On the $B^{*'} \to B$ transition

Antoine Gérardin

In collaboration with Benoît Blossier, John Bulava and Michael Donnellan

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LPT Orsay, LATTICE 2013

Introduction

- $g_{B^* B\pi}$ is now precisely computed on the lattice.
- But light-cone sum rule determination for $g_{D^* D\pi}$ and $g_{B^* B\pi}$ failed to reproduce the experimental data

  $\hookrightarrow$ this problem can be solved assuming an explicit *negative* contribution from the first excited state on the hadronic side of the sum rule [Becirevic et al., JHEP 0301, 009 (2003)]
- Lattice simulations can handle with excited states, so what can we say about $g_{B^{*\prime} B\pi}$?

$g_{B^{*\prime}B\pi}$ coupling

The coupling is defined by the following on-shell matrix element :

$$\langle B^0(p)\pi^+(q)|B^{*'+}(p',\epsilon^{(\lambda)})\rangle = -g_{B^{*\prime}B\pi}(q^2)\times q_\mu \epsilon^{(\lambda)\mu}(p')$$

Performing an LSZ reduction of the pion field and using PCAC relation, we are left with the following matrix element parametrized by three form factors :

$$\langle B^{*'+}(p',\epsilon^{(\lambda)})|\mathcal{A}_\mu|B^0(p)\rangle = 2m_{B^{*\prime}}A_0(q^2)\frac{\epsilon^{(\lambda)}\cdot q}{q^2}q^\mu + (m_B+m_{B^{*\prime}})A_1(q^2)\left(\epsilon^{(\lambda)\mu} - \frac{\epsilon^{(\lambda)}\cdot q}{q^2}q^\mu\right)$$
$$+ A_2(q^2)\frac{\epsilon^{(\lambda)}\cdot q}{m_B+m_{B^{*\prime}}}\left[(p_B+p_{B^{*\prime}})^\mu + \frac{m_B^2-m_{B^{*\prime}}^2}{q^2}q^\mu\right]$$

In the HMχPT at leading order (static limit and chiral limit) and using the normalization of states $\langle B(\vec{p})|B(\vec{p})\rangle_{\mathrm{HQET}} = 1$, we just need to calculate $A_1(q^2_{\mathrm{max}})$, $q^2_{\mathrm{max}} = (m_B^{*'} - m_B)^2$ and :

$$g_{12} = \frac{g_{B^{*\prime}B\pi}}{2\sqrt{m_B m_{B^{*\prime}}}}f_\pi \quad \Leftrightarrow \quad g_{12} = \langle B^{*'}(\epsilon^{(\lambda)})|\mathcal{A}_3|B\rangle_{\mathrm{HQET}} \qquad \epsilon^{(\lambda)} = (0,0,0,1)$$

Correlation functions to be computed on the lattice

2-points correlation functions (pseudoscalar and vector mesons)

$$C_P^{(2)}(t) = \left\langle \sum_{\vec{y},\vec{x}} P(y)P^\dagger(x) \right\rangle\Big|_{y_0=x_0+t} \quad , \quad P(x) = \overline{h}(x)\gamma_5\psi_l(x)$$

$$C_V^{(2)}(t) = \frac{1}{3}\sum_{i=1}^{3}\left\langle \sum_{\vec{y},\vec{x}} V_i(y)V_i^\dagger(x) \right\rangle\Big|_{y_0=x_0+t} \quad , \quad V_i(x) = \overline{h}(x)\gamma_i\psi_l(x)$$

- We work in the static limit. Heavy Quark Symmetry $\Rightarrow$ $C_P^{(2)} = C_V^{(2)}$
- We used $N=4$ interpolating fields : $\mathcal{O}^{(i)} = \overline{h}\ \gamma_5\ (1+\kappa_G a^2\Delta)^{R_i}\ \psi_l$ (Gaussian smearing) $\longrightarrow \kappa_G = 0.1,\ r_i = 2a\sqrt{\kappa_G R_i} \leq 0.6$ fm
- Finally, we compute a matrix of correlators : $C_{ij}^{(2)}(t) = \langle \mathcal{O}^{(i)}(t)\mathcal{O}^{(j)\dagger}(t)\rangle$, $(i,j) \in [1..N]$

3-points correlation function

$$C_{ij}^{(3)}(t_z-t_x, t_y-t_x) = \left\langle \sum_{\vec{z},\vec{y},\vec{x}} V_3^{(i)}(z)\ \mathcal{A}_3(y)\ P^{(j)\dagger}(x) \right\rangle\Big|_{t_x<t_y<t_z} \quad , \quad \mathcal{A}_\mu = Z_\mathcal{A} \times \overline{\psi_l}(x)\gamma_\mu\gamma_5\psi_l(x)$$

- All-to-all propagators were estimated stochastically
- Full time dilution
- The renormalisation constant $Z_\mathcal{A}$ was determined non perturbatively by the ALPHA collaboration [Nucl.Phys. B865 (2012) 397-429]

Generalized Eigenvalue Problem (GEVP)

We solve the generalized eigenvalue problem (GEVP)

$$C^{(2)}(t)v_n(t,t_0) = \lambda_n(t,t_0)C^{(2)}(t_0)v_n(t,t_0)$$

- $\lambda_n(t,t_0) = e^{-E_n^{\text{eff}}(t,t_0)} \quad , \quad E_n^{\text{eff}}(t,t_0) \xrightarrow[t\gg 1, t_0 \gg 1]{} E_n$
- $\alpha_n(t,t_0) \times (v_n(t,t_0),\mathcal{O})^\dagger \, |0\rangle \; = \; |B_n\rangle \; + \; corrections$

$$\alpha_n(t) = \frac{\lambda_n(t+1,t)^{-t/2}}{\left(v_n(t,t-1), C^{(2)}(t)v_n(t,t-1)\right)^{1/2}}$$

$\mathcal{O}^{(i)}$ are the interpolating fields

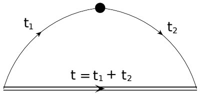
$$(v_n(t,t_0),\mathcal{O}) = \sum_i v_n^{(i)*}\mathcal{O}^{(i)}(t)$$

The sign of the eigenvectors is fixed by imposing the positivity of the decay constant :

$$f_{B_n} = \langle B_n|\mathcal{O}_0|0\rangle = A_n(t,t_0) \times \left(C_{0i}^{(2)}(t)v_i^*(t,t_0)\right) > 0 \qquad (\mathcal{O}_0 \text{ local interpollating field})$$

Matrix element

$$g_{nm} = \langle B_n | \mathcal{A}_3 | B^*_m \rangle$$



- $R^{\mathrm{GEVP}}_{mn}(t_2, t_1) = \alpha_m(t_2)\alpha_n(t_1) \times \langle v_m(t_2, t_2-1) | C^{(3)}(t_1+t_2, t_1) | v_n(t_1, t_1-1) \rangle$

with $\alpha_n(t) = \dfrac{\lambda_n(t+1,t)^{-t/2}}{\left(v_n(t,t-1), C^{(2)}(t) v_n(t,t-1)\right)^{1/2}}$ [JHEP 1201 (2012) 140]

$$R^{\mathrm{GEVP}}_{mn} \xrightarrow{t_1 \gg 1, t_2 \gg 1} g_{nm} + \mathcal{O}\left(e^{-\Delta_{N+1,m} t_1}, e^{-\Delta_{N+1,n} t_2}\right) \quad , \quad \Delta_{N+1,m} = E_{N+1} - E_m$$

$\hookrightarrow$ We choose $t_1 = t_2$ (best convergence).

- We can improve the convergence by summing over the insertion time t_1 :

$$R^{\mathrm{sGEVP}}_{mn}(t, t_0) = -\partial_t \left(\frac{\left(v_m(t,t_0), \left[K(t,t_0)/\lambda_n(t,t_0) - K(t_0,t_0)\right] v_n(t,t_0)\right)}{\left(v_n(t,t_0), C(t_0) v_n(t,t_0)\right)^{1/2} \left(v_m(t,t_0), C(t_0) v_m(t,t_0)\right)^{1/2}} e^{\Sigma(t_0,t_0) t_0/2} \right)$$

with : $K_{ij}(t, t_0) = \sum_{t_1} e^{-(t-t_1)\Sigma(t,t_0)} C^{(3)}_{ij}(t, t_1) \qquad \Sigma(t,t_0) = E_n(t,t_0) - E_m(t,t_0)$

$$R^{\mathrm{sGEVP}}_{mn} \xrightarrow[t_0 = t-1]{t \gg 1} g_{nm} + \mathcal{O}\left(t e^{-\Delta_{N+1,n} t}\right) \quad n < m$$

$$\xrightarrow[t_0 = t-1]{t \gg 1} g_{nm} + \mathcal{O}\left(e^{-\Delta_{N+1,m} t}\right) \quad n > m$$

"summed GEVP"

Results

Lattice setup

Lattice discretization

- $N_f = 2$ $O(a)$ improved Wilson-Clover Fermions
- HYP2 discretization for the static quark action $(1.0, 1.0, 0.5)$

Discretization effects

- 3 lattice spacings a :

  (0.048, 0.065, 0.075) < 0.1 fm

Light quark mass chiral extrapolation

- different pion masses in the range [310 MeV, 440 MeV]

CLS based

$\Rightarrow$ **total of 4 ensembles**

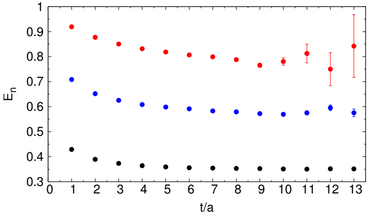
Error analysis

- full jackniffe analysis

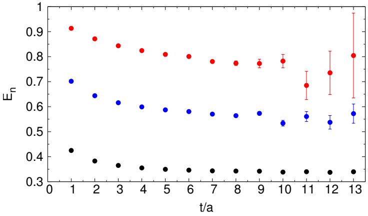
Numerical results

- Examples of effective mass plots :

CLS ensemble E5
($a = 0.065$ fm, $m_\pi = 435$ MeV)

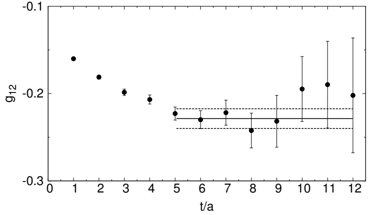


CLS ensemble F6
($a = 0.065$ fm, $m_\pi = 310$ MeV)

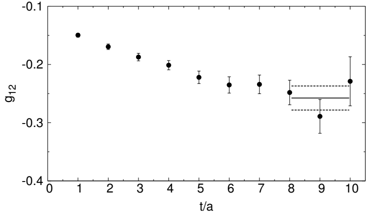


- Examples of plateaux for matrix elements :

CLS ensemble E5



CLS ensemble F6



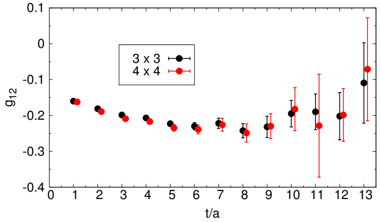
GEVP stability

- We have checked the dependance of g_{12} on the size of the GEVP (3 or 4 levels of smearing)

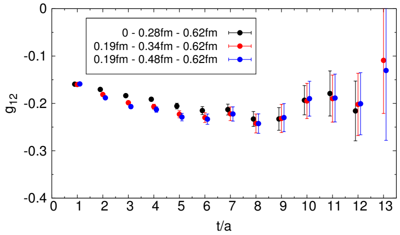
Example of plateau for ensemble $E5$

($32^3 \times 64$, $a = 0.065$ fm, $m_\pi = 435$ MeV)

$\hookrightarrow$ we will use 3 levels of smearing



- We have checked the dependance on the radius of the wave function



3×3 GEVP for different radius of smearing

sGEVP vs GEVP

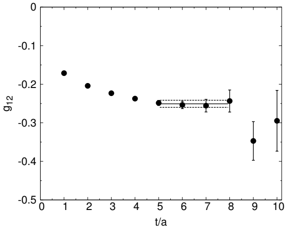
- $$R_{mn}^{\text{GEVP}} \xrightarrow{t_1>>1, t_2>>1} g_{nm} + \mathcal{O}\left(e^{-\Delta_{N+1,m} t_1}, e^{-\Delta_{N+1,n} t_2}\right)$$

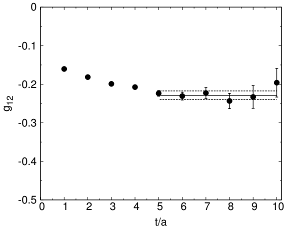
[JHEP 1201 (2012) 140]

- $$R_{mn}^{\text{sGEVP}} \xrightarrow[t_0=t-1]{t>>1} g_{nm} + \mathcal{O}\left(t e^{-\Delta_{N+1,m} t}, t e^{-\Delta_{N+1,n} t}\right)$$

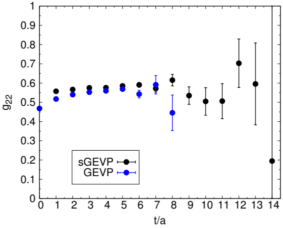
GEVP

sGEVP





A better plateau is observed in the case of sGEVP



sGEVP seems better to deal with excited state.

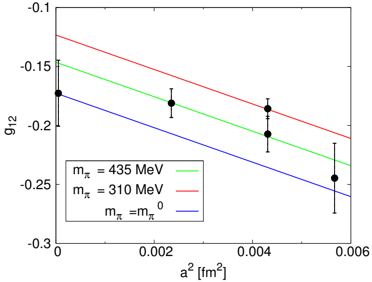
Continuum and chiral extrapolations

Inspired by HMχPT and thanks to $\mathcal{O}(a)$ improved lattice action, we tried two fit formulae :

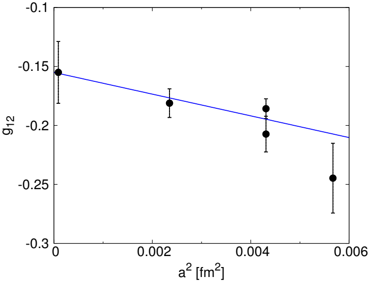
- one with continuum and chiral extrapolations to the physical point
- one with only a continuum extrapolation

$$g_{12} = C_0 + C_1 \left(\frac{a}{a_{\beta=5.3}}\right)^2 + C_2 \left(\frac{m_\pi}{m_\pi^0}\right)^2$$

$$g_{12} = C_0' + C_1' \left(\frac{a}{a_{\beta=5.3}}\right)^2$$



$\hookrightarrow \quad g_{12} = -0.173(28)$



$\hookrightarrow \quad g_{12} = -0.155(26)$

$$g_{12} = -0.17(3)(2)_\chi$$

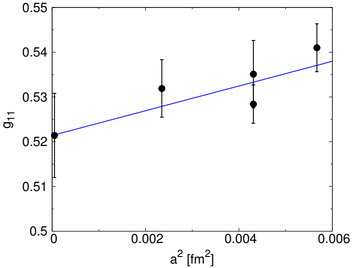
$\Rightarrow$ Negative value of the coupling (assuming positive decay constants)

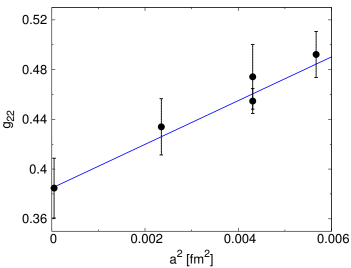
Results 2

Multi-hadron thresholds

- $B^{*'} \to B^*(\vec{p})\pi(-\vec{p})$ $\hookrightarrow$ kinematically forbidden because $L < 3$ fm
- $B^{*'} \to B_1^*\pi$ $\hookrightarrow$ using our lattice results for $m_{B^{*\prime}} - m_B$ and other lattice results (with similar lattice parameters) for $m_{B_1^*} - m_B$ this is also forbidden

- We can also estimate g_{11} and g_{22} and check our results [PoS LATTICE2010 (2010) 303] :





$g_{11} = 0.52(2)$

$g_{22} = 0.38(4)$

$\hookrightarrow$ the total error include the statistical errors and the systematic errors

Conclusion

- This is the first estimate of $g_{12} \propto g_{B^{*\prime} B\pi}$ coupling.

  Our result is : $g_{12} = -0.17(3)(2)$

- We obtain a negative value of the coupling
  $\hookrightarrow$ This could explain the small value of $g_{D^* D\pi}$ in the sum rule approach.

- Our results for g_{11} and g_{22} are in excellent agreement with previous works.
  [PoS LATTICE2010 (2010) 303]

Perspectives

- Add a lattice ensemble with a lower pion mass to improve the chiral fit.

- Extract $A_1(q^2 = 0)$ from the distribution in r of the axial density $f_A(r) = \langle B^{*'} | \mathcal{A}_i(r) | B \rangle$.

Thank you !

Multi-Hadron Thresholds

$$B^{*'} \to H\pi$$

- partiy conservation : $P_{B^{*'}} = P_H \times P_\pi \times (-1)^L$
- momentum conservation : $J_{B^{*'}} = L + J_H$

$B^{*'} \to B^*(\vec{p})\pi(-\vec{p})$ (case $L = 1$)

- in our study : $L < 3$ fm and $m_\pi \leq 440$ MeV
  $\Rightarrow p = \frac{2\pi}{L} \geq 500$ MeV

$B^{*'} \to B_1^*\pi$ (case $L = 0$)

- Our study : $230 \text{ MeV} \leq m_{B^{*'}} - m_B - m_\pi \leq 360 \text{ MeV}$
- [JHEP 1008 (2010) 009] : $400 \text{ MeV} \leq m_{B_1^*} - m_B \leq 500 \text{ MeV}$
  $\hookrightarrow$ pion mass in the range [280-500] MeV and lattice spacings $a \in [0.05 - 0.08]$ fm

Convergence of the sGEVP

We treat the GEVP ($N \times N$) perturbatively
$\rightarrow$ the order "0" correspond to the case where only N states contribute, it can be solve exactly.

- $C_{ij}(t) = C_{ij}^{(0)}(t) + \epsilon C_{ij}^{(1)}(t) = \sum_{n=1}^{N} e^{-E_n t} \psi_{ni}\psi_{nj} + \sum_{n=N+1}^{\infty} e^{-E_n t} \psi_{ni}\psi_{nj}$
- $v_n(t, t_0) = v_n^{(0)} + \epsilon v_n^{(1)}(t, t_0)$
- $\lambda_n(t, t_0) = \lambda_n^{(0)} + \epsilon \lambda_n^{(1)}(t, t_0)$

Our results are (first order) :

$$R_{mn}^{\text{sGEVP}} \xrightarrow[t_0=t-1]{t>>a} g_{nm} + \mathcal{O}\left(t e^{-\Delta_{N+1,n} t}\right) \qquad n > m$$

$$R_{mn}^{\text{sGEVP}} \xrightarrow[t_0=t-1]{t>>a} g_{nm} + \mathcal{O}\left(e^{-\Delta_{N+1,m} t}\right) \qquad n < m$$

We have tested our results with the toy model introduced in [JHEP 1201 (2012) 140] :

$$\psi = \langle 0|\mathcal{O}_i|n\rangle = \begin{pmatrix} 0.92 & 0.03 & -0.10 & -0.01 & -0.02 \\ 0.84 & 0.40 & 0.03 & -0.06 & 0.00 \\ 0.56 & 0.56 & 0.47 & 0.26 & 0.04 \end{pmatrix} , \; M_{nn} = 0.7\frac{6}{n+5} , \; M_{n,m+n} = \frac{M_{nn}}{3m}$$