Chiral behaviour of the pion decay constant in $N_f = 2$ QCD

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ALPHA Collaboration

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Lattice 2013 - Mainz

August 2, 2013

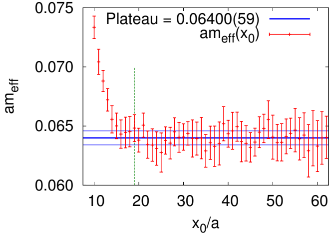
Introduction

- Two-flavour lattice QCD
  - probe for low-energy regime
  - smaller and smaller masses accessible
- Recently, new CLS ensembles with
  - low m_π
  - small lattice spacing
- This is a study of f_π:
  - chiral behaviour
  - $\rightarrow$ $SU(2)$ chiral PT low-energy constants

  (So far, ALPHA analysed the chiral properties of f_K [NPB 865, 397 (2012)])

Ensembles, statistics

- $\mathcal{O}(a)$-improved $N_f = 2$ Wilson lattice action, implementing
  - Domain-Decomposition [Lüscher 2005]
  - Mass-Preconditioning [Marinkovic, Schäfer 2010]
- New CLS ensembles with lighter pions
  - 10 to 20 stochastic sources/config. for correlators
  - increased statistics on already-existing ensembles

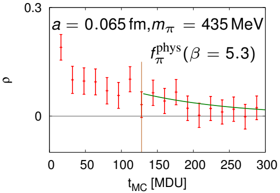


$$m_\pi = 192 \text{ MeV}$$
$$a = 0.065 \text{ fm}$$
$$\frac{L_T}{a} = 128$$

- Renormalisation factors as in [ALPHA 2012]:
  - b-factors from one-loop PT [Sint, Weisz 1997]
  - Z_A, Z_P determined nonperturbatively.

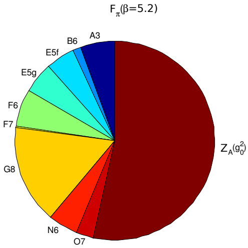
- Correlations are propagated to the final quantities
- Sophisticated error analysis includes slow-mode contributions [ALPHA 2010]:

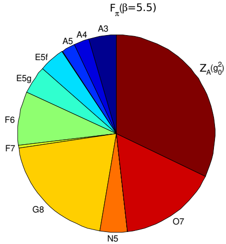
slow modes $\sim \tau_{\mathrm{exp}}$ (previously measured) $\Rightarrow$



$\Rightarrow$ final error

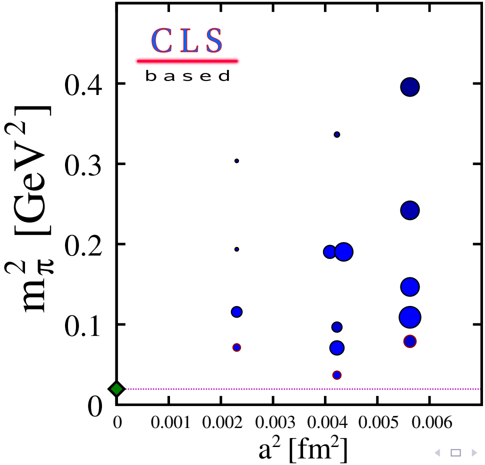
- A large contribution to the error from Z_A factor, e.g. for F_π:





Ensembles

- $m_\pi L$ range is: 4.0 – 7.7
- Lowest m_π is 192 MeV
- Largest point has $N_{\mathrm{cnfg}}/\tau_{\mathrm{exp}} = 122$
- All systems have $L_T = 2L$ and periodic b.c.



Fit generalities

- Global fits with range cut to

$$m_\pi < 650,\ 500,\ 390,\ 345\ \ \mathrm{MeV}$$

- Systematic errors from:
  - different fit functions
  - fit-cutoff dependence
  - higher orders estimate
- Variable employed:

$$\widetilde{y}_1 = \frac{m_\pi^2}{8\pi^2 f_\pi^2} \in [0.026 : 0.142] \qquad \text{(phys. point: } \widetilde{y}_\pi \simeq 0.01353)$$

f_π: $SU(2)$ Chiral perturbation theory

- Strategy: find $F_\pi^{\text{phys}}(\beta) = a(\beta) f_\pi^{\text{phys}}$ to set the scale

  (here, $f_\pi^{\text{phys}} \sim 130.4$ MeV)

- Expected chiral behaviour:

$$f_\pi(\widetilde{y}_1) = f\{1 + \alpha_4 \widetilde{y}_1 - \widetilde{y}_1 \log \widetilde{y}_1 + \ldots\}$$

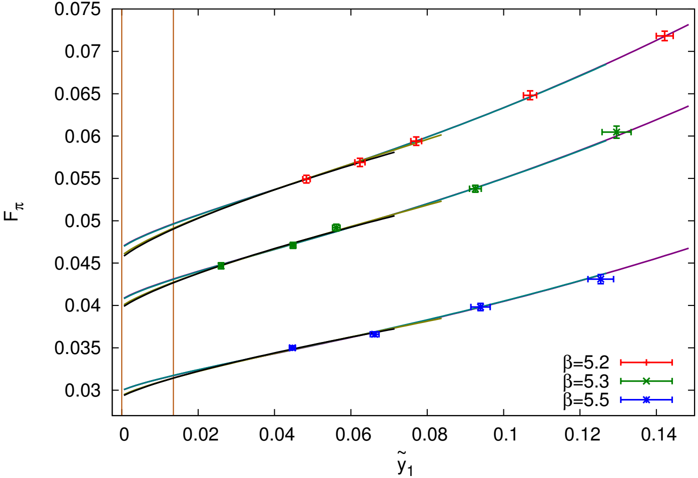
[Gasser & Leutwyler '80s]

- $\alpha_4 \to \bar{\ell}_4$, one of the low-energy constants of $SU(2)$ χPT:

$$\bar{\ell}_i \equiv \log \frac{\Lambda_i^2}{\mu^2} \qquad (\text{here } \mu \leftarrow m_\pi^{\text{phys}})$$

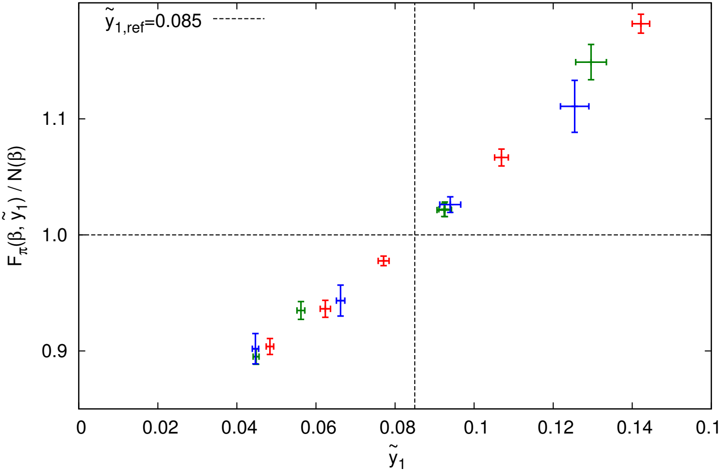
f_π, fits

$$F_\pi(\widetilde{y}_1) = F_\pi^{\mathrm{phys}}\{1 + \alpha_4(\widetilde{y}_1 - \widetilde{y}_\pi) - \widetilde{y}_1 \log \widetilde{y}_1 + \widetilde{y}_\pi \log \widetilde{y}_\pi + B(\widetilde{y}_1 - \widetilde{y}_\pi)^2\}$$



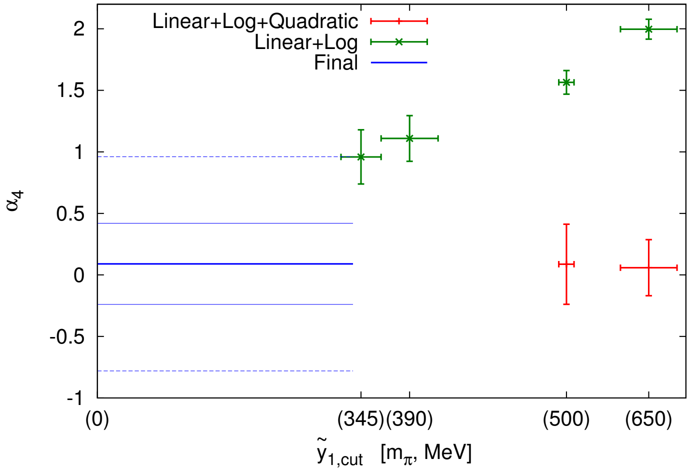
f_π, absence of am_q-effects

The three curves F_π collapse well:



f_π, fit range-dependence

The $\sim \tilde{y}_1^2$ term has a 15% effect along F_π: *not negligible*



f_π, results

- Set $F_\pi^{\mathrm{phys}} = a(\beta) f_\pi^{\mathrm{phys}}$: spacing agrees with [ALPHA 2012, f_K]

β	a [fm]	f_K, 2012	f_K, update
5.2	0.0750(9)(10)	0.0755(9)(7)	0.0751(8)
5.3	0.0652(6)(7)	0.0658(7)(7)	0.0653(6)
5.5	0.0480(5)(5)	0.0486(4)(5)	0.0483(4)

- Ratio

$$\frac{f_\pi}{f} = 1.061(6)(16)$$

- From α_4, the SU(2) low-energy constant

$$\bar{\ell}_4 = \frac{\alpha_4}{1 - \alpha_4 \widetilde{y}_\pi + \widetilde{y}_\pi \log \widetilde{y}_\pi} - \log \widetilde{y}_\pi = 4.4(4)(9)$$

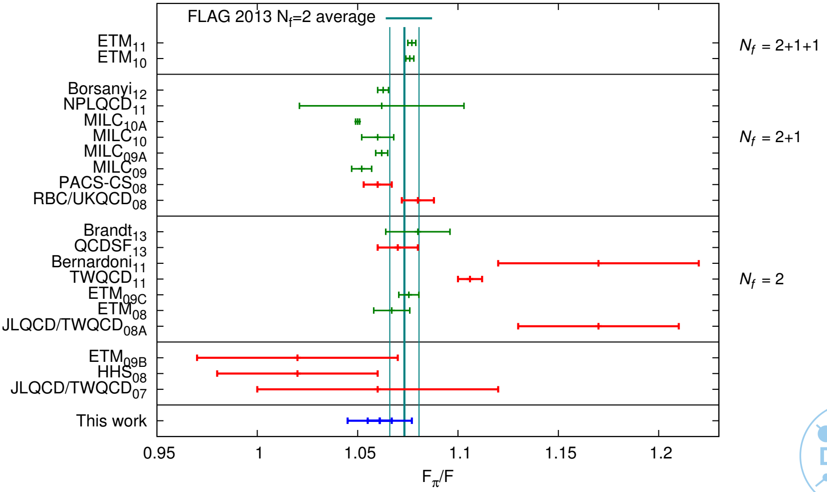
- A continuum extrapolation of $f_\pi r_0$ in a^2 gives:

$$r_0^{\mathrm{phys}} = 0.485(7)(7)\ \mathrm{fm}$$

cf. [ALPHA 2012, f_K]: $r_0^{\mathrm{phys}} = 0.503(10)$ fm

f_π, world results [from FLAG 2013]

F_π/F carries, to NLO, the same information as $\bar{\ell}_4$:



m_π^2/M_R: expectation from χPT

$$\frac{m_\pi^2}{4M_R} = \frac{\Sigma_0}{f^2}\left\{1 + \alpha_3 \widetilde{y}_1 + \frac{1}{2}\widetilde{y}_1 \log \widetilde{y}_1 + \dots\right\}$$

[Gasser & Leutwyler ’80s]

- Amplitude in front rewritten to yield

$$\sqrt[3]{S_0} = \frac{\sqrt[3]{\Sigma_{0,\mathrm{cont}}}}{f_{\pi,\mathrm{phys}}}$$

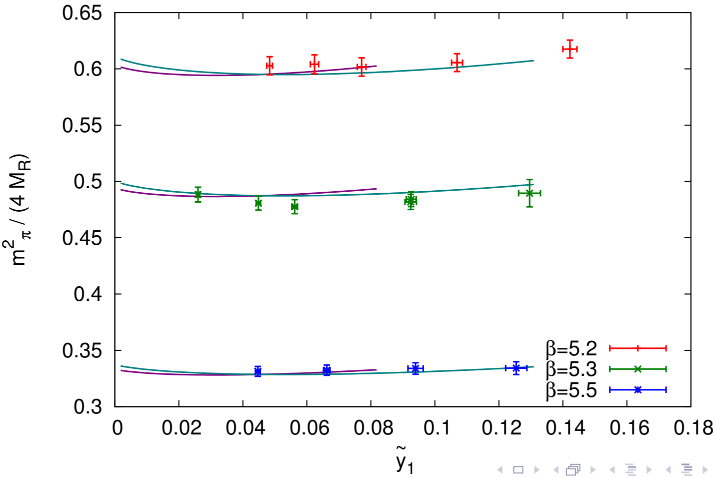
through an a^2-linear continuum limit

- α_3 encodes the LEC

$$\bar{\ell}_3 = -2\alpha_3 - \log \widetilde{y}_\pi$$

m_π^2 / M_R, fits

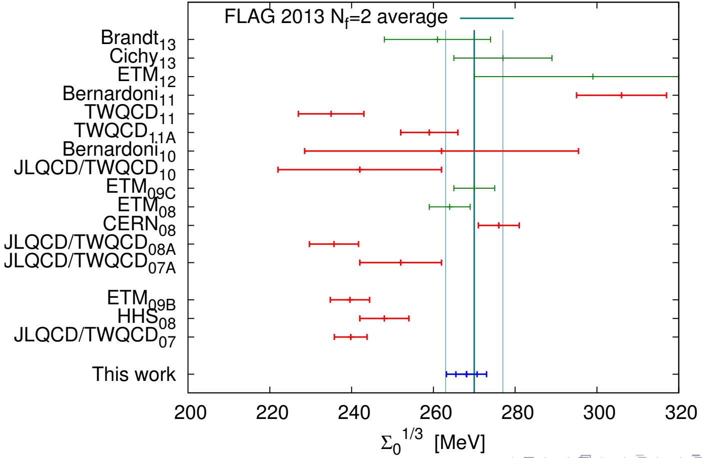
$$\frac{m_\pi^2}{M_R} = \left[(S_0 + a^2 S_1) \frac{F^3_{\pi,\mathrm{phys}}}{F^2} \right] \left\{ 1 + \alpha_3 \widetilde{y}_1 + \frac{1}{2} \widetilde{y}_1 \log \widetilde{y}_1 + B \widetilde{y}_1^2 \right\}$$



m_π^2/M_R, chiral condensate

In the $\overline{\mathrm{MS}}$ scheme at scale $\mu = 2$ GeV,

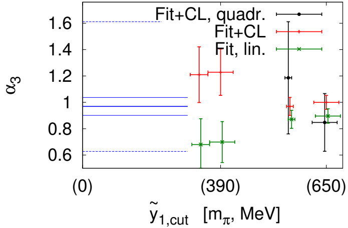
$$\Sigma_0^{1/3}\Big|_{\mathrm{cont}} = 268.1(2.6)(4.9)\ \mathrm{MeV}$$



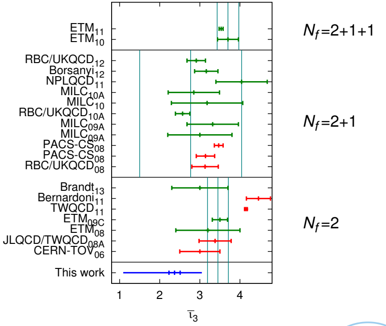
m_π^2/M_R, low-energy constant $\bar{\ell}_3$

$$\bar{\ell}_3 = 2.4(1)(^{+0.7}_{-1.3})$$

large systematic uncertainties



(systematic error assessment)

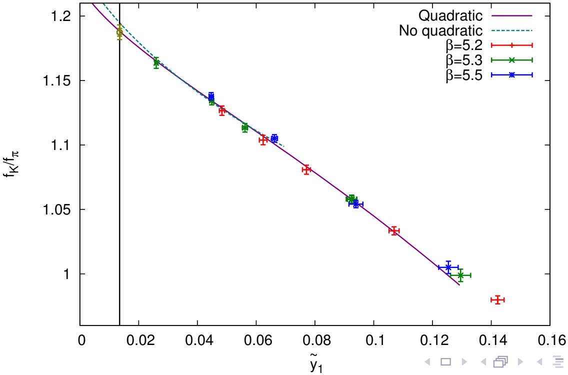


(comparison with world results)

f_K/f_π: set-up

- Trajectory in $(\kappa_1 = \kappa_{\rm sea}, \kappa_3)$ defined by: $\frac{m_K}{f_K} = \frac{m_K}{f_K}\Big|_{\rm phys}$
- Chiral behaviour dictated by that of f_K and f_π
- We also allow for a (hardly necessary) a^2-term

$$\frac{f_K}{f_\pi} = R\left\{1 + c_1\tilde{y}_1 + \frac{\tilde{y}_1}{2}\log\tilde{y}_1 - \frac{\tilde{y}_1}{8}\log\left(2\frac{\tilde{y}_K}{\tilde{y}_1} - 1\right) + c_2\tilde{y}_1^2\right\} \quad , \ \tilde{y}_K \simeq 0.182$$



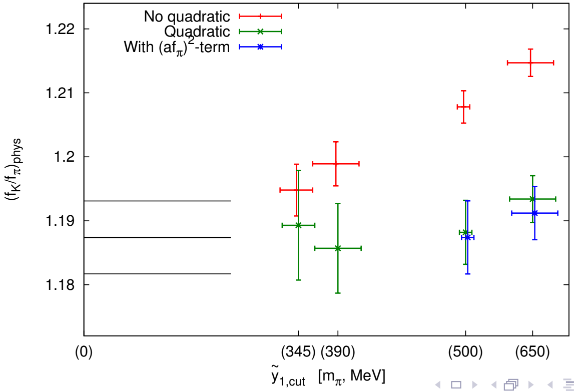
f_K / f_π, value at the physical point

Very stable value thanks to the ensemble at $m_\pi = 192$ MeV.

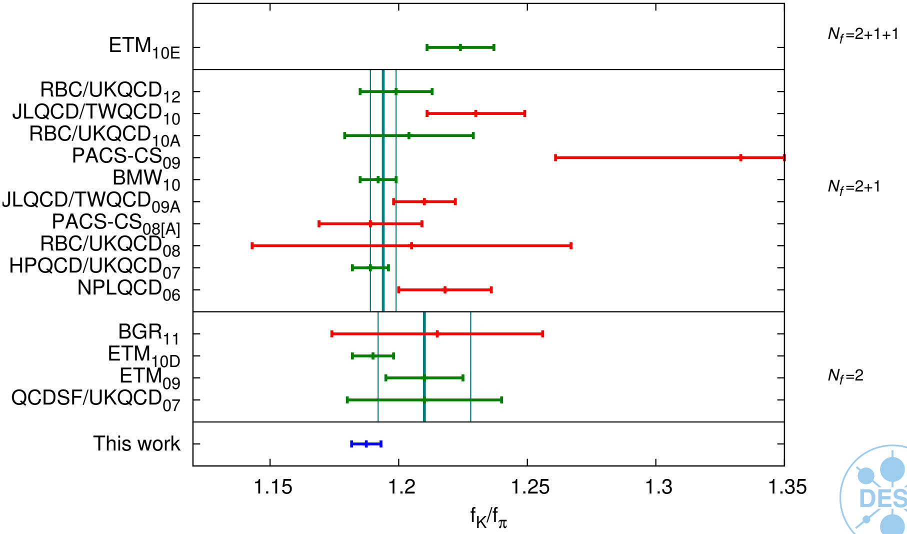
$$\left.\frac{f_K}{f_\pi}\right|_{\text{phys}} = 1.187(6)(3) \quad \Rightarrow \quad |V_{us}| = 0.2263(13)$$

Unitarity check: $|V_{ud}|^2 + |V_{us}|^2 = 1.0004(13)$

using exp. values [Antonelli & al. '10; Hardy, Towner '09]



f_K/f_π, world results



Conclusions

- Good signal on pionic quantities
  - χPT NNLO terms important (as opposed to those $\sim am_q$)
  - data at $m_\pi = 192$ MeV crucial in stabilising
- Results compare well with most other lattice results
- Future plans:
  - Different trajectory for f_K
  - Use t_0 (gradient flow scale) for less uncertainties [M. Bruno's talk]
  - ...then move on to $N_f = 2 + 1$!

f_π / f	1.061(6)(16)
$\bar{\ell}_4$	4.4(4)(9)
$\Sigma_0^{1/3}$	268.1(2.6)(4.9) MeV
$\bar{\ell}_3$	$2.4(1)(^{+0.7}_{-1.3})$
f_K / f_π	1.187(6)(13)
$\lvert V_{us} \rvert$	0.2263(12)
r_0	0.485(7)(7) fm


The end

Slow modes in autocorrelation analysis

(c f. [Schäfer, Sommer, Virotta 2010] for details)

Integrated autocorrelation time "converts naïve error into actual one":

$$\sigma^2 = \frac{2\tau_{\rm int}^0}{N}\Gamma(0) \quad ; \quad \tau_{\rm int}^0 = \frac{1}{2} + \sum_{t=1}\rho(t) \quad \text{with} \quad \rho(t) = \Gamma(t)/\Gamma(0)$$

In practice, signal on ρ soon lost: danger!

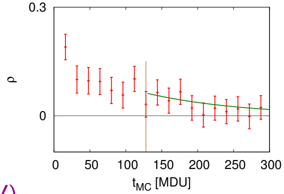
(despite trying to balance uncertainty in ρ and remainder of the tail)

Observable can couple, undetected, to a slow mode

$\Rightarrow$ underestimation of error!

Solution: where the measured ρ vanishes ($t \equiv W$), attach an exponential tail

- It represents the observable's coupling to slow modes
- Its decay rate is measured (or estimated) previously



Use rather: $\tau_{\rm int} = \tau_{\rm int}^0 + \tau_{\rm exp}\rho(W)$

Ensemble table

Tag	β, a [fm]	m_π [MeV]	$m_\pi L$	$N_{\mathrm{cnfg.}}$	$\frac{N_{\mathrm{cnfg.}}}{\tau_{\mathrm{int}}}$	$\frac{N_{\mathrm{cnfg.}}}{\tau_{\mathrm{exp}}}$	N_{src}	trj.	Alg.	r_0?
A2	5.2	629	7.7	1000	817	120.6	10	8	DD	X
A3	(0.075)	492	6.0	1004	633	121.1	10	8	DD	X
A4		383	4.7	1012	861	122.0	10	8	DD	X
A5		330	4.0	1001	1100	163.5	10	4	MP	X
B6		281	5.2	636	639	51.9	20	2	MP	X
E4	5.3	580	6.2	156	97	9.3	10	16	DD	-
E5f	(0.065)	436	4.6	1000	354	59.9	10	16	DD	-
E5g		436	4.7	1000	571	119.8	10	16	DD	X
F6		311	5.0	600	410	35.9	10	8	DD	X
F7		266	4.3	1177	569	70.5	10	8	DD	X
G8		192	4.1	557	457	22.6	20	2	MP	X
N4	5.5	551	6.5	469	30	4.2	10	8	DD	X
N5	(0.048)	440	5.2	476	82	4.2	10	8	DD	X
N6		340	4.0	2010	386	40.2	10	4	MP	X
O7		267	4.2	980	211	19.6	20	4	MP	X

- All systems have $L_T = 2L$ and periodic b.c.

From 2-point functions to PCAC masses

Define

$$P^{rs} = \overline{\psi}_r \gamma_5 \psi_s \quad ; \qquad A_0^{rs} = \overline{\psi}_r \gamma_0 \gamma_5 \psi_s \quad :$$

with the 2-point functions

$$f_{PP}^{rs} = \int \langle P^{rs}(x) P^{sr}(0) \rangle \quad ; \qquad f_{AP}^{rs} = \int \langle A_0^{rs}(x) P^{sr}(0) \rangle \; .$$

one finds

$$\frac{\frac{1}{2}(\partial_0 + \partial_0^*) f_{AP}(x_0) + c_A a \partial_0^* \partial_0 f_{PP}(x_0)}{2 f_{PP}(x_0)} \to m_{rs} \quad , \quad x_0 \to \infty \quad ,$$

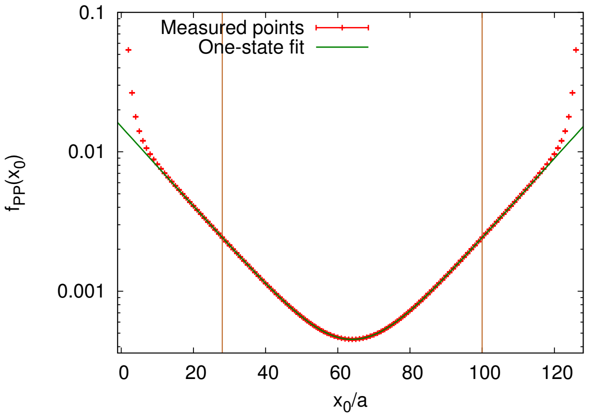
renormalised then as:

$$m_R^{rs} = \frac{Z_A}{Z_P} \frac{(1 + \overline{b}_A a m_{\mathrm{sea}} + \widetilde{b}_A a m_{rs})}{(1 + \overline{b}_P a m_{\mathrm{sea}} + \widetilde{b}_P a m_{rs})} m_{rs} \quad .$$

From 2-point functions to m_{PS} and f_{PS}

$$f_{PP}(x_0) = c_1 \left[e^{-m_{PS} x_0} + (echo) \right] + higher\ states$$

$$f_{PS}^{\rm bare} = 2\sqrt{2c_1}\, m_{rs} m_{PS}^{-3/2} \quad , \quad f_{PS} = Z_A(1 + \overline{b}_A a m_{\rm sea} + \widetilde{b}_A a m_{rs}) f_{PS}^{\rm bare} \quad .$$



2-point functions: stochastic evaluation

- Generate $U(1)$ random noise source on timeslice t: $\eta_t(x)$
- Solve $\zeta_t^r = a^{-1}\left(D + m_{0,r}\right)^{-1}\gamma_5\eta_t$
- Estimator for the two-point functions:

$$
\begin{aligned}
a^3 f_{PP}^{rs}(x_0) &= \sum_{\mathbf{x}} \left\langle\!\!\left\langle\, \zeta_t^r(x_0+t,\mathbf{x})^\dagger \quad \zeta_t^s(x_0+t,\mathbf{x}) \,\right\rangle\!\!\right\rangle \quad ; \\
a^3 f_{AP}^{rs}(x_0) &= \sum_{\mathbf{x}} \left\langle\!\!\left\langle\, \zeta_t^r(x_0+t,\mathbf{x})^\dagger \gamma_5 \zeta_t^s(x_0+t,\mathbf{x}) \,\right\rangle\!\!\right\rangle \quad ,
\end{aligned}
$$

(averages are over sources and configurations).