Leading-order hadronic contributions to $g_\mu - 2$

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for the

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Outline

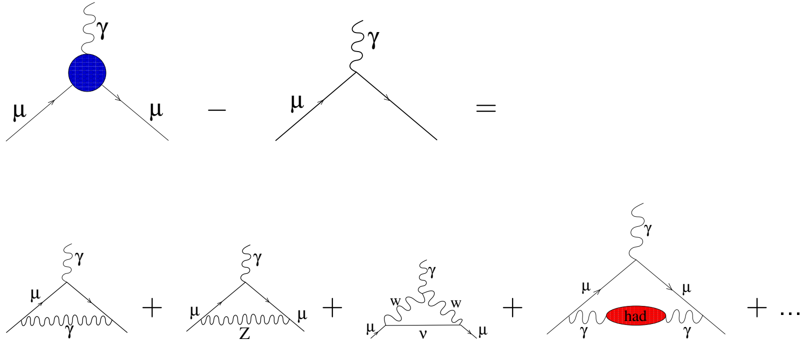
- Introduction
- Lattice calculation
- Results
- Conclusions

Introduction

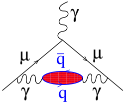
- $a_\mu \equiv \frac{(g_\mu - 2)}{2}$ measured experimentally to ~ 500 parts per billion.
- Similar precision in SM prediction.
- Is $\Delta a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(63)(49) \times 10^{-11}$ a hint of new physics?
- Errors on SM prediction are dominated by hadronic contribution.
- Estimates of hadronic contributions come from $e^+e^- \to$ hadrons or $\tau \to \nu +$hadrons, with $\sim 1\%$ errors.
- Can lattice calculations be competitive?

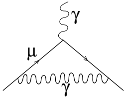
Introduction

$$a_\mu^{\mathrm{SM}} = \frac{g_\mu - 2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{had}}$$



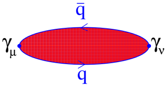
Leading-order (one-loop) hadronic contribution

Contribution to a_μ from 

can be broken into the EM loop: 

$$a_\mu^{(1)} = \frac{\alpha}{\pi}\int_0^\infty dQ^2 f(Q^2),$$

$$f(Q^2) = \frac{m_\mu^2 Q^2 Z(Q^2)^3\left(1-Q^2 Z(Q^2)\right)}{1+m_\mu^2 Q^2 Z(Q^2)^2} \text{ and } Z = -\frac{Q^2-\sqrt{Q^4+4m_\mu^2 Q^2}}{2m_\mu^2 Q^2}$$

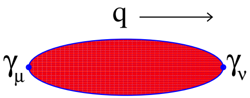
And the insertion of the hadronic blob†: 

$\hat{\Pi}(Q^2)$

$$a_\mu(\mathrm{HVP}) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 f(Q^2)\hat{\Pi}(Q^2),$$

† See T. Blum, Phys.Rev.Lett. 91 (2003) 052001

Hadronic vacuum polarization



$$\equiv i\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle J_\mu(x) J_\nu(y) \rangle,$$

with $J_\mu(x)$ being the electromagnetic current:

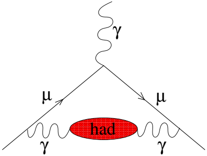
$$J_\mu(x) = \sum_{i=1}^{N_f} Q_i \overline{\psi}^i(x)\gamma_\mu\psi^i(x)$$

The HVP scalar $\Pi(q^2)$ is defined through

$$\Pi_{\mu\nu}(q) = \left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)\Pi(q^2)$$

$$\hat{\Pi}(q^2) = 4\pi\alpha\left(\Pi(q^2) - \Pi(0)\right)$$

Leading-order (one-loop) hadronic contribution



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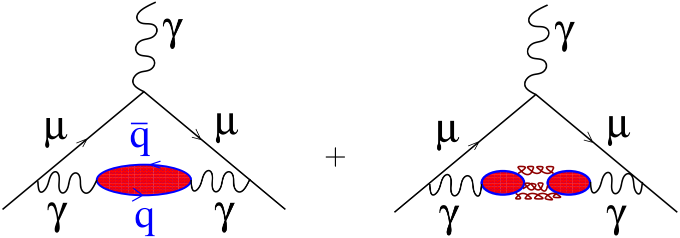
Connected + disconnected parts:



Ensembles

Hex-smeared clover fermions

2-hex ($N_f = 2 + 1$):

am_{ud}^{bare}	am_s^{bare}	volume	# cfgs	M_π (GeV)	n_{tw}
		$\beta = 3.31$, $a^{-1} = 1.697$ GeV			
-0.09933	-0.0400	$48^3 \times 48$	928	0.136(2)	
-0.09300	-0.0400	$24^3 \times 48$	210	0.255(2)	
		$\beta = 3.5$, $a^{-1} = 2.131$ GeV			
-0.05294	-0.0060	$64^3 \times 64$	83	0.130(2)	
-0.04900	-0.0120	$32^3 \times 64$	216	0.250(2)	
-0.04900	-0.0060	$32^3 \times 64$	110	0.258(2)	
-0.04630	-0.0120	$32^3 \times 64$	212	0.308(2)	
		$\beta = 3.61$, $a^{-1} = 2.561$ GeV			
-0.03000	-0.0042	$32^3 \times 48$	188	0.332(4)	0.5, 0.25, 0.1
		$\beta = 3.7$, $a^{-1} = 3.026$ GeV			
-0.02700	0.0000	$64^3 \times 64$	208	0.182(2)	

3-hex ($N_f = 1 + 1 + 1 + 1$):

am_u^{bare}	am_d^{bare}	am_s^{bare}	am_c^{bare}	volume	# cfgs	M_π (GeV)
		$\beta = 3.2$, $a^{-1} = 1.897$ GeV				
-0.0806	-0.0794	-0.033	0.71	$32^3 \times 64$	240	0.250

Lattice calculation

We measure the correlator:

$$\Pi^{f}_{\mu\nu}(\hat{q}) = Z_V \sum_{y} \langle V^{\mathrm{loc},f}_{\mu}(x) V^{\mathrm{cvc},f}_{\nu}(y)\rangle e^{iq(x-y-\frac{a\hat{\nu}}{2})},$$

with the local vector current at the source

$$V^{\mathrm{loc},f}_{\mu}(x) = \overline{\psi}_f(x)\gamma_\mu\psi_f(x)$$

and conserved vector current at the sink

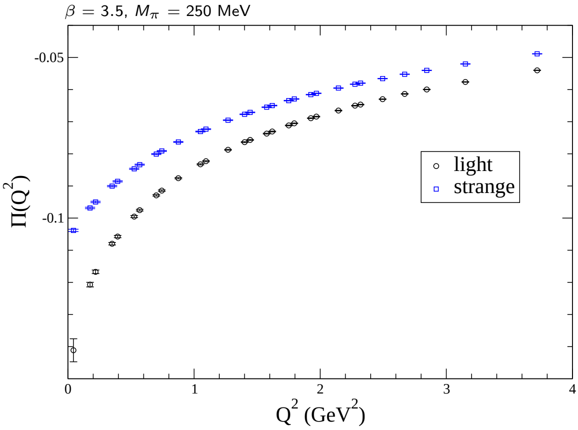
$$V^{\mathrm{cvc},f}_{\nu}(y) = \frac{1}{2}\left[\bar{\psi}_f(y+a\hat{\nu})(1+\gamma_\nu)U^{\dagger}_{\nu}(y)\psi_f(y) - \bar{\psi}_f(y)(1-\gamma_\nu)U_\nu(y)\psi_f(y+a\hat{\nu})\right].$$

$$\hat{q}_\nu = \tfrac{2}{a}\sin\left(\tfrac{aq_\nu}{2}\right) \quad \text{and} \quad q_\nu = \tfrac{2\pi n_\nu}{L_\nu}$$

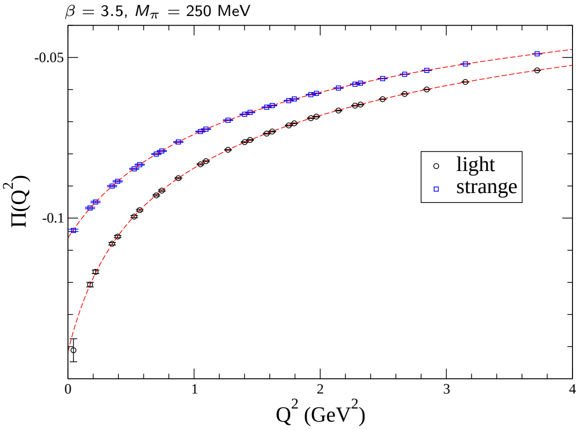
CVC obeys the Ward Identity:

$$\hat{q}_\nu \Pi_{\mu\nu}(\hat{q}) = 0$$

Sample unsubtracted HVP scalar data



Fit to favorite form(s) for integration



Fit ansatz

We fit the HVP scalar lattice data for each flavor to a 2-state monopole form:

$$\Pi(Q^2) = A + \frac{b_1}{Q^2 + c_1} + \frac{b_2}{Q^2 + c_2}$$

Vector dominance model:

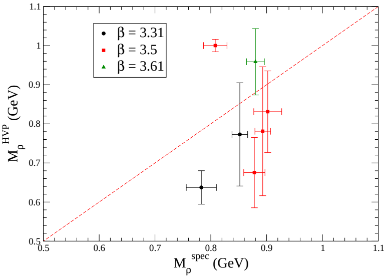
$$\Pi^{\rm tree}(Q^2) = \frac{2}{3}\frac{f_V^2}{Q^2 + m_V^2}$$

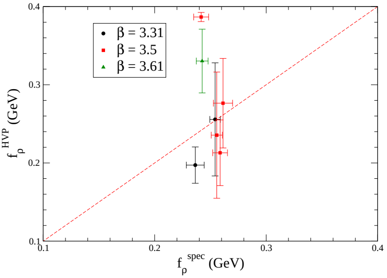
so we expect $b_1 \approx \frac{2}{3} f_\rho^2$ and $c_1 \approx m_\rho^2$

† (see Aubin & Blum, Phys. Rev. D **75**, 114502 (2007))

Fit parameters

Sanity/consistency check

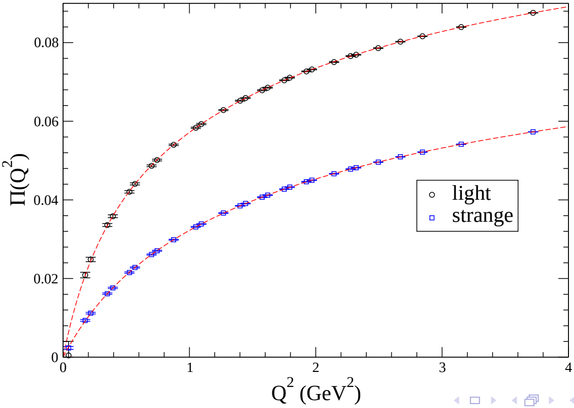




Vacuum subtraction

Subtract $Q^2 = 0$ value:

$$\Pi(Q^2) \longrightarrow \Pi(Q^2) - \Pi(0)$$



Integration

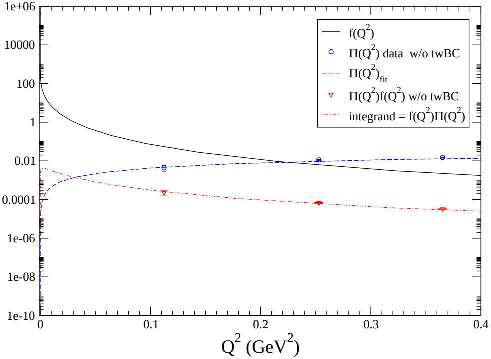
We combine the single-flavor HVP scalar fit functions and integrate

$$\Pi(Q^2) = \sum_{i=1}^{N_f} \Pi_i(Q^2) q_i^2,$$

with q_i the EM charge of the i^{th} quark flavor.
Integrate:

$$a_\mu(\mathrm{HVP}) = 4\alpha^2 \int_0^\infty dQ^2 f(Q^2)\Pi(Q^2)$$

Data missing the integrand peak → Twisted BCs



The function $f(Q^2)$ diverges sharply at the origin. $\longrightarrow$ $f(Q^2)\hat{\Pi}(Q^2)$ is peaked at $Q^2 \sim 0.01$ GeV^2.

Our longest lattices give us access to momenta $Q^2 \sim 0.025$ GeV^2.

Twisted BCs

Twist the spatial boundary conditions in the valence quark and anti-quark fields by a relative angle

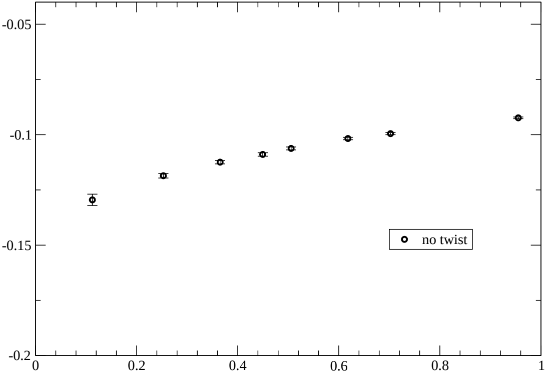
$$\theta^{\mathrm{tw}} = 2\pi n_{\mathrm{tw}}$$

$$\psi(x + L\mu) = e^{i\theta^{\mathrm{tw}}_{\mu}}\psi(x)$$

Momenta transform as

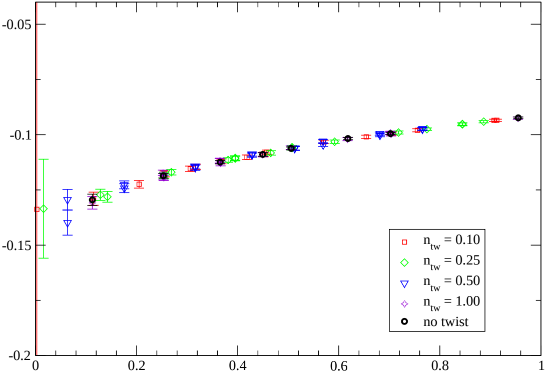
$$q_\mu \to q_\mu - \theta^{\mathrm{tw}}_{\mu}/L_\mu.$$

Twisted BCs



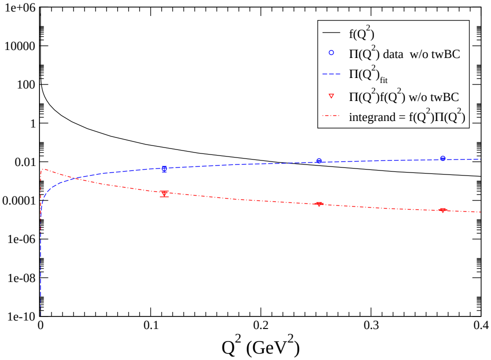
$\beta = 3.61$, $M_\pi = 332$ MeV, $L = 64$, $a^{-1} = 3.026$ GeV

Twisted BCs

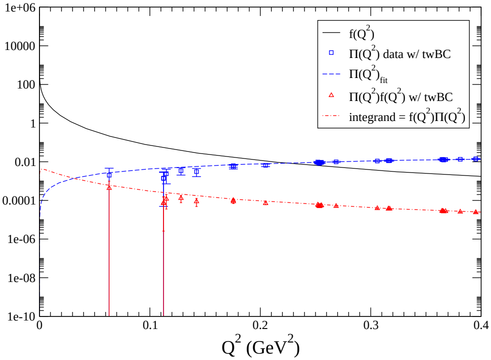


$\beta = 3.61$, $M_\pi = 332$ MeV, $L = 64$, a^{-1} =3.026 GeV

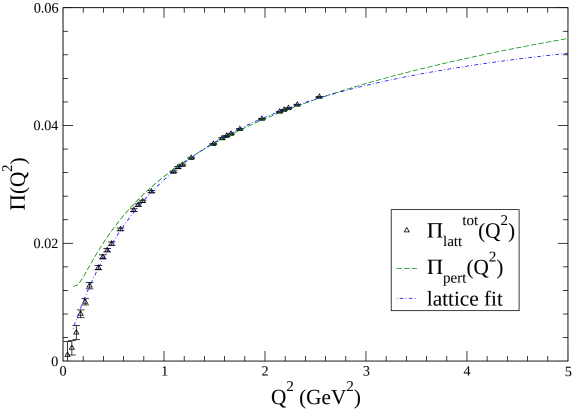
Twisted BCs


1e+06
10000
100
1
0.01
0.0001
1e-06
1e-08
1e-10
0
0.1
0.2
0.3
0.4
Q^2 (GeV^2)
$f(Q^2)$
$\Pi(Q^2)$ data w/o twBC
$\Pi(Q^2)_{fit}$
$\Pi(Q^2)f(Q^2)$ w/o twBC
integrand = $f(Q^2)\Pi(Q^2)$

Twisted BCs


f(Q^2)
Π(Q^2) data w/ twBC
Π(Q^2)_fit
Π(Q^2)f(Q^2) w/ twBC
integrand = f(Q^2)Π(Q^2)
1e+06
10000
100
1
0.01
0.0001
1e-06
1e-08
1e-10
0
0.1
0.2
0.3
0.4
Q^2 (GeV^2)

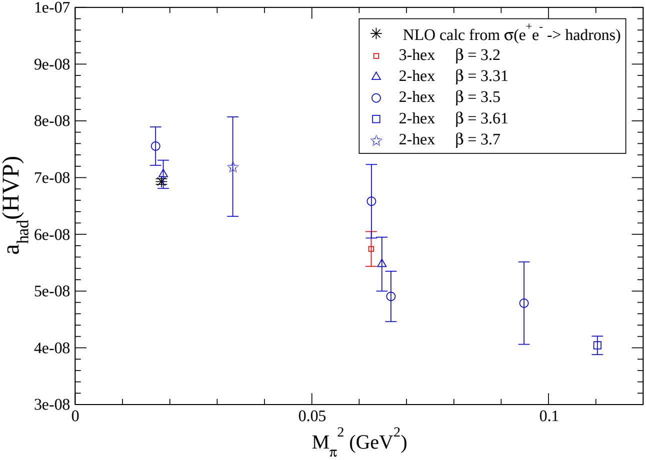
Matching to perturbative QCD



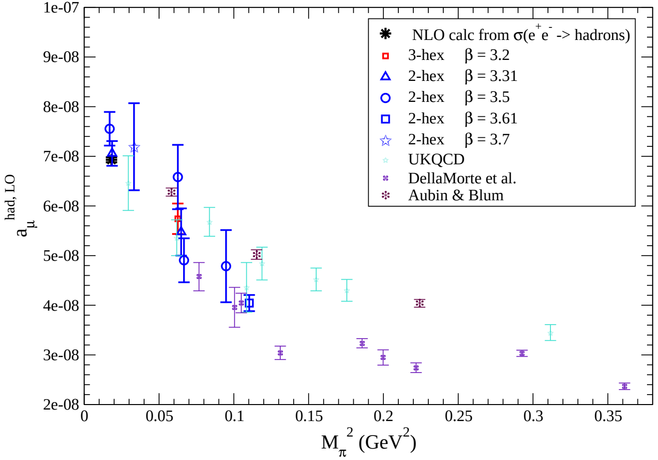
Not yet incorporated into integral.

Use 3-loop polarization function from Chetyrkin, Kühn & Steinhauser, (1996).

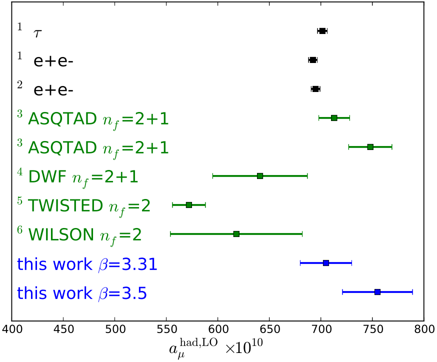
Results


NLO calc from σ(e+e- -> hadrons)
3-hex β = 3.2
2-hex β = 3.31
2-hex β = 3.5
2-hex β = 3.61
2-hex β = 3.7
a_had(HVP)
1e-07
9e-08
8e-08
7e-08
6e-08
5e-08
4e-08
3e-08
0
0.05
0.1
M_π^2 (GeV^2)

Results


NLO calc from σ(e+e- -> hadrons)
3-hex β = 3.2
2-hex β = 3.31
2-hex β = 3.5
2-hex β = 3.61
2-hex β = 3.7
UKQCD
DellaMorte et al.
Aubin & Blum
a_μ^had, LO
M_π^2 (GeV^2)
1e-07
9e-08
8e-08
7e-08
6e-08
5e-08
4e-08
3e-08
2e-08
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Results



[1] M. Davier, *et al.*(2011)

[2] K. Hagiwara, *et al.*(2011)

[3] C. Aubin and T. Blum (2007)

[4] P. Boyle,*et al.*(2012)

[5] X. Feng, *et al.* (2011)

[6] M. Della Morte *et al.* (2012)

Blue points are the two ensembles at the physical pion mass (stat. errors only).

Conclusions & Outlook

Conclusions

- We have made a preliminary calculation of the connected part of $a_\mu^{\rm had}$(HVP) on the lattice with pions at the physical mass.
- Results are consistent with e^+e^- and τ decay data.

Future directions

- Disconnected contribution calculation.
- Different fit forms
- Reduce errors

BACKUP

Light quark $\Pi(Q^2)$ fits

β	M_π (GeV)	M_ρ (GeV)	$M_\rho^{(\Pi\mathrm{fit})}$ (GeV)	f_ρ (GeV)	$f_\rho^{(\Pi\mathrm{fit})}$ (GeV)	χ^2/dof
3.31	0.136(2)	0.783(27)	0.637(43)	0.2364(80)	0.197(23)	3.3
3.31	0.255(2)	0.852(14)	0.77(130)	0.2542(47)	0.256(72)	1.18
3.5	0.130(2)	0.808(21)	1.000(16)	0.2417(67)	0.3867(58)	1.15
3.5	0.250(2)	0.878(19)	0.675(90)	0.2587(66)	0.213(42)	0.32
3.5	0.258(2)	0.902(25)	0.831(100)	0.2613(86)	0.276(57)	0.36
3.5	0.308(2)	0.893(14)	0.780(160)	0.2557(50)	0.235(81)	0.13
3.61	0.332(4)	0.880(16)	0.959(85)	0.2426(53)	0.330(41)	0.017

Twisted BCs

Twist the spatial boundary conditions in the valence quark and anti-quark fields by a relative angle

$$\theta^{\rm tw} = 2\pi n_{\rm tw}$$

$$\psi(x + L\mu) = e^{i\theta^{\rm tw}_\mu/2}\psi(x) \text{ and } \overline{\psi}(x + L\mu) = e^{-i\theta^{\rm tw}_\mu/2}\overline{\psi}(x)$$

In practice the twist is spread across the lattice by transforming gauge fields

$$U_\mu(x) \longrightarrow U_\mu(x)e^{\pm i\frac{\theta_\mu}{2L_\mu}}$$

(Sign $\pm$ depends on whether inverting for q or $\overline{q}$.)
Momenta transform as $q_\mu \to q_\mu - \theta^{\rm tw}_\mu/L_\mu$.

Integral fraction