Tests of the vacuum polarization fits for muon $g-2$

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Previous Work

- T. Blum, Phys. Rev. Lett. **91**, 052001 (2003)
- C. Aubin and T. Blum, Phys. Rev. D **75**, 114502 (2007)
- X. Feng, K. Jansen, M. Petschlies and D. B. Renner, Phys. Rev. Lett. **107**, 081802 (2011)
- P. Boyle, L. Del Debbio, E. Kerrane and J. Zanotti, Phys. Rev. D **85**, 074504 (2012)
- M. Della Morte, B. Jager, A. Juttner and H. Wittig, JHEP **1203**, 055 (2012)
- G. M. de Divitiis, R. Petronzio and N. Tantalo, Phys. Lett. B **718**, 589 (2012)
- X. Feng, S. Hashimoto, G. Hotzel, K. Jansen, M. Petschlies and D. B. Renner, arXiv:1305.5878 [hep-lat].
- A. Francis, B. Jaeger, H. B. Meyer and H. Wittig, arXiv:1306.2532 [hep-lat].
- etc, etc... (Apologies to all those I missed)

"IN GOD WE TRUST,

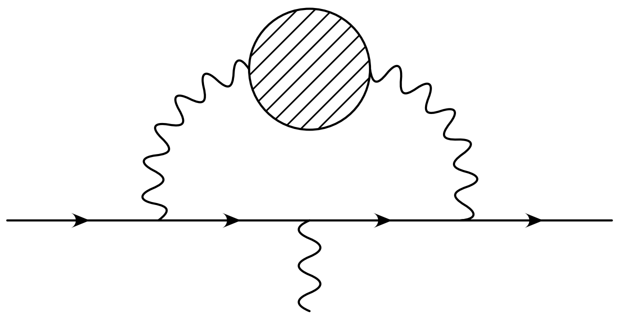
ALL THE OTHERS MUST BRING DATA."

(W. Edwards Deming, American Statistician, 1900-1993.)

Introduction

Lautrup-de Rafael '69

Blum '02



$$(g-2)_\mu^{HVP} \sim \int_0^\infty dQ^2 \underbrace{f(Q^2)}_{\text{known}} \left[\Pi(Q^2) - \Pi(0)\right]$$

$$\Pi(Q^2) - \Pi(0) \Longrightarrow (g-2)_\mu^{HVP}$$

integrand strongly peaked at $Q^2 \sim m_\mu^2/4$ $\sim 0.003\ \mathrm{GeV}^2$

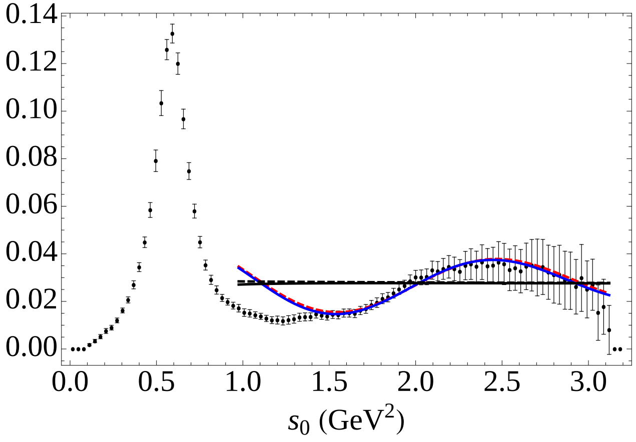
(g-2) integrand

if no good data in region of curvature $\Longrightarrow$ possibly wrong results !

(even with good χ^2)

how to test this theoretical error?

A τ-based model for $I = 1$ contributions



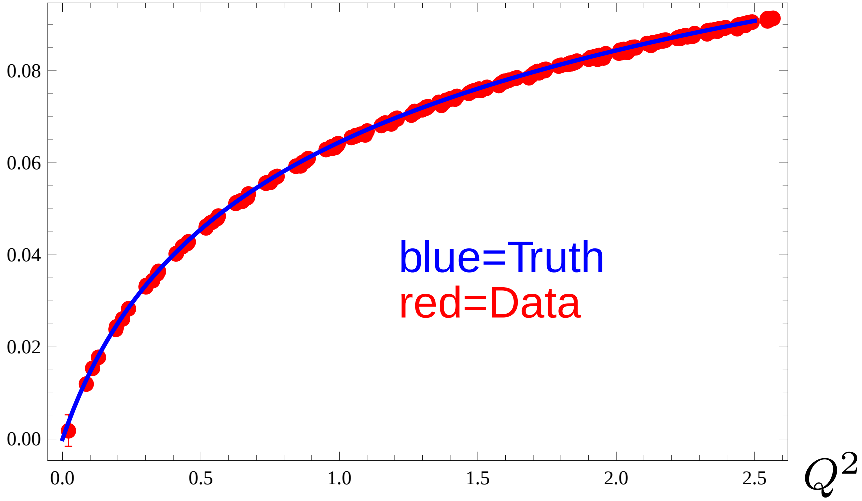
Boito, Cata, Golterman, Jamin, Mahdavi, Maltman, Osborne, SP '11 + '12

$$\mathrm{Im}\Pi \Longrightarrow \Pi(Q^2) - \Pi(0) \Longrightarrow (g-2)_\mu^{HVP}$$

Goal: test fitting ansatze accuracy in lattice determinations

- Take typical lattice Q^2 values + lattice Covariance matrix (e.g., Aubin et al. '12, $64^3 \times 144$ lattice, a = 0.06 fm, periodic BCs).
$\Longrightarrow$ generate fake lattice data for $\Pi(Q^2) - \Pi(0)$ and compare with true answer from model

$\Pi(Q^2) - \Pi(0)$



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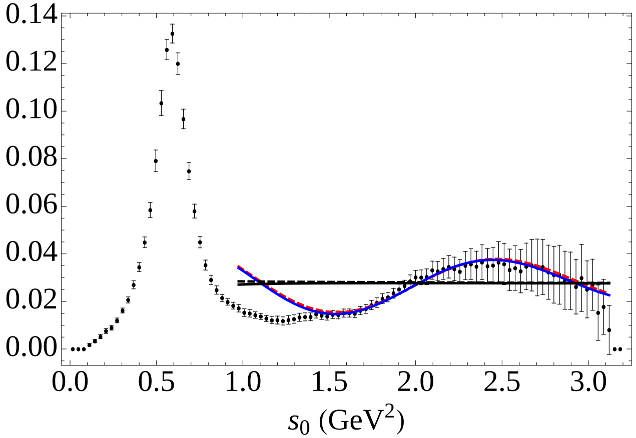

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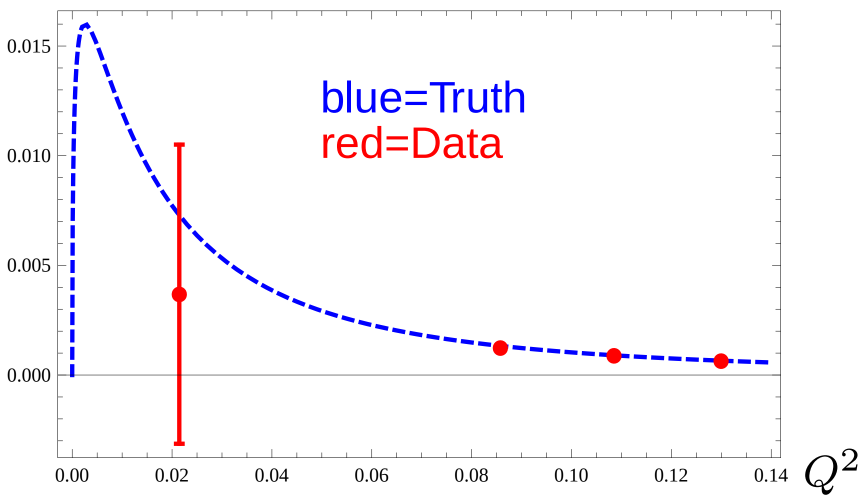
$(g-2)$ integrand



Fitting functions

Aubin, Blum, Golterman, SP '12

★ Padés, model independent, they enjoy a convergence theorem for $N \to \infty$:

$$\Pi(Q^2) = \Pi(0) + Q^2 \underbrace{\left(a_0 + \sum_{r=1}^{N} \frac{a_r}{Q^2 + b_r} \right)}_{\text{Pade}}$$

$\Pi(0)$, $a's$ and $b's$ are fitting parameters.

★ VMD is not a Pade, since you fix $b_1 = M_\rho^2$. (true $\Pi(Q^2)$ has cut starting at $4m_\pi^2$...)

We have: $a_0 \neq 0 \Longrightarrow [N, N]$ Pade; $a_0 = 0 \Longrightarrow [N-1, N]$ Pade.

For instance:

- $\frac{a_1}{Q^2+b_1}$ is a [0,1] Pade $\Longrightarrow \Pi(Q^2) = \Pi(0) + Q^2 \left(\frac{a_1}{Q^2+b_1} \right)$
- $a_0 + \frac{a_1}{Q^2+b_1}$ is a [1,1] Pade $\Longrightarrow \Pi(Q^2) = \Pi(0) + Q^2 \left(a_0 + \frac{a_1}{Q^2+b_1} \right)$

etc...

Model Fits

Golterman, Maltman, SP , work in progress

"Exact result": $(g-2)_{\mu}^{HVP}|_{Q^2 \leq 1\ \mathrm{GeV}^2} = 1.2059 \times 10^7$.

Fit interval $0 < Q^2 \leq 1$ GeV^2 , (49 points).

Pull = (exact - fit) / error

	$(g-2)_{\mu} \times 10^7$	Error$\times 10^7$	χ^2/dof	Pull
VMD			2189/47 ×	
VMD+			67.4/46	
$[0,1]$			285/46	
$[1,1]$			61.4/45	
$[1,2]$			55.0/44	
$[2,2]$			54.6/43	

VMD "flavors" :

- VMD: [0,1] Pade with $b_1 = M_{\rho}^2$.
- VMD+: [1,1] Pade with $b_1 = M_{\rho}^2$ (i.e. VMD + linear polynomial)

Model Fits

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	$(g-2)_\mu \times 10^7$	Error$\times 10^7$	χ^2/dof	Pull
VMD	1.3225	0.0052	2189/47	-
VMD+			67.4/46	
$[0,1]$			285/46	
$[1,1]$			61.4/45	
$[1,2]$			55.0/44	
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VMD "flavors" :

- VMD has a bad χ^2 and $(g-2)$.

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	$(g-2)_\mu \times 10^7$	Error$\times 10^7$	χ^2/dof	Pull
VMD	1.3225	0.0052	2189/47	-
VMD+	1.0678	0.0076	67.4/46	18
$[0,1]$			285/46	
$[1,1]$			61.4/45	
$[1,2]$			55.0/44	
$[2,2]$			54.6/43	

- VMD has a bad χ^2 and $(g-2)$.
- VMD+ also gets it wrong although the χ^2 is good $\Longrightarrow$ DANGER !

Model Fits

Golterman, Maltman, SP , work in progress

"Exact result": $(g-2)_{\mu}^{HVP}|_{Q^2 \leq 1 \text{ GeV}^2} = 1.2059 \times 10^7$.

Fit interval $0 < Q^2 \leq 1$ GeV^2 , (49 points).

Pull = (exact - fit) / error

	$(g-2)_{\mu} \times 10^7$	Error$\times 10^7$	χ^2/dof	Pull
VMD	1.3225	0.0052	2189/47	-
VMD+	1.0678	0.0076	67.4/46	18
$[0, 1]$	0.87191	0.0093	285/46	-
$[1, 1]$	1.1176	0.022	61.4/45	4
$[1, 2]$	1.1838	0.043	55.0/44	0.5
$[2, 2]$	1.1785	0.057	54.6/43	0.5

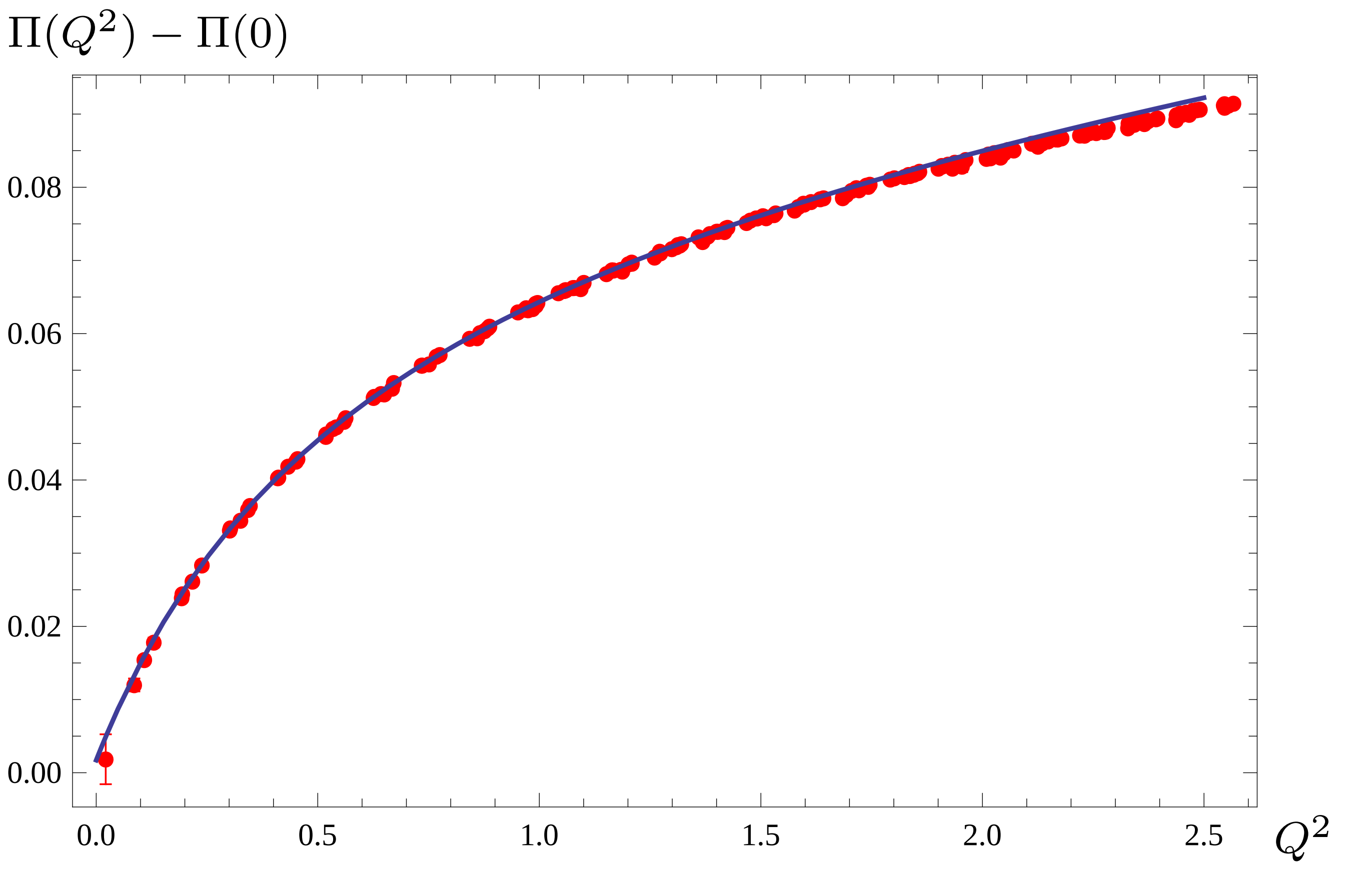
- VMD has a bad χ^2 and $(g-2)$.
- VMD+ also gets it wrong although the χ^2 is good $\Longrightarrow$ DANGER !
- Pades [1,2] and [2,2] get it right, but the error is $\sim 4\%$.

Moral

Take, e.g., the VMD+ case

You may think this is a good fit for an accurate $(g-2)_\mu$:

solid line=VMD+ Fit

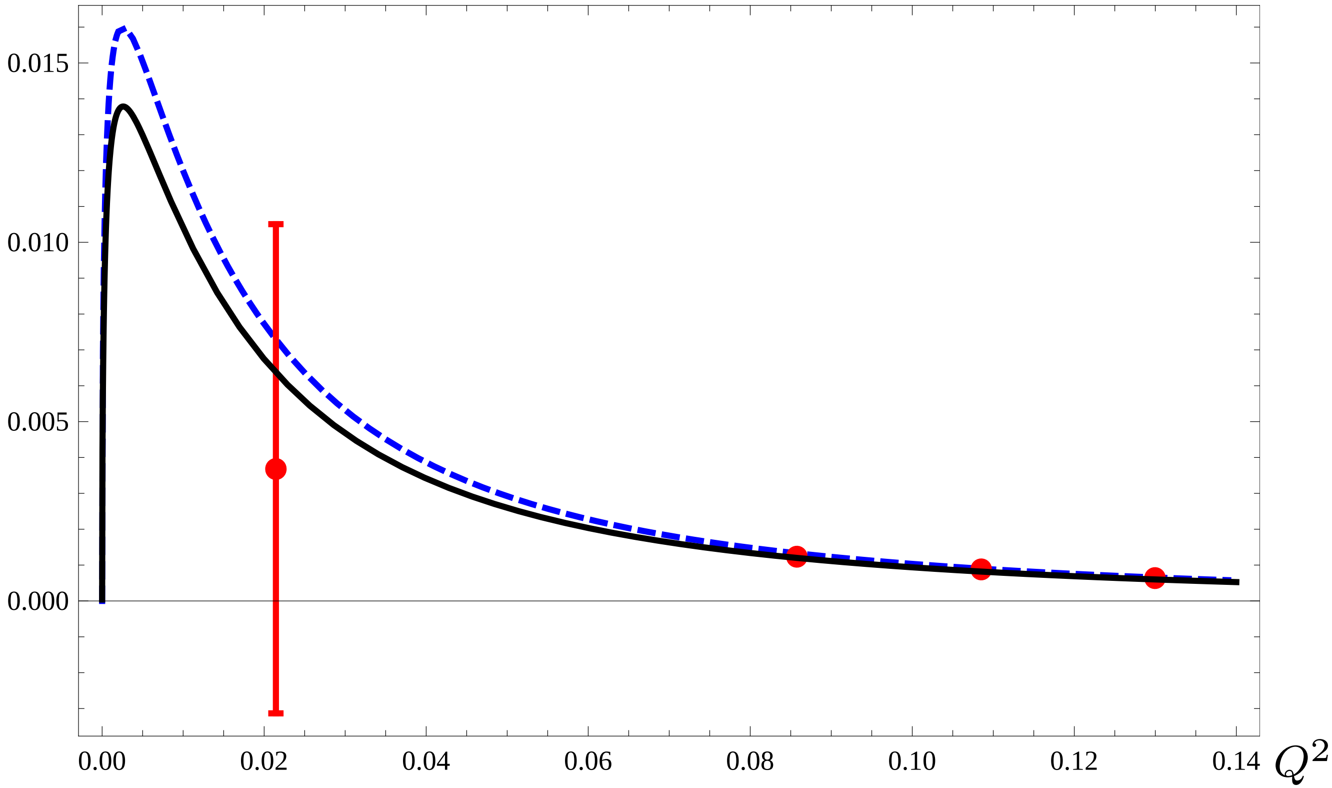


Moral

while, in fact, this is what you should be looking at:

dashed blue=Truth
solid line=VMD+ Fit

(g-2) Integrand



Science Fiction

(Recall exact value: $(g-2)_\mu^{HVP}|_{Q^2\leq 1\ \mathrm{GeV}^2} = 1.2059 \times 10^7$.)

Reduce the previous covariance matrix by 10^4:

	$(g-2)_\mu \times 10^7$	Error$\times 10^7$	χ^2/dof	Pull
VMD	1.3210	0.00005	$2 \times 10^7/47$	-
VMD+	1.0732	0.00008	$7 \times 10^4/46$	-
$[0,1]$	0.89774	0.00010	$2 \times 10^7/46$	-
$[1,1]$	1.1011	0.0002	$5 \times 10^4/45$	-
$[1,2]$	1.1644	0.0004	$1340/44$	-
$[2,2]$	1.1884	0.0015	$76.4/43$ (?)	12
$[2,3]$	1.1987	0.0028	$42.0/42$	2.6

- VMD-type fits are very bad.
- Pade fits are eventually better, but need good data around the peak for χ^2 errors to be reliable.
- $[2,3]$ reaches error comparable to present e^+e^- and τ-data based determination.
- To reduce "Pull", need twisting (see Aubin later in this session) or larger volumes to have very good data in the region of curvature *of the integrand.* See also the strategies in de Divitiis et al. '13 and Feng et al. '13.

Uncorrelated Fits

(Recall exact value: $(g-2)_\mu^{HVP}|_{Q^2\leq 1\ \mathrm{GeV}^2} = 1.2059\times 10^7$.)

	$(g-2)_\mu \times 10^7$	Error$\times 10^7$	$\mathcal{Q}^2/\mathrm{dof}$	Pull
VMD	1.2145	0.0082	$75.2/47$	-1.1
VMD+	1.085	0.017	$15.0/46$	7
$[0,1]$	0.999	0.023	20.1/46	9
$[1,1]$	1.17	0.074	13.8/45	0.4
$[1,2]$	1.30	0.32	13.6/44	-0.3

- “Error" column is the result of linear propagation

Conclusions

- VMD-type fits turn out to be not reliable for an accuracy in $(g-2)_\mu$ of few per cent.
- Do not necessarily trust the χ^2 of your fit for assessing the accuracy in $(g-2)_\mu$, if you don't have very good data in the region of curvature of the integrand.
- Blow up the region of the integrand around $Q^2 \sim m_\mu^2$. Showing plots of $\Pi(Q^2)$ for large Q^2 ranges is very misleading.
- Benchmark your fitting method:

  Should try this exercise on your Q^2 values and Cov. matrix to get a good check on your systematic error. (If you are interested, we can try to help.)
- Getting $(g-2)_\mu$ with accuracy of $\lesssim 1\%$ won't be a rose garden.

BACK-UP SLIDES