A determination of the average up-down, strange and charm quark masses at $N_f = 2+1+1$

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Outline

- Simulation details
- Plateaux
- $m_{u/d}, m_s, m_s/m_{u/d}, m_u/m_d$
- $m_c, m_c/m_s$
- Summary of the results

Simulation details

- Twisted Mass action at maximal twist at $N_f = 2 + 1 + 1$
- Osterwalder-Seiler valence quark action
- Iwasaki gluonic action

PRA027
"QCD simulations for flavor physics in the Standard Model and beyond" (35 milions of core-hours at the BG/P system in Julich from December 2010 to March 2011)

- Three values of the lattice spacing (approximately $0.06fm$,$0.08fm$ and $0.09fm$).
- Pion masses in the range $210 - 440MeV$.

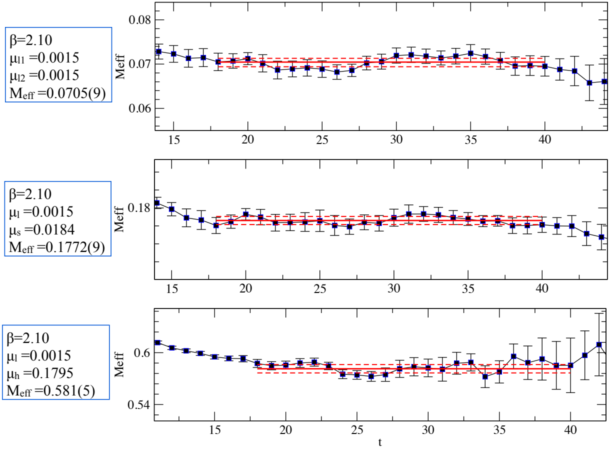
Simulation details (2)

β	V	$a\mu_{sea}$	N_{cfg}	$a\mu_s$	$a\mu_c$
1.90	$32^3 \times 64$	0.0030	150	0.0145, 0.0185, 0.0225	0.1800, 0.2200, 0.2600, 0.3000, 0.3600, 0.4400
		0.0040	90		
		0.0050	150		
	$24^3 \times 48$	0.0040	150		
		0.0060	150		
		0.0080	150		
		0.0100	150		
1.95	$32^3 \times 64$	0.0025	150	0.0141, 0.0180, 0.0219	0.1750, 0.2140, 0.2530, 0.2920, 0.3510, 0.4290
		0.0035	150		
		0.0055	150		
		0.0075	75		
	$24^3 \times 48$	0.0085	150		
2.10	$48^3 \times 96$	0.0015	60	0.0118, 0.0151, 0.0184	0.1470, 0.1795, 0.2120, 0.2450, 0.2945, 0.3595
		0.0020	90		
		0.0030	90		

β	$Z_P^{\overline{MS}}(2\,\text{GeV})(M_1)$	$Z_P^{\overline{MS}}(2\,\text{GeV})(M_2)$	r_0/a	$a(fm)$
1.90	0.521(7)	0.564(6)	5.31(8)	0.0885(36)
1.95	0.506(4)	0.537(4)	5.77(6)	0.0815(30)
2.10	0.513(3)	0.540(2)	7.60(8)	0.0619(18)

β	$L(fm)$	M_π(MeV)	$M_\pi L$
1.90	2.84	245.41	3.53
		282.13	4.06
		314.43	4.53
1.90	2.13	282.13	3.05
		343.68	3.71
		396.04	4.27
		442.99	4.78
1.95	2.61	238.67	3.16
		280.95	3.72
		350.12	4.64
		408.13	5.41
1.95	1.96	434.63	4.32
2.10	2.97	211.18	3.19
		242.80	3.66
		295.55	4.46

Plateaux in m_{eff} as a function of time



M_π^2 and M_K^2 analysis

Analysis of the dependence of M_π^2 and M_K^2 on m_l and a^2

- Discretization effects: data in units of either r_0 or $M_{\langle ss \rangle}$

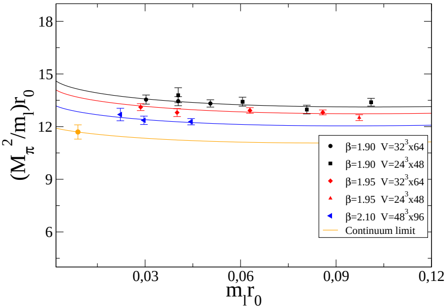
$M_{\langle ss \rangle}$: meson mass corresponding to a reference strange quark with mass near its physical value.

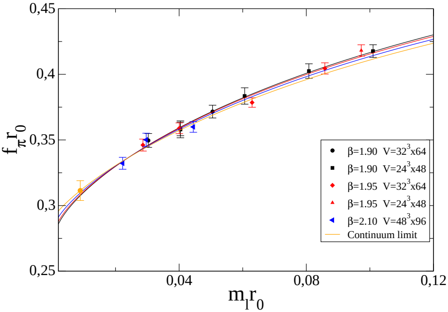
- Extrapolation to the physical point: $SU(2)$ χPT fit and polynomial fit.
- Finite Size Effects: evaluated with CWW ($SU(2)$) formulas [1] for pions and CDH ($SU(3)$) formulas [2] for kaons (see L. Riggio's talk).

[1] G. Colangelo,U. Wenger, J.M.S. Wu: arXiv:1003.0847 [hep-lat]

[2] G. Colangelo,S. Durr,C. Haefeli: Nucl.Phys.B721:136-174,2005

Example of π analysis - Chiral fit in units of r_0



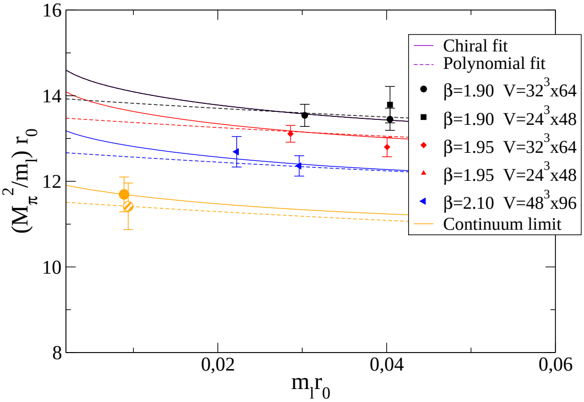


Scale setting done with $f_{\pi^+} = 130.41\,\mathrm{MeV}$

$$(\mathrm{M}_\pi r_0)^2 = 2(B_0 r_0)(m_l r_0)\left(1 + \xi_l log(\xi_l) + P_3\xi_l + P_4\frac{a^2}{r_0^2} + \frac{4C_2}{(4\pi f_0)^2}\frac{a^2}{r_0^2}log(\xi_l)\right)K_M^{FSE}$$

$$\mathrm{f}_\pi r_0 = f_0\left(1 - 2\xi_l log(\xi_l) + P_6\xi_l + P_7\frac{a^2}{r_0^2} - \frac{4C_2}{(4\pi f_0)^2}\frac{a^2}{r_0^2}log(\xi_l)\right)K_f^{FSE} \qquad \xi_l = \frac{2B_0 m_l}{(4\pi f_0)^2}$$

Comparing Chiral and Polynomial analyses



- Chiral fit $(M_\pi r_0)^2 = 2(B_0 r_0)(m_l r_0)\left(1 + \xi_l log(\xi_l) + P_3\xi_l + P_4\frac{a^2}{r_0^2} + \frac{4C_2}{(4\pi f_0)^2}\frac{a^2}{r_0^2}log(\xi_l)\right) K_M^{FSE}$
- Polynomial fit $(M_\pi r_0)^2 = 2(B_0 r_0)(m_l r_0)\left(1 + P_3\xi_l + P_4\frac{a^2}{r_0^2} + P_5\xi_l^2\right) K_M^{FSE}$

$$m_{u/d}(2\,\mathrm{GeV}) = 3.70(17)\,\mathrm{MeV}$$

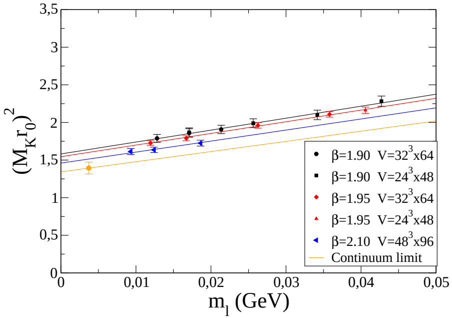
more in detail

$$m_{u/d}(2\,\mathrm{GeV}) = 3.70(13)_{stat+fit}(6)_{chir}(5)_{disc}(5)_{Z_P}(4)_{FSE}\,\mathrm{MeV}$$

Systematic errors are estimated from the spread between different analyses:

- $()_{stat+fit}$: includes statystical error and the uncertainty induced by the fitting procedure.
- $()_{chir}$: $SU(2)$ ChPT or polynomial fit.
- $()_{disc}$: analysis in units of r_0 or $M_{\langle ss\rangle}$.
- $()_{Z_P}$: different methods in calculating RI-MOM renormalization constants Z_P.
- $()_{FSE}$: CWW corrections or no corrections.

M_K^2: Chiral fit (in units of r_0)



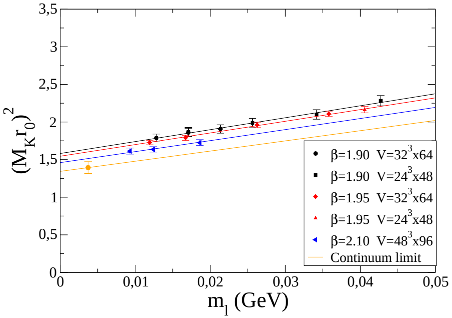
According to $SU(2)$ ChPT there is no chiral log at NLO in M_K^2

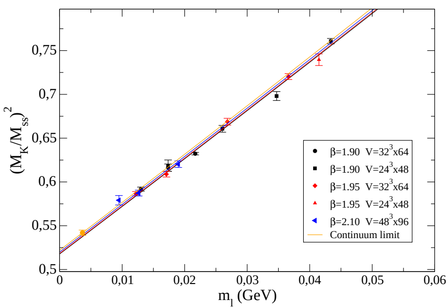
$$(M_K r_0)^2 = P_1(m_l r_0 + m_s r_0) \left(1 + P_2 \xi_l + P_3 \frac{a^2}{r_0^2}\right) K_M^{FSE}$$

All M_K^2 data interpolated to m_s so that we obtain M_K^{exp} at $a = 0$ and $m_{u/d}$.

$$(M_K^{exp})^2 = \frac{M_{K^+}^2 + M_{K^0}^2}{2} - \frac{(1+\epsilon)}{2}(M_{\pi^+}^2 - M_{\pi^0}^2) \simeq (494.4\,\text{MeV})^2 \qquad \epsilon \simeq 0.7$$

Comparing r_0 and $M_{\langle ss \rangle}$ units





- units of r_0: 9% discretization effects
- units of $M_{\langle ss \rangle}$: 0.4% discretization effects

  But they are large in the continuum extrapolation of $M_{\langle ss \rangle}$

$$m_s(2\,\mathrm{GeV}) = 99.2(4.0)\,\mathrm{MeV}$$

$$m_s(2\,\mathrm{GeV}) = 99.2(3.4)_{stat+fit}(0.6)_{chir}(1.1)_{disc}(1.5)_{Z_P}(0.5)_{FSE}\,\mathrm{MeV}$$

Systematic errors are estimated from the spread between different analyses:

- $()_{stat+fit}$: includes statystical error and the uncertainty induced by the fitting procedure.
- $()_{chir}$: $SU(2)$ ChPT or polynomial (up to $O(m_l^3)$) fit. Each fit uses the respective scale set by the Pion analysis.
- $()_{disc}$: analysis in units of r_0 or $M_{\langle ss \rangle}$.
- $()_{Z_P}$: different methods in calculating RI-MOM renormalization constants Z_P.
- $()_{FSE}$:CDH corrections or no corrections.

$m_s/m_{u/d}$ and m_u/m_d

By analyzing M_K^2 we also obtain

Indirect determination from m_s and $m_{u/d}$

$$m_s/m_{u/d} = 27.0(1.3)$$

From $M_{K^0}^2 - M_{K^+}^2$ we obtain

$$m_u/m_d = 0.49(5)$$

M_{D_s} analysis

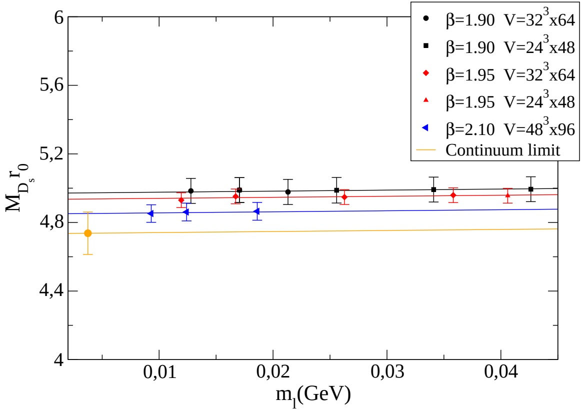
Small interpolation of M_{D_s} data to m_s and m_c and analysis of the dependence on m_l and a^2

- Discretization effects: data in units of either r_0 or $M_{\langle cs \rangle}$.

$M_{\langle cs \rangle}$: meson mass corresponding to reference strange and charm quark with masses near their physical values.

- Extrapolation to the physical point: linear and quadratic fit.

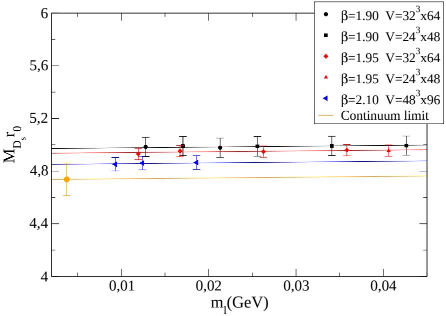
Example of M_{D_s} analysis: linear fit (in units of r_0)

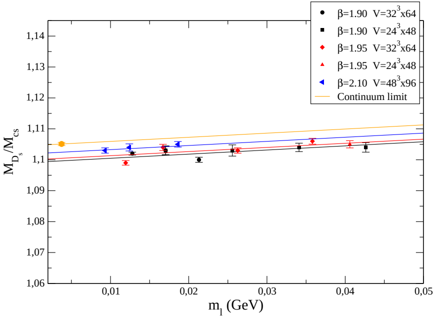


$$M_{D_s} r_0 = P_1 + P_2 m_l + P_3 \frac{a^2}{r_0^2}$$

Same technique as with the kaon: interpolation to m_s and m_c that allows us to get $M_{D_s} = 1.969\,\text{GeV}$ at the physical and continuum limit.

Comparing r_0 and $M_{\langle cs \rangle}$ units.





- units of r_0: 3% discretization effects
- units of $M_{\langle cs \rangle}$: 0.3% discretization effects

$$m_c(m_c) = 1.350(46)\,\text{GeV}$$

$$m_c(m_c) = 1.350(43)_{stat+fit+scale}(3)_{chir}(8)_{disc}(19)_{Z_P}(5)_{m_s}\,\text{GeV}$$

- $()_{stat+fit+scale}$: includes statystical errors and those induced by the fitting procedure. It also includes the uncertainty associated with the error on the scale.
- $()_{chir}$: linear or quadratic fit.
- $()_{disc}$: analysis in units of r_0 or $M_{\langle cs \rangle}$.
- $()_{Z_P}$: different methods in calculating RI-MOM renormalization constants Z_P.
- $()_{m_s}$: uncertainty associated with the error on m_s.

Indirect determination from m_c and m_s

$$\frac{m_c}{m_s} = 11.86(59)$$

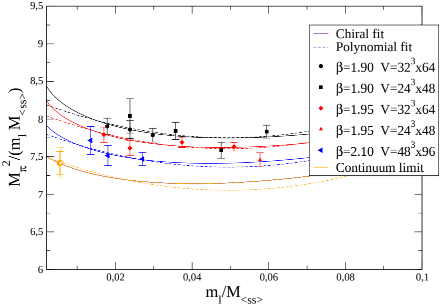
Summary

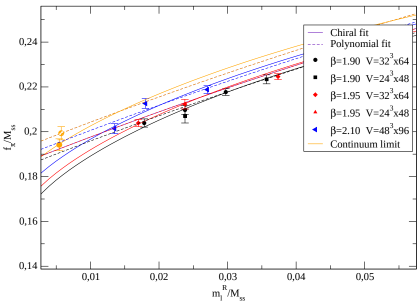
	Our results	FLAG$_{N_f=2}$	FLAG$_{N_f=2+1}$
$m_{u/d}$(MeV)	3.70(17)	3.6(2)	3.42(9)
m_s(MeV)	99.2(4.0)	101(3)	93.8(2.4)
$m_s/m_{u/d}$	27.0(1.3)	28.1(1.2)	27.5(4)
m_u/m_d	0.49(5)	0.50(4)	0.46(2)
m_c(GeV)	1.350(46)		
m_c/m_s	11.86(59)		

Thanks for the attention!

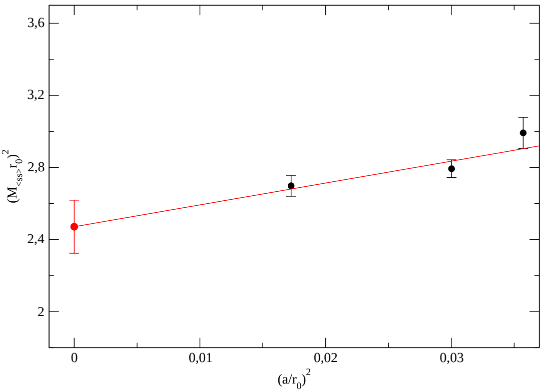
BACKUP SLIDES

π analysis: Chiral and polynomial fit (units of $M_{\langle ss\rangle}$)





Continuum extrapolation of the quantity $(aM_{\langle ss \rangle})^2$



Results of π analysis

	r_0 Analysis		$M_{<ss>}$ Analysis	
Quantity	Chiral Fit	Polynomial Fit	Chiral Fit	Polynomial Fit
m_l(MeV)	3.72(13)	3.87(17)	3.66(10)	3.75(13)
$r_0(GeV^{-1})$	2.39(6)	2.42(7)	-	-
$r_0(fm)$	0.470(12)	0.477(14)	-	-
$M_{ss}(GeV)$	-	-	0.672(9)	0.654(10)
a($\beta = 1.90$)(fm)	0.0886(27)	0.0899(31)	0.0868(33)	0.0892(34)
a($\beta = 1.95$)(fm)	0.0815(21)	0.0827(25)	0.0799(27)	0.0820(28)
a($\beta = 2.10$)(fm)	0.0619(11)	0.0628(13)	0.0607(14)	0.0623(15)
B_0(MeV)	2515(90)	-	2551(73)	-
f_0(MeV)	121.1(2)	-	121.3(2)	-
$\bar{l}_3$	3.24(25)	-	2.94(20)	-
$\bar{l}_4$	4.69(10)	-	4.65(8)	-

Quantity	No Correction	GL	CDH	CWW
m_l(MeV)	3.68(14)	3.76(14)	3.73(13)	3.72(13)
$r_0(fm)$	0.464(12)	0.466(12)	0.468(12)	0.470(12)
$\bar{l}_3$	3.42(20)	3.35(20)	3.34(21)	3.24(25)
$\bar{l}_4$	4.83(9)	4.77(9)	4.76(9)	4.69(10)
B_0(MeV)	2548(99)	2497(97)	2500(93)	2515(90)
f_0(MeV)	120.8(1)	120.9(1)	120.9(1)	121.1(2)

FSE effects in π analysis

	GL	CDH	CWW	Lattice data M_{32}/M_{24}
$FSE_{M\pi,32}/FSE_{M\pi,24}$	0.994	0.985	0.981	0.972(13)

	GL	CDH	CWW	Lattice data f_{32}/f_{24}
$FSE_{f\pi,32}/FSE_{f\pi,24}$	1.023	1.040	1.054	1.050(19)

$\Delta_{corr}/\Delta_{raw}$	GL	CDH	CWW
M_π	0.80093	0.43535	0.32101
f_π	0.51765	0.18235	−0.05882

	GL	CDH	CWW
$R_{M\pi,24}$	0.0070	0.0187	0.0243
$R_{f\pi,24}$	−0.0280	−0.0469	−0.0632

Average values of some quantities calculated from π analysis.

$$r_0 = 0.474(14)\,fm = 0.474(13)_{stat+fit}(3)_{Chiral}(6)_{FSE}\,fm$$

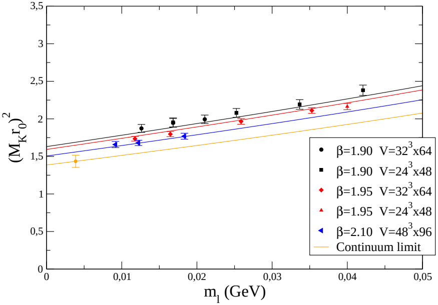
$$B_0 = 2571(97)\,MeV = 2571(80)_{stat+fit}(22)_{Disc.}(33)_{FSE}(38)_{Z_p}\,MeV$$

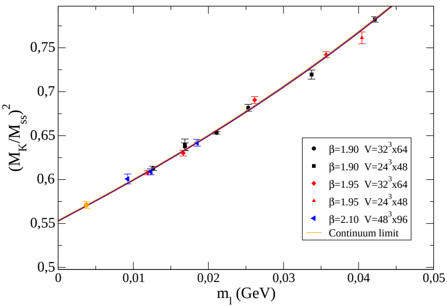
$$f_0 = 121.2(4)\,MeV = 121.2(2)_{stat+fit}(1)_{Disc.}(3)_{FSE}\,MeV$$

$$\bar{l}_3 = 3.11(34) = 3.11(23)_{stat+fit}(17)_{Disc.}(18)_{FSE}$$

$$\bar{l}_4 = 4.69(17) = 4.69(9)_{stat+fit}(2)_{Disc.}(14)_{FSE}$$

Comparing r_0 and $M_{\langle ss \rangle}$ units: Polynomial fit





Results of K analysis

	r_0 Analysis		M_{ss} Analysis	
Quantity	Chiral Fit	Polynomial Fit	Chiral Fit	Polynomial Fit
m_s(MeV)	101.6(4.4)	102.5(3.9)	99.4(2.9)	100.8(3.2)
f_K(MeV)	153.8(2.5)	154.2(1.9)	156.6(1.3)	155.1(1.6)
f_K at m_{up}(MeV)	152.3(2.6)	153.3(2.0)	155.2(1.4)	154.2(1.7)
f_K/f_π	1.179(20)	1.182(15)	1.201(09)	1.189(12)
f_K/f_π at m_{up}	1.168(20)	1.175(16)	1.190(11)	1.182(13)

Quantity	No Correction	GL	CDH
m_s(MeV)	101.1(4.4)	101.1(4.4)	101.6(4.4)
f_K(MeV)	151.8(2.6)	152.2(2.6)	152.3(2.6)
f_K/f_π	1.164(20)	1.167(20)	1.168(20)

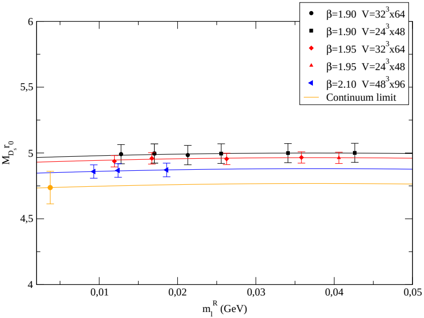
$FSE_{M_K,32}/FSE_{M_K,24}$	GL	CDH	Lattice data M_{32}/M_{24}
Fit CDH	1	0.991	0.990(7)
Fit GL	1		0.990(7)

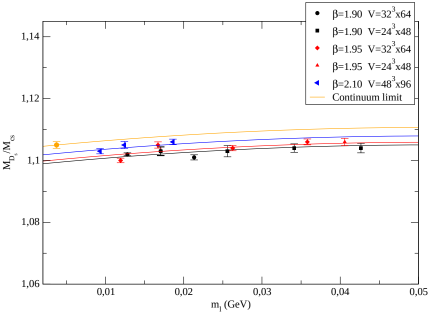
$FSE_{f_K,32}/FSE_{f_K,24}$	GL	CDH	Lattice data f_{32}/f_{24}
Fit CDH	1.007	1.028	1.020(13)
Fit GL	1.007		1.020(13)

$\Delta_{corr}/\Delta_{raw}$	GL	CDH
M_K	1	0.128
f_K	0.628	-0.414

$R_{M_K,24}$	GL	CDH
Fit GL	0	
Fit CDH	0	0.0102
$R_{f_K,24}$	GL	CDH
Fit GL	-0.009	
Fit CDH	-0.009	-0.032

Comparing r_0 and $M_{\langle cs\rangle}$ units: quadratic fit





	r_0 Analysis		M_{cs} Analysis	
	Linear Fit	Quadratic Fit	Linear Fit	Quadratic Fit
$\bar{m}_c$(GeV)	1.198(37)	1.201(37)	1.189(38)	1.190(38)