A $N_f = 2+1+1$ "twisted" determination of m_b and of f_{B_s} and f_{B_s}/f_B

Eleonora Picca (ETM Collaboration)
In collaboration with:
N.Carrasco Vela, P.Dimopoulos, R.Frezzotti,
V.Giménez, V.Lubicz, G.C.Rossi, F.Sanfilippo, S.Simula and
C.Tarantino

Università Roma Tre, INFN Roma Tre

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Outlook

- Some details on the ETMC simulations for $N_f = 2 + 1 + 1$ with the smearing technique
- Description of the analysis performed: w optimal method and ratio method
- Preliminary results for m_b, f_{B_s} and f_{B_s}/f_B

B physics on the lattice

- UV cut off: numerical simulations with relativistic quarks only feasible for heavy quarks lighter than the b $\rightarrow$ extrapolation from the charm region to the b quark mass region
- The signal/noise ratio prevents us from extracting the mass from two point correlation functions $\rightarrow$ smearing technique and *w optimal method*

ETMC ensambles with $N_f = 2+1+1$

- Simulation with $N_f = 2+1+1$ at three values of the lattice spacings
- $m_s/8 < \mu_l < m_s/2$ (210 MeV $< M_\pi <$ 440 MeV) $\rightarrow$ chiral extrapolation to the physical light quark mass is needed!
- $m_c < \mu_h < 3m_c$
- Two point correlation functions with smearing sources

β	1/a (GeV)	$Z_P(\overline{MS}, 2\text{ GeV})$	V/a^4	$a\mu_{sea}$	N_{conf}	$a\mu_s$	$a\mu_h$
1.90	2.224(68)	0.5210(18)	$32^3 \times 64$	0.0030	150	0.01800	0.25000
				0.0040	90	0.02200	... 0.77836
				0.0050	150	0.02600	
			$24^3 \times 48$	0.0060	150		
				0.0080	150		
				0.0100	150		
1.95	2.416(63)	0.5060(11)	$32^3 \times 64$	0.0025	150	0.01550	0.22000
				0.0035	150	0.01900	... 0.68495
				0.0055	150	0.02250	
				0.0075	75		
			$24^3 \times 48$	0.0085	150		
2.10	3.182(59)	0.5130(08)	$48^3 \times 96$	0.0015	60	0.01230, 0.01500	0.17000
				0.0030	90	0.01770	... 0.52928

Lattice Action

- *Gauge field*: Iwasaki action
- *Sea quark action*: Twisted mass action at maximal twist
  - u/d quarks: mass degenerate light doublet
  - s/c quarks: non degenerate mass doublet
- *Valence quark action*: Osterwalder-Seiler at maximal twist

Smearing technique

Using of smeared sources $\rightarrow$ reduced coupling of the operators to the excited states

- Gaussian smearing for the fermion fields
- APE smearing for the gauge field

$$C_{SL} \equiv \langle \mathcal{O}_S \mathcal{O}_L \rangle \qquad C_{SS} \equiv \langle \mathcal{O}_S \mathcal{O}_S \rangle$$

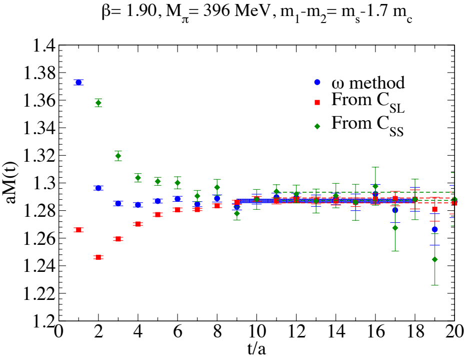
Plateau at earlier time?

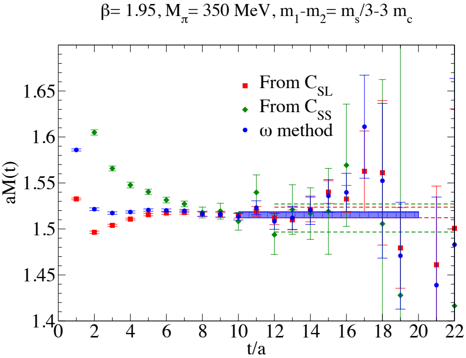
Meson masses: w optimal method

Construction of a new operator in order to maximize the coupling to the ground state (safe plateau at earlier time)

$$\mathcal{O}_w \sim w\mathcal{O}_L + (1-w)\mathcal{O}_s$$

$$\langle 0|\, \mathcal{O}_w \mathcal{O}_s \,|0\rangle = C_w(t) \sim w\ C_{SL} + (1-w) C_{SS}$$





m_b and decay constants calculation: ratio method[1]

An extrapolation from the charm region to the b quark mass is needed

Construct HQET-inspired ratios of the observables of interest at successive values of the heavy quark mass

- The heavy quark mass are choosen to have fixed ratio

$$\mu_h = \lambda \cdot \mu_{h-1}.$$

  The first mass μ_1, *triggering mass*, is chosen in the charm region
- Ratios have small discretization effects
- Known static limit ($\mu_h \to \infty$): better interpolation from the charm-region to m_b

[1] See talk by P. Dimopoulos on Tuesday for $N_f = 2$

b quark mass calculation

$$\lim_{\mu_h^{pole}\to\infty}\left(\frac{M_{hl}}{\mu_h^{pole}}\right) = const \quad (HQET)$$

$$\mu_h^{pole} = \rho(\mu^*, \mu_h)\cdot\mu_h$$

$$(N^3LO)$$

- Define

$$y(\mu_h;\mu_l;a) = \frac{M_{h,l}}{M_{h-1,l}}\frac{\mu_{h-1}^{pole}}{\mu_h^{pole}} = \lambda^{-1}\frac{M_{h,l}}{M_{h-1,l}}\frac{\rho(\mu^*,\mu_{h-1})}{\rho(\mu^*,\mu_h)}, \qquad \frac{\mu_h}{\mu_{h-1}} = \lambda$$

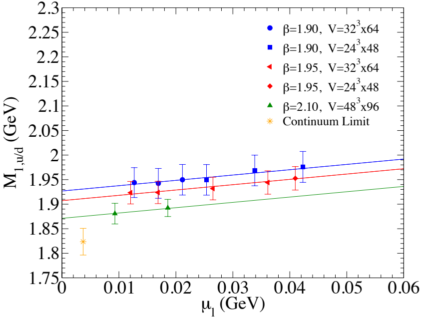
$$\lim_{\mu_h^{pole}\to\infty} y(\mu_h;\mu_{u,d};a=0) = 1$$

- Find m_b from a chain of ratios: $m_b = \mu_1\cdot\lambda^{K_b}$

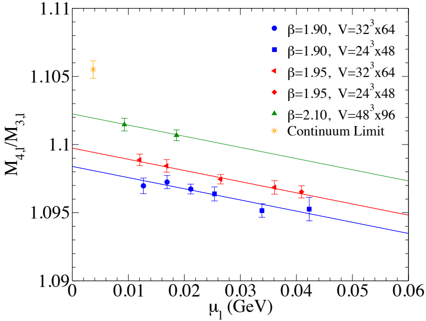
$$y(\mu_2)\cdot y(\mu_3)\cdot\ldots\cdot y(\mu_{K_b+1}) = \lambda^{K_b}\frac{M_{K_b+1,u/d}}{M_{1,u/d}}\left[\frac{\rho(\mu^*,\mu_1)}{\rho(\mu^*,\mu_{K_b+1})}\right]$$

fixing $M_{K_b+1,u/d}\equiv M_B^{exp} = 5.279$ GeV so that K_b is an integer.

Triggering point meson mass: discretization effect $\sim 3\%$.

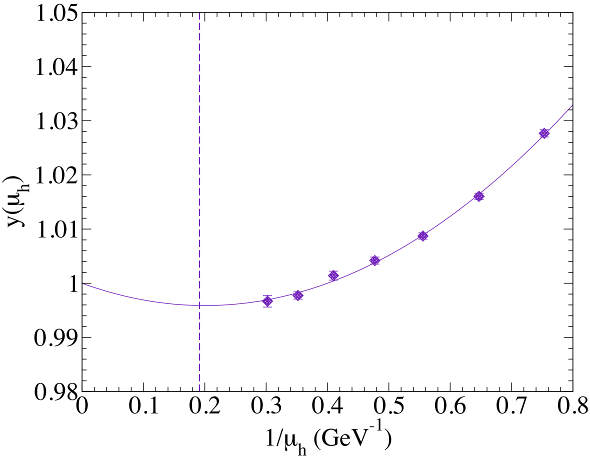


Fit of ratios discretization errors $\sim 0.3 - 0.8\%$



$$y(\mu_h) = 1 + \frac{C_1}{\mu_h} + \frac{C_2}{\mu_h^2}$$

$\chi^2/dof \sim 0.6$



- Choose λ in such a way that K_b is an integer:

$$m_{tr} = 1.14 \text{ GeV}, \quad \lambda = 1.1644, \quad K_b = 10$$

b quark mass result (preliminary)

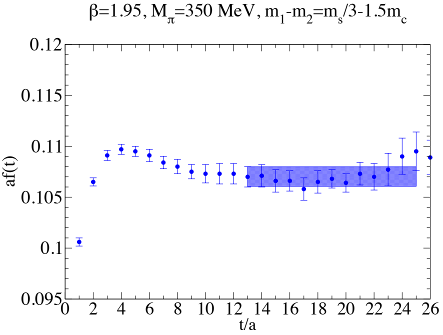
$$m_b(\overline{MS}, m_b)_{N_f=4} = 4.29(8)(9)(6) \text{ GeV} = 4.29(13) \text{ GeV}$$

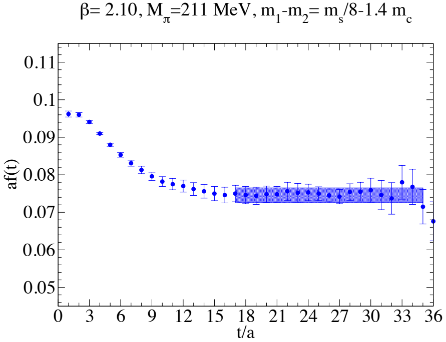
- First error is statistical error (fit error, lattice spacing and renormalization constants statistical error)
- The other contributions are respectively from the lattice spacing ($\sim 2\%$) and renormalization constants ($\sim 1.5\%$) systematic uncertainties
- Changing the triggering mass gives a perfectly compatible result

A more detailed investigation of systematic uncertainties is in progress

Pseudoscalar decay constants

$$f_{PS} = (m_1 + m_2) \cdot \frac{\langle 0 | \mathcal{O}_L | PS \rangle}{M \sinh M}, \quad \langle 0 | \mathcal{O}_L | PS \rangle \sim \frac{C_{SL}}{\sqrt{C_{SS}}}$$





No w optimal method (in progress): safe plateau only for smallest heavy valence quark (in this case $m_c < \mu_h < 1.7 m_c$)

f_{B_s} and f_{B_s}/f_B calculation

$$\lim_{\mu_h^{pole}\to\infty} f_{hl(s)}\sqrt{\mu_h^{pole}} = \ const \quad (HQET)$$

- Construct ratios with known static limit

$$z_{s(l)}(\mu_h) = \lambda^{1/2}\frac{f_{h,s(l)}}{f_{h-1,s(l)}} \cdot \frac{C_A(\mu^*,\mu_{h-1})}{C_A(\mu^*,\mu_h)}\sqrt{\frac{\rho(\mu^*,\mu_h)}{\rho(\mu^*,\mu_{h-1})}}$$

$$C_A(\mu^*,\mu_h) \ \ (NLO); \qquad \rho(\mu^*,\mu_h) \ \ (N^3LO)$$

For f_{B_s} calculation:

$$\lim_{\mu_h^{pole}\to\infty} z_s(\mu_h;\mu_{u/d};a=0) = 1$$

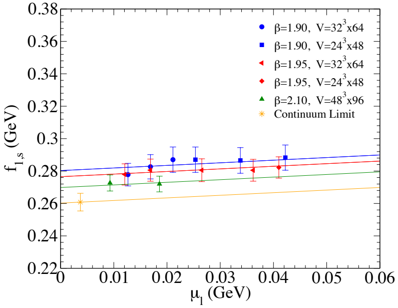
For f_{B_s}/f_B calculation:

$$\lim_{\mu_h^{pole}\to\infty} \frac{z_s(\mu_h;\mu_{u/d};a=0)}{z_l(\mu_h;\mu_{u/d};a=0)} = 1$$

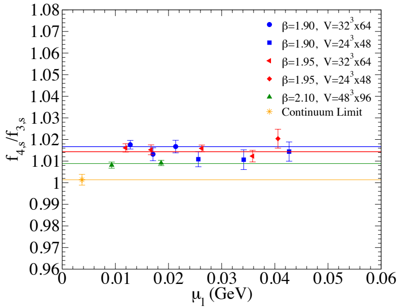
f_{B_s}

f_{B_s} is determined using a chain of ratios. λ and K_b are obtained in the previous analysis

$$z_s(\mu_2)\cdot\ldots\cdot z_s(\mu_{K_b+1}) = \lambda^{K_b/2}\frac{f_{K_b+1,s}}{f_{1,s}}\frac{C_A(\mu^*,\mu_1)}{C_A(\mu^*,\mu_{K_b+1})}\sqrt{\frac{\rho(\mu^*,\mu_{K_b+1})}{\rho(\mu^*,\mu_1)}}$$



Discretization effects $\sim 5\%$

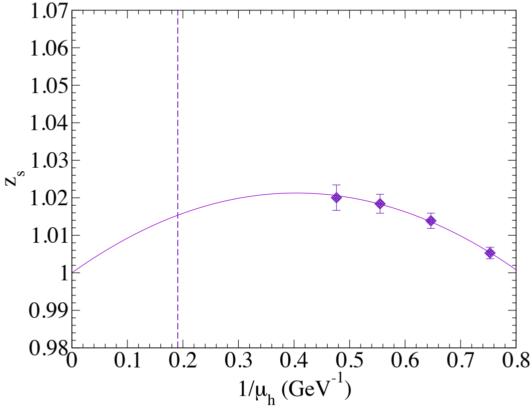


Discretization effects $\sim 0.5\% - 1\%$

Negligible dependance on μ_l

$$1 + \frac{\eta_1}{\mu_h} + \frac{\eta_2}{\mu_h^2}$$

$\chi^2/dof \sim 0.02$



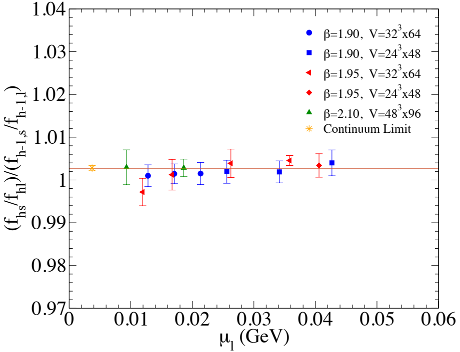
$$\boxed{f_{B_s} = 235(8)(5)\ \text{MeV} = 235(9)\ \text{MeV}} \quad \text{(preliminary)}$$

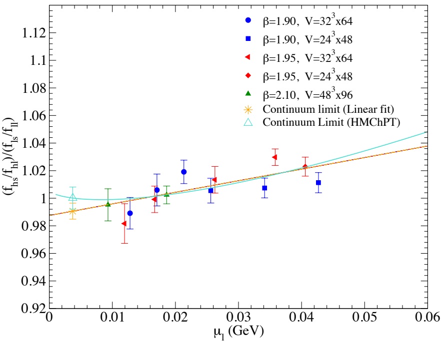
- Statistical + lattice spacing systematics
- Less heavy quark masses than for the b quark mass calculation: w optimal method and better estimate of the systematical uncertainties in progress

f_{B_s}/f_B

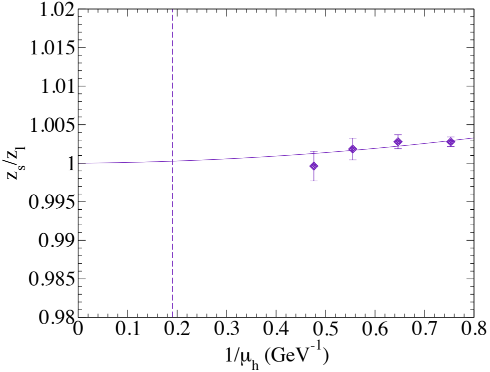
- Using the double ratios z_s/z_l: constant fit
- f_{B_s}/f_B at triggering point: Linear + log (HMChPT) dependance on μ_l (counted in the systematics).
  Using $(f_{hs}/f_{hl})/(f_{ls}/f_{ll})$ the systematical error is strongly reduced (from $\sim 2\%$ to 0.5%).

$$\chi^2_{lin}/dof = 1.19; \quad \chi^2_{HMChPT}/dof = 1.36$$





$\chi^2/dof \sim 0.6$



(preliminary)

$\boxed{f_{B_s}/f_B = 1.201(21)(6)}$ $\rightarrow$ $\boxed{f_B = 196(7)(4) \text{ MeV} = 196(8) \text{ MeV}}$

Statistical (also from f_K/f_π $\sim 0.4\%$) + systematic from linear vs HMChPT fit

$f_K/f_\pi = 1.193(5)$ [by FLAG]

Statistical + systematic from lattice spacing.
Systematic from linear vs HMChPT is negligible ($\sim 0.5\%$)

Summary and Conclusions

- ETMC simulations with $N_f = 2+1+1$ with smearing sources
- Application of the w optimal method for the meson masses
- Use of ratio method in order to reduce statistical errors

	Preliminary $N_f = 2+1+1$	$N_f = 2^{[2]}$
$m_b(\overline{MS}, m_b)$	4.29(13) GeV	4.29(12) GeV
f_{B_s}	235(9) MeV	228(8) MeV
f_{B_s}/f_B	1.201(22)	1.206(24)
f_B	196(8) MeV	189(8) MeV

Future plans:

- Use of the w optimal method for the decay constants
- Better estimate of the systematic uncertainties

[2] Presented by P.Dimopoulos on Tuesday

Thanks for the attention!

Backup

Some other details on the simulations

β	$L(fm)$	M_π(MeV)	$M_\pi L$
1.90	2.84	245.41	3.53
		282.13	4.06
		314.43	4.53
1.90	2.13	282.13	3.05
		343.68	3.71
		396.04	4.27
		442.99	4.78
1.95	2.61	238.67	3.16
		280.95	3.72
		350.12	4.64
		408.13	5.41
1.95	1.96	434.63	4.32
2.10	2.97	211.18	3.19
		242.80	3.66
		295.55	4.46

Smearing technique

Gaussian smearing

$$\mathcal{O}_S(t,\vec{x}) = \left[\frac{1}{1+6\alpha_S}\left(\delta_{\vec{x},\vec{x}'} + \alpha_s H_{\vec{x},\vec{x}'}[U']\right)^{N_s}\right]\mathcal{O}_L(t,\vec{x}')$$

N_s is the number of iterations and α_S is the coupling between nearby lattice sites

$$H_{\vec{x},\vec{x}'}[U] = \sum_{\mu=1}^{3}\left[U_{\vec{x},\vec{x}+\mu}\cdot\delta_{\vec{x}',\vec{x}+\mu} + U^{\dagger}_{\vec{x}-\mu,\vec{x}}\cdot\delta_{\vec{x}',\vec{x}-\mu}\right]$$

APE smearing

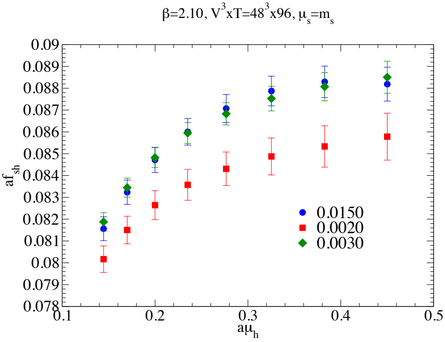
$$U'_{x,x+\mu} \equiv U^{(N_G)}_{x,x+\mu} = P_{SU(3)}\left\{U^{(N_G-1)}_{x,x+\mu} + \right.$$

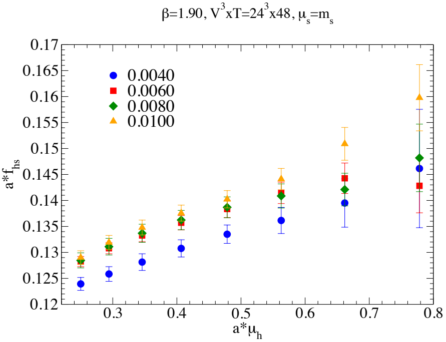
$$\left. + \frac{\alpha_{APE}}{6}\sum_{\nu=\pm1,\pm2,\pm3}^{\nu\neq\pm\mu} U^{(N_G-1)}_{x,x+\nu}\; U^{(N_G-1)}_{x+\nu,x+\nu+\mu}\; U^{(N_G-1)}_{x+\nu+\mu,x+\mu}\right\}$$

w optimal method: details

$$C_w(t) = w \cdot \frac{C_{SL}(t)}{C_{SL}(1)} + (1-w) \cdot \frac{C_{SS}(t)}{C_{SS}(1)}$$

β	t_{norm}/a	$L^3 \times T$	*Time intervals* (t/a)	$[\frac{t_{min}}{a}, \frac{t_{max}}{a}]_w$	$[\frac{t_{min}}{a}, \frac{t_{max}}{a}]$
1.90	5	$24^3 \times 48$	[7,10] [8, 11] [9,12] [10, 13] [11, 14]	[9,18]	[11,22]
		$32^3 \times 64$		[9,20]	[11,28]
1.95	6	$24^3 \times 48$	[9, 12] [10, 13] [11,14] [12,15] [13,16]	[10,18]	[12,22]
		$32^3 \times 64$		[10,20]	[12,28]
2.10	8	$48^3 \times 96$	[12, 16] [13, 17] [14, 18] [15, 19] [16, 20]	[13,30]	[16,40]

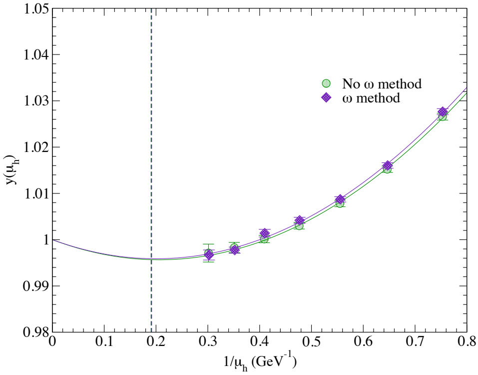

β=2.10, V^3xT=48^3x96, μ_s=m_s
af_sh
aμ_h
0.0150
0.0020
0.0030


β=1.90, V^3xT=24^3x48, μ_s=m_s
a*f_hs
a*μ_h
0.0040
0.0060
0.0080
0.0100

m_b without w optimal method

Discretization effects:

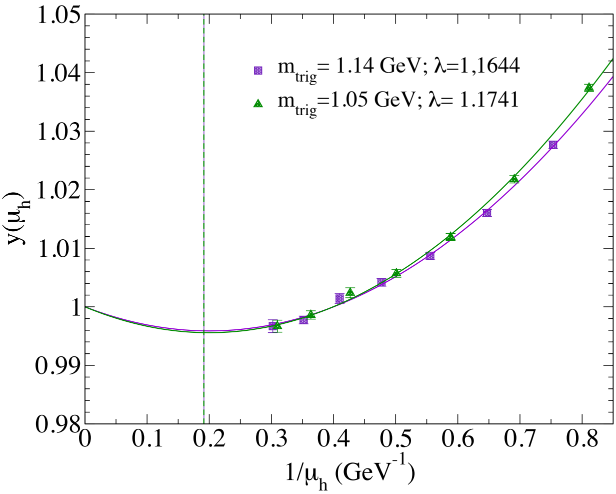
- mass at triggering point $\sim 3\%$
- ratios $0.2\% - 0.8\%$ (using w optimal method errors are smaller especially for heavier quark masses)



$$m_b(\overline{MS}, m_b) = 4.31(9)(9)(6)\ \text{GeV} = 4.31(14)\ \text{GeV}$$

$$(\text{w } \mathit{optimal\ method}:\ m_b(\overline{MS}, m_b) = 4.29(13)\ \text{GeV})$$

Changing the triggering mass



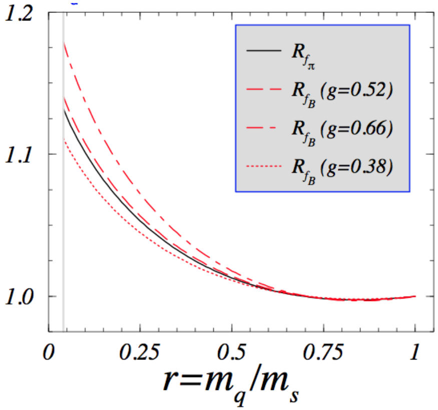
Practically the same result

HMChPT for decay constant ratio

$$\hat{g} = 0.61(7)$$

$$f_{hl} = A_h \left[1 - \frac{3(1+3\hat{g}^2)}{4} \xi\mu_l \log(\xi\mu_l) + B_h \cdot \xi\mu_l + C_h \cdot a^2 \right], \qquad \xi = \frac{2B_0}{(4\pi f_0)^2}$$

$$f_{hs} = D_h(1 + E_h\mu_l + G_h a^2)$$

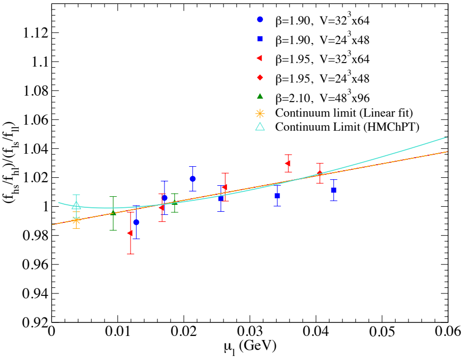


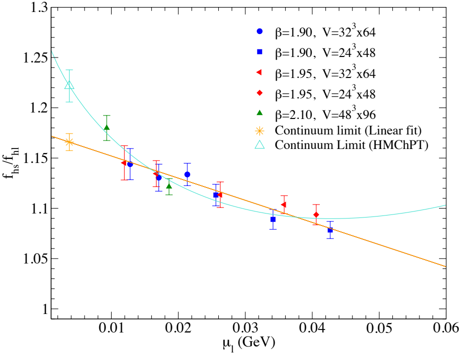
[3]

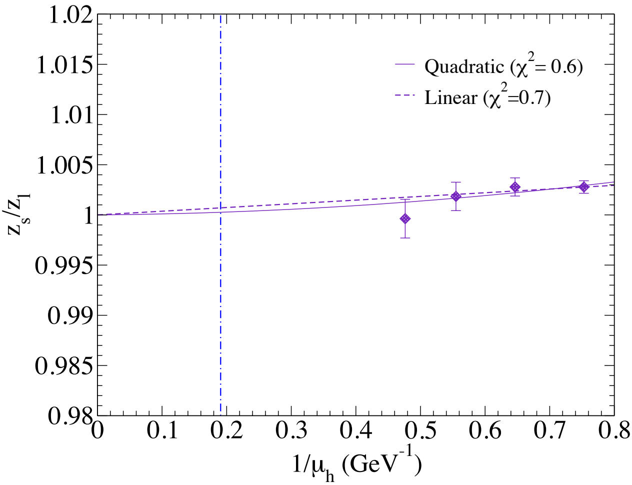
[3] D. Becirevic, S. Fajfer, S. Prelovsek, J. Zupan, [hep-ph/0211271]

Confrontation with/without f_K/f_π

	Using f_K/f_π	Not using f_K/f_π
f_{B_s}/f_B	1.201(21)(6)=1.201(22)	1.207(21)(28)=1.207(35)
f_B	196(8)(4)(/) MeV=196(8) MeV	195(7)(4)(5) MeV= 195(9)MeV







	Quadratic	Linear
f_{B_s}/f_B	1.201(21)(6)=1.201(22)	1.206(14)(6)=1.206(15)
f_B	196(8)(4) MeV=196(8) MeV	195(6)(4) MeV= 195(7)MeV