Chiral behavior of pion properties from lattice QCD

Alfonso Sastre

CPT, Marseille

Budapest-Marseille-Wuppertal collaboration

Mainz, August 2, 2013

Motivation

$SU(3)_c$ + two massless quarks (u,d)

$$SU(2)_R \times SU(2)_L \times U(1)_V \xrightarrow[S\chi SB]{} \begin{array}{c} SU(2)_V \times U(1)_V \\ \text{3 Goldstone Bosons } (\pi^{\pm}, \pi^0) \end{array}$$

Real World: Quarks masses are small but different from zero

Pseudo Goldstone Bosons (PGB) with $M_\pi > 0$

Chiral Perturbation Theory (χPT)

- Effective theory of PGBs.
- ✓ Perturbative expansion in term s of PGB momenta and masses.
- × Infinite new parameters.

Motivation

$SU(3)_c$ + two massless quarks (u,d)

$$SU(2)_R \times SU(2)_L \times U(1)_V \xrightarrow[S\chi SB]{} \begin{array}{c} SU(2)_V \times U(1)_V \\ \text{3 Goldstone Bosons } (\pi^{\pm}, \pi^0) \end{array}$$

Real World: Quarks masses are small but different from zero

Pseudo Goldstone Bosons (PGB) with $M_\pi > 0$

Chiral Perturbation Theory (χPT)

- Effective theory of PGBs.
- ✓ Perturbative expansion in term s of PGB momenta and masses.
- × Infinite new parameters.
- ✓ Only a finite number at each order.
- ✓ All these parameters are determined by QCD.

SU(2) Chiral Perturbation Theory (χPT)

Gasser,Leutwyler Nucl.Phys.B(85), Ann.Phys.158(84)

Leading Order (LO) $\mathcal{O}(p^2)$

$$F = \lim_{m_q \to 0} F_\pi \text{ and } B = \lim_{m_q \to 0} \frac{\langle 0 | \bar{q}q | 0 \rangle}{F_\pi^2}$$

Next to Leading Order (NLO) $\mathcal{O}(p^4)$

Seven new terms in the Lagrangian $\mathcal{O}(p^4)$: $\bar{l}_i,\ i = 1, .., 7$

Next to Next to Leading Order (NNLO) $\mathcal{O}(p^6)$

Many other terms ....

Are we able to compute these numbers from QCD?

Different approaches to LEC determination.

$N_f = 2$

- ETM, MILC, JLQCD/TWQCD, CERN, HHS.

$N_f = 2 + 1$

- MILC, JLQCD/TWQCD, RBC/UKQCD, PACS-CS, BMWc.
- Necessary to fix the m_s dependence.

$N_f = 2 + 1 + 1$

- ETM
- Necessary to fix the m_s, m_c dependence.

Results are reviewed by FLAG Eur.Phys.J. C71 (2011) 1695

Apologies if I forget someone

Quark-mass dependence of M_π^2 and F_π

Colangelo, Gasser, Leutwyler Nucl.Phys.B603(01)

x-expansion $\left(x = \frac{M^2}{(4\pi F)^2}\right)$, $M^2 = 2Bm_{ud}$

$$M_\pi^2 = M^2\left(1 - \frac{x}{2}\log\left(\frac{\Lambda_3^2}{M^2}\right) + x^2\left(\frac{17}{8} log(\frac{\Lambda_M^2}{M^2}) + k_M\right) + \mathcal{O}(x^3)\right)$$

$$F_\pi = F\left(1 + x\log\left(\frac{\Lambda_4^2}{M^2}\right) + x^2\left(-\frac{5}{4} log(\frac{\Lambda_F^2}{M^2}) + k_F\right) + \mathcal{O}(x^3)\right)$$

$$\log\left(\frac{\Lambda_M^2}{M^2}\right) = \frac{1}{51}\left(4\left(7\log\left(\frac{\Lambda_1^2}{M^2}\right) + 8\log\left(\frac{\Lambda_2^2}{M^2}\right)\right) - 9\log\left(\frac{\Lambda_3^2}{M^2}\right) + 49\right)$$

$$\log\left(\frac{\Lambda_F^2}{M^2}\right) = \frac{1}{30}\left(2\left(7\log\left(\frac{\Lambda_1^2}{M^2}\right) + 8\log\left(\frac{\Lambda_2^2}{M^2}\right)\right) + 6\left(\log\left(\frac{\Lambda_3^2}{M^2}\right) - \log\left(\frac{\Lambda_4^2}{M^2}\right)\right)\right)$$

$$\bar{l}_i \equiv \log\left(\frac{\Lambda_i^2}{M_\pi^2}\right)$$

Quark-mass dependence of M_π^2 and F_π

Colangelo,Gasser,Leutwyler Nucl.Phys.B603(01)

ξ-expansion $\left(\xi = \frac{M_\pi^2}{(4\pi F_\pi)^2}\right)$

$$M^2 = M_\pi^2 \left(1 + \frac{\xi}{2} \log\left(\frac{\Lambda_3^2}{M_\pi^2}\right) + \xi^2 \left(log(\frac{\Omega_M^2}{M_\pi^2}) + \kappa_M\right) + \mathcal{O}(\xi^3)\right)$$

$$F = F_\pi \left(1 - \xi \log\left(\frac{\Lambda_4^2}{M^2}\right) + \xi^2 \left(log(\frac{\Omega_F^2}{M_\pi^2}) + \kappa_F\right) + \mathcal{O}(\xi^3)\right)$$

$$\log\left(\frac{\Omega_M^2}{M^2}\right) = \frac{1}{15}\left(-\left(4\log\left(7\frac{\Lambda_1^2}{M^2}\right) + 8\log\left(\frac{\Lambda_2^2}{M^2}\right)\right) - 33\log\left(\frac{\Lambda_3^2}{M^2}\right) - 12\log\left(\frac{\Lambda_4^2}{M^2}\right) + 52\right)$$

$$\log\left(\frac{\Omega_F^2}{M^2}\right) = \frac{1}{3}\left(-\left(7\log\left(\frac{\Lambda_1^2}{M^2}\right) + 8\log\left(\frac{\Lambda_2^2}{M^2}\right)\right) + 18\log\left(\frac{\Lambda_4^2}{M^2}\right) - \frac{29}{2}\right)$$

$$\bar{l}_i \equiv \log\left(\frac{\Lambda_i^2}{M_\pi^2}\right)$$

BMWc 2HEX. Action

Gauge Action

Lüscher, Weisz Phys.Lett.B158(85)

$$S_G = \frac{1}{\beta}\left[\frac{c_0}{3}\sum_{plaq}\mathrm{ReTr}\,(1 - U_{plaq}) + \frac{c_1}{3}\sum_{rect}\mathrm{ReTr}(1 - U_{rect})\right]$$

Fermionic action

Sheikholeslami, Wohlert Nucl.Phys.B259(85)

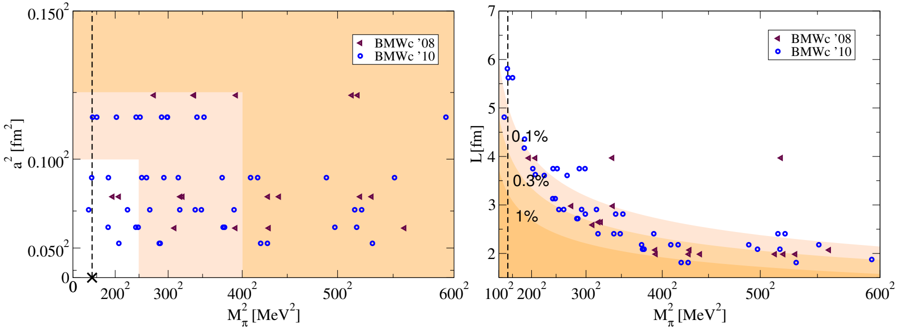
$$S_F = S_{\mathrm{Wilson}} - \frac{c_{SW}}{2}\sum_x\sum_{\mu<\nu}\left(\bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}[V]\psi\right)(x)$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and c_i are set to their tree level values.

- V refers to links which have undergone two HEX smearing.

Hasenfratz, Knechtli Phys.Rev.D64(01)
Morningstar, Peardon Phys.Rev.D69(04)
Capitani, Durr, Hölbling JHEP11(06)

BMWc 2HEX. Landscape



⇒ 47 points at five different lattice spacing: $a^{-1} \sim (1.697, 2.131, 2.561, 3.026, 3.662)$ GeV.

⇒ Pion masses around the physical point: $M_\pi^{\mathrm{MIN}} \sim (136, 131, 120, 182, 219)$ MeV.

⇒ Spacial volumes as large as $\sim (6\text{ fm})^3$ and temporal extents up ~ 8 fm.

What do we need from the lattice to compute LEC?

Inputs for χPT theory

- (am_{ud}), (aM_π), (aF_π)

Strange mass corrections

- (aM_{SS})

Scale Setting

- lattice spacing: a

Renormalization

- Z_S and Z_A

Quark masses

Improved ratio-different method

BMWc Phys.Lett.B701(11),JHEP 1108(11)

$$r = \frac{am_s^{PCAC}}{am_{ud}^{PCAC}}, \quad d = am_s^{\text{bare}} - am_{ud}^{\text{bare}}$$

$$r^{imp} = r, \quad d^{imp} = d\left(1 - \frac{d}{2}\frac{r+1}{r-1}\right)$$

$$am_{ud}^{\text{scheme}} = \frac{1}{Z_S^{\text{scheme}}}\frac{d^{imp}}{r^{imp}-1}, \quad am_s^{\text{scheme}} = \frac{1}{Z_S^{\text{scheme}}}\frac{r^{imp}d^{imp}}{r^{imp}-1}$$

$$am_1^{PCAC} + am_2^{PCAC} = \frac{\sum_{\vec{x}}\langle \partial_\mu[A_\mu(x)]\mathcal{O}(0)\rangle}{\sum_{\vec{x}}\langle P(x)\mathcal{O}(0)\rangle}$$

M_π and F_π constant from the Lattice

M_π and F_π are extracted from the combined fit of correlators

$$\sum_{\vec{x}} \left\langle (\bar{d}\gamma_5 u)(\vec{x},t)(\bar{u}\gamma_5 d)(0) \right\rangle = C_{PP} \cosh\left((aM_\pi)\left(\frac{T}{2} - t\right)\right)$$

$$\sum_{\vec{x}} \left\langle (\bar{d}\gamma_0\gamma_5 u)(\vec{x},t)(\bar{u}\gamma_5 d)(0) \right\rangle = C_{AP} \sinh\left((aM_\pi)\left(\frac{T}{2} - t\right)\right)$$

F_π

$$(aF_\pi) = \frac{C_{AP}}{\sqrt{2(aM_\pi)C_{PP}}} e^{\frac{(aM_\pi)T}{4}}$$

Strange quark effects and scale setting

$M_{SS}^2 \equiv 2M_k^2 - M_\pi^2$

$$\sum_{\vec{x}} \langle (\bar{s}\gamma_5 u)(\vec{x}, t)(\bar{u}\gamma_5 s)(0)\rangle = C'_{PP} \cosh\left((aM_K)\left(\frac{T}{2} - t\right)\right)$$

Scale setting

- $\frac{M_\pi}{M_\Omega}, \frac{M_K}{M_\Omega} \rightarrow$ Experimental value

Renormalization Z_S and Z_A

- RI/MOM scheme

Martinelli, Pittori, Sachrajda, Testa, Vladikas Nucl.Phys.B445(95)

Fitting procedure

Chiral Fits

LECs are computed by a fully correlated combined fit:

- x-expansion: $B_\pi \equiv \frac{M_\pi^2}{2m_{ud}}$ and F_π in terms of m_{ud}
- ξ-expansion: $B_\pi^{-1} \equiv \frac{2m_{ud}}{M_\pi^2}$ and F_π in terms of M_π^2

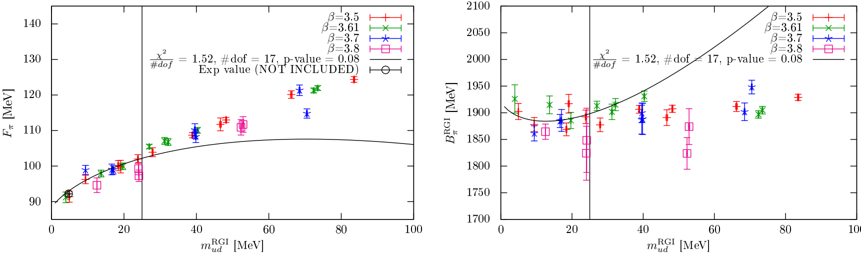
Statistical errors:

- 2000 bootstrap samples are used in each fit

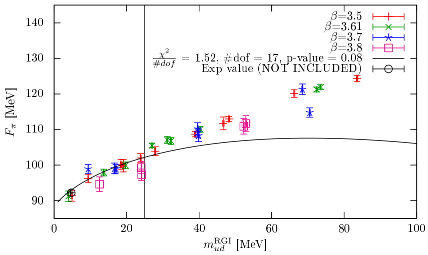
Systematics errors: Final results: $2 \times 2 \times 3 \times 6 \times 2 \times 2 = 288$

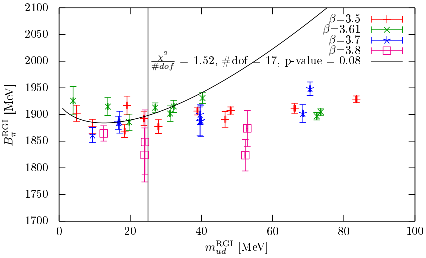
- 2 different time fitting ranges for correlators.
- 2 M_π cuts for interpolating scale fixing quantity M_Ω (380/480 MeV).
- 3 Z_A determinations and 6 Z_S determinations.
- 2 M_π cuts for chiral fits (250/300 MeV).
- 2 chiral expansions: x, ξ

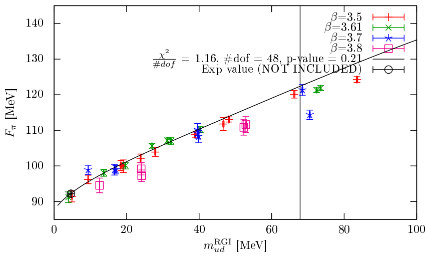
Typical Fit x-expansion

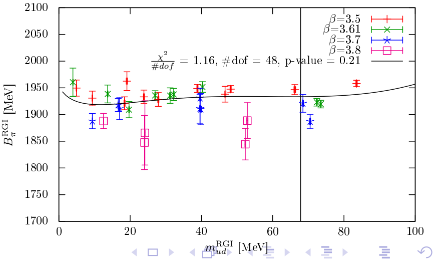


Typical Fit x-expansion

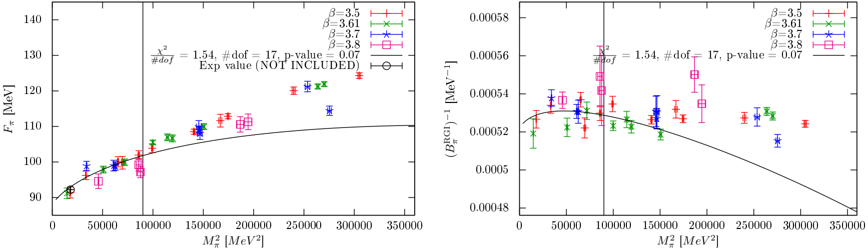




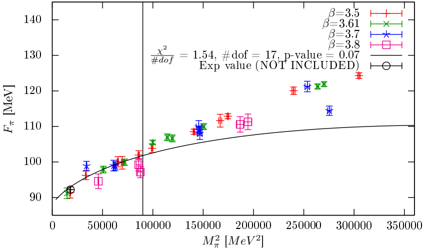


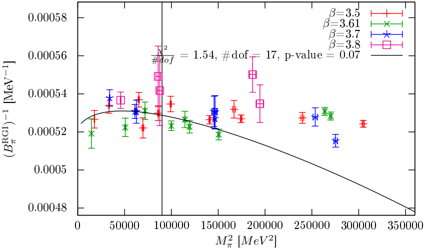


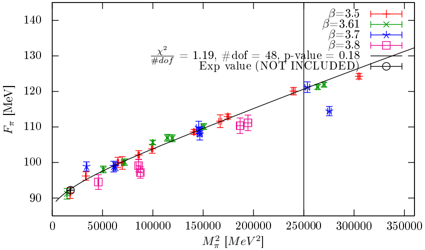
Typical Fit ξ-expansion

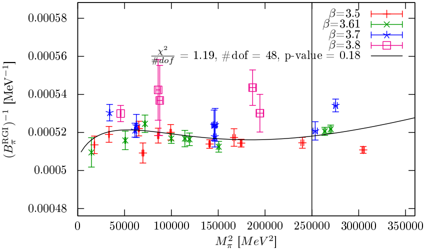


Typical Fit ξ-expansion

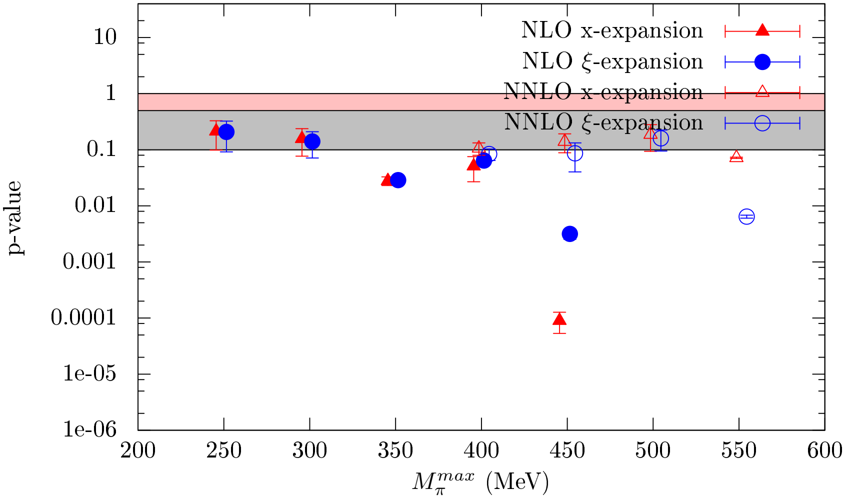




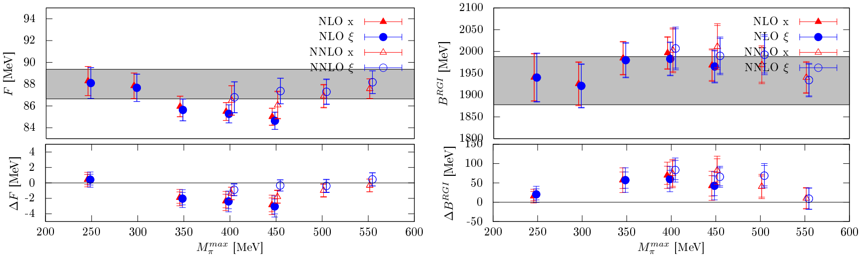




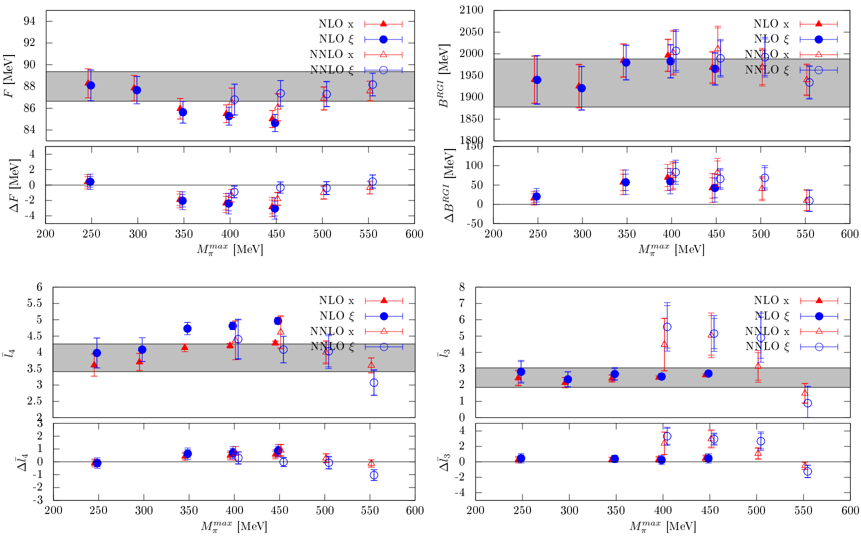
Fit quality in terms of M_π^{max}



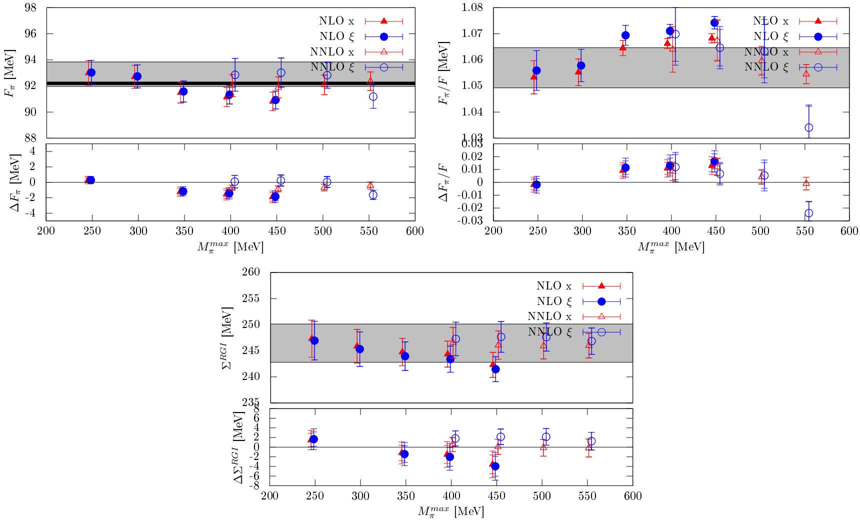
LO LEC in terms of M_π^{max}



LO LEC in terms of M_π^{max}



F_π and other interesting quantities in terms of M_π



LECs from x,ξ expansions

	ξ-expansion	x-expansion
	LO	
B^{RGI} [GeV]	$1.93 \pm 0.05 \pm 0.02$	$1.93 \pm 0.05 \pm 0.01$
F [MeV]	$87.9 \pm 1.4 \pm 0.3$	$88.1 \pm 1.3 \pm 0.3$
$[\Sigma^{RGI}]^{1/3}$ [MeV]	$246 \pm 4 \pm 1$	$247 \pm 3 \pm 1$
	NLO	
l_3	$2.6 \pm 0.6 \pm 0.3$	$2.3 \pm 0.4 \pm 0.2$
l_4	$4.0 \pm 0.4 \pm 0.1$	$3.7 \pm 0.3 \pm 0.1$
	Other quantities	
F_π [MeV]	$92.9 \pm 0.9 \pm 0.2$	$92.9 \pm 0.9 \pm 0.2$
F_π/F	$1.057 \pm 0.007 \pm 0.001$	$1.054 \pm 0.006 \pm 0.001$

Combined results

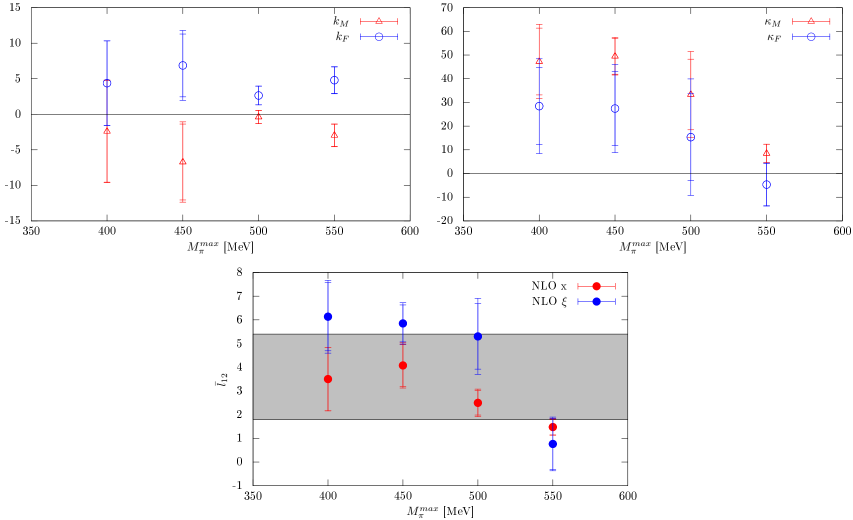
	Combined	FLAG + PDG
	LO	
B^{RGI} [GeV]	$1.93 \pm 0.05 \pm 0.01$	
F [MeV]	$88.0 \pm 1.3 \pm 0.3$	86.4 ± 0.7
$[\Sigma^{RGI}]^{1/3}$ [MeV]	$246 \pm 4 \pm 1$	246 ± 14
	NLO	
$\bar{\ell}_3$	$2.5 \pm 0.5 \pm 0.3$	3.2 ± 0.7
$\bar{\ell}_4$	$3.8 \pm 0.4 \pm 0.2$	4.4 ± 0.6
	Other quantities	
F_π [MeV]	$92.9 \pm 0.9 \pm 0.2$	$92.2 \pm 0.02 \pm 0.14$
F_π / F	$1.057 \pm 0.007 \pm 0.003$	1.067 ± 0.009

Conclusion and outlook

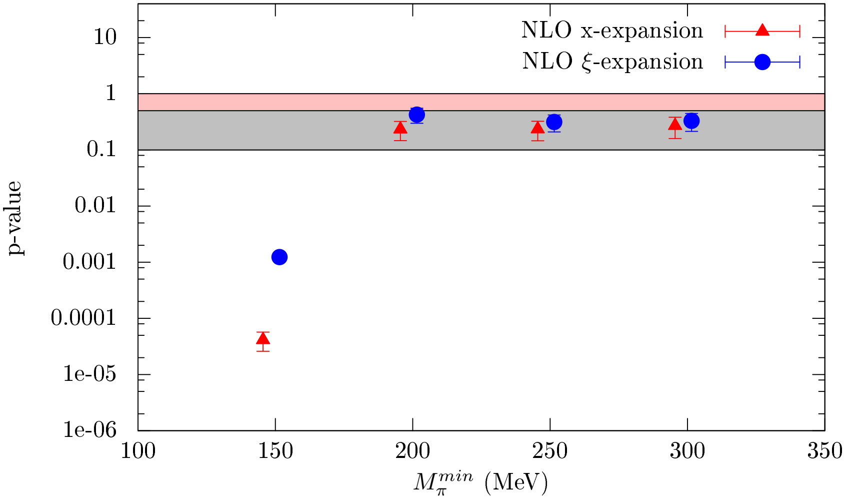
- SU(2) χPT has been compared to lattice QCD simulations all the way down to the physical pion mass point [see also Borsanyi et al, arXiv:1205.0788 for an analysis with staggered fermion simulations]
- Results for both x and ξ expansions look promising:
  - $\rightarrow$ Within our O(1%) statisttical errors, NLO SU(2) χPT describes the F_π and M_π^2 well up to $M_\pi \approx 300$ MeV
  - $\rightarrow$ LO and NLO LECs generally consistent with expectations, but some errors are substantially reduced
- The era of precision on Lattice QCD has started
- Ab-initio computations of QCD at low energy are possible now

thank you!

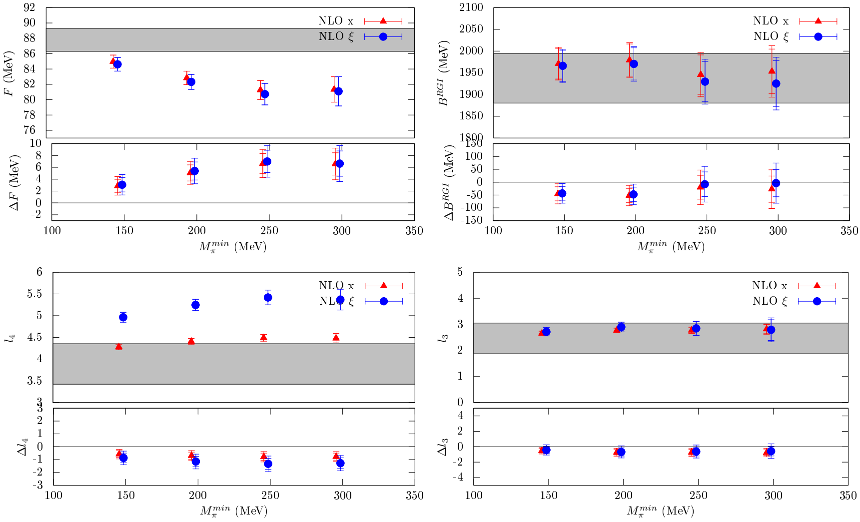
l_{12} and NNLO LEC



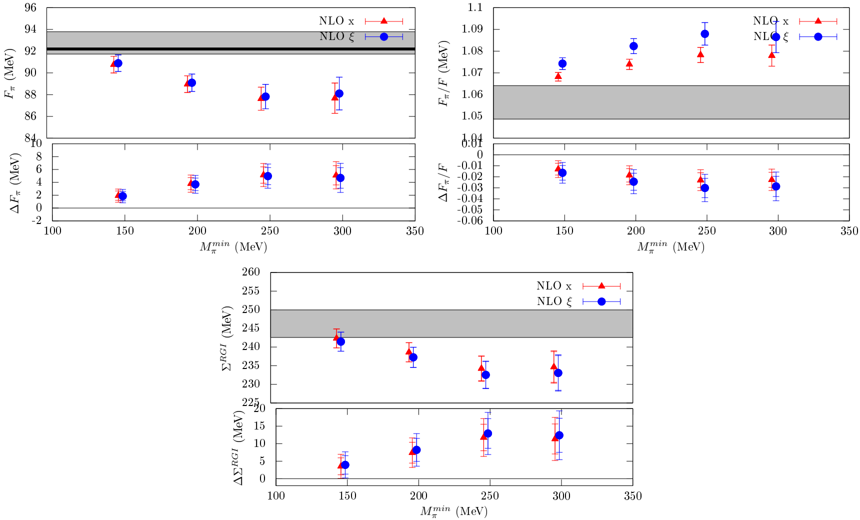
Fit quality in terms of M_π^{max}



LO LEC in terms of M_π^{max}



F_π and other interesting quantities in terms of M_π



Log?

Fitting χPT formulae with 2HEX data

x-expansion

$$\chi^2 = \sum_i^{N_{points}} X_i^T S_i^{-1} X_i + \sum_\beta \left(\frac{(a^p_\beta - a_\beta)^2}{\sigma^2_{a_\beta}} + \frac{(Z^p_{S,\beta} - Z_{S,\beta})^2}{\sigma^2_{Z_{S,\beta}}} + \frac{(Z^p_{A,\beta} - Z_{A,\beta})^2}{\sigma^2_{Z_{A,\beta}}} \right)$$

$$X = \begin{pmatrix} am_{ud} - a^p Z^p_S m^p_{ud}(1 - \gamma_1 (a^p)^2) \\ (aM_{SS})^2 - (a^p)^2 (M^p_{SS})^2 \\ \frac{(aM_\pi)^2}{2(am_{ud})} - (a^p Z^p_S)(1 + \gamma^p_1 (a^p)^2 + \gamma^p_2((M^p_{SS})^2 - M^2_{\Phi,SS}))B_\pi(m^p_{ud}, B^p, F^p, \Lambda^p_3 | l_{12}, k_M, k_F) \\ (aF_\pi) - \frac{a^p}{Z^p_a}(1 + \gamma^p_3((M^p_{SS})^2 - M^2_{\Phi,SS}))F_\pi(m^p_{ud}, B^p, F^p, \Lambda^p_4 | l_{12}, k_M, k_F) \end{pmatrix}$$

$$S_{ab} = \sum_{m=1}^{N_{BOOTS}} \left(y^m_a - y^0_a\right)\left(y^m_b - y^0_b\right), \quad y^t = \left(am_{ud}, (aM_{SS})^2, \frac{(aM_\pi)^2}{2(am_{ud})}, aF_\pi\right)$$

- A term, $\gamma_1 \alpha a + \gamma_2 a^2 + \gamma_3((M^p_{SS})^2 - M^2_{\Phi,SS})$, was tried for each component of X.
- $l_{12} = \frac{7}{15}\log\left(\frac{\Lambda_1^2}{M_\pi^2}\right) + \frac{8}{15}\log\left(\frac{\Lambda_2^2}{M_\pi^2}\right)$, k_M and k_F are included only for NNLO fits
- Fits are repeated for each bootstrap sample.

Fitting χPT formulae with 2HEX data

ξ-expansion

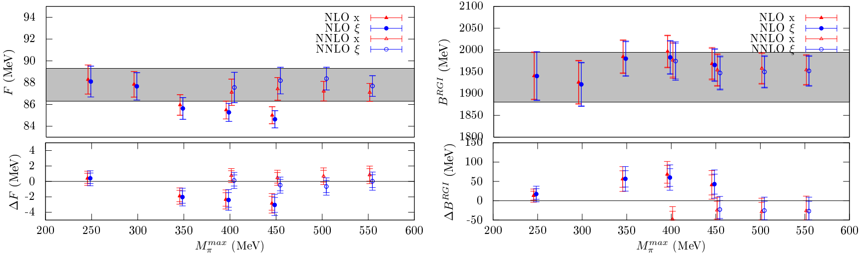
$$\chi^2 = \sum_i^{N_{points}} X_i^T S_i^{-1} X_i + \sum_\beta \left(\frac{(a^p_\beta - a_\beta)^2}{\sigma^2_{a_\beta}} + \frac{(Z^p_{S,\beta} - Z_{S,\beta})^2}{\sigma_{Z^2_{S,\beta}}} + \frac{(Z^p_{A,\beta} - Z_{A,\beta})^2}{\sigma^2_{Z_{A,\beta}}} \right)$$

$$X = \begin{pmatrix} (aM_\pi)^2 - (a^p)^2 (M^p_\pi)^2 \\ (aM_{SS})^2 - (a^p)^2 (M^p_{SS})^2 \\ \frac{2(am_{ud})}{(aM_\pi)^2} - \frac{1}{(a^p Z^p_S)} (1 + \gamma^p_1 (a^p)^2 + \gamma^p_2 ((M^p_{SS})^2 - M^2_{\Phi,SS})) B^{-1}_\pi ((M^p_\pi)^2, B^p, F^p, \Lambda^p_3 | l_{12}, \kappa_M, \kappa_F) \\ (aF_\pi) - \frac{a^p}{Z^p_a} (1 + \gamma^p_3 ((M^p_{SS})^2 - M^2_{\Phi,SS})) F_\pi ((M^p_\pi)^2, B^p, F^p, \Lambda^p_4 | l_{12}, \kappa_M, \kappa_F) \end{pmatrix}$$

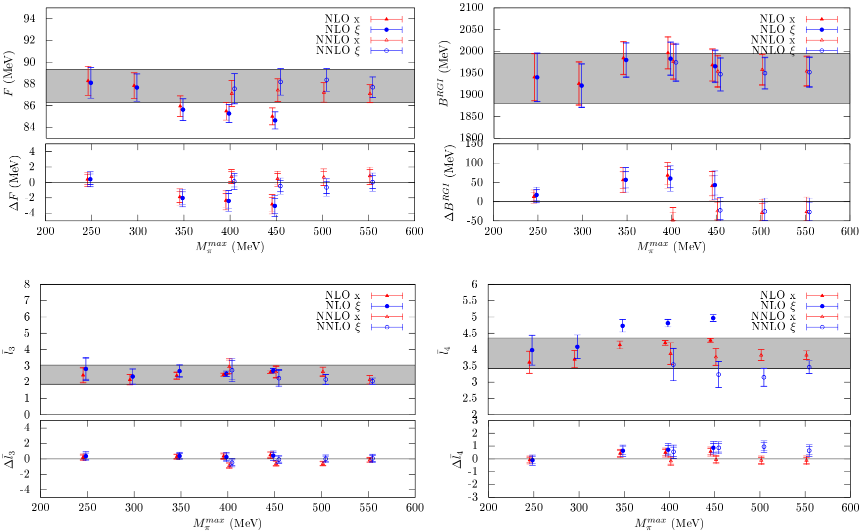
$$S_{ab} = \sum_{m=1}^{N_{BOOTS}} \left(y^m_a - y^0_a\right)\left(y^m_b - y^0_b\right), \quad y^t = \left((aM_\pi)^2, (aM_{SS})^2, \frac{2(am_{ud})}{(aM_\pi)^2}, aF_\pi \right)$$

- A term, $\gamma_1 \alpha a + \gamma_2 a^2 + \gamma_3 ((M^p_{SS})^2 - M^2_{\Phi,SS})$, was tried for each component of X.
- $l_{12} = \frac{7}{15} \log\left(\frac{\Lambda^2_2}{M^2_\pi}\right) + \frac{8}{15} \log\left(\frac{\Lambda^2_1}{M^2_\pi}\right)$, κ_M and κ_F are included only for NNLO fits
- Fits are repeated for each bootstrap sample.

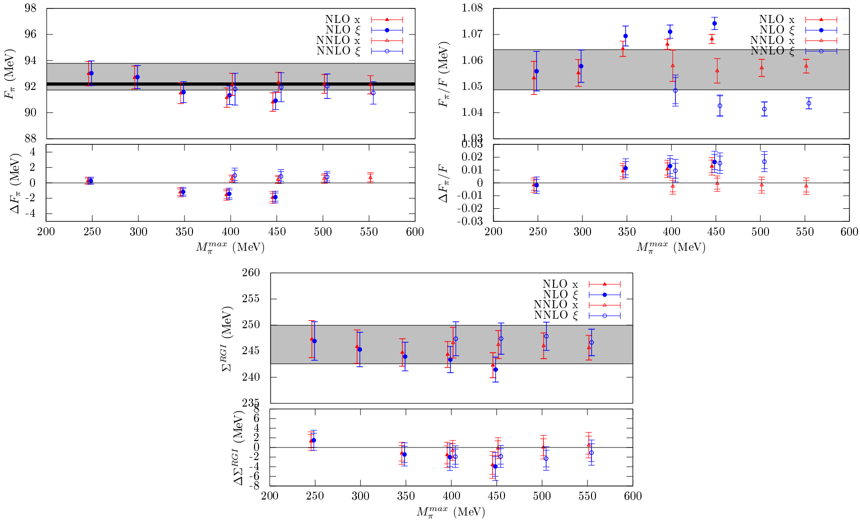
LO LEC in terms of M_π (prior $\bar{l}_1$ and $\bar{l}_2$)



LO LEC in terms of M_π (prior $\bar{l}_1$ and $\bar{l}_2$)



F_π and other interesting quantities in terms of M_π (prior $\bar{l}_1$ and $\bar{l}_2$)



l_{12} and NNLO LEC