Non-perturbative fermion mass generation in Wilson lattice QCD

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Outline and main ideas of the talk

1. O(\Lambda_S) contribution to \( m_{cr} \) seen in lattice QCD with plain Wilson quarks: a dynamically generated fermion mass? Not really: it comes entangled with the 1/\( a \)-divergent mass term, their separation neither well defined nor “natural”. Conjecture: a mechanism reproducing such an O(\( \Lambda_S \)) mass term in chiral WTI’s.

2. Toy model of two fermion species subjected to a non-Abelian gauge interaction and coupled to scalars by Wilson-like and Yukawa terms: given the model symmetries, we conjecture a non-perturbative (NP) mechanism analogous to that in item 1 yielding a well defined and natural O(\( \Lambda_S \)) fermion mass term.

3. Outlook: applications of the conjectured NP mechanism for dynamical mass generation to fermion mass hierarchy and inclusion of electroweak-interactions:
   - each fermion species (except for neutrinos) receives a mass of the order of the \( \Lambda \)-parameter of the strongest gauge interaction in the model scaled by powers of the coupling of the interaction connecting it to the superstrong sector;
   - upon including SM-like weak gauge interactions weak gauge bosons naturally acquire via a “NP analog of the Higgs mechanism” a mass of the order of the superstrong \( \Lambda \)-parameter times \( g_W \) and an appropriate loop factor.

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$m_{cr}$ in Lattice QCD (LQCD) with Wilson quarks

$m_{cr} = m_{cr}(g_0^2)$: $m_0$–value(s) at which chiral symmetries are recovered up to $O(a)$

best determined from (flavour non-singlet) axial Ward-Takahashi identities (WTI):

$$\nabla_\mu \hat{A}_\mu^i = 2m_0 \hat{P}^i + a \hat{O}_5^i , \quad a O_5^i = \delta_A^i (\text{Wilson term}) \quad i = 1, 2, ... N_f - 1$$

mixing $$a \hat{O}_5^i = Z_{5,k}^{-1}(\hat{O}_{5,k}^i)_R - 2 \bar{M} \hat{P}^i - (Z_A - 1) \partial_\mu A_\mu^i \Rightarrow \text{renormalized WTI}$$

$$\nabla_\mu (Z_A \hat{A}_\mu^i) = 2(m_0 - \bar{M}) \hat{P}^i + O(a) , \quad Z_A = Z_A(g_0^2) , \quad a \bar{M} = w(g_0^2, am_0)$$

$$w(g_0^2, am_{cr}) = am_{cr} \quad \text{determines} \quad am_{cr} = f(g_0^2) = c_0 + c_1 a \Lambda_S + c_2 (a \Lambda_S)^2 + O(a^3)$$

$c_i$ polynomials in $g_0^2$, $a \Lambda_S \approx \exp(-1/(2b_0 g_0^2))$ non-perturbative contribution(s)

one operator ($\hat{P}^i$) associated to $\bar{M}$: any chiral symmetry recovery condition fixes

$m_{cr} = c_0/a + c_1 \Lambda_S + O(a) \Rightarrow \text{no symmetry-based criterion to isolate } c_1 \Lambda_S$. 

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• Strong evidence of $c_1 \Lambda_S$ term in $m_{cr}$ for plain ($c_{SW} = 0$) Wilson fermions:

$am_{cr}$ appears linear in $a/r_0$ in a wide $a/r_0$–“window”, with $-c_1 \Lambda_S \in [0.7, 1.0]$ GeV

“window”: as $a \to 0$ terms polynomial in $g_0^2$, as $a \to \infty$ higher-$a$ orders dominating

• Suggestive: “theory” with $m_0 = c_0/a \Rightarrow \nabla_\mu A^i_\mu = 2(-c_1 \Lambda_S)\hat{P}^i + O(a)$

but $m_0 = c_0/a$ is most “unnatural” and not well defined within LQCD ...
Understanding & modelling the $c_1\Lambda_S$ term in $m_{cr}$

- Do non-Abelian gauge models exist where a symmetry criterion allows to fix the bare parameters so as to “naturally” obtain an effective fermion mass $\sim \Lambda_S$?
- Try to understand the NP mechanism responsible for the $c_1\Lambda_S$ term of $m_{cr}$ in LQCD, then possibly “export” it to a theory where the answer may be positive.

- $am_{cr} = c_0(g_0^2) + c_1(g_0^2)a\Lambda_S + \ldots$: look for an $O(a)$ correction wrt leading term

- In Wilson’s LQCD $O(a)$ corrections arise due to chiral (Ch) symmetry breaking induced by the Wilson action term, can be described via Symanzik’s LEL (SLEL) approach – $O(a\Lambda_S)$ provided dynamical ChSSB takes place in continuum QCD

- If $a\Lambda_S^2 \gg m_0 - m_{cr}, \mu_q \to 0$ the Wilson term dominates and acts as a seed of ChSSB: observed numerically, reflected in lattice ChPT (Aoki, Sharpe, Bär, …)

$\Rightarrow$ the SLEL description of e.g. $O = S^1 S^1$ reads $\langle S^1(x) S^1(0) \rangle_{m_0 = m_{cr}, \mu_q = 0}^L = \langle S^1(x) S^1(0) \rangle_{m_q = 0, \text{sgn}(r)}^C = \langle S^1(x) S^1(0) \rangle_{m_q = 0, \text{sgn}(r)}^C - a \int_y \langle L_5(y) S^1(x) S^1(0) \rangle_{m_q = 0, \text{sgn}(r)}^C + O(a^2)$ with $L_5|_{m_q = \mu_q = 0} = b_{SW}(r)\bar{q}(i\sigma \cdot F)q + b_{D2}(r)\bar{q}(-\nabla \nabla)q, \quad b_{SW,D2}$ odd in $r$. 

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• In fixed gauge, for $a\Lambda_S^2 \gg m_0 - m_{cr}, \mu_q \to 0$, expect $O(a)$ corrections to

$$\langle G^b_\mu(x) G^c_\nu(0) \rangle |_{m_0=m_{cr}, \mu_q=0}^L \quad \text{and} \quad \langle q_L(x) \bar{q}_L(0) \rangle |_{m_0=m_{cr}, \mu_q=0}^L$$

of the form

$$\Delta \langle G^b_\mu(x) G^c_\nu(0) \rangle |_{m_0=m_{cr}, \mu_q=0}^L = -a \int_y \langle L_5(y) G^b_\mu(x) G^c_\nu(0) \rangle |_{m_0=m_{cr}, \mu_q=0}^C \text{sgn}(r) + O(a^2)$$

$$\Delta \langle q_L(x) \bar{q}_L(0) \rangle |_{m_0=m_{cr}, \mu_q=0}^L = -a \int_y \langle L_5(y) q_L(x) \bar{q}_L(0) \rangle |_{m_0=m_{cr}, \mu_q=0}^C \text{sgn}(r) + O(a^2)$$

• $O(a\Lambda_S)$ corrections in propagators may yield a mixing of $\hat{O}_5^i$ with $\Lambda_S \hat{P}^i$...

Conjecture: NP corrections to the mixing of $\hat{O}_5^i$ can effectively be described by lattice PT with augmented Feynman rules for gluon and $q_L/R$ propagators

$$\Delta \Gamma^{bc}_{\mu\nu}(k) |_{m_0=m_{cr}, \mu_q=0}^L = a\Lambda_S \alpha_S(\Lambda_S) \delta^{bc} \Pi^{\mu\nu}_{\mu\nu}(k) k^{-2} f_{AA}(\Lambda_S^2/k^2)$$

$$\Delta S_{LL}(k) |_{m_0=m_{cr}, \mu_q=0}^L = a\Lambda_S \alpha_S(\Lambda_S) \frac{i k_\mu (\gamma_\mu)_{LL}}{k^2 + ((ar/2)k^2)^2} f_{q\bar{q}}(\Lambda_S^2/k^2)$$

where $f_{AA}(\Lambda_S^2/k^2) \xrightarrow{k^2 \to a^{-2}} h_{AA} \neq 0$ and $f_{q\bar{q}}(\Lambda_S^2/k^2) \xrightarrow{k^2 \to a^{-2}} h_{q\bar{q}} \neq 0$

region $k^2 \sim a^{-2}$: relevant for the leading NP corrections to the mixing of $\hat{O}_5^i$ ...
• Within our model (conjecture above): leading NP effects in the mixing of $a \hat{O}_5^i$ stem from $O(\alpha_S^2)$ “diagrams” of the kind of the following ones

* the loop integral is dominated by loop momenta $k^2 \sim a^{-2}$, yielding a contribution $a^{-2}(\text{loop UV divergency}) \times a(\text{insertion of } a \hat{O}_5^i) \times a\Lambda_S(\text{propagator correction}) \sim O(\Lambda_S)$

* the bare $a \hat{O}_5^i$ undergoes a mixing with $O(\alpha_S^2 \Lambda_S) \hat{P}^i$ that can not be canceled by any insertion of $m_{cr} \hat{P}^i$ (such insertions can give NP contributions to $O(\alpha_S^3 \Lambda_S)$ at most).

• For $k^2 \sim a^{-2}$ the $\Delta(\text{Feynman rules})|_{m_0=m_{cr},\mu_q=0}$ above are just as if derived from $\Delta L(y) = a\Lambda_S\alpha_S(\Lambda_S)[h_{AA}(F^G \cdot F^G) + h_{q\bar{q}}(\bar{q}\not\!D q)](y)$

• This “model” of the NP mixing phenomenon for $a \hat{O}_5^i$ can be extended to theories where NP (SSB) effects are expected to give rise to new, peculiar operators in the mixing pattern of the operator of interest.
A (toy) gauge model with scalar fields and Wilson-like term

Model with two fermion species $Q = (Q_1, Q_2)^t$ coupled minimally to a $SU(N)$ gauge field $G_\mu$ and through Wilson-like and Yukawa terms to a complex scalar doublet $\phi$.

$$L_{\text{toy}}(Q, G, \Phi) = L_{\text{kin}}(Q, G, \Phi) + V(\Phi) + L_{\text{Wil}}(Q, G, \Phi) + L_{\text{Yuk}}(Q, \Phi),$$

$$L_{\text{kin}}(Q, G, \Phi) = \frac{1}{4} F_{\mu\nu}^a G^a G^\mu G^\nu + \bar{Q}_L D^G Q_L + \bar{Q}_R D^G Q_R + \frac{1}{2} \left[ \partial_\mu \Phi^\dagger \partial_\mu \Phi \right]_{\text{tr}}$$

$$V(\Phi) = \frac{\mu_0^2}{2} \left[ \Phi^\dagger \Phi \right]_{\text{tr}} + \frac{\gamma_0}{4} \left( \left[ \Phi^\dagger \Phi \right]_{\text{tr}} \right)^2,$$

$$L_{\text{Yuk}}(Q, \Phi) = \eta_0 \left( \bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L \right)$$

$$L_{\text{Wil}}(Q, G, \Phi) = \frac{b^2}{2} \rho \left( \bar{Q}_L \tilde{D}_\mu^G \Phi D_\mu^G Q_R + \bar{Q}_R \tilde{D}_\mu^G \Phi^\dagger D_\mu^G Q_L \right)$$

with $\Phi = [\phi, i\tau^2 \phi^*]$, $\phi = (\phi_1, \phi_2)^t$, $D_\mu^G = \partial_\mu - ig_0 G_\mu$; $[M_{2\times2}]_{\text{tr}} = \frac{1}{2} \text{tr}(M_{2\times2})$.

Renormalizable: $1/b = \Lambda_{UV}$. No W-interactions (yet): $\rho$ free parameter.

Symmetries: Poincaré group, C, P, T, gauge $SU(N)$ and $\chi_L \times \chi_R$, where

$\chi_L$: $\tilde{\chi}_L \otimes (\Phi \to \Omega_L \Phi) \otimes (\Phi^\dagger \to \Phi^\dagger \Omega_L^\dagger)$, $\Omega_L \in SU(2)_L$,

$\chi_R$: $\tilde{\chi}_R \otimes (\Phi \to \Phi \Omega_R^\dagger) \otimes (\Phi^\dagger \to \Omega_R \Phi^\dagger)$, $\Omega_R \in SU(2)_R$,

$\tilde{\chi}_{L,R}$: $Q_{L,R} \to \Omega_{L,R} Q_{L,R}$, $\bar{Q}_{L,R} \to \bar{Q}_{L,R}^\dagger \Omega_{L,R}^\dagger$
• Exact $\chi_L \times \chi_R$ symmetry: $\partial J^L_i(x) = 0$ ($i = 1, 2, 3$), with conserved currents

$$
J^L_i = \bar{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{1}{2} \left[ \Phi^\dagger \frac{\tau^i}{2} \partial_\mu \Phi + \text{h.c.} \right]_{\text{tr}} - \frac{b^2}{2} \rho \left( \bar{Q}_L \frac{\tau^i}{2} \Phi \mathcal{D}_\mu^G Q_R + \text{h.c.} \right)
$$

$$
J^R_i = \bar{Q}_R \gamma_\mu \frac{\tau^i}{2} Q_R + \frac{1}{2} \left[ \frac{\tau^i}{2} \Phi^\dagger \partial_\mu \Phi + \text{h.c.} \right]_{\text{tr}} + \frac{b^2}{2} \rho \left( \bar{Q}_L \mathcal{D}_\mu^G \frac{\tau^i}{2} \Phi Q_R + \text{h.c.} \right)
$$

• In general $\tilde{\chi}_L$ and $\tilde{\chi}_R$ transformations are no symmetry: broken bare WTI's

$$
\partial \tilde{J}^L_i(x) = -\eta \left( \bar{Q}_L \frac{\tau^i}{2} \Phi Q_R - \text{h.c.} \right)(x) - \frac{b^2}{2} \rho \left( \bar{Q}_L \mathcal{D}_\mu^G \frac{\tau^i}{2} \Phi \mathcal{D}_\mu^G Q_R - \text{h.c.} \right)(x)
$$

$$
\partial \tilde{J}^R_i(x) = \eta \left( \bar{Q}_L \Phi \frac{\tau^i}{2} Q_R - \text{h.c.} \right)(x) + \frac{b^2}{2} \rho \left( \bar{Q}_L \mathcal{D}_\mu^G \Phi \frac{\tau^i}{2} \mathcal{D}_\mu^G Q_R - \text{h.c.} \right)(x)
$$

with non-conserved currents (differing from $J^L/R_i$ by the absence of $\Phi$-bilinear terms)

$$
\tilde{J}^L_i = \bar{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2} \rho \left( \bar{Q}_L \frac{\tau^i}{2} \Phi \mathcal{D}_\mu^G Q_R + \text{h.c.} \right)
$$

$$
\tilde{J}^R_i = \bar{Q}_R \gamma_\mu \frac{\tau^i}{2} Q_R + \frac{b^2}{2} \rho \left( \bar{Q}_L \mathcal{D}_\mu^G \Phi \frac{\tau^i}{2} Q_R + \text{h.c.} \right)
$$

• For given $\mu^2_0$, $\gamma_0$, $\rho_0$: a family of models parameterized by $\eta_0$; just one model (with $\eta_0 = \eta_{cr}$) selected by requiring “maximal $\tilde{\chi}$–symmetry enhancement”.

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\( \tilde{\chi} \) symmetry enhancement (restoring) for \( \mu_R^2 > 0 \)

- Recovery of \( \tilde{\chi} \) symmetry has to do with the mixings of \( d = 6 \) oper.s in bare WTI

\[
O^L_i = \frac{\rho}{2} (\bar{Q}_L \overleftarrow{D}_\mu \frac{G \tau^i}{2} \Phi D_\mu^G Q_R - \text{h.c.}) , \quad O^R_i = -\frac{\rho}{2} (\bar{Q}_L \overrightarrow{D}_\mu \frac{G \tau^i}{2} D_\mu^G Q_R - \text{h.c.})
\]

which to all orders in PT reads

\[
O^L_{6 \text{ bare}} = O^L_{6 \text{ sub}} + \frac{Z^{-1}}{b^2} \partial_\mu \tilde{J}^L_\mu - \frac{c_0}{b^2} \left[ \bar{Q}_L \frac{\tau^i}{2} \Phi Q_R - \text{h.c.} \right] \\
O^R_{6 \text{ bare}} = O^R_{6 \text{ sub}} + \frac{Z^{-1}}{b^2} \partial_\mu \tilde{J}^R_\mu - \frac{c_0}{b^2} \left[ \bar{Q}_R \frac{\tau^i}{2} \Phi^\dagger Q_L - \text{h.c.} \right]
\]

- If \( \mu_R^2 > 0 \) the mixing patterns above are expected not to be altered by NP effects.

As \( \langle [\Phi]_{tr} \rangle = 0, \ \Phi = \sigma 1 + i \vec{\pi} \vec{\tau}, \) no contribution from \( \mathcal{L}_{Wil} \) to \( Q \)-propagator \( \Rightarrow \)

no seed of dynamical \( \tilde{\chi} \) SSB \( \Rightarrow \) at variance with the situation in Wilson LQCD

- At \( \eta_0 = \eta_{cr} = c_0 : \ \partial_\mu Z^i_j \tilde{J}^L_\mu = O(b^2), \ \partial_\mu Z^i_j \tilde{J}^R_\mu = O(b^2) \)

\( \tilde{\chi}_L \times \tilde{\chi}_R \) restored \( \iff \) \( \Phi \) decoupled \( \iff \) up to (negligible) \( O(b^2) \)

\( \eta_{cr} \) can be determined e.g. by \( \partial_\mu \langle \tilde{J}^L_\mu (x) \left( \bar{Q}_L \frac{\tau^i}{2} \Phi Q_R - \text{h.c.} \right) \rangle \big|_{\eta_0 = \eta_{cr}} = 0 \)

\( \eta_{cr} = f_{cr}(\rho_0; g_0^2, \gamma_0, b^2 \mu_R^2 Z_{\Phi^\dagger}^\Phi) \) is odd in \( \rho_0 \) and essentially \( \mu_R \)–independent
\(\tilde{\chi}\) symmetry enhancement (no restoring?) for \(\mu_R^2 < 0\)

- If \(\mu_R^2 < 0\), \(\chi\) SSB takes place. Without loss of generality choose the vacuum where
  \[\langle [\Phi]_{tr} \rangle = \nu, \quad \Phi = (\nu + \sigma)1 + i \vec{\pi} \vec{\tau}, \quad \pi^{1,2,3}\text{ massless Goldstone bosons and} \]
  \[m_\sigma^2 \sim \nu^2 \sim |\mu_R^2|/\gamma_R.\] Moreover we work with bare parameters s.t. \(\nu \gg \Lambda_S\)

- \(\mathcal{L}_{\text{Wil}}\) contributes to the \(Q\)-propagator:
  \[i \frac{k'}{k^2} + \frac{b}{2} \nu^2 \text{ in mom. space} \Rightarrow \mathcal{L}_{\text{Wil}}\]
  provides the seed for dynamical \(\tilde{\chi}\) SSB – a NP phenomenon, as in Wilson LQCD

- Expect \(O(b^2 \Lambda_S)\) relative corrections in correlators (non-vanishing even) at \(\eta_0 = \eta_{cr}:\)
  \[\langle O(x, y, ..) \rangle \bigg|^{1/b}_{\eta_0 = \eta_{cr}} = \left[ \langle O(x, y, ..) \rangle - b^2 \int_w \langle L_6(w)O(x, y, ..) \rangle + O(b^4) \right]^{C}_{\delta \eta \to 0 \text{ sgn}(\rho \nu)}\]
  \[L_6 = L_6^{\text{inv}} + \left[b_{SW}(\bar{Q}_L \Phi i \sigma \cdot F_{QR} + \text{h.c.}) + b_{D2}(\bar{Q}_L \vec{\mathcal{D}} G \Phi \mathcal{D} G Q_R + \text{h.c.}) \right]\]
  owing to the \(\tilde{\chi}\)-violating piece of \(L_6\) & dynamical \(\tilde{\chi}\) SSB (order parameter \(\langle [\bar{Q} Q]_{\text{sub}} \rangle \sim \Lambda_S^3\))

  e.g. for \(O(x, y, ..) \to G^a_\mu(x) G^a_\nu(y), \ G^a_\mu(x) G^a_\nu(y) \sigma(z), \ Q_L(x) \bar{Q}_L(y), \ Q_L(x) \bar{Q}_L(y) \sigma(z)\)
• Conjecture: NP corrections to the mixings of $b^2 O^L_6 i$, $b^2 O^R_6 i$ can effectively be described via PT with Feynman rules augmented as dictated by

$$\Delta L(y) = b^2 \alpha_S(\Lambda_S) \left\{ h_{AA} \Lambda_S [U^\dagger \Phi + \Phi^\dagger U]_\text{tr} (F^G \cdot F^G) + h_{qq} \Lambda_S [U^\dagger \Phi + \Phi^\dagger U]_\text{tr} (\bar{q} \gamma \tau q) \right\}(y)$$

with $U(\sigma, \vec{\pi})$: $U \rightarrow \Omega_L U \Omega_R^\dagger$ under $\chi_L \times \chi_R$ so as to preserve $\chi$ invariance

$U(\sigma, 0) = 1$ ($\vec{\pi} = 0$, no phases) $\Rightarrow$ unique $U(\sigma, \vec{\pi}) = \exp(i \text{Arg}(\Phi))_{\text{non-Abelian}}$

• Extra “diagrams” contribute to the mixings of $b^2 O^L_6 / R i$ extra $O(\alpha_S^0)$ terms

\[
\begin{align*}
\Rightarrow \quad & \partial_\mu Z_j \tilde{J}^L_\mu |_{\eta_0=\eta_{cr}} = -c_1 \Lambda_S (\tilde{Q}_L \frac{\tau_i^i}{2} U Q_R - \text{h.c.})|_{\eta_0=\eta_{cr}} + O(b^2), \quad c_1 = O(\alpha_S^2) \\
\star \quad & \text{r.h.s. is RG-invariant} \ (U \ \text{unique}): \ \text{a natural, NP.ly generated mass} \ \tilde{m}_Q(1/b) = -c_1 \Lambda_S \\
\star \quad & \Phi \ \text{NP.ly coupled at} \ \eta_0 = \eta_{cr} \ \text{if} \ \mu_R^2 < 0: \ \text{\tilde{\chi} symmetry broken by} \ O(\alpha_S^0) \ \text{terms} \ & \text{c}_1 \ \text{depending on the} \ 1/b\text{-scale details, order of magnitude of} \ m_Q(1/b) \ "\text{universal}"
\star \quad & \text{n}o \ \text{term} \sim \Lambda_S (\tilde{Q}_L U Q_R + \text{h.c.}) \ \text{can be added to} \ L_{\text{toy}}: \ \text{only polynomial of local fields compatible with} \ (\text{power counting}) \ \text{renormalizability}
\end{align*}
\]
Outlook 1: fermion mass hierarchy

To describe $t$, $b$ quark (and $W$, $Z$) masses need a superstrong force with $\Lambda_T \gg \Lambda_{QCD}$

Simplest case: $m_q/m_Q$ in model with strong and superstrong gauge fields ($A$ and $G$):

$$L_{\text{toy}}^{(qQAG)} = (F^A \cdot F^A) + (F^G \cdot F^G) + (\bar{q}D^A q) + (\bar{Q}D^G,A, q) + [\partial_\mu \Phi^\dagger \partial_\mu \Phi] + V(\Phi) +$$

$$+ L_{\text{Wil}}[q, A, \Phi] + L_{\text{Wil}}[Q, A, G, \Phi] + L_{\text{Yuk}}[q, \Phi] + L_{\text{Yuk}}[Q, \Phi], \quad \mu_0^2 : \mu_R^2 < 0$$

$$\bar{m}_Q \sim -c_{1Q}^{(2)} \alpha_T(b^{-1})\alpha_T(\Lambda_T)\Lambda_T + O(\Lambda_S) \quad \text{at} \quad \eta_Q = \eta_{Q,cr}$$

$$\bar{m}_q \sim -c_{1q}^{(2)} \alpha_S(b^{-1})\alpha_S(\Lambda_T)\Lambda_T + O(\Lambda_S) \quad \text{at} \quad \eta_q = \eta_{q,cr}$$

Assuming $\alpha_T(\mu) \sim \alpha_S(\mu)$ for $\nu \leq \mu \leq b^{-1}$ (coupling unification) and similar coefficients for the $d = 6$ Wilson-like terms $L_{\text{Wil}}[q, A, \Phi]$ and $L_{\text{Wil}}[Q, A, G, \Phi]$

$$\bar{m}_q/\bar{m}_Q \sim \alpha_S(\Lambda_T)/\alpha_T(\Lambda_T) \sim 0.1 \quad \text{essentially independent on UV-scale details}$$

Taking $|c_{1Q}^{(2)}|$ as in Wilson LQCD & $\rho_{cr} \sim 0.2$ (Outlook 2) $\Rightarrow \bar{m}_Q \sim \Lambda_T \sim$ a few TeV
Outlook 2: weak interactions

Three massless Goldstones $\pi^{1,2,3}$ in the toy model(s) above: this calls for introducing weak interactions, global $\chi_L$ symmetry being ready to be gauged

Once this is done ($g_W > 0$) e.g. in the toy model specified by $\mathcal{L}_{\text{toy}}^{(qQAG)}$: $W^{1,2,3}$ weak gauge bosons appear in $(F^W \cdot F^W)$ term and in cov. derivatives acting on $\chi_L$-doublets

No $U_Y(1)$ gauge interaction yet, $\mu_R^2 < 0$: custodial $SU(2)_Y$ preserved by $\chi$ SSB.

A new term $\sim ig_w \left[ \Phi^\dagger [\frac{\tau^i}{2}, W_{\mu}] D^W_\mu \Phi + \text{h.c.} \right]_{\text{tr}}$ in the r.h.s. of broken bare $\tilde{\chi}_L$–WTI $\leftrightarrow$ a new mixing of $b^2 O_6^{L,i}$: maximal $\tilde{\chi}$ enhancement implies $(\rho, \eta_0) = (\rho_{cr}, \eta_{cr})$, with

$$\partial_\mu Z_j J^{L,i}_{\mu} = -ig_w c_1 W^L \Lambda_T \left[ U^\dagger [\frac{\tau^i}{2}, W_{\mu}] D^W_\mu \Phi + \text{h.c.} \right]_{\text{tr}} + O(c_1 Q, 1 q \Lambda_T) + O(b^2)$$

dynamically generated $\bar{m}_W \sim g_w \Lambda_T O(\frac{\alpha_T(\Lambda_T)}{4\pi^2}) \sim 100 \text{ GeV}$, while $\bar{m}_{Q,q}$ as above

Due to $WQ\bar{Q}$ coupling, $WW$ bound states with binding energies $O(50)$ GeV: LHC “Higss boson”? If yes, get low ($\ll \Lambda_T$) energy effective theory very similar to SM ...

$W_{\mu}^i$ couples to conserved currents $J^{L,i}_{\mu}$, with a non-SM-like piece: $J^{L,i}_{\mu}$-matrix elem.s in principle differ by $O(\alpha(b^{-1})\rho_{cr}^2)$ from their SM counterparts, tiny effect if $\rho_{cr} \sim 0.2$ ...