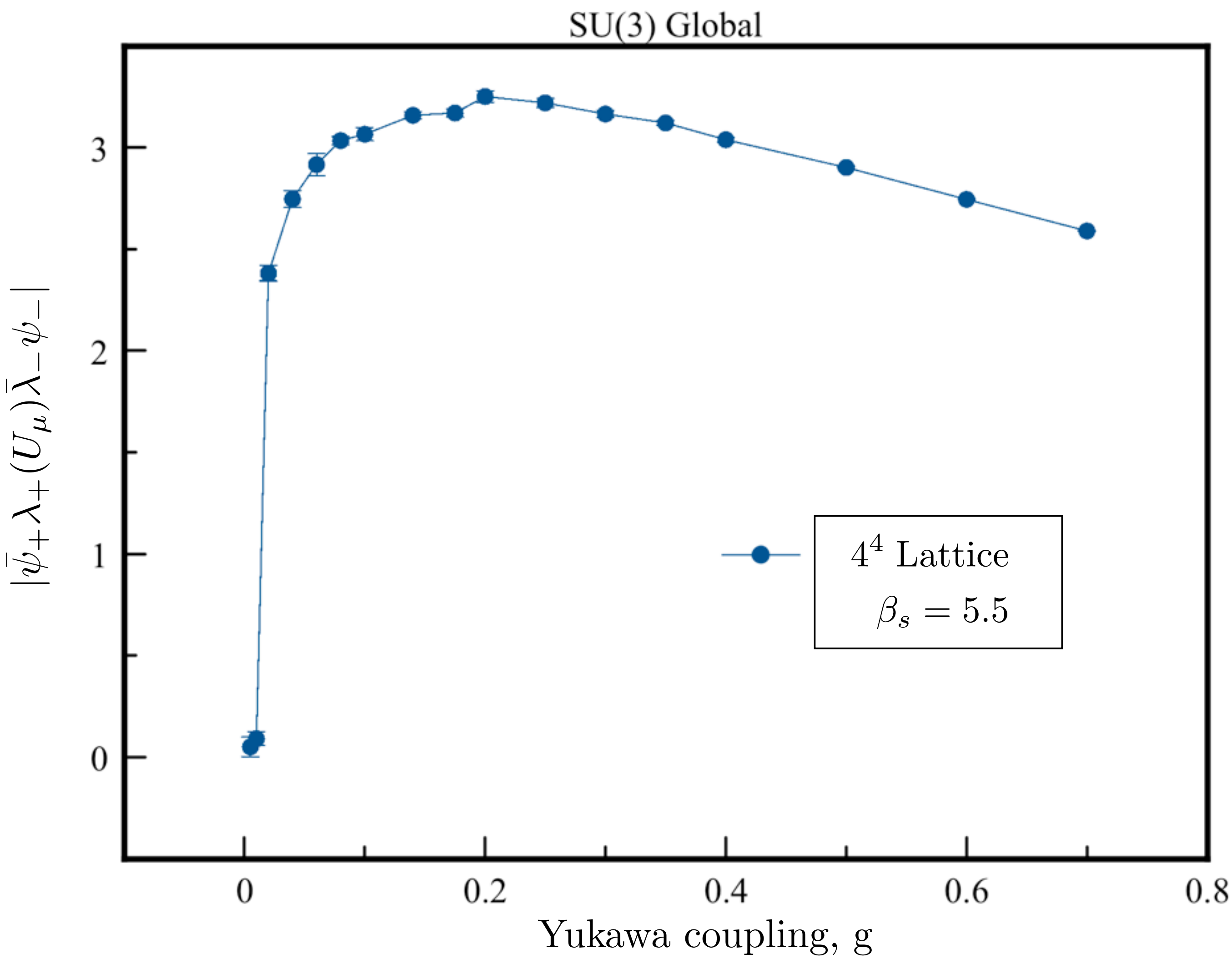


Tells us about confinement under strong dynamics

Condensate Vs Strong coupling
(Global Symmetry)



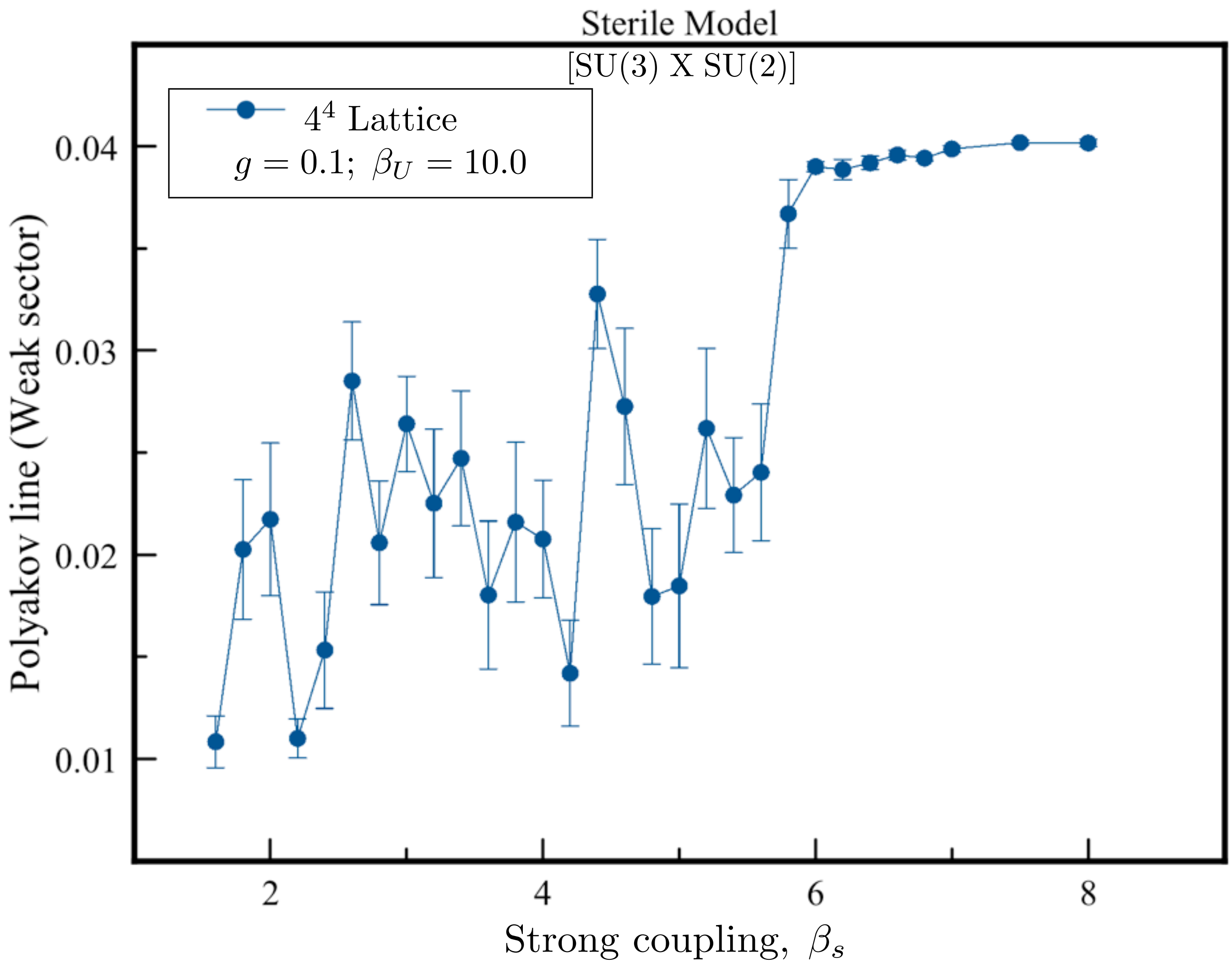
Condensate Vs Yukawa coupling
(Global Symmetry)



Local SU(2) Gauge Symmetry

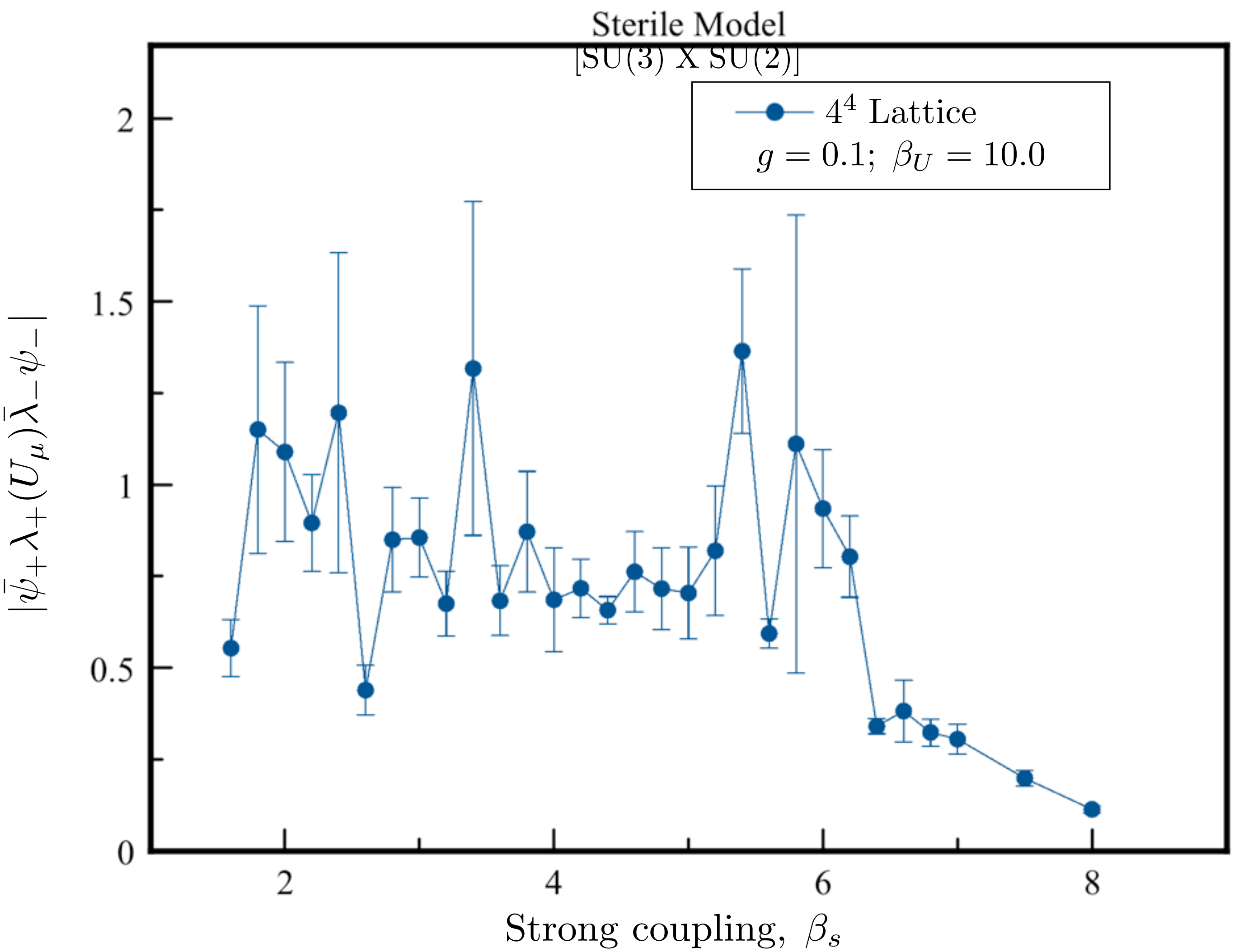
Weak Polyakov Line Vs Strong Coupling

(Local gauge symmetry)

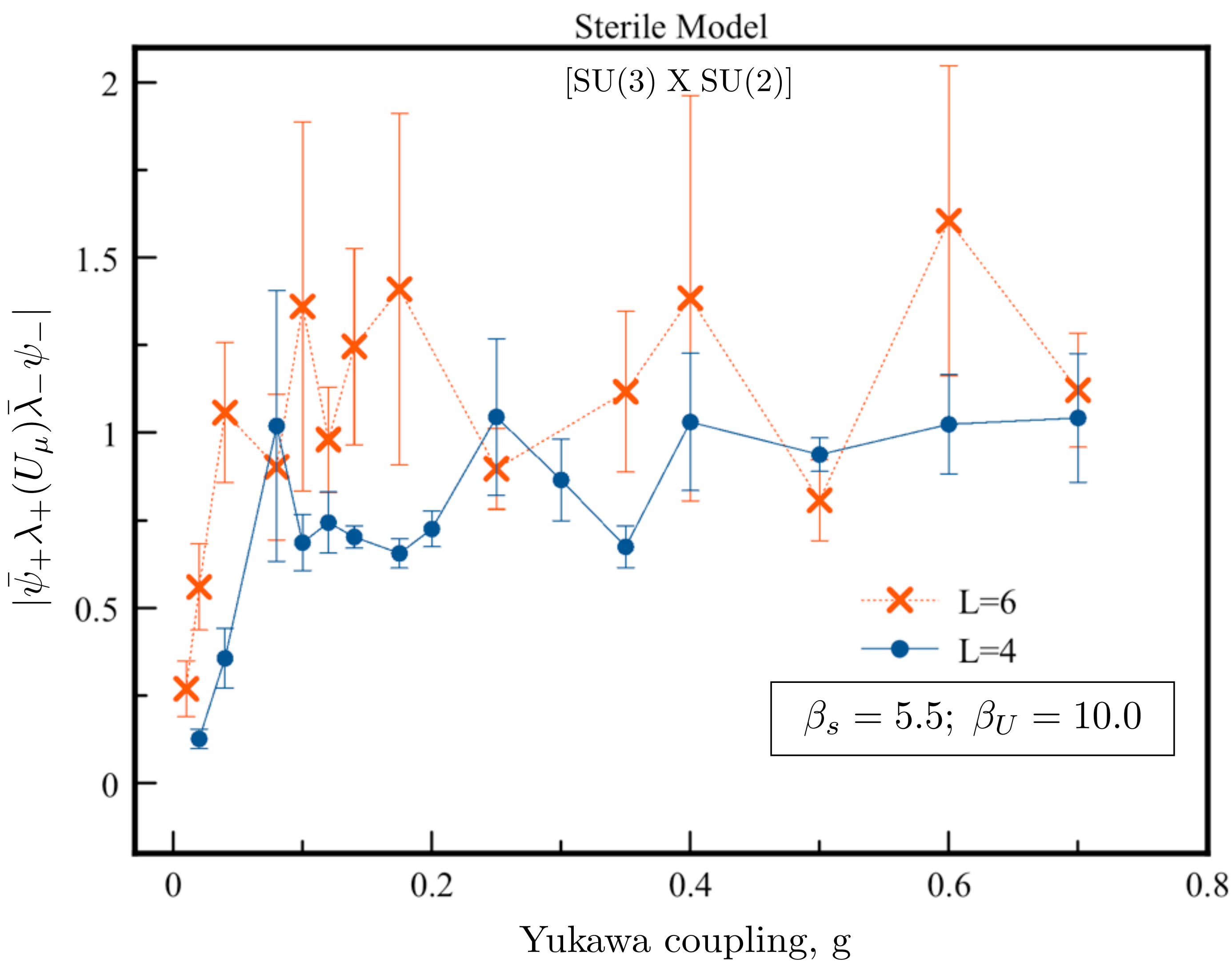


Reveals the existence of the Higgs phase

Condensate Line Vs Strong coupling
(Local Gauge Symmetry)

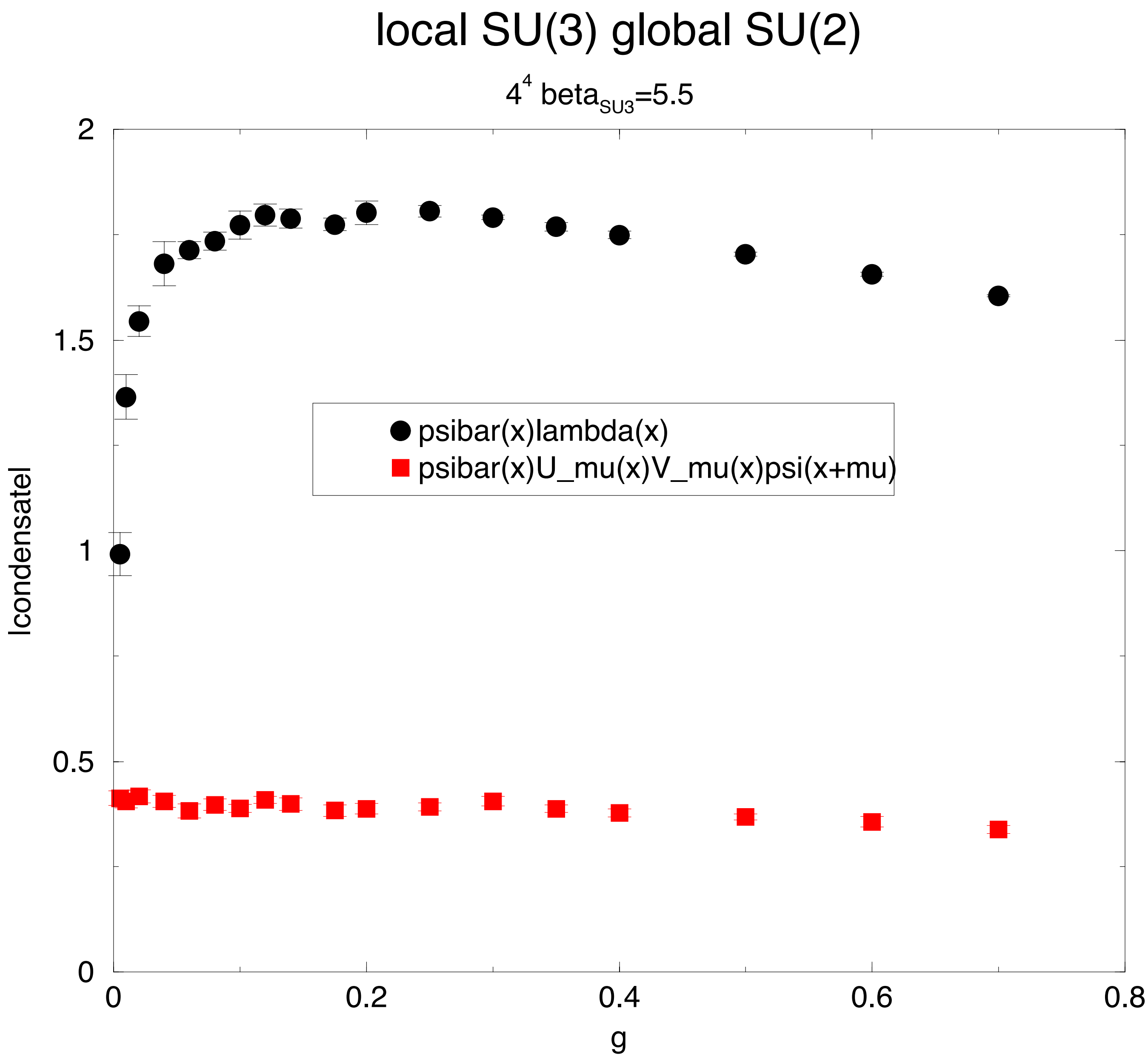


Condensate Vs Yukawa coupling
(Local Gauge Symmetry)



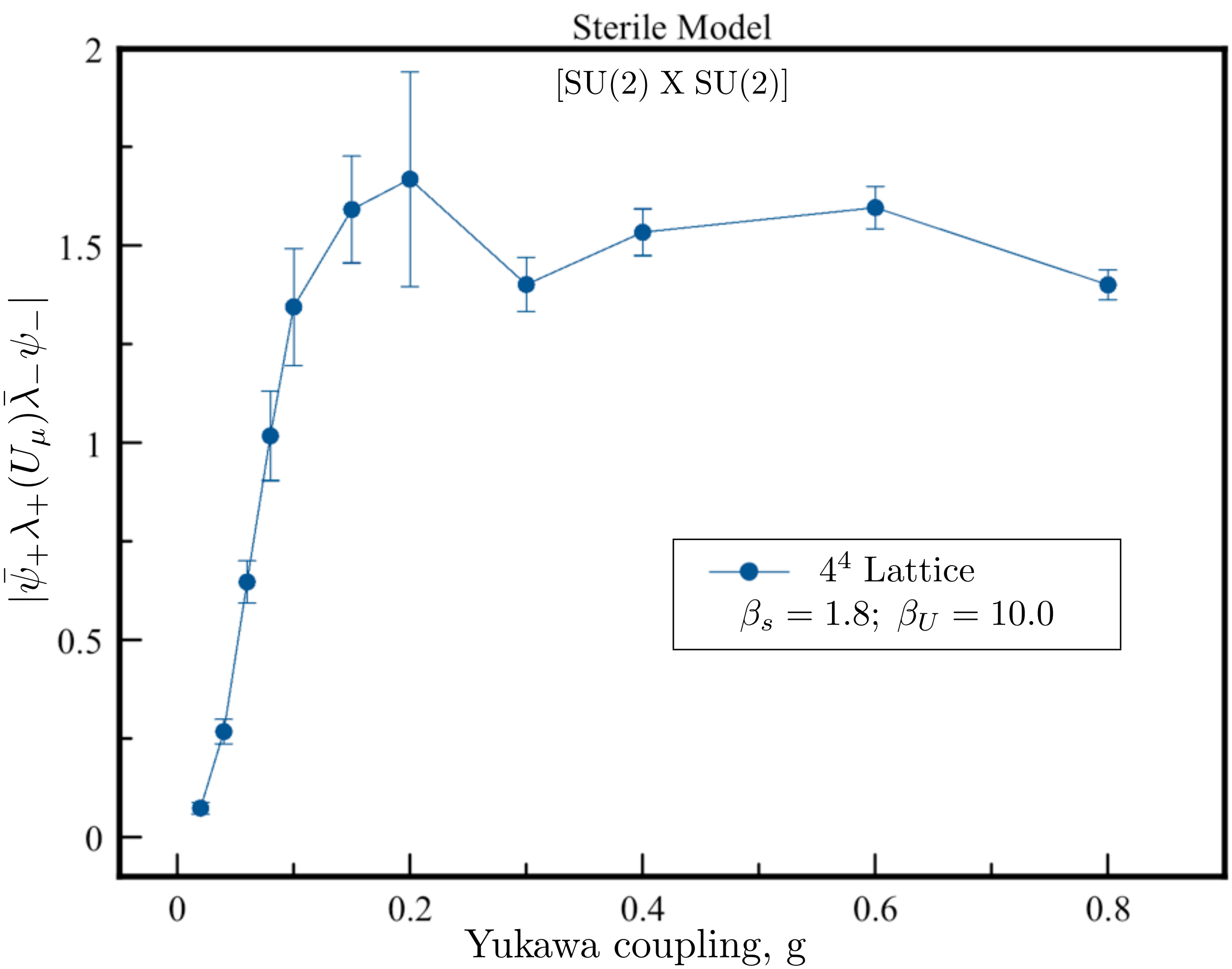
Conclusions

- Constructed a vector like lattice theory with reduced staggered fermions in which strong dynamics Higgses a weakly coupled sector with exact gauge symmetry.
- Key: This theory does not permit single site gauge invariant mass terms. Gauge invariant one-link terms are permitted but the corresponding condensate is observed to be small.
- Symmetry breaking occurs via the formation of a four-fermion condensate with drives the Higgs mechanism.
- This condensate appears to breaks chiral symmetries in the continuum (like QCD).



local SU(3) global SU(2)
4^4 beta_SU3=5.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
g
lcondensatel
psibar(x)lambda(x)
psibar(x)U_mu(x)V_mu(x)psi(x+mu)

Backup



Sterile Model
[SU(2) X SU(2)]
$|\bar{\psi}_+ \lambda_+ (U_\mu) \bar{\lambda}_- \psi_-|$
Yukawa coupling, g
4^4 Lattice
$\beta_s = 1.8;\ \beta_U = 10.0$
0
0.5
1
1.5
2
0
0.2
0.4
0.6
0.8