Computation of the chiral condensate using Nf=2 and Nf=2+1+1 twisted mass fermions at maximal twist

Elena García Ramos
in collaboration with
K. Cichy, K. Jansen, A. Shindler

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Index

Chiral Condensate and Banks-Casher Relation

- In the continuum: (Banks & Casher, 1980)

$$\frac{\Sigma}{\pi} = \lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\rho(\lambda, m)$$

$$\rho(\lambda, m) = \frac{1}{V}\sum_{k=1}^{\infty}\langle\delta(\lambda-\lambda_k)\rangle\,, \qquad \Sigma = -\lim_{m\to 0}\lim_{V\to\infty}\langle\bar{u}u\rangle$$

- mode number $\nu \rightsquigarrow$ average number of eigenmodes of $D_m^\dagger D_m$ with $\lambda \leq M^2$

$$\nu(M, m) = V\int_{-\Lambda}^{\Lambda} d\lambda\rho(\lambda, m), \qquad \Lambda = \sqrt{M^2 - m^2}$$

$\nu(M, m) = \nu_R(M_R, m_R)$ $\rightsquigarrow$ renormalization-group invariant (Giusti & Lüscher, 2008)

- For non-vanishing mass and finite volume

$$\Sigma_R \propto \frac{\partial}{\partial M_R}\nu_R$$

- Direct relation between ν and spectral sum σ_k

$$\sigma_k(\mu, m) = \int_0^\infty dM\,\nu(M, m)\frac{2kM}{(M^2+\mu^2)^{k+1}}, \qquad \sigma_k(\mu, m) = \left\langle \mathrm{Tr}\left\{(D^\dagger D + \mu^2)^{-k}\right\}\right\rangle$$

Mode number and Spectral Projectors

- Spectral Projector $\mathbb{P}_M$ to compute $\nu(M, m)$ (Giusti & Lüscher, 2008)

$$\nu(M, m_q) = \langle \mathrm{Tr}\{\mathbb{P}_M\} \rangle$$

- Approximation of $\mathbb{P}_M$:

$$\mathbb{P}_M \approx h(\mathbb{X})^4, \qquad \mathbb{X} = 1 - \frac{2M_\star^2}{D_m^\dagger D_m + M_\star^2}, \qquad M_\star \approx M$$

⇝ $h(x)$ is an approximation to the step function $\theta(-x)$ in the interval $[-1, 1]$.

$$h(x) = \frac{1}{2}\{1 - xP(x^2)\}$$

where $P(x)$ is the polynomial which minimizes

$$\delta = \max_{\epsilon \leq y \leq 1} \|1 - \sqrt{y}P(y)\|$$

-

$$\nu(M, m_q) = \langle \mathcal{O}_N \rangle \,, \quad \mathcal{O}_N = \frac{1}{N} \sum_{k=1}^{N} (\eta_k, \mathbb{P}_M \eta_k)$$

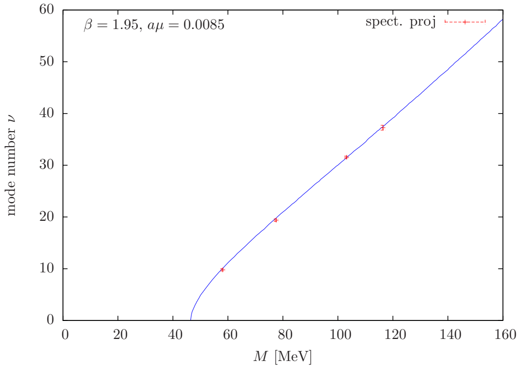
η_k sources generated randomly.

Index

M* for chiral condensate

We want to compute the mode number in the linear region to extract the chiral condensate.
(Giusti & Lüscher, 2009)

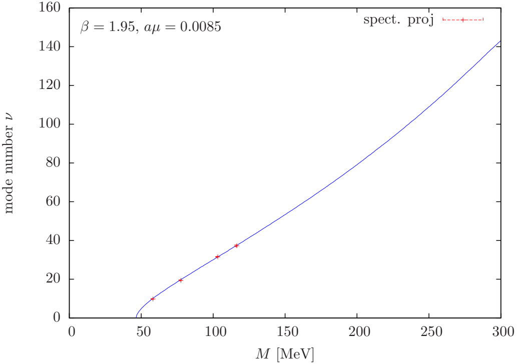
$$\Sigma_R = \frac{\pi}{2V}\sqrt{1-\left(\frac{m_R}{M_R}\right)^2}\,\frac{\partial}{\partial M_R}\nu_R$$



avoid values $\approx m_q$ and $>> m_q$

M* for chiral condensate

We want to compute the mode number in the linear region.



avoid values $\approx m_q$ and $>> m_q$

Index

Configurations setup

- Wilson Twisted Mass Action at maximal twist (Frezotti & Rossi, 2004)
- Tree-Level Symanzik Gauge Action (Weisz, 1982), (Lüscher & Weisz, 1985)
- $N_f = 2$ dynamical fermions (Boucaud et al., 2007,2008), (Baron et al.,2009)

Ensemble	β	lattice	$a\mu$	μ_R (MeV)	κ_c	L (fm)
b30.32	3.90	$32^3 \times 64$	0.003	16	0.160856	2.7
b40.16	3.90	$16^3 \times 32$	0.004	21	0.160856	1.4
b40.20	3.90	$20^3 \times 40$	0.004	21	0.160856	1.7
b40.24	3.90	$24^3 \times 48$	0.004	21	0.160856	2.0
b40.32	3.90	$32^3 \times 64$	0.004	21	0.160856	2.7
b64.24	3.90	$24^3 \times 48$	0.0064	34	0.160856	2.0
b85.24	3.90	$24^3 \times 48$	0.0085	45	0.160856	2.0
c30.32	4.05	$32^3 \times 64$	0.003	19	0.157010	2.1
c60.32	4.05	$32^3 \times 64$	0.006	37	0.157010	2.1
c80.32	4.05	$32^3 \times 64$	0.008	49	0.157010	2.1
d20.48	4.20	$48^3 \times 96$	0.002	15	0.154073	2.6
d65.32	4.20	$32^3 \times 64$	0.0065	47	0.154073	1.7

Configurations setup

- Wilson Twisted Mass Action at maximal twist (Frezzotti, Rossi, 2003, 2004)
- Iwasaki Gauge Action (Iwasaki, 1985)
- $N_f = 2 + 1 + 1$ dynamical fermions (Baron et al., 2010, 2011)

Ensemble	β	lattice	$a\mu_l$	$\mu_{l,R}$ (MeV)	κ_c	L (fm)
A30.32	1.90	$32^3 \times 64$	0.0030	13	0.163272	2.8
A40.20	1.90	$20^3 \times 40$	0.0040	17	0.163270	1.7
A40.24	1.90	$24^3 \times 48$	0.0040	17	0.163270	2.1
A40.32	1.90	$32^3 \times 64$	0.0040	17	0.163270	2.8
A50.32	1.90	$32^3 \times 64$	0.0050	22	0.163267	2.8
A60.24	1.90	$24^3 \times 48$	0.0060	26	0.163265	2.1
A80.24	1.90	$24^3 \times 48$	0.0080	35	0.163260	2.1
B25.32	1.95	$32^3 \times 64$	0.0025	13	0.161240	2.5
B35.32	1.95	$32^3 \times 64$	0.0035	18	0.161240	2.5
B55.32	1.95	$32^3 \times 64$	0.0055	28	0.161236	2.5
B75.32	1.95	$32^3 \times 64$	0.0075	38	0.161232	2.5
B85.24	1.95	$24^3 \times 48$	0.0085	45	0.161231	1.9
D15.48	2.10	$48^3 \times 96$	0.0015	9	0.156361	2.9
D20.48	2.10	$48^3 \times 96$	0.0020	12	0.156357	2.9
D30.48	2.10	$48^3 \times 96$	0.0030	19	0.156355	2.9

Index

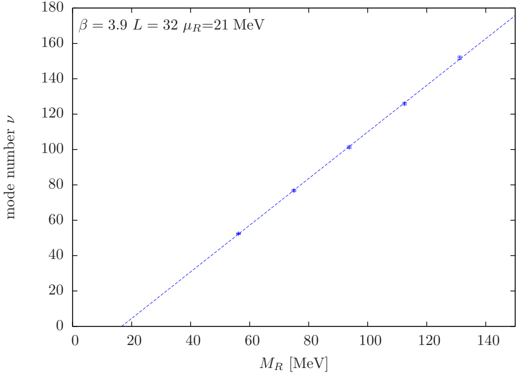
Extracting Σ_R from ν_R

(Giusti & Lüscher, 2008)

$$\Sigma_R = \frac{\pi}{2V}\sqrt{1-\left(\frac{m_R}{M_R}\right)^2}\,\frac{\partial}{\partial M_R}\nu_R$$

$\nu(M,m) = \nu_R(M_R,m_R)$ $\rightsquigarrow$ renormalization-group invariant

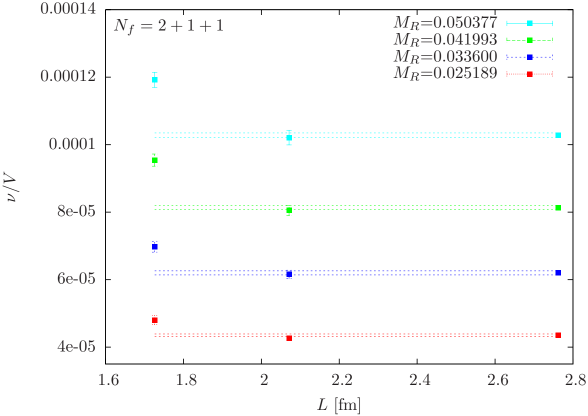
- We extract the term $\frac{\partial}{\partial M_R}\nu_R$ through the slope of a linear fit.



Finite Volume Effects $\frac{\nu}{vol}$ $N_f = 2+1+1$

Study of the finite size effects for the chiral condensate Σ.

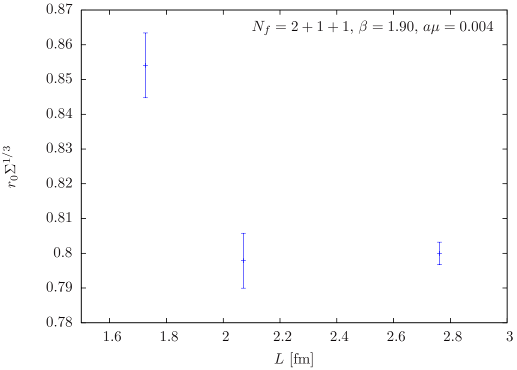
$\frac{\nu}{vol} = const$



Analytical calculation finite volume corrections for ν (Necco & Shindler, 2011)

Finite Volume Effects Σ $N_f = 2 + 1 + 1$

Study of the finite size effects for the chiral condensate Σ. $\frac{\nu}{vol} = const$



$\mathcal{O}(a)$ improvement

- ⋆ Spectral sum as density chain

$$\sigma_3(\mu, m) = -a^{24} \sum_{x_1,\dots,x_5} \left\langle P^+_{12}(x_1) P^-_{23}(x_2) P^+_{34}(x_3) P^-_{45}(x_4) P^+_{56}(x_5) P^-_{61}(0) \right\rangle$$

$\rightsquigarrow$ even under $\mathcal{R}^1_5$ transformations.

- ⋆ Symanzik expansion at maximal twist:

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^+_{12}(x_1) P^-_{23}(x_2) P^+_{34}(x_3) P^-_{45}(x_4) P^+_{56}(x_5) P^-_{61}(0) \right\rangle =$$

$$= \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^+_{12}(x_1) P^-_{23}(x_2) P^+_{34}(x_3) P^-_{45}(x_4) P^+_{56}(x_5) P^-_{61}(0) \right\rangle_0 + \begin{array}{c}\text{contact}\\ \text{terms}\end{array} + \mathcal{O}(a^2)$$

⋆ Contacts terms $\longrightarrow$ OPE

$\downarrow$

Only $P^+_{ab}(x) P^-_{bc}(0)$ leads to $\mathcal{O}(a)$ terms

$\downarrow$

$$P^+_{ab}(x) P^-_{bc}(0) \sim^{x\to 0} C(x) S^{\uparrow}_{ac}(0) + \dots, \qquad S^{\uparrow}_{ac} = \overline{\psi}_a \frac{1}{2}(\mathbb{1} + \tau^3)\psi_c$$

- Contact terms take the form:

$$\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle S^{\uparrow}_{13}(x_2) P^+_{34}(x_3) P^-_{45}(x_4) P^+_{56}(x_5) P^-_{61}(0) \right\rangle_0$$

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$\rightsquigarrow$ even under $\mathcal{R}_5^1$ transformations.

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$$\sigma_3(\mu, m) = -a^{24} \sum_{x_1,\ldots,x_5} \left\langle P_{12}^+(x_1)P_{23}^-(x_2)P_{34}^+(x_3)P_{45}^-(x_4)P_{56}^+(x_5)P_{61}^-(0)\right\rangle$$

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Only $P_{ab}^+(x)P_{bc}^-(0)$ leads to $\mathcal{O}(a)$ terms

$\downarrow$

$$P_{ab}^+(x)P_{bc}^-(0) \sim^{x\to 0} C(x)S_{ac}^{\uparrow}(0) + \ldots, \qquad S_{ac}^{\uparrow} = \overline{\psi}_a \frac{1}{2}(\mathbb{1} + \tau^3)\psi_c$$

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$\downarrow$

Only $P^+_{ab}(x)P^-_{bc}(0)$ leads to $\mathcal{O}(a)$ terms

$\downarrow$

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$\mathcal{O}(a)$ improvement

⋆ non-singlet axial Ward-Takahashi identity:

$$\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle S^{\uparrow}_{13}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) P^{-}_{61}(0) \right\rangle + \tag{1}$$
$$+ \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^{-}_{23}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) S^{\uparrow}_{62}(0) \right\rangle$$
$$= 2m_q \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \int d^4x_1 \left\langle P^{+}_{12}(x_1) P^{-}_{23}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) P^{-}_{61}(0) \right\rangle$$

⋆ Substitute in Symanzik expansion

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1) P_{23}(x_2) P_{34}(x_3) P_{45}(x_4) P_{56}(x_5) P_{61}(0) \rangle =$$
$$= \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1) P_{23}(x_2) P_{34}(x_3) P_{45}(x_4) P_{56}(x_5) P_{61}(0) \rangle_0$$
$$+ a6c_2 m_q \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1) P_{23}(x_2) P_{34}(x_3) P_{45}(x_4) P_{56}(x_5) P_{61}(0) \rangle_0$$
$$+ \mathcal{O}(a^2)$$

⋆ At maximal twist $m_q = 0 \longrightarrow$ we recover $\mathcal{O}(a)$ improvement.

$\mathcal{O}(a)$ improvement

- non-singlet axial Ward-Takahashi identity:

$$\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle S^{\uparrow}_{13}(x_2)P^{+}_{34}(x_3)P^{-}_{45}(x_4)P^{+}_{56}(x_5)P^{-}_{61}(0)\right\rangle + \quad (1)$$
$$+ \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^{-}_{23}(x_2)P^{+}_{34}(x_3)P^{-}_{45}(x_4)P^{+}_{56}(x_5)S^{\uparrow}_{62}(0)\right\rangle$$
$$= 2m_q \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \int d^4x_1 \left\langle P^{+}_{12}(x_1)P^{-}_{23}(x_2)P^{+}_{34}(x_3)P^{-}_{45}(x_4)P^{+}_{56}(x_5)P^{-}_{61}(0)\right\rangle$$

- Substitute in Symanzik expansion

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(0)\right\rangle =$$
$$= \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(0)\right\rangle_0$$
$$+ a6c_2m_q \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(0)\right\rangle_0$$
$$+ \mathcal{O}(a^2)$$

- At maximal twist $m_q = 0 \longrightarrow$ we recover $\mathcal{O}(a)$ improvement.

$\mathcal{O}(a)$ improvement

- non-singlet axial Ward-Takahashi identity:

$$\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle S^{\uparrow}_{13}(x_2)P^{+}_{34}(x_3)P^{-}_{45}(x_4)P^{+}_{56}(x_5)P^{-}_{61}(0)\right\rangle + \tag{1}$$

$$+ \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^{-}_{23}(x_2)P^{+}_{34}(x_3)P^{-}_{45}(x_4)P^{+}_{56}(x_5)S^{\uparrow}_{62}(0)\right\rangle$$

$$= 2m_q \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \int d^4x_1 \left\langle P^{+}_{12}(x_1)P^{-}_{23}(x_2)P^{+}_{34}(x_3)P^{-}_{45}(x_4)P^{+}_{56}(x_5)P^{-}_{61}(0)\right\rangle$$

- Substitute in Symanzik expansion

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(0)\rangle =$$

$$= \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(0)\rangle_0$$

$$+ a6c_2 m_q \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1)P_{23}(x_2)P_{34}(x_3)P_{45}(x_4)P_{56}(x_5)P_{61}(0)\rangle_0$$

$$+ \mathcal{O}(a^2)$$

- At maximal twist $m_q = 0 \longrightarrow$ we recover $\mathcal{O}(a)$ improvement.

$\mathcal{O}(a)$ improvement

- non-singlet axial Ward-Takahashi identity:

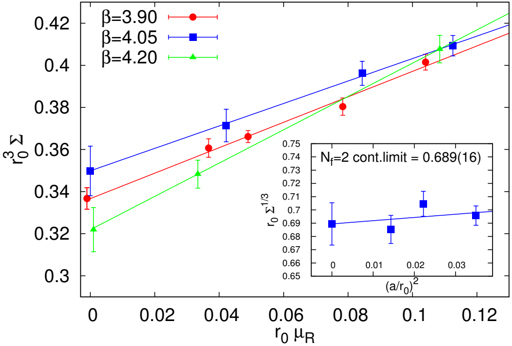
$$\int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle S^{\uparrow}_{13}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) P^{-}_{61}(0) \right\rangle + \tag{1}$$
$$+ \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \left\langle P^{-}_{23}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) S^{\uparrow}_{62}(0) \right\rangle$$
$$= 2m_q \int d^4x_2 d^4x_3 d^4x_4 d^4x_5 \int d^4x_1 \left\langle P^{+}_{12}(x_1) P^{-}_{23}(x_2) P^{+}_{34}(x_3) P^{-}_{45}(x_4) P^{+}_{56}(x_5) P^{-}_{61}(0) \right\rangle$$

- Substitute in Symanzik expansion

$$\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1) P_{23}(x_2) P_{34}(x_3) P_{45}(x_4) P_{56}(x_5) P_{61}(0) \rangle =$$
$$= \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 d^4x_5 \langle P_{12}(x_1) P_{23}(x_2) P_{34}(x_3) P_{45}(x_4) P_{56}(x_5) P_{61}(0) \rangle_0$$
$$+ \mathcal{O}(a^2)$$

- At maximal twist $m_q = 0 \longrightarrow$ we recover $\mathcal{O}(a)$ improvement.

Chiral and Continuum Limit of Σ $\quad N_f = 2$



⋆ Chiral extrapolation ⇝ (Giusti & Lüscher, 2008)

β	$r_0\Sigma^{1/3}$
3.9	0.6957 (35)(37)(52)(186)
4.05	0.7046 (78)(30)(42)(206)
4.2	0.6853 (73)(59)(49)(265)

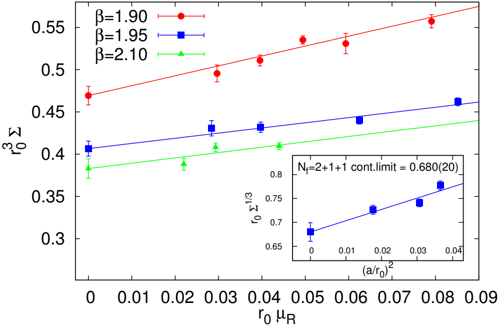
errors→ (stat)(Z_P)(r_0)(fit)

⋆ Continuum limit

$$r_0\Sigma^{1/3} = 0.689(16)(29)$$

errors→ (combined)(fit)

Chiral and Continuum Limit of Σ $\quad N_f = 2 + 1 + 1$



⋆ Chiral extrapolation ⇝ (Giusti & Lüscher, 2008)

β	$r_0\Sigma^{1/3}$
1.9	0.7772 (61)(44)(56)(157)
1.95	0.7408 (55)(25)(53)(112)
2.1	0.7262 (72)(14)(56)(75)

errors→ (stat)(Z_P)(r_0)(fit)

⋆ Continuum limit

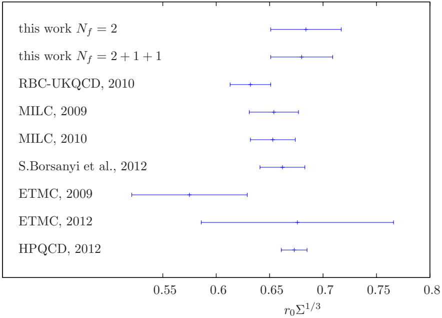
$$r_0\Sigma^{1/3} = 0.680(20)(21)$$

errors→ (combined)(fit)

Comparison of continuum results for $r_0\Sigma^{1/3}$

Result	method	N_f	fermions	$r_0\Sigma^{1/3}$
this work	spectral proj.	2	twisted mass	0.689(16)(29)
this work	spectral proj.	2+1+1	twisted mass	0.680(20)(21)
RBC-UKQCD (1)	chiral fits	2+1	domain wall	0.632(15)(12)
MILC (2)	chiral fits	2+1	staggered	0.654(14)(18)
MILC (3)	chiral fits	2+1	staggered	0.653(18)(11)
(4)	chiral fits	2+1	staggered	0.662(5)(20)
ETMC (5)	chiral fits	2	twisted mass	0.575(14)(52)
ETMC (6)	quark propagator	2	twisted mass	0.676(89)(14)
HPQCD (7)	quark propagator	2+1+1	staggered	0.673(5)(11)

(1) Aoki et al., 2010
(2) Bazavov et al., 2009
(3) Bazavov et al., 2010
(4) S. Borsanyi et al., 2012
(5) Baron et al., 2009
(6) Burger et al., 2012
(7) McNeile et al., 2012

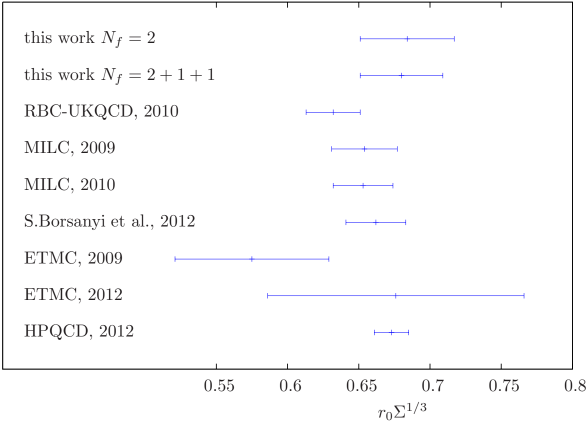


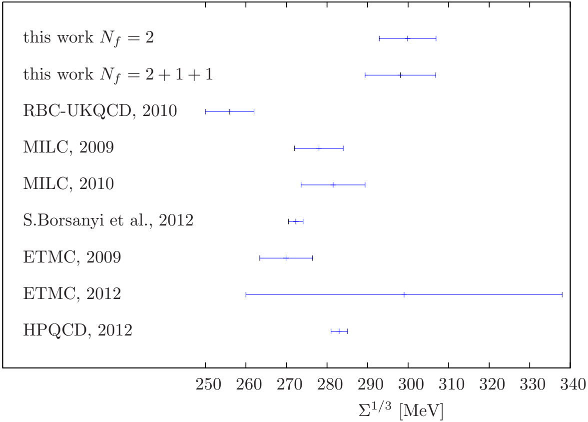
Index

Conclusions and outlook

- We have applied the spectral projector method using $N_\mathrm{f} = 2$ and $N_\mathrm{f} = 2+1+1$ twisted mass ensembles generated by ETMC.
- We have analyzed the finite volume effects for the mode number and the chiral condensate.
- We have computed the continuum limit of the chirally extrapolated chiral condensate.
  - ⋆ $N_\mathrm{f} = 2$ using different quark masses for 3 different lattice spacings.
  - ⋆ $N_\mathrm{f} = 2+1+1$ using different quark masses for 3 different lattice spacings.
  - ⋆ Our results are consistent with the results of other groups.
- We have applied this method to compute other quantities: See Krzysztof Cichy talk!
  - ⋆ χ_top: quenched & dynamical
  - ⋆ Z_P/Z_S

Thank you for your attention!


this work $N_f = 2$
this work $N_f = 2+1+1$
RBC-UKQCD, 2010
MILC, 2009
MILC, 2010
S.Borsanyi et al., 2012
ETMC, 2009
ETMC, 2012
HPQCD, 2012
0.55
0.6
0.65
0.7
0.75
0.8
$r_0\Sigma^{1/3}$


this work $N_f = 2$
this work $N_f = 2+1+1$
RBC-UKQCD, 2010
MILC, 2009
MILC, 2010
S.Borsanyi et al., 2012
ETMC, 2009
ETMC, 2012
HPQCD, 2012
250 260 270 280 290 300 310 320 330 340
$\Sigma^{1/3}$ [MeV]