# Topological susceptibility from twisted mass fermions using spectral projectors

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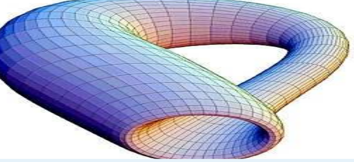


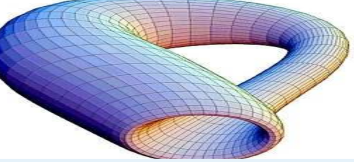
# Topological susceptibility



- the topological susceptibility expresses fluctuations of the topological charge
- as such, it describes non-trivial topological properties of the underlying quantum vacuum
- such properties have far-reaching phenomenological implications
- the most prominent example: flavour-singlet pseudoscalar $\eta'$ meson $\rightarrow$ Witten-Veneziano relation

Definition and computation on the lattice:

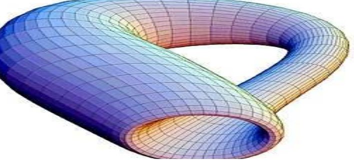
- notoriously difficult
- long debate in the literature about the validity of different approaches
- clean definition: index of overlap Dirac operator $\rightarrow$ very costly
- another clean definition: from density chain correlators
  [L. Giusti, G.C. Rossi, M. Testa 2004], [M. Lüscher 2004]
  using spectral projectors $\rightarrow$ subject of this talk
  [L. Giusti, M. Lüscher 2008], [M. Lüscher, F. Palombi 2010]



# Presentation outline



1. Theoretical introduction:
   - spectral projectors, topological susceptibility, $Z_P/Z_S$
2. Simulation setup
3. Results for $N_f = 2$ and $N_f = 2+1+1$
   - chiral fits of topological susceptibility – chiral condensate
4. Results in the quenched case ($N_f = 0$)
   - Witten-Veneziano formula
5. Summary



# Spectral projectors

- Introduced in: [L. Giusti, M. Lüscher 2008]
- First application for the computation of the topological susceptibility: (quenched case) [M. Lüscher, F. Palombi 2010]
- $\mathbb{P}_M$ – spectral projector $\longrightarrow$ [talk by E. García Ramos]
- $\mathrm{Tr}\,\mathbb{P}_M$ can be represented stochastically by:
  $\mathrm{Tr}\,\mathbb{P}_M = \frac{1}{N}\sum_{j=1}^{N}(\eta_j, \mathbb{P}_M \eta_j)$, where $\eta_1, \ldots, \eta_N$ are pseudo-fermion fields added to the theory.
- One can also evaluate other traces of this kind, e.g. $\mathrm{Tr}\,\gamma_5 \mathbb{P}_M$.
- In practice, one constructs an approximation to the projector $\mathbb{P}_M$ – we denote it by $\mathbb{R}_M$.

- For Ginsparg-Wilson fermions:

$$\mathrm{Tr}\left\{\gamma_5 \mathbb{P}_M\right\} = Q_{top}$$

.

- The topological susceptibility is in general given by:

$$\chi = \frac{\langle Q_{top}^2 \rangle}{V}$$

.

- Hence:

$$\chi = \frac{Z_S^2}{Z_P^2} \frac{\langle \mathrm{Tr}\{\gamma_5 \mathbb{R}_M^2\} \mathrm{Tr}\{\gamma_5 \mathbb{R}_M^2\} \rangle}{V}.$$

- The renormalization constants ratio $Z_P/Z_S$ and the absence of short-distance singularities in this definition can be inferred from density chain correlators.

# Computation of topological susceptibility



Introduce the following observables: [M. Lüscher, F. Palombi 2010]

$$\mathcal{A} = \frac{1}{N}\sum_{k=1}^{N}\left(\mathbb{R}_M^2\eta_k, \mathbb{R}_M^2\eta_k\right),$$

$$\mathcal{B} = \frac{1}{N}\sum_{k=1}^{N}\left(\mathbb{R}_M\gamma_5\mathbb{R}_M\eta_k, \mathbb{R}_M\gamma_5\mathbb{R}_M\eta_k\right),$$

$$\mathcal{C} = \frac{1}{N}\sum_{k=1}^{N}\left(\mathbb{R}_M\eta_k, \gamma_5\mathbb{R}_M\eta_k\right),$$

Topological susceptibility:

$$\chi = \frac{Z_S^2}{Z_P^2}\frac{\langle\mathcal{C}^2\rangle - \frac{\langle\mathcal{B}\rangle}{N}}{V} \quad \leftrightarrow \quad \chi = \frac{\langle Q_{top}^2\rangle}{V}$$

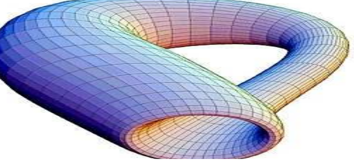
$Z_P/Z_S$:

$$\frac{Z_S^2}{Z_P^2} = \frac{\langle\mathcal{A}\rangle}{\langle\mathcal{B}\rangle}$$

**$\mathcal{C}$ – estimator of topological charge**

Expect:
- $\langle\mathcal{C}\rangle = 0$ for long enough MC history
- $\mathcal{C}$ Gaussian distributed, distribution width $\rightarrow \chi$
- $\mathcal{C}$ a sensitive measure of autocorrelations

$\rightarrow$ freezing of topological charge for increasing $\beta$



# Simulation setup

- We use dynamical twisted mass configurations generated by ETMC
  - ⋆ $N_f = 2$ [P. Boucaud et al., 2007, 2008], [R. Baron et al., 2009],
  - ⋆ $N_f = 2+1+1$ [R. Baron et al., 2010, 2011].
- We also generated $N_f = 0$ configurations.
- Gauge action: $$S_G[U] = \frac{\beta}{3} \sum_x \Big( b_0 \sum_{\mu,\nu=1} \mathrm{Re\,Tr}\big(1 - P^{1\times1}_{x;\mu,\nu}\big) + b_1 \sum_{\mu\neq\nu} \mathrm{Re\,Tr}\big(1 - P^{1\times2}_{x;\mu,\nu}\big)\Big),$$

  $N_f = 2$ – tree-level Symanzik improved action [P. Weisz, 1982], i.e. $b_1 = -\frac{1}{12}$.
  $N_f = 2+1+1$ and $N_f = 0$ – Iwasaki action [Y. Iwasaki, 1985], i.e. $b_1 = -0.331$, $b_0 = 1 - 8b_1$,
- Wilson twisted mass fermion action for the light sector
  [R. Frezzotti, P.A. Grassi, G.C. Rossi, S. Sint, P. Weisz, 2000-2004]
  $$S_l[\psi, \bar{\psi}, U] = a^4 \sum_x \bar{\chi}_l(x) \big(D_W + m_{0,l} + i\mu_l \gamma_5 \tau_3\big) \chi_l(x),$$
  $\chi_l = (\chi_u, \chi_d)$, $m_{0,l}$ and $\mu_l$ are the bare untwisted and twisted light quark masses.
- Twisted mass action for the heavy doublet [R. Frezzotti, G.C. Rossi, 2003, 2004]
  $$S_h[\psi, \bar{\psi}, U] = a^4 \sum_x \bar{\chi}_h(x) \big(D_W + m_{0,h} + i\mu_\sigma \gamma_5 \tau_1 + \mu_\delta \tau_3\big) \chi_h(x),$$
  $\chi_h = (\chi_c, \chi_s)$, $m_{0,h}$ – bare untwisted heavy quark mass, $\mu_\sigma$ – bare twisted mass with the twist along the $\tau_1$ direction, $\mu_\delta$ – mass splitting along the $\tau_3$ direction that makes the strange and charm quark masses non-degenerate.
  Renormalized strange and charm quark masses $m_R^{s,c} = Z_P^{-1}\,(\mu_\sigma \mp (Z_P/Z_S)\mu_\delta)$.

# Ensembles used

| Ensemble | $\beta$ | lattice | $a\mu_l$ | $\mu_{l,R}$ [MeV] | $\kappa_c$ | L [fm] | $m_\pi L$ | a ]fm] | $Z_P/Z_S$ | $r_0/a$ |
|---|---|---|---|---|---|---|---|---|---|---|
| A30.32 | 1.90 | $32^3 \times 64$ | 0.0030 | 13 | 0.163272 | 2.8 | 4.0 | 0.0863 | 0.699(13) | 5.231(38) |
| A40.20 | 1.90 | $20^3 \times 40$ | 0.0040 | 17 | 0.163270 | 1.7 | 3.0 | 0.0863 | 0.699(13) | 5.231(38) |
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| A50.32 | 1.90 | $32^3 \times 64$ | 0.0050 | 22 | 0.163267 | 2.8 | 5.1 | 0.0863 | 0.699(13) | 5.231(38) |
| A60.24 | 1.90 | $24^3 \times 48$ | 0.0060 | 26 | 0.163265 | 2.1 | 4.2 | 0.0863 | 0.699(13) | 5.231(38) |
| A80.24 | 1.90 | $24^3 \times 48$ | 0.0080 | 35 | 0.163260 | 2.1 | 4.8 | 0.0863 | 0.699(13) | 5.231(38) |
| B25.32 | 1.95 | $32^3 \times 64$ | 0.0025 | 13 | 0.161240 | 2.5 | 3.4 | 0.0779 | 0.697(7) | 5.710(41) |
| B35.32 | 1.95 | $32^3 \times 64$ | 0.0035 | 18 | 0.161240 | 2.5 | 4.0 | 0.0779 | 0.697(7) | 5.710(41) |
| B55.32 | 1.95 | $32^3 \times 64$ | 0.0055 | 28 | 0.161236 | 2.5 | 5.0 | 0.0779 | 0.697(7) | 5.710(41) |
| B75.32 | 1.95 | $32^3 \times 64$ | 0.0075 | 38 | 0.161232 | 2.5 | 5.8 | 0.0779 | 0.697(7) | 5.710(41) |
| B85.24 | 1.95 | $24^3 \times 48$ | 0.0085 | 45 | 0.161231 | 1.9 | 4.7 | 0.0779 | 0.697(7) | 5.710(41) |
| D20.48 | 2.10 | $48^3 \times 96$ | 0.0020 | 12 | 0.156357 | 2.9 | 3.9 | 0.0607 | 0.740(5) | 7.538(58) |
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| D45.32 | 2.10 | $32^3 \times 64$ | 0.0045 | 29 | 0.156315 | 1.9 | 3.9 | 0.0607 | 0.740(5) | 7.538(58) |
| b40.16 | 3.90 | $16^3 \times 32$ | 0.004 | 21 | 0.160856 | 1.4 | 2.5 | 0.085 | 0.6390(32) | 5.35(4) |
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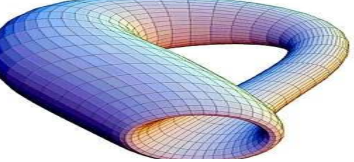
| c30.20 | 4.05 | $20^3 \times 40$ | 0.003 | 19 | 0.157010 | 1.3 | 2.4 | 0.067 | 0.6820(23) | 6.71(4) |
| d20.24 | 4.20 | $24^3 \times 48$ | 0.002 | 15 | 0.154073 | 1.3 | 2.4 | 0.054 | 0.7130(29) | 8.36(6) |
| e17.32 | 4.35 | $32^3 \times 64$ | 0.00175 | 16 | 0.151740 | 1.3 | 2.4 | 0.042 | 0.7398(33) | 5.35(4) |

$Z_P/Z_S$ from [C. Alexandrou et al., 2012], [K. Cichy, K. Jansen, P. Korcyl, 2012], [D. Palao, private communication]



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# Ensembles used – lattice spacings

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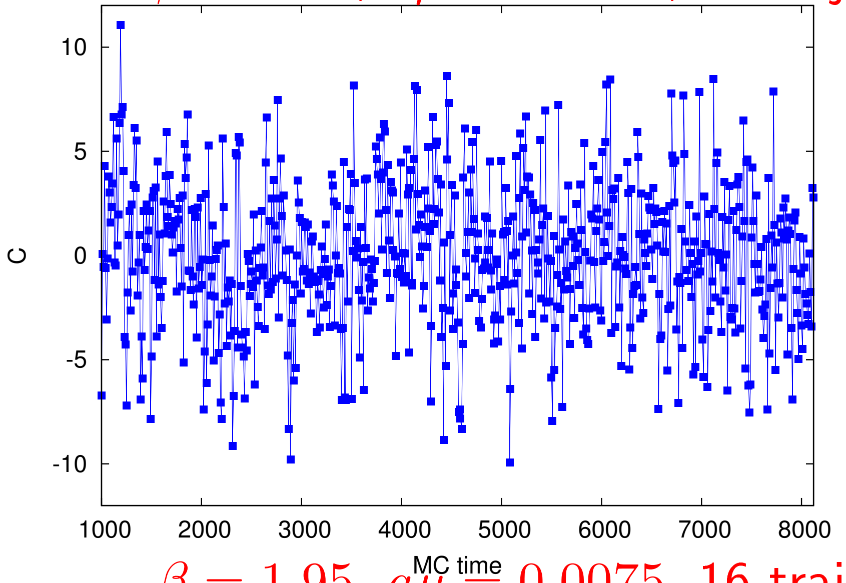
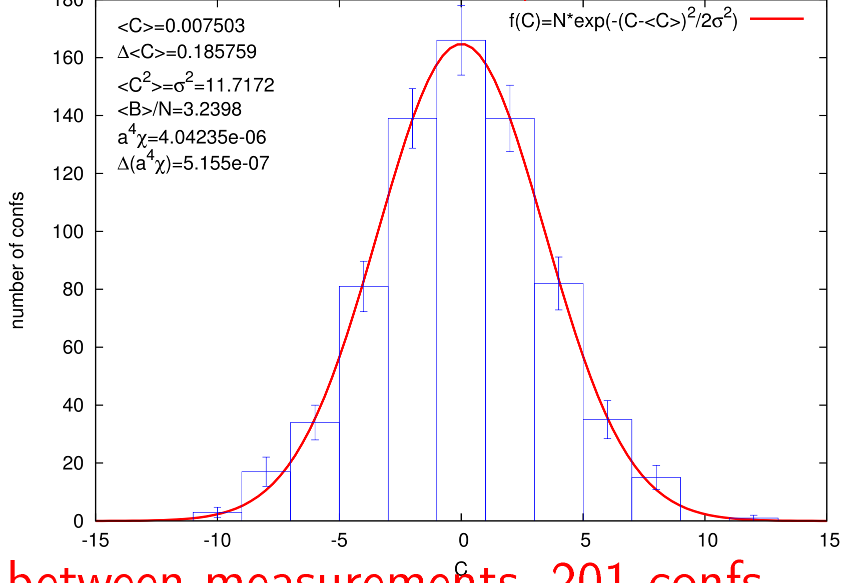
| D20.48 | 2.10 | $48^3 \times 96$ | 0.0020 | 12 | 0.156357 | 2.9 | 3.9 | 0.0607 | 0.740(5) | 7.538(58) |
| D30.48 | 2.10 | $48^3 \times 96$ | 0.0030 | 19 | 0.156355 | 2.9 | 4.7 | 0.0607 | 0.740(5) | 7.538(58) |
| D45.32 | 2.10 | $32^3 \times 64$ | 0.0045 | 29 | 0.156315 | 1.9 | 3.9 | 0.0607 | 0.740(5) | 7.538(58) |
| b40.16 | 3.90 | $16^3 \times 32$ | 0.004 | 21 | 0.160856 | 1.4 | 2.5 | 0.085 | 0.6390(32) | 5.35(4) |
| b40.20 | 3.90 | $20^3 \times 40$ | 0.004 | 21 | 0.160856 | 1.7 | 2.8 | 0.085 | 0.6390(32) | 5.35(4) |
| b40.24 | 3.90 | $24^3 \times 48$ | 0.004 | 21 | 0.160856 | 2.0 | 3.3 | 0.085 | 0.6390(32) | 5.35(4) |
| b40.32 | 3.90 | $32^3 \times 64$ | 0.004 | 21 | 0.160856 | 2.7 | 4.3 | 0.085 | 0.6390(32) | 5.35(4) |
| b64.24 | 3.90 | $24^3 \times 48$ | 0.0064 | 34 | 0.160856 | 2.0 | 4.1 | 0.085 | 0.6390(32) | 5.35(4) |
| b85.24 | 3.90 | $24^3 \times 48$ | 0.0085 | 45 | 0.160856 | 2.0 | 4.7 | 0.085 | 0.6390(32) | 5.35(4) |
| c30.20 | 4.05 | $20^3 \times 40$ | 0.003 | 19 | 0.157010 | 1.3 | 2.4 | 0.067 | 0.6820(23) | 6.71(4) |
| d20.24 | 4.20 | $24^3 \times 48$ | 0.002 | 15 | 0.154073 | 1.3 | 2.4 | 0.054 | 0.7130(29) | 8.36(6) |
| e17.32 | 4.35 | $32^3 \times 64$ | 0.00175 | 16 | 0.151740 | 1.3 | 2.4 | 0.042 | 0.7398(33) | 5.35(4) |

$Z_P/Z_S$ from [C. Alexandrou et al., 2012], [K. Cichy, K. Jansen, P. Korcyl, 2012], [D. Palao, private communication]

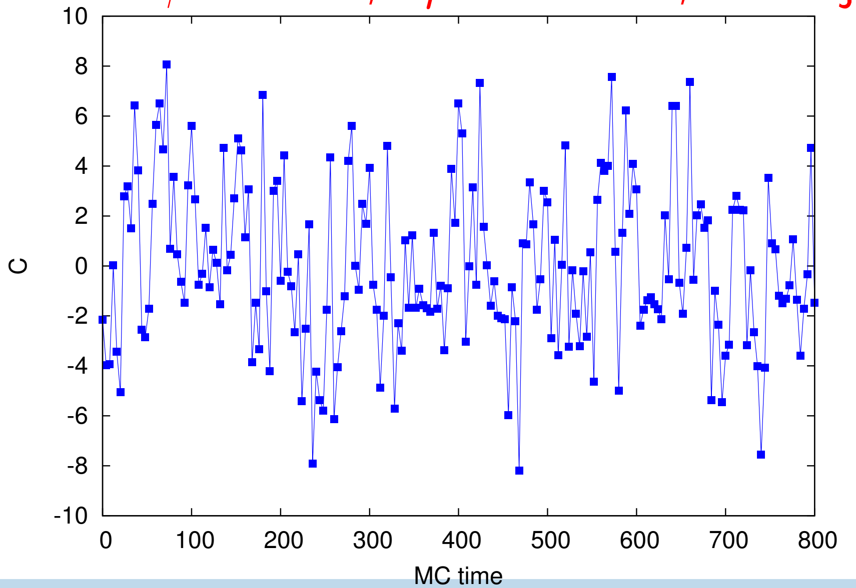
# Results – $N_f = 2$ and $N_f = 2+1+1$

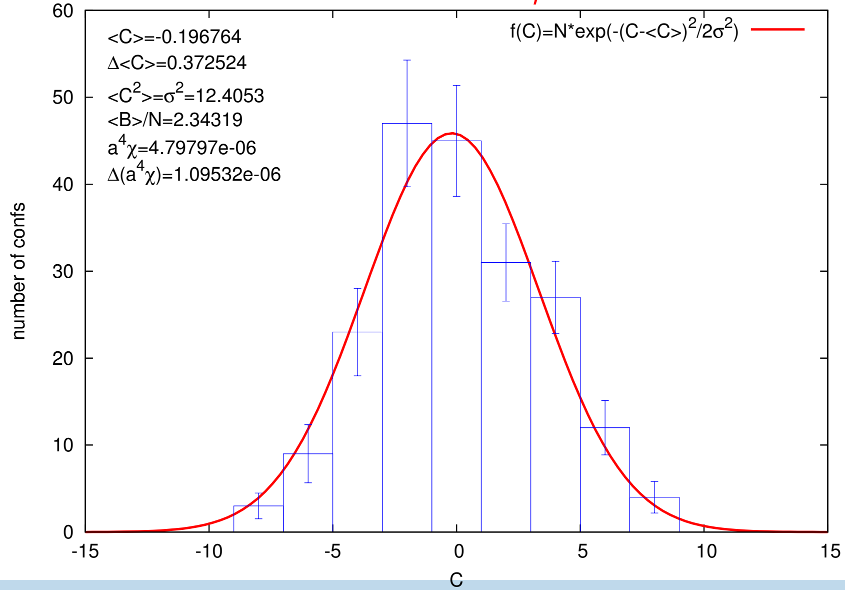
$\beta = 1.95$, $a\mu = 0.0055$, 20 traj. between measurements, 712 confs



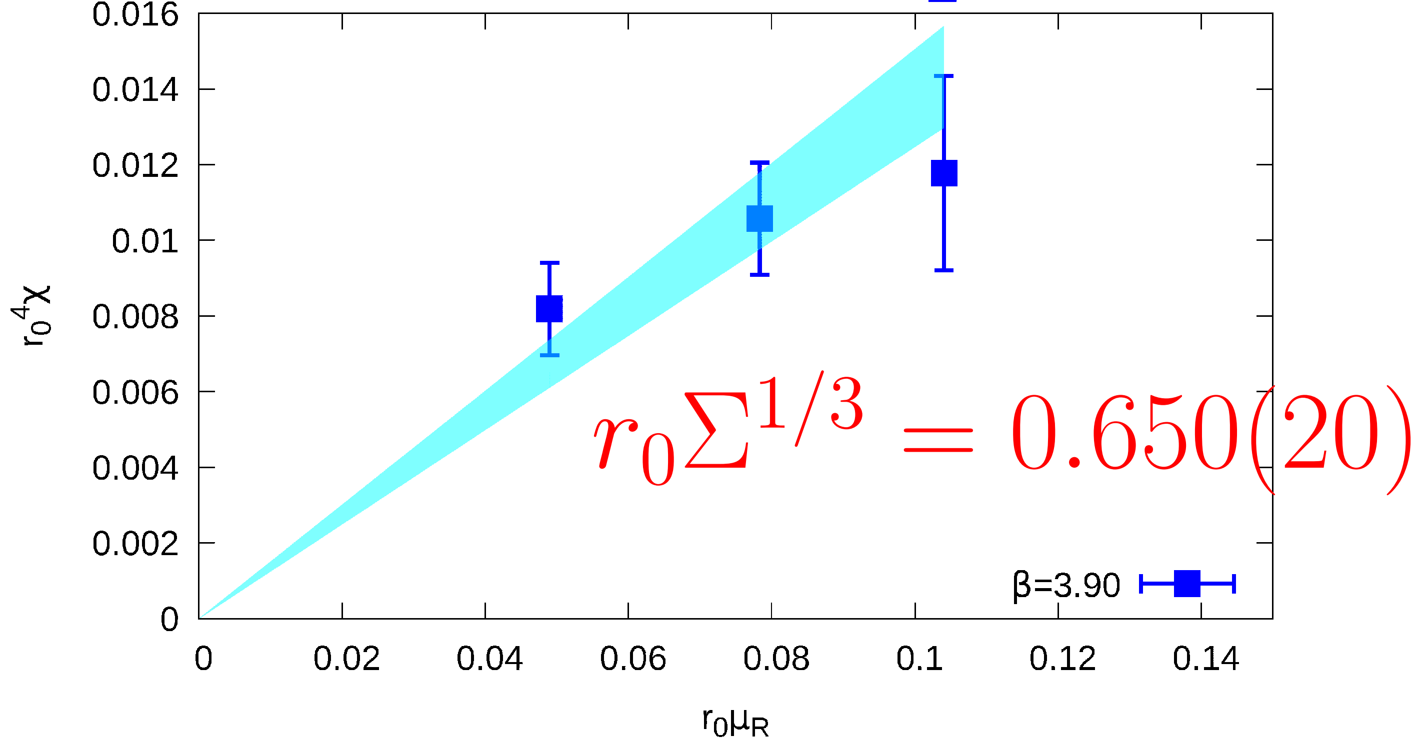




$\beta = 1.95$, $a\mu = 0.0075$, 16 traj. between measurements, 201 confs



tree-level formula of $\chi$PT: $r_0^4\chi = \frac{r_0^3\Sigma\cdot r_0\mu_R}{2}$

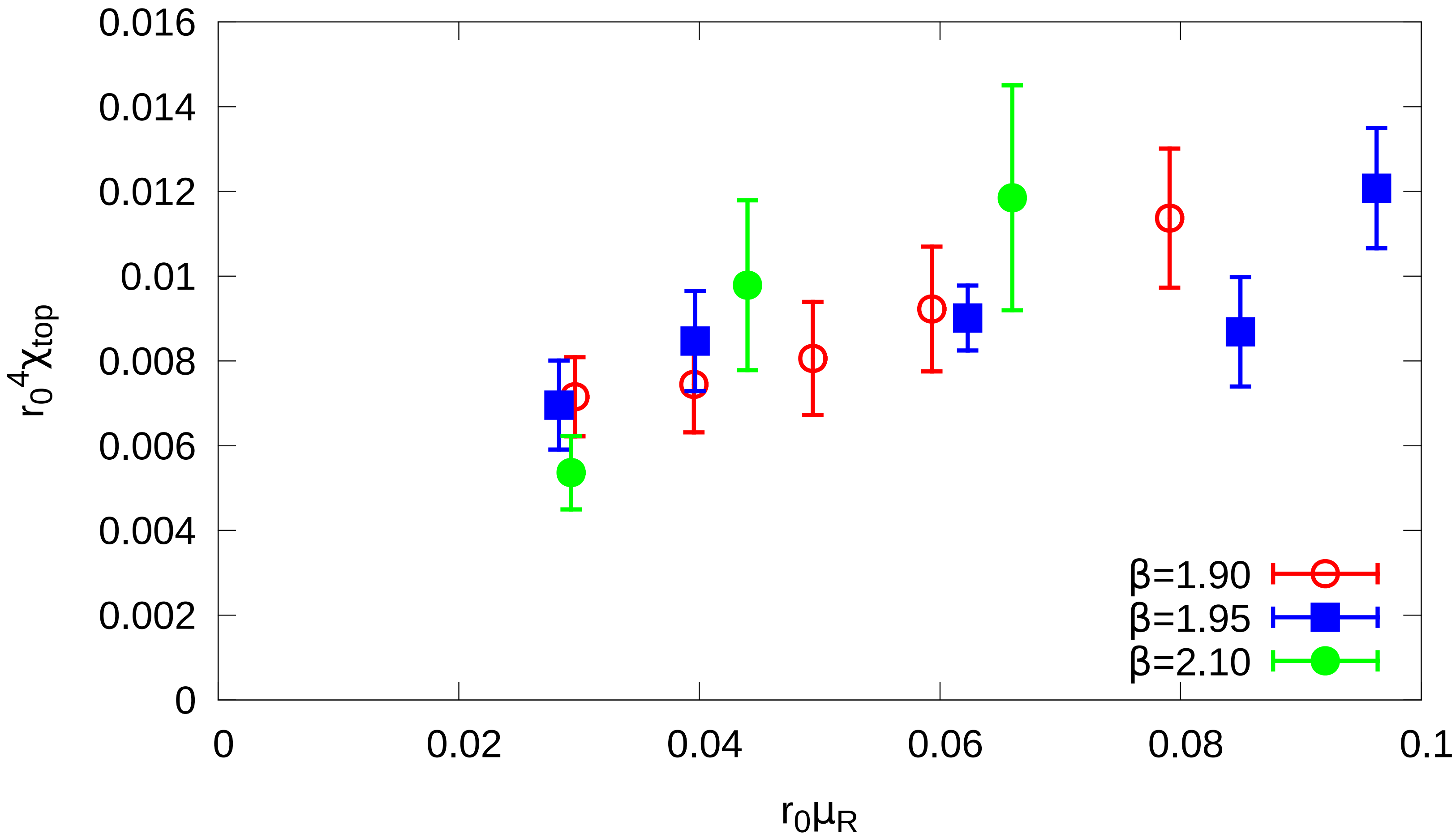



compare to direct determination [talk by E. García Ramos]

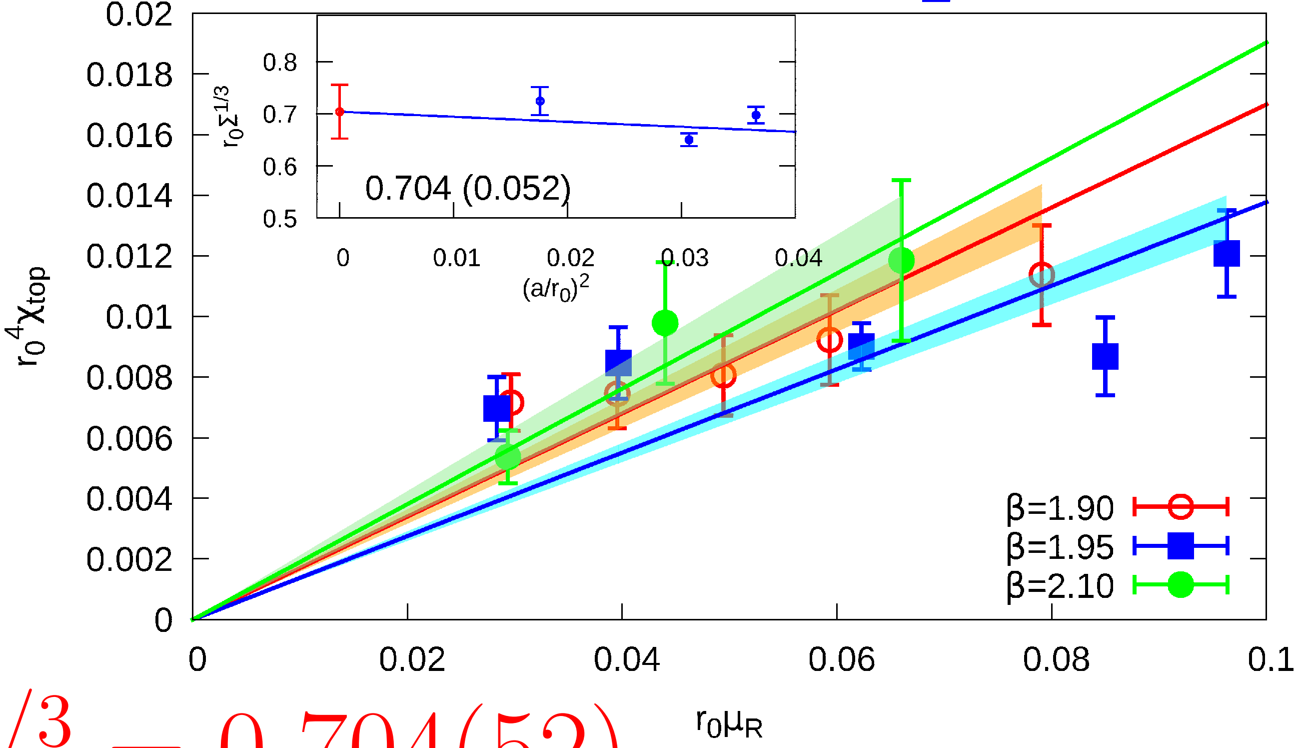
$$r_0\Sigma^{1/3}_{\beta=3.9} = 0.696(20), \quad r_0\Sigma^{1/3}_{cont} = 0.689(33)$$

# All data for $N_f = 2+1+1$





tree-level formula of $\chi$PT: $r_0^4\chi = \frac{r_0^3\Sigma\cdot r_0\mu_R}{2}$



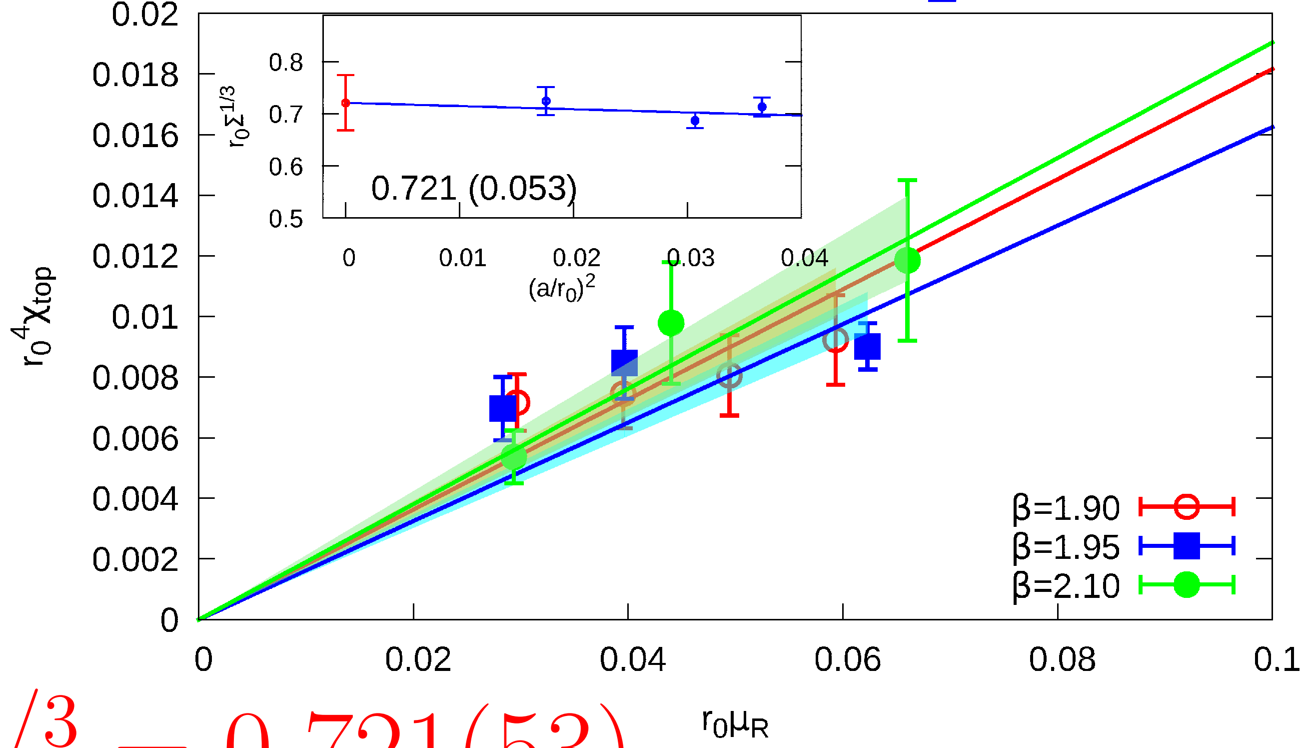

$$r_0\Sigma^{1/3} = 0.704(52)$$

compare to direct determination [talk by E. García Ramos]

$$r_0\Sigma^{1/3}_{cont,N_f=2+1+1} = 0.680(29)$$

# Fit excluding pion masses >400 MeV

tree-level formula of $\chi$PT: $r_0^4\chi = \frac{r_0^3\Sigma \cdot r_0\mu_R}{2}$




$$r_0\Sigma^{1/3} = 0.721(53)$$

compare to direct determination [talk by E. García Ramos]

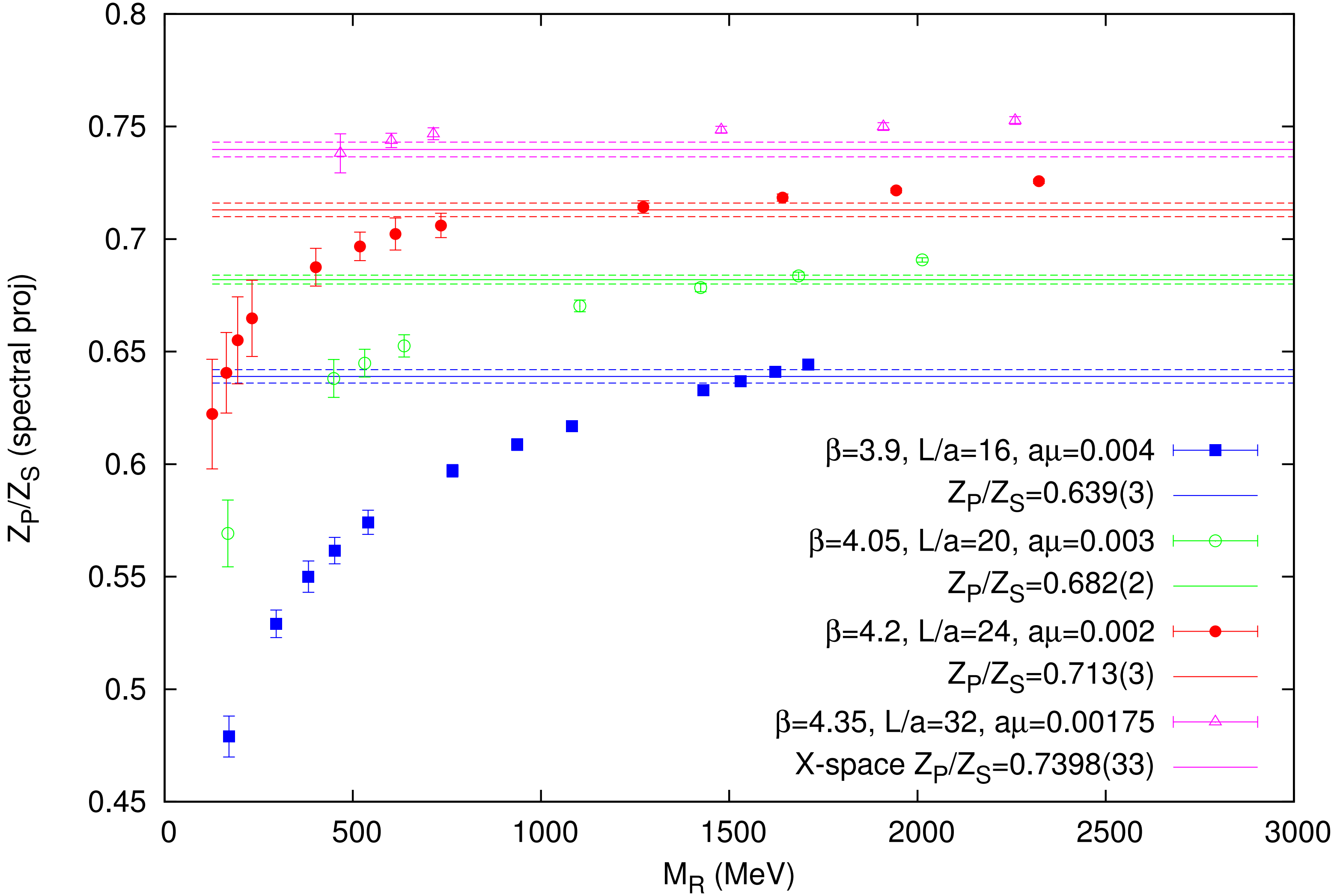
$$r_0\Sigma^{1/3}_{cont,N_f=2+1+1} = 0.680(29)$$

# Results – quenched

| $\beta$ | $\kappa_c$ | $(L/a)^3 \times T/a$ | $a\mu$ | $r_0/a$ | lat.spac. [fm] |
|---|---|---|---|---|---|
| 2.37 | 0.158738 | $20^3 \times 40$ | 0.0087 | 3.593(35) | 0.139 |
| 2.48 | 0.154928 | $24^3 \times 48$ | 0.0074 | 4.233(55) | 0.118 |
| 2.67 | 0.150269 | $32^3 \times 64$ | 0.0055 | 5.691(32) | 0.088 |
| 2.85 | 0.147180 | $40^3 \times 80$ | 0.0043 | 7.290(68) | 0.069 |

$N_f = 0$ configurations generated with the Iwasaki gauge action

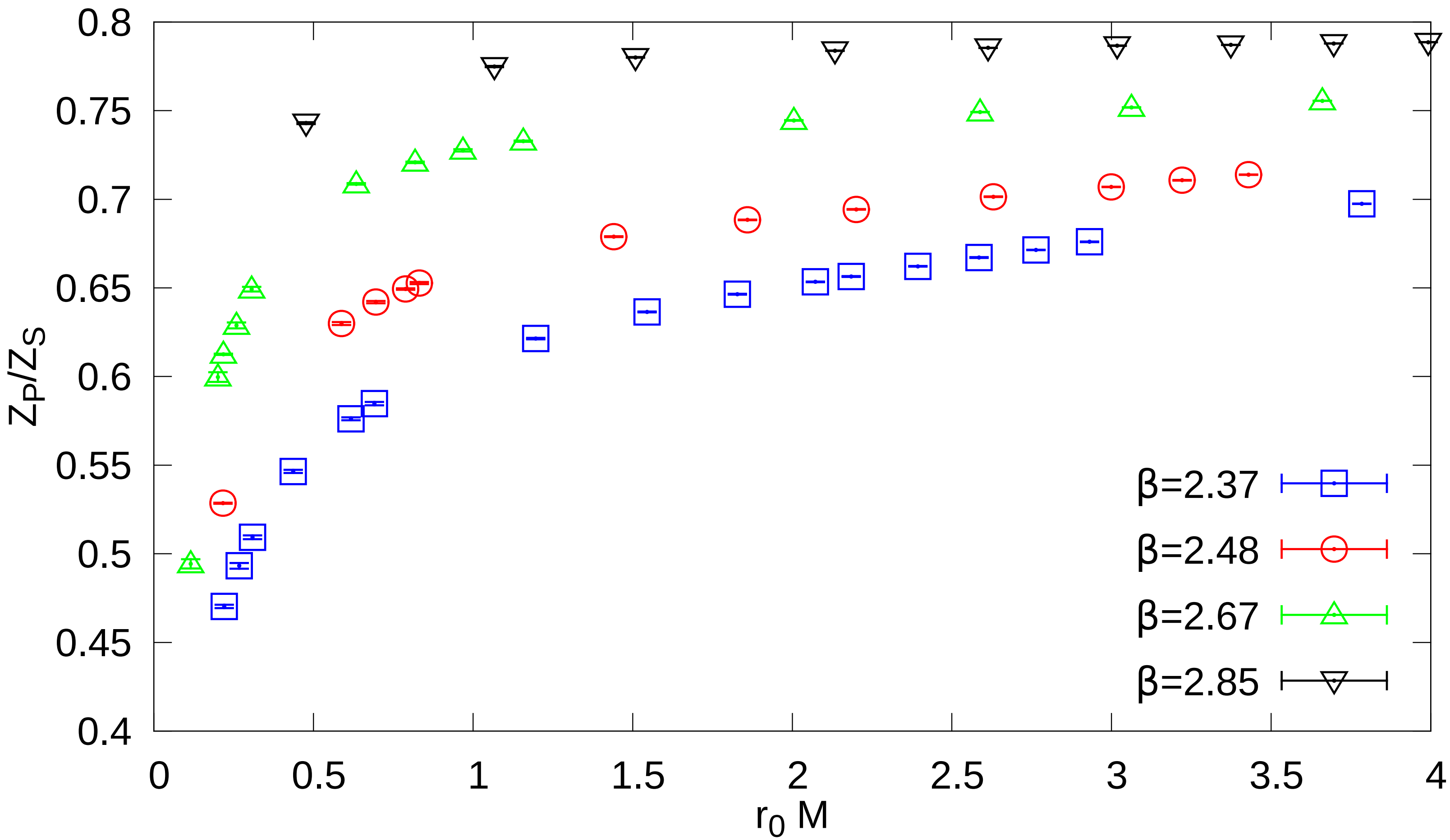
# $Z_P/Z_S$ – $M_R$-dependence for $N_f = 2$




RI-MOM results ($\beta = 3.9,\ 4.05,\ 4.2$) [C. Alexandrou et al. 2012]

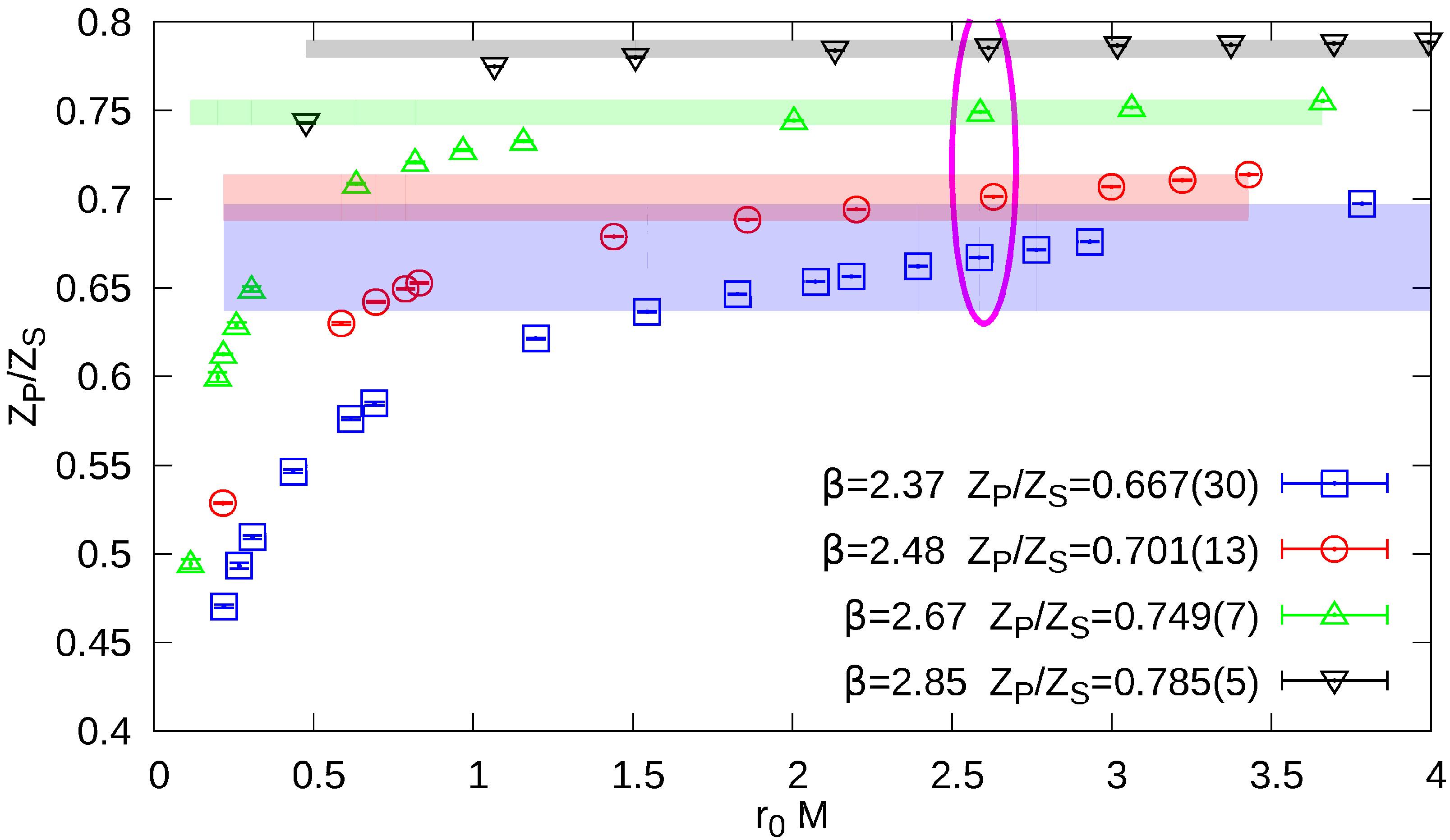
X-space result ($\beta = 4.35$) [K. Cichy, K. Jansen, P. Korcyl 2012]

# $Z_P/Z_S$ – $M_R$-dependence for $N_f = 0$



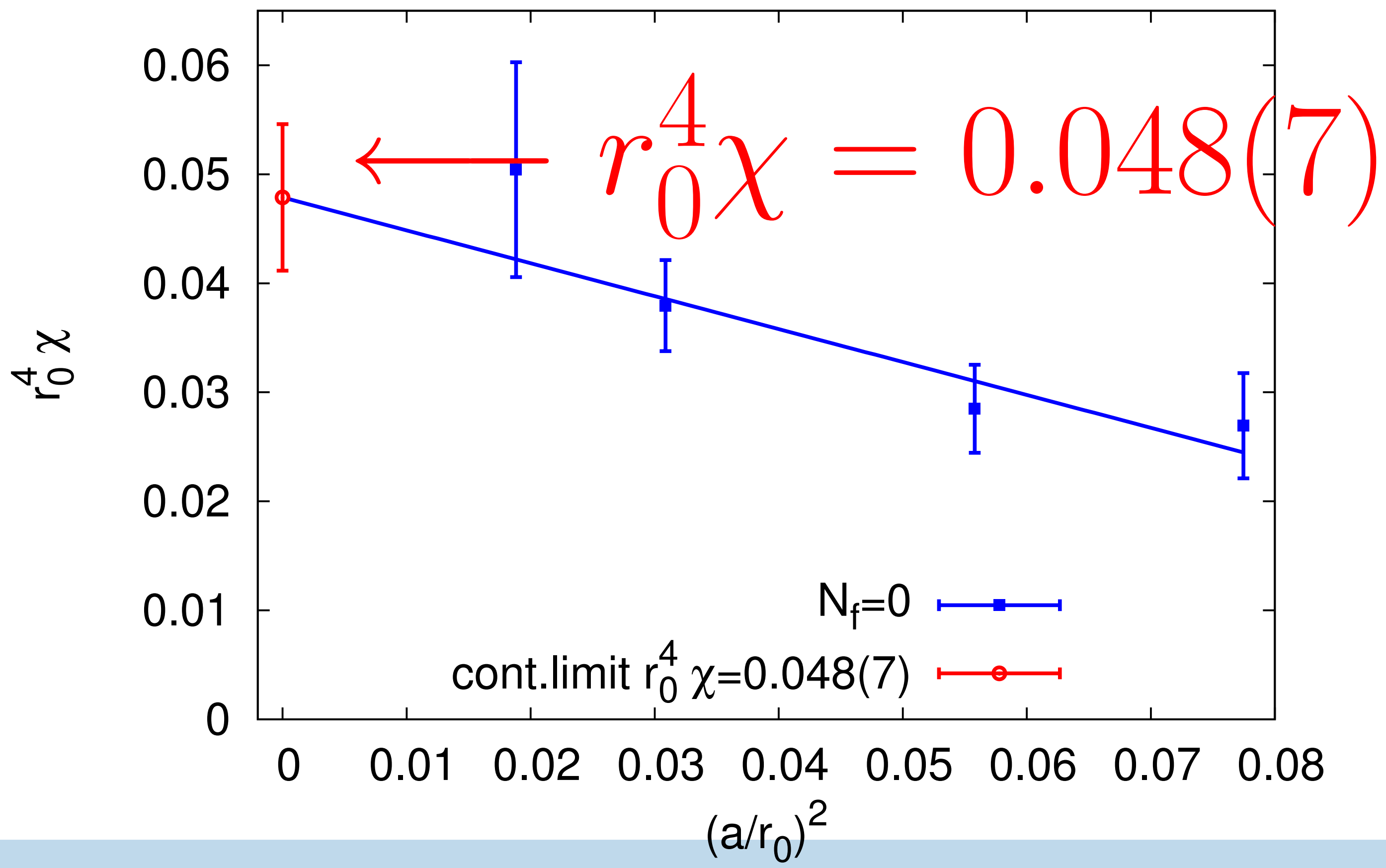
# $Z_P/Z_S$ – $M_R$-dependence for $N_f = 0$





# Results – $N_f = 0$

| $\beta$ | $a^4\chi$ | $r_0/a$ | $Z_P/Z_S$ | $r_0^4\chi$(stat.)$(r_0/a)(Z_P/Z_S)$ |
|---|---|---|---|---|
| 2.37 | 7.19(36)e-5 | 3.593(35) | 0.667(30) | 0.0269(14)(10)(24) |
| 2.48 | 4.36(23)e-5 | 4.233(55) | 0.701(13) | 0.0285(15)(15)(11) |
| 2.67 | 2.03(14)e-5 | 5.691(32) | 0.749(7) | 0.0379(27)(9)(7) |
| 2.85 | 1.10(16)e-5 | 7.290(68) | 0.785(5) | 0.0504(71)(19)(10) |



Final result: $r_0^4\chi = 0.048(7)$

using $r_0 = 0.5$ fm: $\chi = (185 \pm 7\ \mathrm{MeV})^4$

compare to: $r_0^4\chi = 0.059(3)$ [L. Del Debbio, L. Giusti, C. Pica, 2005]

using $r_0 f_K$ to set the scale: $\chi = (191 \pm 5\ \mathrm{MeV})^4$
or using $r_0 = 0.5$ fm: $\chi = (194.5 \pm 2.4\ \mathrm{MeV})^4$

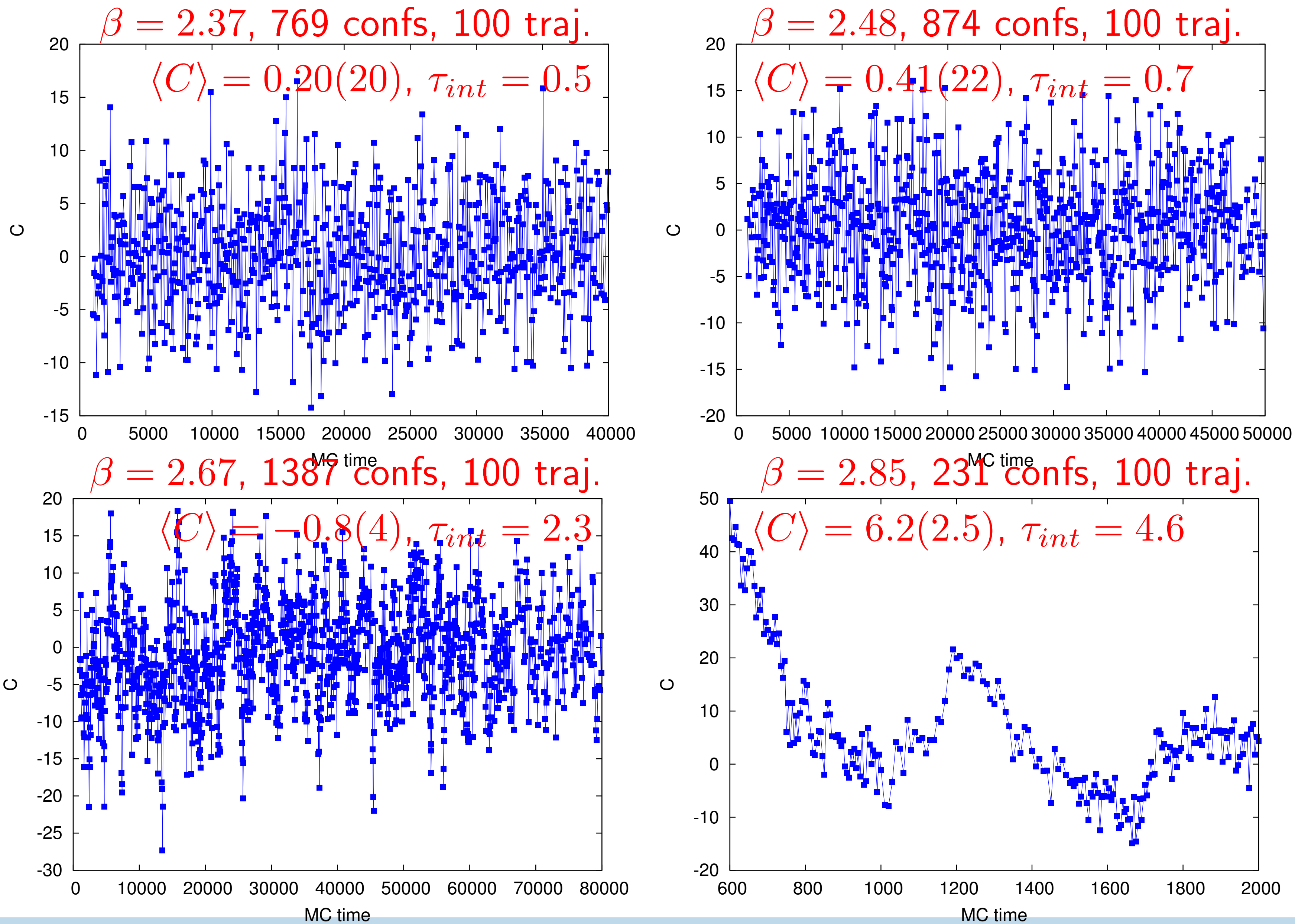
$r_0^4\chi = 0.061(6)$ [M. Lüscher, F. Palombi, 2010]
using $r_0 = 0.5$ fm: $\chi = (196.5 \pm 5.1\ \mathrm{MeV})^4$

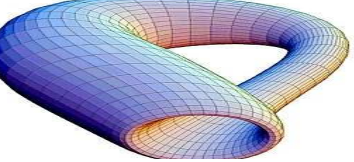
Witten-Veneziano formula [E. Witten, 1979], [G. Veneziano, 1979]
explains the origin of the mass of the $\eta'$ meson (non-zero in the chiral limit)
real-world $m_{\eta'} = 957.66(24)$ MeV

$$\frac{f_\pi^2}{6}\left(m_\eta^2 + m_{\eta'}^2 - 2m_K^2\right) = \chi_\infty$$

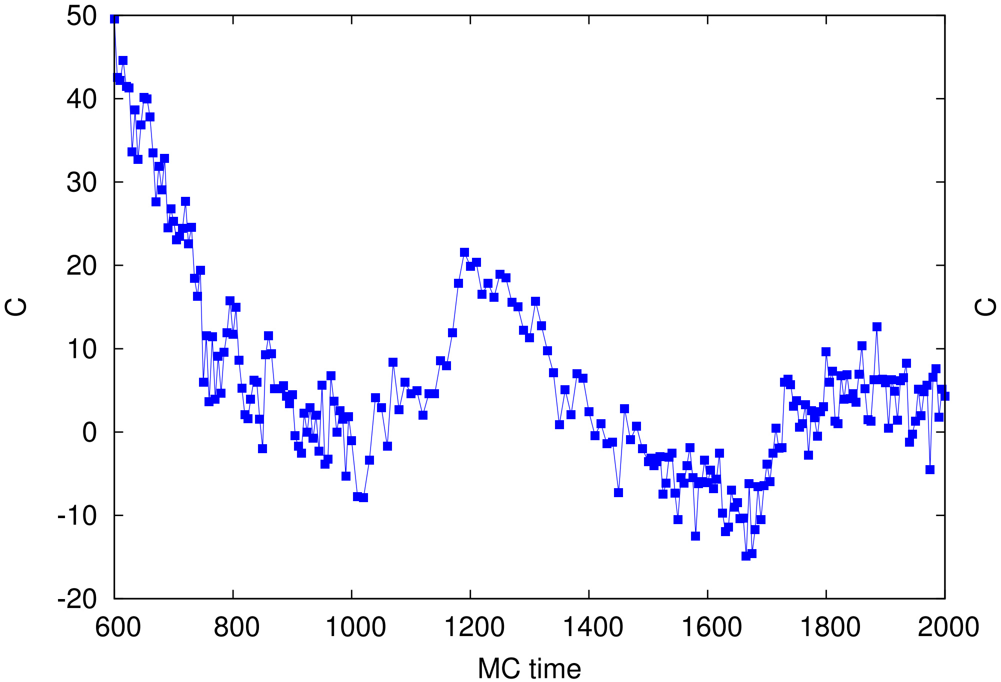
leading order in 't Hooft's large-$N_c$ limit ($N_c \to \infty$, $g \to 0$, $\lambda = g^2 N_c$ fixed)
LHS: computed in full QCD, experimental: $(180\ \mathrm{MeV})^4$
RHS: computed in the quenched approximation

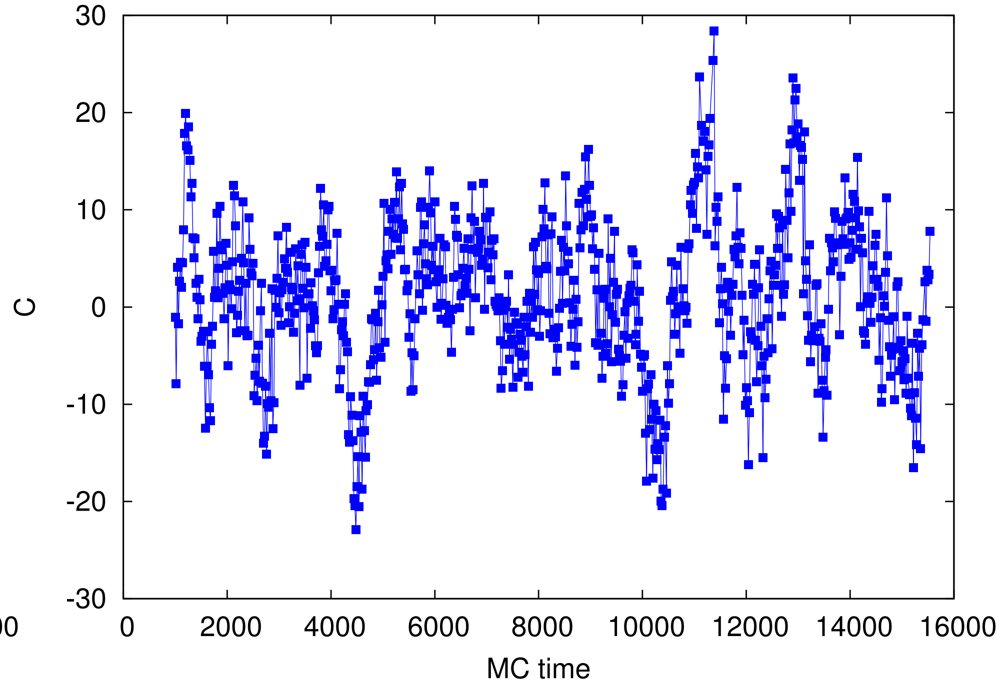
# Autocorrelations





# Autocorrelations



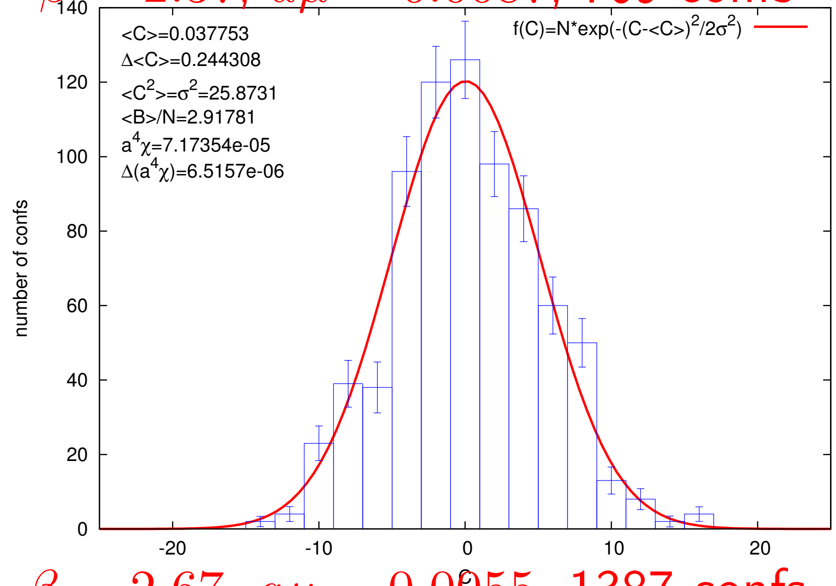




$\beta = 2.85$, 231 confs, 100 traj.
$\langle C \rangle = -6.2(2.5)$, $\tau_{int} = 4.6$

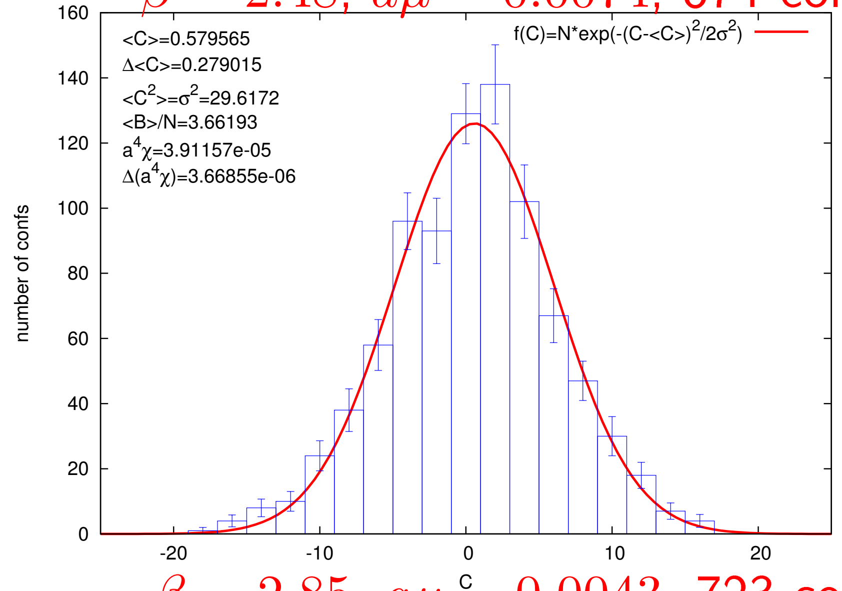
$\beta = 2.85$, 723 confs, 400 traj.
$\langle C \rangle = 1.1(7)$, $\tau_{int} = 3.6$

# Histograms of topological charge

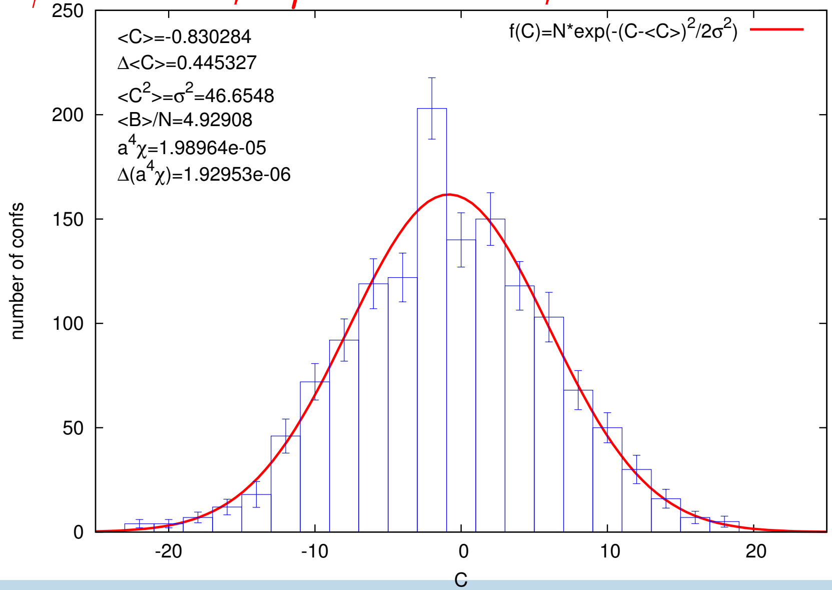
$\beta = 2.37,\ a\mu = 0.0087$, 769 confs



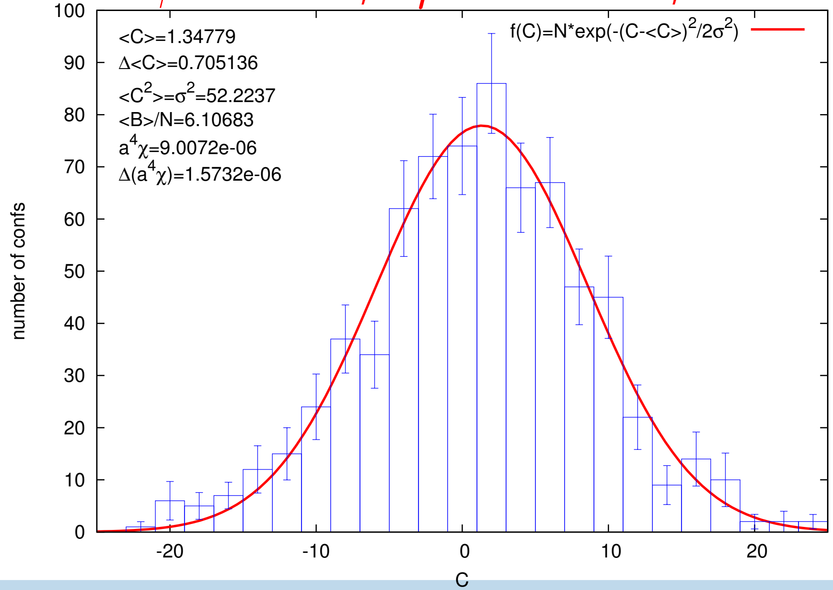

$\beta = 2.48,\ a\mu = 0.0074$, 874 confs




$\beta = 2.67,\ a\mu = 0.0055$, 1387 confs




$\beta = 2.85,\ a\mu = 0.0043$, 723 confs



# Conclusions

- Computing the topological susceptibility for dynamical simulations is very difficult:
  - ⋆ one has to sample properly the topological charge distribution, such that one obtains a Gaussian distribution and $\langle Q_{\rm top} \rangle = 0$,
  - ⋆ long autocorrelations at small lattice spacings
  - ⋆ with spectral projectors: statistics of $\mathcal{O}(200)$ confs gives a statistical error of $\mathcal{O}(10-20)\%$ (for $\Sigma$ it is only $\mathcal{O}(1-2)\%$)
- Still, the spectral projector method seems to be a very promising approach:
  - ⋆ especially if one can afford longer runs
  - ⋆ much cheaper than the index method
  - ⋆ and other methods have theoretical problems (e.g. gluonic definition of $Q_{\rm top}$)
- $\Sigma$ extracted from $\chi$ vs. $\mu$ dependence agrees with the one from direct calculation (but: rather large error and neglecting higher orders of $\chi$PT)
- Quenched case: $\chi_\infty$ agrees with earlier determinations; good agreement with the Witten-Veneziano relation

Thank you for attention!