# Leading-order hadronic contribution to the anomalous magnetic moment of the muon from $N_f = 2 + 1 + 1$ twisted mass fermions

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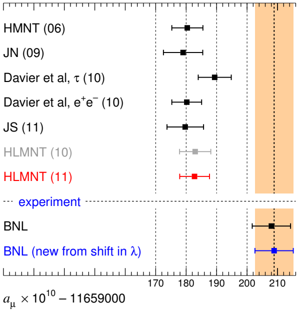
# Why the muon's anomalous magnetic moment?

- The anomalous magnetic moment of the muon, $a_\mu$, can be measured very precisely: [B. Lee Roberts, Chinese Phys. C 34, 2010]

$$a_\mu^{\text{exp}} = 116592089(63) \times 10^{-11}$$
$$a_\mu^{\text{SM}} = 116591828(49) \times 10^{-11}$$

[Hagiwara et al.,J. Phys. G38, 2011]




There is a $\approx 3\sigma$ discrepancy between $a_\mu^{\text{exp}}$ and $a_\mu^{\text{SM}}$:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(80) \times 10^{-11}$$

# Charm quark necessary to reach required precision

## Current discrepancy [Hagiwara et al.,J. Phys. G38, 2011]

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(80) \times 10^{-11}$$

- charm quark contribution computed in perturbation theory [Bodenstein, Dominguez, Schilcher, Phys.Rev. D85, 2012]

$$a_\mu^{\text{hvp,c}} = 144(1) \times 10^{-11}$$

- comparable to hadronic light-by-light scattering contribution [Prades, de Rafael, Vainshtein, arXiv:0901.0306 [hep-ph], 2009]

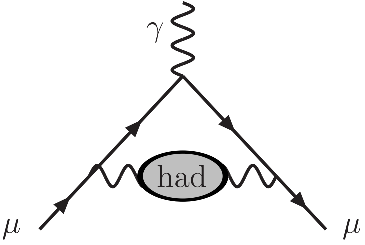
$$a_\mu^{\text{hlbl}} = 105(26) \times 10^{-11}$$

- and also to electroweak contribution [Jegerlehner, Nyffeler, Phys. Rept. 477 , 2009]

$$a_\mu^{\text{EW}} = 153(2) \times 10^{-11}$$

# Leading hadronic contribution $a_\mu^{\rm hvp}$

$$a_\mu^{\rm QCD} = a_\mu^{\rm lo,hvp} + a_\mu^{\rm ho,hvp} + a_\mu^{\rm lbl}$$




- can be computed directly in Euclidean space-time [T. Blum, PRL 91, 2003]

$$a_\mu^{\rm hvp} = \alpha^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{m_\mu^2}\right) \Pi_{\rm R}(Q^2)$$

where $\Pi_{\rm R}(Q^2) = \Pi(Q^2) - \Pi(0)$

- main ingredient: hadronic vacuum polarisation tensor

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot(x-y)} \langle J_\mu^{\rm em}(x) J_\nu^{\rm em}(y)\rangle = (Q_\mu Q_\nu - Q^2 g_{\mu\nu})\Pi(Q^2)$$

with

$$J_\mu^{\rm em}(x) = \tfrac{2}{3}\overline{u}(x)\gamma_\mu u(x) - \tfrac{1}{3}\overline{d}(x)\gamma_\mu d(x) + \tfrac{2}{3}\overline{c}(x)\gamma_\mu c(x) - \tfrac{1}{3}\overline{s}(x)\gamma_\mu s(x)$$

# Mixed-action set-up

- configurations generated by ETMC [Baron et al., JHEP 1006, 2010]
- light quarks: twisted mass action for mass-degenerate fermion doublet [Frezzotti, Rossi, JHEP 0408, 2004]

$$S_F[\chi, \overline{\chi}, U] = \sum_x \overline{\chi}(x) \left[ D_W + m_0 + i\mu_q \gamma_5 \tau^3 \right] \chi(x)$$

- heavy sea quarks: twisted mass action for non-degenerate fermion doublet [Frezzotti, Rossi, Nucl. Phys. Proc. Suppl. 128, 2004]

$$S_F[\chi_h, \overline{\chi}_h, U] = \sum_x \overline{\chi}_h(x) \left[ D_W + m_0 + i\mu_\sigma \gamma_5 \tau^1 + \mu_\delta \tau^3 \right] \chi_h(x)$$

- heavy valence quarks: Osterwalder-Seiler action

[Frezzotti, Rossi, JHEP 0410, 2004]

$$S_F[\chi_h, \overline{\chi}_h, U] = \sum_x \overline{\chi}_h(x) \left[ D_W + m_0 + i \begin{pmatrix} \mu_c & 0 \\ 0 & -\mu_s \end{pmatrix} \gamma_5 \right] \chi_h(x)$$

- tune bare mass parameters $\mu_{c/s}$ such that physical kaon and D-meson masses are reproduced

# The $N_f = 2+1+1$ ensembles

| Ensemble | $\beta$ | $a$[fm] | $L^3 \times T$ | $m_{PS}$[MeV] | $L$[fm] |
|---|---|---|---|---|---|
| D15.48 | 2.10 | 0.061 | $48^3 \times 96$ | 227 | 2.9 |
| D30.48 | 2.10 | 0.061 | $48^3 \times 96$ | 318 | 2.9 |
| D45.32sc | 2.10 | 0.061 | $32^3 \times 64$ | 387 | 1.9 |
| B25.32t | 1.95 | 0.078 | $32^3 \times 64$ | 274 | 2.5 |
| B35.32 | 1.95 | 0.078 | $32^3 \times 64$ | 319 | 2.5 |
| B35.48 | 1.95 | 0.078 | $48^3 \times 96$ | 314 | 3.7 |
| B55.32 | 1.95 | 0.078 | $32^3 \times 64$ | 393 | 2.5 |
| B75.32 | 1.95 | 0.078 | $32^3 \times 64$ | 456 | 2.5 |
| B85.24 | 1.95 | 0.078 | $24^3 \times 48$ | 491 | 1.9 |
| A30.32 | 1.90 | 0.086 | $32^3 \times 64$ | 283 | 2.8 |
| A40.32 | 1.90 | 0.086 | $32^3 \times 64$ | 323 | 2.8 |
| A50.32 | 1.90 | 0.086 | $32^3 \times 64$ | 361 | 2.8 |

# First $N_f = 2$ configurations at the physical point

More details: Talk by Bartosz Kostrzewa, Monday, 16:50

- again use Iwasaki action in gauge sector
- add clover-term to twisted mass action for non-degenerate fermion doublet

$$S_F[\chi, \overline{\chi}, U] = \sum_x \overline{\chi}(x) \left[D_W + m_0 + i\mu_q\gamma_5\tau^3\right] \chi(x) + c_{\mathrm{SW}} \sum_x \overline{\chi}(x) \left[\frac{i}{4}\sigma_{\mu\nu}\mathcal{F}_{\mu\nu}\right] \chi(x)$$

- very preliminary parameters of first ensemble:

| $\beta$ | $c_{\mathrm{SW}}$ | $a[\mathrm{fm}]$ | $L^3 \times T$ | $m_{PS}$[MeV] | $L$[fm] |
|---|---|---|---|---|---|
| 2.10 | 1.57551 | 0.096 | $48^3 \times 96$ | 128 | 4.6 |

# How the observables are determined

- use conserved (point-split) vector current

$$J^C_\mu(x) = \frac{1}{2}\Big(\overline{\chi}(x+\hat{\mu})(\mathbb{1}+\gamma_\mu)U^\dagger_\mu(x)Q_{\mathrm{el}}\chi(x) - \overline{\chi}(x)(\mathbb{1}-\gamma_\mu)U_\mu(x)Q_{\mathrm{el}}\chi(x+\hat{\mu})\Big)$$

  where $Q_{\mathrm{el}} = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3})$
- use redefinition [Feng, Jansen, Petschlies, Renner, PRL 107, 2011]

$$a^{\mathrm{hvp}}_{\overline{\mu}} = \alpha^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{H^2}\frac{H^2_{\mathrm{phys}}}{m^2_\mu}\right)\Pi_{\mathrm{R}}(Q^2)$$

  which goes to $a^{\mathrm{hvp}}_\mu$ for $m_{PS} \to m_\pi$, i.e. when $H \to H_{\mathrm{phys}}$
- effectively, redefinition of muon mass

$$m_{\overline{\mu}} = m_\mu \cdot \frac{H}{H_{\mathrm{phys}}}$$

- in the following will always use $H = m_V$ - $\rho$-meson mass

# Fitting the hadronic vacuum polarisation function

- have $\Pi(\hat{Q}^2)$ depending on discrete momenta
- to obtain smooth function fit this for each flavour to

$$\Pi(Q^2) = (1 - \theta(Q^2 - Q^2_{\mathrm{match}}))\Pi_{\mathrm{low}}(Q^2) + \theta(Q^2 - Q^2_{\mathrm{match}})\Pi_{\mathrm{high}}(Q^2)$$

with

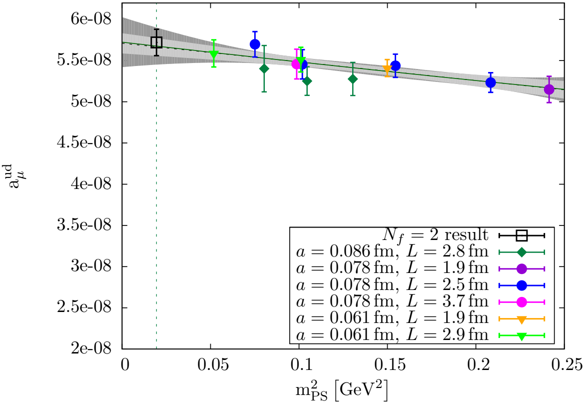
$$\Pi_{\mathrm{low}}(Q^2) = \sum_{i=1}^{M} g_i^2 \frac{m_i^2}{Q^2 + m_i^2} + \sum_{j=0}^{N-1} a_j (Q^2)^j$$

and

$$\Pi_{\mathrm{high}}(Q^2) = \sum_{k=0}^{C-1} c_k (Q^2)^k + \left( \sum_{l=0}^{B-1} b_l (Q^2)^l \right) \cdot \log(Q^2)$$

- different matching conditions and functions possible
- Padé approximants [Aubin, Blum, Golterman, Peris, Phys.Rev. D86, 2012] under investigation

# Light quark contribution on $N_f = 2 + 1 + 1$ sea



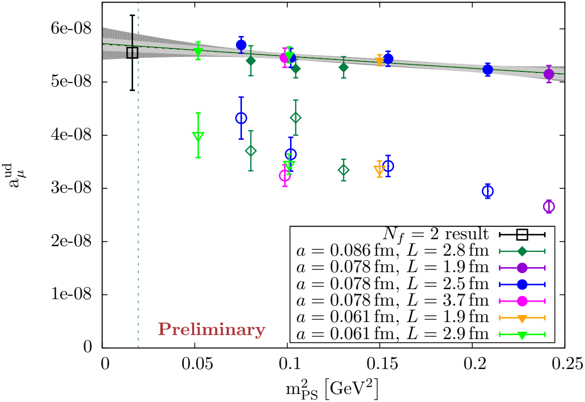

$H = m_V$

- $N_f = 2 + 1 + 1$ result: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.67(11) \cdot 10^{-8}$
- $N_f = 2$ result: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.72(16) \cdot 10^{-8}$

[Feng, Jansen, Petschlies, Renner, PRL 107, 2011]

# Light quark contribution on $N_f = 2+1+1$ sea

## Comparing to preliminary $N_f = 2$ result at physical pion mass



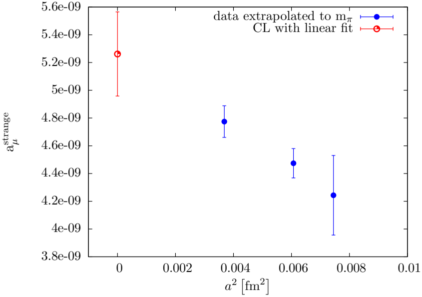

$H = m_V$

$H = 1$

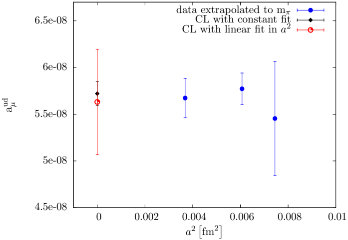
- $N_f = 2+1+1$ result: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.67(11) \cdot 10^{-8}$
- new $N_f = 2$ result at physical point: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.55(70) \cdot 10^{-8}$

# Adding the strange quark in the valence sector

- for strange quark lattice artefacts have to be taken into account




light sector: cannot discriminate $a^2$ effects

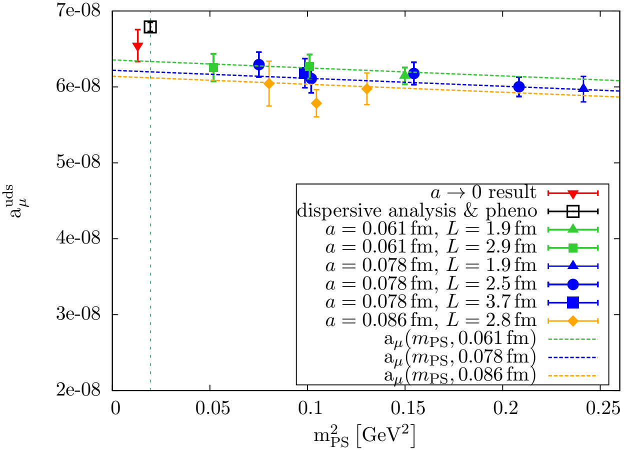



- for combined chiral and continuum extrapolation will use

$$a_{\mu,\mathrm{uds}}^{\mathrm{hvp}}(m_{\mathrm{PS}}, a) = A + B\, m_{\mathrm{PS}}^2 + C\, a^2$$

with fit parameters $A$, $B$, $C$

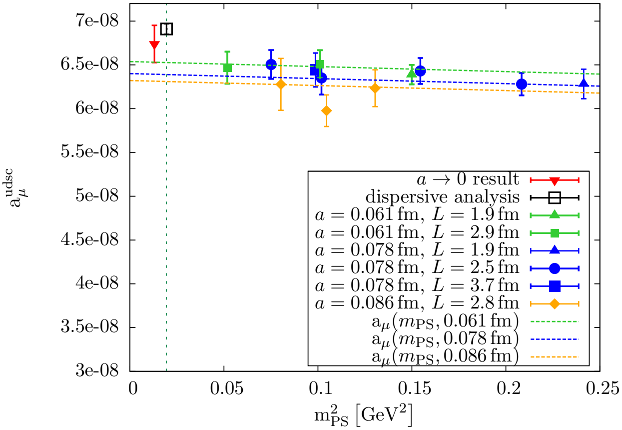
# Three-flavour contribution on $N_f = 2+1+1$ sea




$H = m_V$

- $N_f = 2+1+1$ result: $a_{\mu,\text{uds}}^{\text{hvp}} = 6.55(21) \cdot 10^{-8}$
- three-flavour result extracted from dispersive analysis of [Jegerlehner, Szafron, Eur. Phys. J C71, 2011]: $a_{\mu,\text{uds}}^{\text{hvp}} = 6.79(05) \cdot 10^{-8}$

# The four-flavour contribution on $N_f = 2 + 1 + 1$ sea




$H = m_V$

- $N_f = 2 + 1 + 1$ result: $a_\mu^{\mathrm{hvp}} = 6.74(21) \cdot 10^{-8}$
- result from dispersive analysis: [Jegerlehner, Szafron, Eur. Phys. J C71, 2011]
  $a_\mu^{\mathrm{hvp}} = 6.91(05) \cdot 10^{-8}$

# Systematic uncertainties

- from choosing fit ranges for vector mesons:

$$\Delta_V = 0.13 \cdot 10^{-8}$$

- from choosing different number of terms in fit function:

$$\Delta_{MNBC} = 0.12 \cdot 10^{-8}$$

- found to be negligible: $m_{PS}L > 3.8$, $m_{PS} < 400\,\mathrm{MeV}$, varying matching momentum between $[1\,\mathrm{GeV}^2, 3\,\mathrm{GeV}^2]$
- not quantified yet: disconnected contributions, wrong sea quark masses
- preliminary final result:

$$a_\mu^{\mathrm{hvp}} = 6.74(21)(18) \cdot 10^{-8}$$

# Comparison of results for $a_\mu^{\mathrm{hvp}}$

- $a_\mu^{\mathrm{hvp}}$ in $10^{-8}$ for different numbers of valence quarks:

| | u,d | u, d, s | u, d, s, c |
|---|---|---|---|
| this work | 5.67(11) | 6.55(21) | 6.74(27) |
| ETMC 2011 | 5.72(16) | - | - |
| Mainz 2011 | 5.46(66) | 6.18(64) | - |
| RBC-UKQCD 2011 | - | 6.41(33) | - |
| HLS estimate 2011 (dispersive analysis + flavour weighting) | 5.59(04) | 6.71(05) | 6.83(05) |

ETMC 2011: [Feng, Jansen, Petschlies, Renner, PRL 107, 2011]

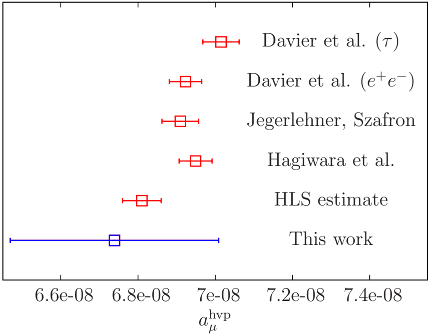
Mainz 2011: [Della Morte, Jäger, Jüttner, Wittig, JHEP 1203, 2012]

RBC-UKQCD 2011: [Boyle, Del Debbio, Kerrane, Zanotti, Phys. Rev. D85, 2012]

HLS estimate 2011: [Benayoun, David, DelBuono, Jegerlehner, Eur. Phys. J. C72, 2012]

# Summary

- First $N_f = 2 + 1 + 1$ lattice calculation of $a_\mu^{\mathrm{hvp}}$ gives compatible result with dispersive analyses.




- Modified method [Feng, Jansen, Petschlies, Renner, PRL 107, 2011] works for $N_f = 2 + 1 + 1$ computation.
- Chiral extrapolation in light sector to be checked with computation at physical pion mass.

# Outlook

- improve data by more statistics, especially in heavy sector, and all-mode-averaging
- use Padé approximants
- disconnected contributions
- more $N_f = 2$ configurations at physical point
- $N_f = 2 + 1 + 1$ configurations at physical point, probably several lattice spacings needed
- include isospin breaking effects
- different observables: $\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}$, Adler function, weak mixing angle, S-parameter
- light-by-light scattering

# Why it works

- redefinition [Feng, Jansen, Petschlies, Renner, PRL 107, 2011]

$$a_{\bar{\mu}}^{\mathrm{hvp}} = \alpha_0^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{H^2}\frac{H_{\mathrm{phys}}^2}{m_\mu^2}\right) \Pi_{\mathrm{R}}(Q^2)$$

which goes to $a_\mu^{\mathrm{hvp}}$ for $m_{PS} \to m_\pi$, i.e. when $H \to H_{\mathrm{phys}}$

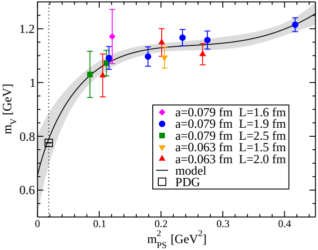
- effectively, redefinition of muon mass

$$m_{\bar{\mu}} = m_\mu \cdot \frac{H}{H_{\mathrm{phys}}}$$

- leading vector meson contribution

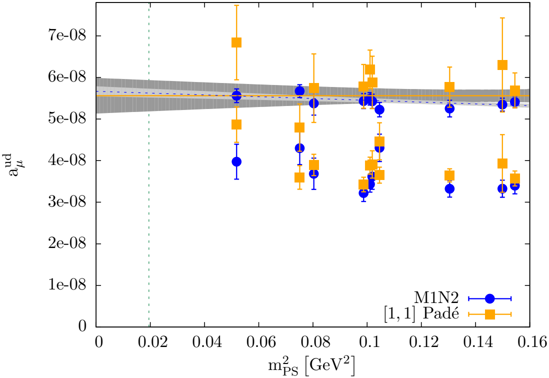
$$a_\mu^{\mathrm{hvp}} \propto \alpha^2 g_V^2 \frac{m_\mu^2}{m_V^2}$$

$\Rightarrow$ strong dependence on $m_{PS}$ via $m_V$



# Comparison with Padé approximants

- Padé approximants model-independent, systematically improvable way of fitting vacuum polarisation [Aubin et al., Phys.Rev. D86, 2012]
- vacuum polarisation integrated up to $Q^2_{\mathrm{max}} = 1.5\,\mathrm{GeV}^2$:




$H = m_V$

$H = 1$

- will use Padé approximants when chiral extrapolation no longer needed

# Example for standard fit