η and η' Masses from Lattice QCD

for the ETM collaboration

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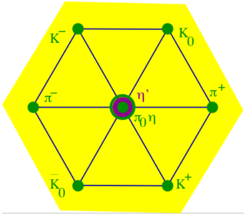
Mainz, Lattice 2013

Flavour Singlet Pseudo-Scalar Mesons

- nine lightest pseudo-scalar mesons show a peculiar spectrum:
  - 3 very light pions (140 MeV)
  - kaons and the η around 500 MeV
  - η' around 1 GeV



- the large mass of the η' meson is thought to be caused by the QCD vacuum structure and the $U_A(1)$ anomaly
- η' meson is not a (would be) Goldstone Boson

$\Rightarrow$ massive even in the SU(3) chiral limit filled symbols: η open: η'

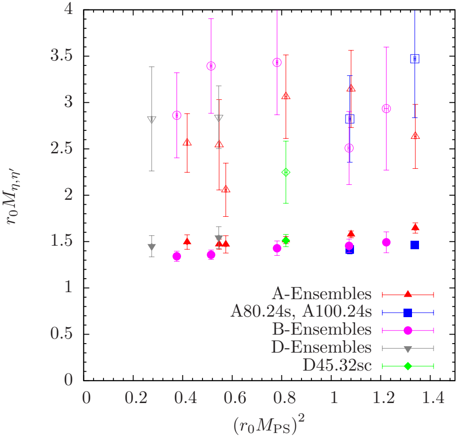
- η' mainly the flavour singlet
- disconnected contributions significant

$\Rightarrow$ hard problem

- chiral extrapolation difficult

$\Rightarrow$ no clear picture

- need for improvement



[C. Michael, K. Ottnad, S. Reker, C.U., JHEP 1211 (2012) 048]

Ensemble-Details

- $2+1+1$ flavour gauge configurations from ETM Collaboration
  [ETMC, R. Baron et. al., JHEP 06 111 (2010)]
- Iwasaki Gauge action
  [Iwasaki, Nucl. Phys. B258, 141]
- three lattice spacings (A, B and D ensembles):
  $a_A = 0.086\ \mathrm{fm}$, $a_B = 0.078\ \mathrm{fm}$ and $a_D = 0.061\ \mathrm{fm}$
- charged pion masses range from $\approx 230\ \mathrm{MeV}$ to $\approx 500\ \mathrm{MeV}$
- $L \geq 3\ \mathrm{fm}$ and $M_\pi \cdot L \geq 3.5$ for most ensembles
- ≈ 600 up to ≈ 2500 gauge configuration per ensemble
- bare m_s and m_c fixed for each lattice spacing
- use $r_0 = 0.45(2)\ \mathrm{fm}$ (from f_π) throughout the talk

- need to estimate correlator matrix

$$\mathcal{C} = \begin{pmatrix} \eta_{\ell\ell} & \eta_{\ell s} & \eta_{\ell c} \\ \eta_{s\ell} & \eta_{ss} & \eta_{sc} \\ \eta_{c\ell} & \eta_{cs} & \eta_{cc} \end{pmatrix}$$

- η_{XY} correlator of appropriate interpolating fields, e.g.

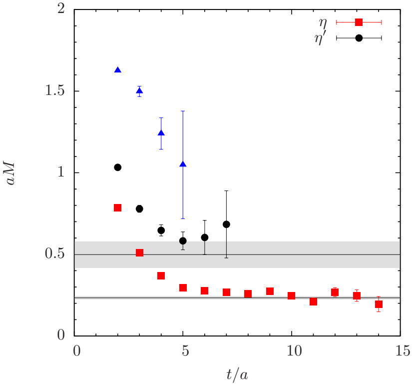
$$\eta_{ss}(t) \quad \equiv \quad \langle \bar{s} i\gamma_5 s(t)\, \bar{s} i\gamma_5 s(0)\rangle$$

  projected to zero momentum

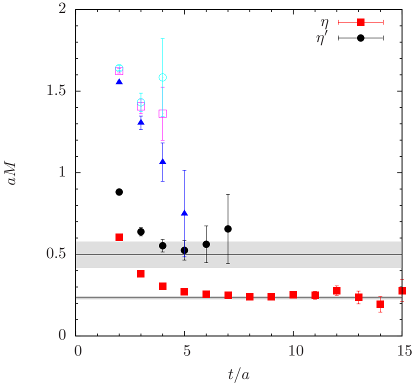
$\Rightarrow$ diagonalise matrix:
masses and pseudo-scalar matrix elements

- η: lowest state, η': first state, η_c ...

3 × 3 matrix



6 × 6 matrix



- ground state η well determined
- η' signal lost in noise before plateau reached

Improved η' Extraction

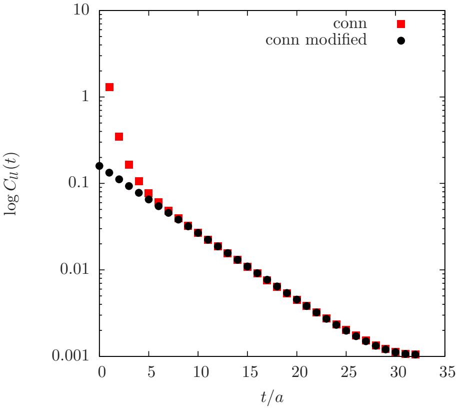
- make model assumption:
  disconnected contributions couple only to η and η' states, not to higher states

  [Neff et al., Phys.Rev.D64 (2001)]

  [K.Jansen, C.Michael, C.U., Eur.Phys.J.C58(2008)]

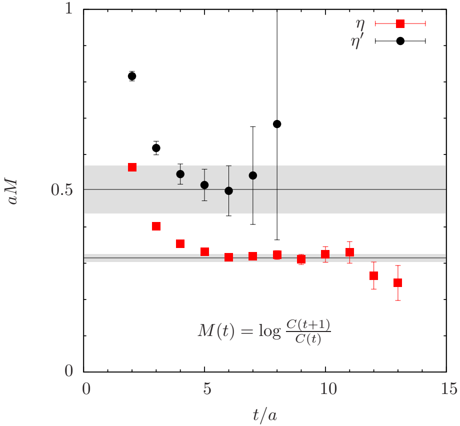
- replace connected contributions by only the ground states

- if model justified:
  there should be a plateau in the effective masses from very low times on!



- we see a plateau from $t/a = 2$ on
- for both η and η'
- η: good agreement with previous results
- η': possibly much better determination
- assumption justified?
- systematic uncertainties?

w/o removal



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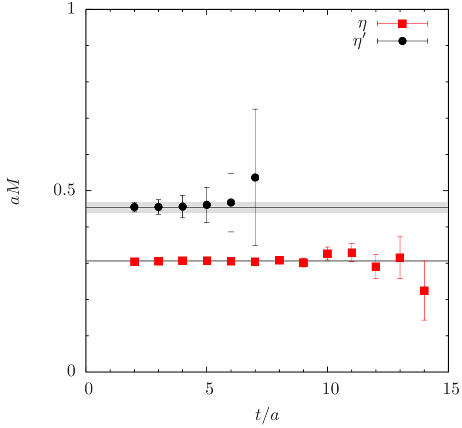

- w/ removal: only two states left $\mathcal{C}$

η:

- masses agree well
- improved precision

η':

- masses determined much better
- always agreement within 2σ
- systematics hard to quantify
- from distribution of differences: assign 7% systematic uncertainty

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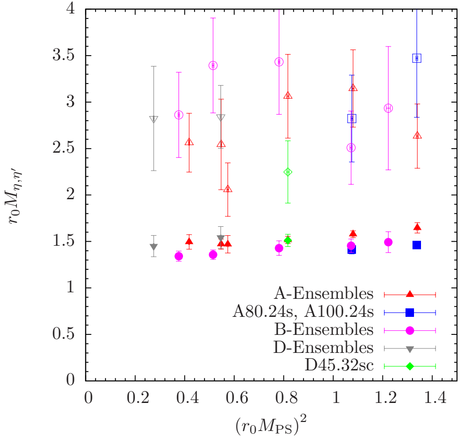

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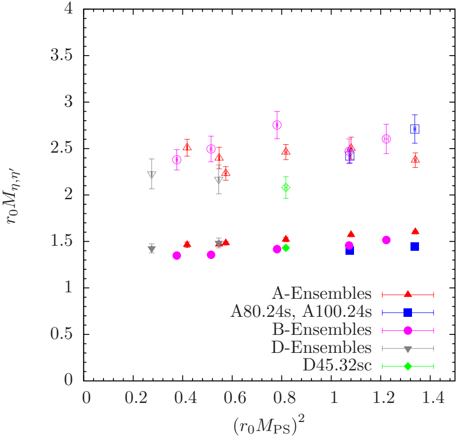


Strange Quark Mass Dependence

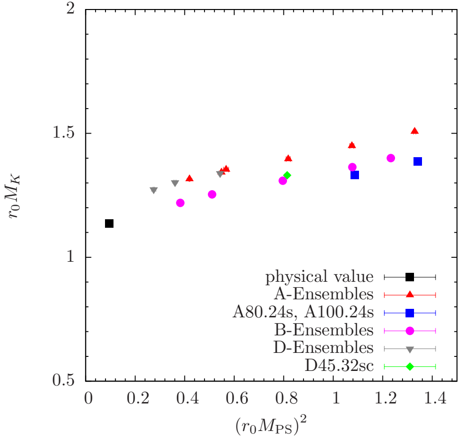
- m_s not perfectly tuned to its physical value
- two re-tuned ensembles for a_A

$\Rightarrow$ can estimate m_s dependence

- estimate

$$D_\eta \equiv \frac{d(aM_\eta)^2}{d(aM_\mathrm{K})^2} = 1.47(11)$$

- now assume: D_η independent of a, m_ℓ, m_s, m_c
- ...correct η masses



Scaling Test for M_η

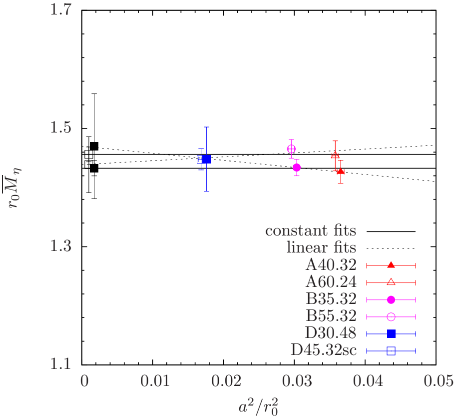
- use two ensembles sets
  ($A60$, $B55$, $D45$)
  ($A40$, $B35$, $D30$)
  with $r_0 M_{\mathrm{PS}} \approx \mathrm{const}$

- correct M_η using D_η linearly in M_{K}^2

$\Rightarrow$ $r_0 M_{\mathrm{K}} = 1.34$ fixed

- compatible with both, constant and linear continuum extrapolation

$\Rightarrow$ assign 5% systematic error from maximal difference



- more ambitious: shift all M_η to physical strange mass
- fit c_1, c_2

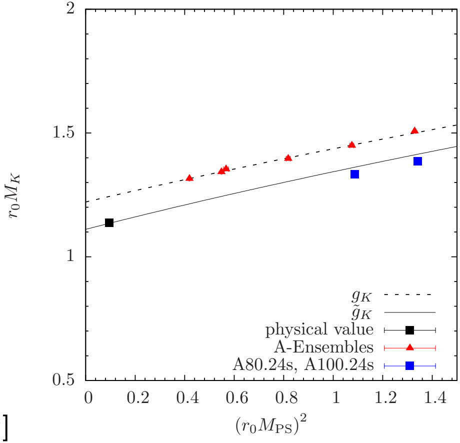
$$g_{\rm K} = c_1 + c_2 (r_0 M_{\rm PS})^2$$

  to data for $(r_0 M_{\rm K})^2$ from *A* ensembles

- adjust c_1 to match physical $M_{\rm K}$ for $M_{\rm PS} = M_\pi \quad \Rightarrow \tilde{g}_{\rm K}$
- compute

$$\delta_{\rm K}[(r_0 M_{\rm PS})^2] = (r_0 M_{\rm K})^2 - \tilde{g}_{\rm K}[(r_0 M_{\rm PS})^2]$$

  for all ensembles



- now correct all $(r_0 M_\eta)^2$ by corresponding

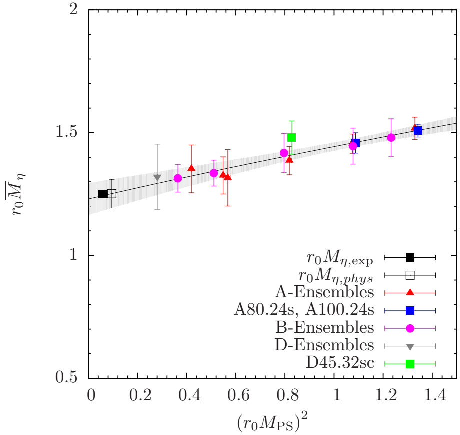
$$D_\eta \cdot \delta_{\mathrm{K}}[(r_0 M_{\mathrm{PS}})^2]$$

$\Rightarrow$ $(r_0 \bar{M}_\eta)^2 \propto (r_0 M_{\mathrm{PS}})^2$

- all *a*-values fall on the same curve!
- extrapolate $(r_0 \bar{M}_\eta)^2$ linearly in $(r_0 M_{\mathrm{PS}})^2$ to $M_{\mathrm{PS}} = M_\pi$
- result

$$M_\eta = 552(10)_{\mathrm{stat}}\ \mathrm{MeV}$$

- similarly with $(\bar{M}_\eta / \bar{M}_{\mathrm{K}})^2$ or GMO relation



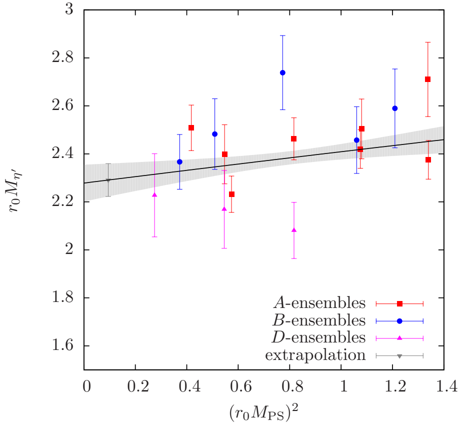
Chiral Extrapolation of $M_{\eta'}$

- no clear dependence on
  - lattice spacing
  - strange quark mass
- errors still significant
- include all data in extrapolation
- $(r_0 M_{\eta'})^2 \propto (r_0 M_{\mathrm{PS}})^2$

⇒ result

$$M_{\eta'} = 1005(54)_{\mathrm{stat}}\ \mathrm{MeV}$$

- fitting *A*, *B* and *D* seperately gives compatible results



Summary

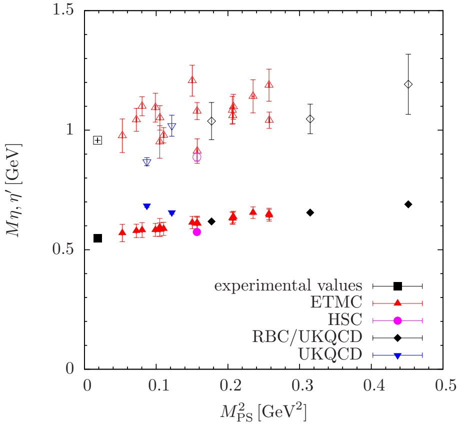
- η and η' for three lattice spacings and various quark mass values
- presented excited state removal method
- η can be extracted precisely

$$M_\eta = 552(10)_{\text{stat}}(28)_{\text{sys}}\ \text{MeV}$$

- η' from excited state removal

$$M_{\eta'} = 1005(54)_{\text{stat}}(86)_{\text{sys}}\ \text{MeV}$$

$\rightarrow$ mixing: talk by Konstantin Ottnad



[HSC, J. J. Dudek et al., Phys. Rev. D83 (2011)]
[RBC/UKQCD, N. Christ et al., Phys. Rev. Lett. 105 (2010)]
[UKQCD, E. B. Gregory et al., Phys.Rev. D86 (2012)]

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