Flavor physics with Λ_b baryons

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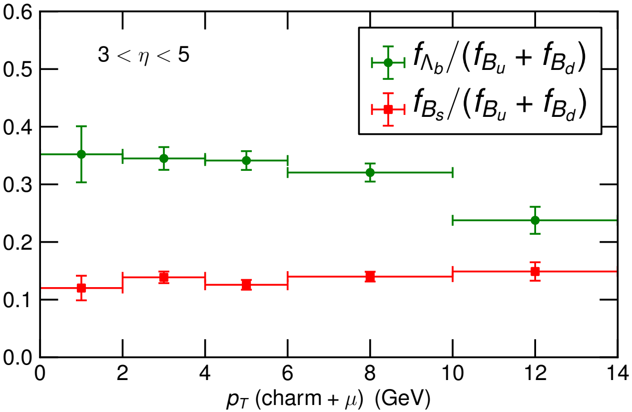
The LHC is a Λ_b factory

(Λ_b = lightest bottom baryon, quark content udb, $J^P = \frac{1}{2}^+$)



[LHCb, arXiv:1111.2357]

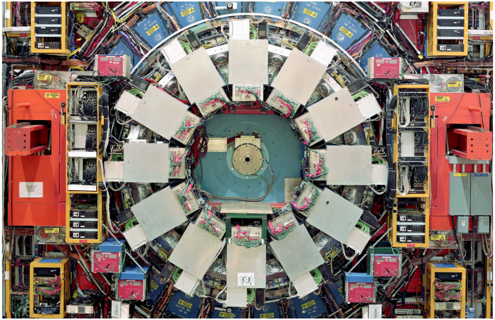
Outline

1. The decay $\Lambda_b \to \Lambda\,\ell^+\ell^-$
2. The decay $\Lambda_b \to p\,\ell^-\bar{\nu}_\ell$
3. Lattice calculation with static b quarks
4. Lattice calculation with relativistic b quarks

The decay $\Lambda_b \to \Lambda\, \ell^+ \ell^-$

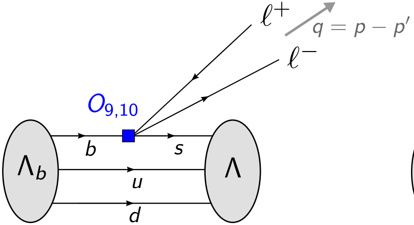
$(\ell = e, \mu, \tau)$

First observed by CDF [arXiv:1107.3753]

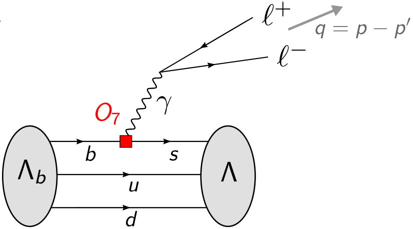


http://www-cdf.fnal.gov/events/CDFPictures.html

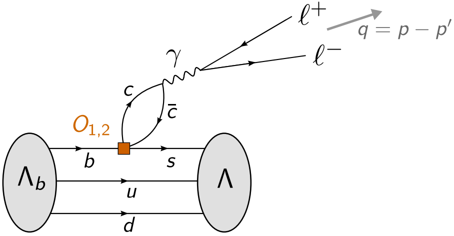
$\Lambda_b \to \Lambda\, \ell^+ \ell^-$



Large contribution for all q^2



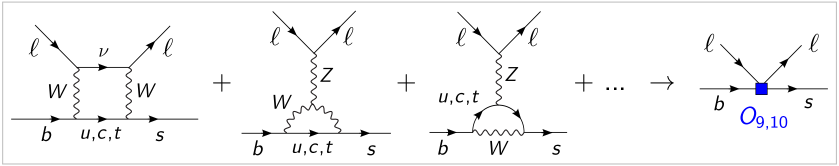
Large contribution near $q^2 = 0$

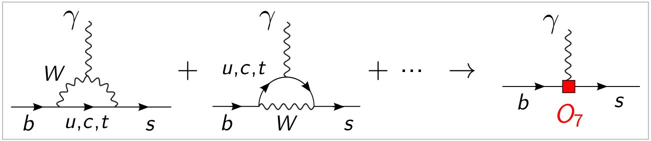


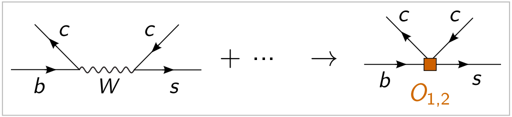
Large contribution near $q^2 = m^2_{J/\psi}, m^2_{\psi'}$, but small otherwise

$b \to s\ell^+\ell^-$ effective Hamiltonian

Standard-Model contributions to effective operators:







$b \to s\ell^+\ell^-$ effective Hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left(C_i O_i + C_i' O_i' \right)$$

with

$$\begin{aligned} O_9^{(\prime)} &= e^2/(16\pi^2)\ \bar{s}\gamma^\mu P_{L(R)} b\ \ \bar{\ell}\gamma_\mu \ell, \\ O_{10}^{(\prime)} &= e^2/(16\pi^2)\ \bar{s}\gamma^\mu P_{L(R)} b\ \ \bar{\ell}\gamma_\mu \gamma_5 \ell, \\ O_7^{(\prime)} &= e/(16\pi^2) m_b\ \bar{s}\sigma^{\mu\nu} P_{R(L)} b\ \ F_{\mu\nu}^{(\text{e.m.})}, \\ O_1 &= \bar{c}^b \gamma^\mu P_L b^a\ \ \bar{s}^a \gamma_\mu P_L c^b, \\ O_2 &= \bar{c}^a \gamma^\mu P_L b^a\ \ \bar{s}^b \gamma_\mu P_L c^b \end{aligned}$$

$[P_{R,L} = (1 \pm \gamma_5)/2]$

Wilson coefficients in the Standard Model (at $\mu = m_b$):

($C_{i,\text{eff}}$ include one-loop matrix elements of $O_1,\ldots,O_6$)

$$\begin{aligned} &C_{9,\text{eff}} \approx 4.2 + Y(q^2), && C_{10,\text{eff}} \approx -4.1, && C_{7,\text{eff}} \approx -0.30, \\ &C'_{9,\text{eff}} \approx 0, && C'_{10,\text{eff}} \approx 0, && C'_{7,\text{eff}} \approx 0 \\ &C_1 \approx -0.26, && C_2 \approx 1.0 && \end{aligned}$$

[Altmannshofer et al, arXiv:0811.1214]

Some new-physics models predict sizable C_i' [see, e.g., Cho and Misiak, hep-ph/9310332; Rizzo, hep-ph/9802401]

$\Lambda_b \to \Lambda\,\ell^+\ell^-$ decay amplitude

Without long-distance contributions from $O_{1,2}$ etc., the decay amplitude is

$$\mathcal{M} = \frac{G_F\, e^2}{4\sqrt{2}\pi^2} V_{tb} V_{ts}^* \Big[\quad \langle\Lambda|\, \bar{s}\gamma^\mu (C_{9,\mathrm{eff}} P_L + C'_{9,\mathrm{eff}} P_R) b\, |\Lambda_b\rangle \;\; \bar{u}_\ell \gamma_\mu v_\ell$$

$$+ \langle\Lambda|\, \bar{s}\gamma^\mu (C_{10,\mathrm{eff}} P_L + C'_{10,\mathrm{eff}} P_R) b\, |\Lambda_b\rangle \;\; \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell$$

$$- \frac{2m_b}{q^2} \;\; \langle\Lambda|\, \bar{s} i\sigma^{\mu\nu} q_\nu (C_{7,\mathrm{eff}} P_R + C'_{7,\mathrm{eff}} P_L) b\, |\Lambda_b\rangle \;\; \bar{u}_\ell \gamma_\mu v_\ell \;\; \Big]$$

$\to$ Relative contributions from C_i vs C'_i depend on polarization of $\langle\Lambda|$ and $|\Lambda_b\rangle$.

[Mannel and Recksiegel, hep-ph/970139; Chen and Geng, hep-ph/0101171; Hiller and Kagan, hep-ph/0108074]

Hadronic matrix elements are written in terms of ten form factors, which are functions of q^2:

$$\langle\Lambda|\, \bar{s}\, \gamma^\mu\, b\, |\Lambda_b\rangle = \bar{u}_\Lambda \left[f_1^V\, \gamma^\mu - f_2^V\, i\sigma^{\mu\nu} q_\nu / m_{\Lambda_b} + f_3^V\, q^\mu / m_{\Lambda_b} \right] u_{\Lambda_b},$$

$$\langle\Lambda|\, \bar{s}\, \gamma^\mu \gamma_5\, b\, |\Lambda_b\rangle = \bar{u}_\Lambda \left[f_1^A\, \gamma^\mu - f_2^A\, i\sigma^{\mu\nu} q_\nu / m_{\Lambda_b} + f_3^A\, q^\mu / m_{\Lambda_b} \right] \gamma_5\, u_{\Lambda_b},$$

$$\langle\Lambda|\, \bar{s}\, i\sigma^{\mu\nu} q_\nu\, b\, |\Lambda_b\rangle = \bar{u}_\Lambda \left[f_1^{TV}\, (\gamma^\mu q^2 - q^\mu \not{q}) / m_{\Lambda_b} - f_2^{TV}\, i\sigma^{\mu\nu} q_\nu \right] u_{\Lambda_b},$$

$$\langle\Lambda|\, \bar{s}\, i\sigma^{\mu\nu} q_\nu\, \gamma_5\, b\, |\Lambda_b\rangle = \bar{u}_\Lambda \left[f_1^{TA}\, (\gamma^\mu q^2 - q^\mu \not{q}) / m_{\Lambda_b} - f_2^{TA}\, i\sigma^{\mu\nu} q_\nu \right] \gamma_5\, u_{\Lambda_b}$$

These need to be calculated in lattice QCD.

$\Lambda_b \to \Lambda\,\ell^+\ell^-$: comparison with mesonic decays

$\Lambda_b \to \Lambda\,\ell^+\ell^-$ is a baryonic analogue of $B \to K^{(*)}\ell^+\ell^-$.

	$B \to K\ell^+\ell^-$	$B \to K^*\ell^+\ell^-$	$\Lambda_b \to \Lambda\,\ell^+\ell^-$
Initial hadron has spin	×	×	✓
Final hadron has spin	×	✓	✓
Final hadron stable in QCD	✓	×	✓
First observation	Belle, 2002	Belle, 2003	CDF, 2011

Polarization of the Λ/K^* influences the angular distribution of the four-body decays

$$\Lambda_b \to \Lambda(\to p^+\pi^-)\,\ell^+\ell^-$$

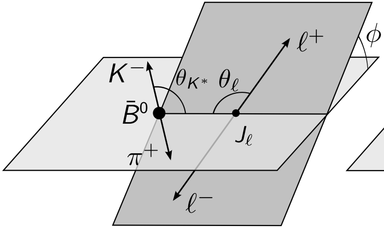
$$B \to K^*(\to K\pi)\,\ell^+\ell^-$$

$B \to K^*\ell^+\ell^-$ form factors: forthcoming paper by
Ron Horgan, Zhaofeng Liu, Stefan Meinel, Matthew Wingate

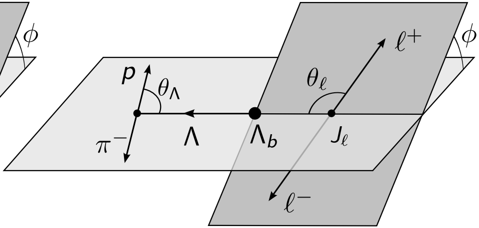
$B \to K\ell^+\ell^-$ form factors:
talks by Andreas Kronfeld (Fri 5:30) and Chris Bouchard (Fri 5:50)

$\Lambda_b \to \Lambda\, \ell^+\ell^-$: comparison with mesonic decays

$\bar{B}^0 \to K^{*0}(\to K^- \pi^+)\, \ell^+\ell^-$



$\Lambda_b \to \Lambda(\to p^+ \pi^-)\, \ell^+\ell^-$



$K^{*0} \to K^- \pi^+$ is a strong decay.

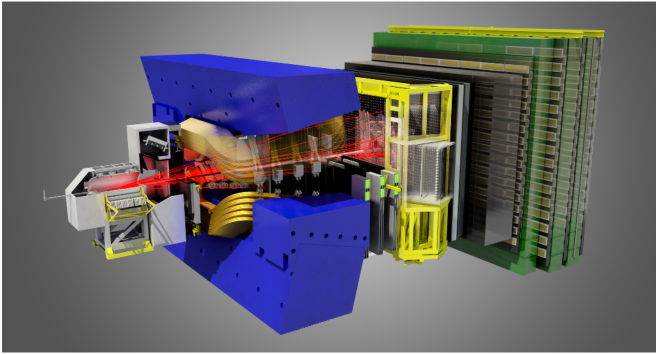
Complicates theory. Strictly speaking, need $B \to K\pi$ form factors (not $B \to K^*$)

[Lü and Wang, arXiv:1111.1513; Bećirević and Tayduganov, arXiv:1207.4004; Matias, arXiv:1209.1525; Blake, Egede, Shires, arXiv:1210.5279; Döring, Meißner, Wang, arXiv:1307.0947].

$\Lambda \to p\,\pi^-$ is a weak decay. Parity-violating. Protons emitted preferentially in direction of Λ spin vector. [Gutsche, Ivanov, Körner, Lyubovitskij, Santorelli, arXiv:1301.3737]

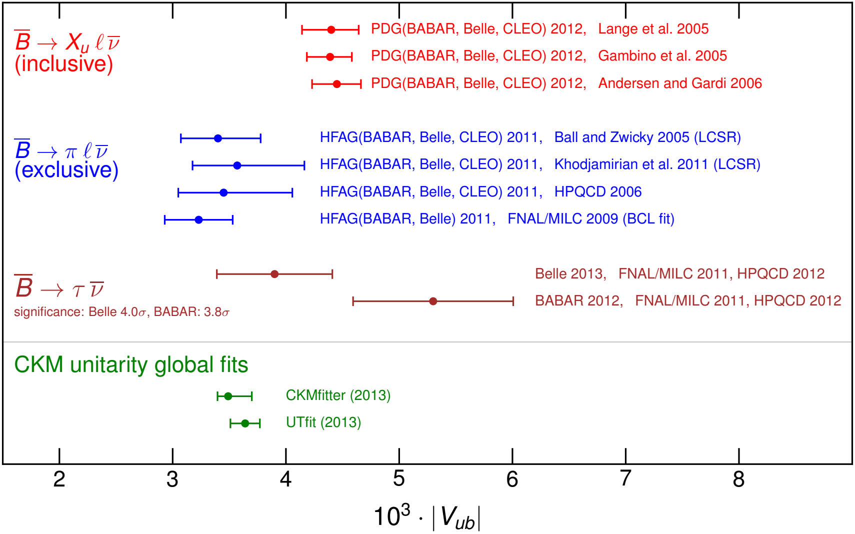
Long lifetime of (electrically neutral) Λ complicates experiment ($c\tau_\Lambda = 7.9$ cm). At LHCb, $\sim$75% of the $\Lambda \to p\,\pi^-$ decays happen outside the vertex locator [LHCb, arXiv:1306.2577]

The decay $\Lambda_b \to p\,\ell^- \bar{\nu}_\ell$



http://lhcb-public.web.cern.ch/lhcb-public/en/lhcb-outreach/multimedia/

The CKM matrix element $|V_{ub}|$



The CKM matrix element $|V_{ub}|$

- So far, all experimental data come from B factories (which are no longer running).

- Extraction of $|V_{ub}|$ from $\bar{B} \to \pi \ell^- \bar{\nu}_\ell$ uses form factors from lattice QCD. New calculations in progress.

  Talks by Taichi Kawanai (Tue 2:20), Daping Du (Fri 4:30), Chris Bouchard (Fri 5:50)
  See also F. Bahr et al., arXiv:1210.3478

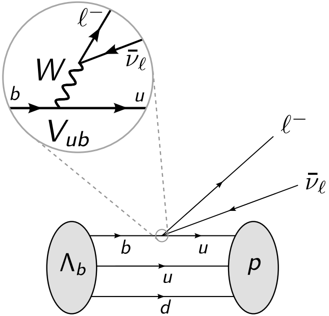
- At LHCb, measurement of $\bar{B} \to \pi \ell^- \bar{\nu}_\ell$ is difficult because of large pion background. Need more distinctive final state.

  Better: $\bar{B}_s \to K \ell^- \bar{\nu}_\ell$

  Talks by Andreas Kronfeld (Fri 5:30) and Chris Bouchard (Fri 5:50)

  Even better: $\Lambda_b \to p \ell^- \bar{\nu}_\ell$ [Egede and Sutcliffe, 2013]

$\Lambda_b \to p \ell^- \bar{\nu}_\ell$



- Analysis of $\Lambda_b \to p \mu^- \bar{\nu}_\mu$ at LHCb in progress. Thanks to Ulrik Egede and William Sutcliffe for discussions!
- To extract $|V_{ub}|$ from experimental data, need lattice QCD calculation of the form factors

$$\begin{aligned}
\langle N^+ |\, \bar{u}\, \gamma^\mu\, b\, | \Lambda_b \rangle &= \bar{u}_N \left[f_1^V\, \gamma^\mu - f_2^V\, i\sigma^{\mu\nu} q_\nu / m_{\Lambda_b} + f_3^V\, q^\mu / m_{\Lambda_b} \right] u_{\Lambda_b}, \\
\langle N^+ |\, \bar{u}\, \gamma^\mu \gamma_5\, b\, | \Lambda_b \rangle &= \bar{u}_N \left[f_1^A\, \gamma^\mu - f_2^A\, i\sigma^{\mu\nu} q_\nu / m_{\Lambda_b} + f_3^A\, q^\mu / m_{\Lambda_b} \right] \gamma_5\, u_{\Lambda_b}
\end{aligned}$$

(f_3^V and f_3^A do not contribute to decay rate in $m_\ell = 0$ approximation)

$|V_{ub}|$ inclusive-exclusive tension: new physics?

Possible new-physics explanation: right-handed current

[Chen and Nam, arXiv:0807.0896; Crivellin, arXiv:0907.2461; Buras, Gemmler, Isidori, arXiv:1007.1993]

$$\begin{aligned}\mathcal{H}_{\rm eff} \quad = \quad & \frac{G_F}{\sqrt{2}} V_{ub}\,(\bar{u}\gamma_\mu b - \bar{u}\gamma_\mu\gamma_5 b)\,(\bar{\ell}\gamma^\mu\nu - \bar{\ell}\gamma^\mu\gamma_5\nu) \\ & + \frac{G_F}{\sqrt{2}} V'_{ub}\,(\bar{u}\gamma_\mu b + \bar{u}\gamma_\mu\gamma_5 b)\,(\bar{\ell}\gamma^\mu\nu - \bar{\ell}\gamma^\mu\gamma_5\nu)\end{aligned}$$

Contributions to matrix elements:

Process	$\bar{u}\gamma_\mu b$	$\bar{u}\gamma_\mu\gamma_5 b$
$\bar{B} \to \pi\,\ell\,\bar{\nu}_\ell$	✓	×
$\bar{B} \to \ell\,\bar{\nu}_\ell$	×	✓
$\bar{B} \to X_u\,\ell\,\bar{\nu}_\ell$	✓	✓
$\Lambda_b \to p\,\ell\,\bar{\nu}_\ell$	✓	✓

Lattice calculation with static b quarks

William Detmold, C.-J. David Lin, Stefan Meinel, Matthew Wingate,

arXiv:1212.4827 $(\Lambda_b \to \Lambda\, \ell^+ \ell^-)$,
arXiv:1306.0446 $(\Lambda_b \to p\, \ell^- \bar{\nu}_\ell)$

Credit: Lawrence Berkeley National Lab - Roy Kaltschmidt, photographer

Λ_b decay form factors with static b quarks

In the following, we discuss the case $\Lambda_b \to \Lambda$. The case $\Lambda_b \to p$ is analogous.

- In the limit $m_b \to \infty$, the hadronic matrix elements with an arbitrary gamma matrix Γ in the current can be written as

  $$\langle \Lambda(p', s') | \, \bar{s} \, \Gamma \, Q \, | \Lambda_Q(v, s) \rangle = \bar{u}_\Lambda(p', s') \, [F_1 + \not{v} \, F_2] \, \Gamma \, u_{\Lambda_Q}(v, s)$$

  where

  $$v = (\text{four-velocity of } \Lambda_Q)$$

  and the two form factors F_1, F_2 are functions of

  $$p' \cdot v$$

  (the energy of the Λ in the Λ_Q rest frame)

  [Mannel, Roberts, Ryzak, NPB **355**, 38 (1991); Hussain, Körner, Kramer, Thompson, Z. Phys. C **51**, 321 (1991)]

- This simplifies the lattice calculation and the data analysis

- Finite-m_b corrections are of order $\mathcal{O}(\Lambda_{\rm QCD}/m_b, \, |\mathbf{p}'|/m_b)$

Static b quarks on the lattice

Set $v = (1, 0, 0, 0)$.

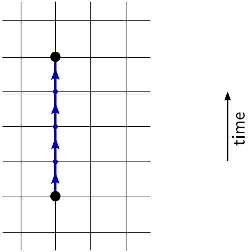
- Action:

$$S = a^3 \sum_x \bar{Q}(x) \left[Q(x) - U_0^\dagger(x - a\hat{0}) Q(x - a\hat{0}) \right]$$

[Eichten and Hill, PLB **240**, 193 (1990)]

- Propagator:

$$\frac{1 + \gamma^0}{2} \times$$



- Exponential improvement of signal-to-noise ratio achieved by smearing U_0

  [Della Morte et al., hep-lat/0307021]

- We use one level of HYP smearing, $(\alpha_1, \alpha_2, \alpha_3) = (1.0,\ 1.0,\ 0.5)$

  [Della Morte, Shindler, Sommer, hep-lat/0506008]

Actions for light quarks and gluons

- RBC/UKQCD ensembles with $a \approx 0.11$ fm (coarse), $a \approx 0.08$ fm (fine), $L \approx 2.7$ fm [Aoki et al., arXiv:1011.0892] Thank you for providing them!
- u, d, s quarks: domain-wall action [Kaplan, hep-lat/9206013; Furman and Shamir, hep-lat/9405004]
- gluons: Iwasaki action [Iwasaki and Yoshie, PLB **143**, 449 (1984)]
- seven data sets, four partially quenched

Set	$L^3 \times T$	$am_{u,d}^{(\mathrm{sea})}$	$am_s^{(\mathrm{sea})}$	a (fm)	$am_{u,d}^{(\mathrm{val})}$	$am_s^{(\mathrm{val})}$	$m_\pi^{(\mathrm{vv})}$ (MeV)
C14	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.001	0.04	245(4)
C24	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.002	0.04	270(4)
C54	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	0.04	336(5)
C53	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	0.03	336(5)
F23	$32^3 \times 64$	0.004	0.03	0.0849(12)	0.002	0.03	227(3)
F43	$32^3 \times 64$	0.004	0.03	0.0849(12)	0.004	0.03	295(4)
F63	$32^3 \times 64$	0.006	0.03	0.0848(17)	0.006	0.03	352(7)

Baryon interpolating fields, static-light current

- baryon interpolating fields:

$$\Lambda_{Q\alpha} = \epsilon^{abc}\,(C\gamma_5)_{\beta\gamma}\,\tilde{d}^a_\beta\,\tilde{u}^b_\gamma\,Q^c_\alpha,$$
$$\Lambda_\alpha = \epsilon^{abc}\,(C\gamma_5)_{\beta\gamma}\,\tilde{u}^a_\beta\,\tilde{d}^b_\gamma\,\tilde{s}^c_\alpha$$

(tilde = Gaussian smearing)

- $\mathcal{O}(a)$ improved current, matched to continuum HQET in $\overline{\text{MS}}$ scheme:

$$J_\Gamma = \underbrace{U(m_b, a^{-1})}_{\text{RG evolution}}\,\mathcal{Z}\left[\left(1 + \frac{c_\Gamma^{(ma)}\,m_s\,a}{1-(w_0^{\text{MF}})^2}\right)\bar{Q}\,\Gamma\,s + c_\Gamma^{(pa)}\,a\,\bar{Q}\,\Gamma\,\gamma\cdot\boldsymbol{\nabla}\,s\right]$$

$\mathcal{Z}$, $c^{(ma)\Gamma}$, $c_\Gamma^{(pa)}$ calculated using tadpole-improved one-loop lattice perturbation theory by Tomomi Ishikawa *et al.* [arXiv:1101.1072]

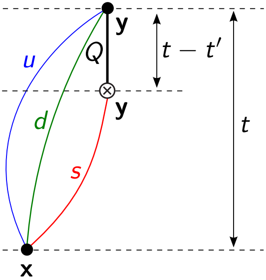
See talk on B^0-$\bar{B}^0$ mixing by Tomomi Ishikawa (Tue, 5:20)

Three-point functions

"Backward" three-point function:

$$C^{(3,\mathrm{bw})}_{\alpha\delta}(\Gamma,\ \mathbf{p}',t,t-t')$$

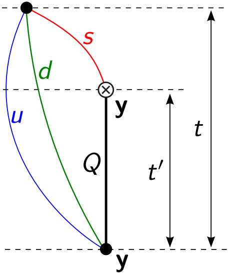
$$=\sum_{\mathbf{y}} e^{-i\mathbf{p}'\cdot(\mathbf{y}-\mathbf{x})}\Big\langle \Lambda_{Q\alpha}(x_0+t,\mathbf{y})\ \ J_\Gamma(x_0+t',\mathbf{y})\ \ \bar{\Lambda}_\delta(x_0,\mathbf{x})\Big\rangle$$



"Forward" three-point function:

$$C^{(3,\mathrm{fw})}_{\delta\alpha}(\Gamma,\ \mathbf{p}',t,t')$$

$$=\sum_{\mathbf{y}} e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}\Big\langle \Lambda_\delta(x_0,\mathbf{x})\ \ J^\dagger_\Gamma(x_0-t+t',\mathbf{y})\ \ \bar{\Lambda}_{Q\alpha}(x_0-t,\mathbf{y})\Big\rangle$$



No sequential DW propagators needed

$\rightarrow$ Inexpensive to do many source-sink separations t

Ratios

Compute the ratio

$$\mathcal{R}(\Gamma, \mathbf{p}', t, t') = \frac{4\,\mathrm{Tr}\left[C^{(3,\mathrm{fw})}(\Gamma,\ \mathbf{p}', t, t')\ \ C^{(3,\mathrm{bw})}(\Gamma,\ \mathbf{p}', t, t-t')\right]}{\mathrm{Tr}\left[C^{(2,\Lambda)}(\mathbf{p}', t)\right]\ \mathrm{Tr}\left[C^{(2,\Lambda_b)}(t)\right]}$$

For Γ = (any product of γ^μ's), this gives

$$\mathcal{R}(\Gamma, \mathbf{p}', t, t') = \begin{cases} \dfrac{E_\Lambda + m_\Lambda}{E_\Lambda}\,[F_1 + F_2]^2\ + (\text{excited-state contribs}), & \text{if}\ \ [\Gamma, \gamma^0] = 0, \\[2ex] \dfrac{E_\Lambda - m_\Lambda}{E_\Lambda}\,[F_1 - F_2]^2\ + (\text{excited-state contribs}), & \text{if}\ \ \{\Gamma, \gamma^0\} = 0. \end{cases}$$

Define

$$\begin{aligned} F_+ &= F_1 + F_2 \\ F_- &= F_1 - F_2 \end{aligned}$$

Ratios

To improve statistics, average over multiple gamma matrices and define

$$\begin{aligned}\mathcal{R}_+(\mathbf{p}', t, t') &= \frac{1}{4}\Big[\mathcal{R}(1, \mathbf{p}', t, t') + \mathcal{R}(\gamma^2\gamma^3, \mathbf{p}', t, t') \\ &\quad + \mathcal{R}(-\gamma^3\gamma^1, \mathbf{p}', t, t') + \mathcal{R}(\gamma^1\gamma^2, \mathbf{p}', t, t')\Big], \\ \mathcal{R}_-(\mathbf{p}', t, t') &= \frac{1}{4}\Big[\mathcal{R}(\gamma^1, \mathbf{p}', t, t') + \mathcal{R}(\gamma^2, \mathbf{p}', t, t') \\ &\quad + \mathcal{R}(\gamma^3, \mathbf{p}', t, t') + \mathcal{R}(-i\gamma_5, \mathbf{p}', t, t')\Big]\end{aligned}$$

For given $|\mathbf{p}'|^2$, then average over the direction of $\mathbf{p}'$; denote the direction-averaged quantities by

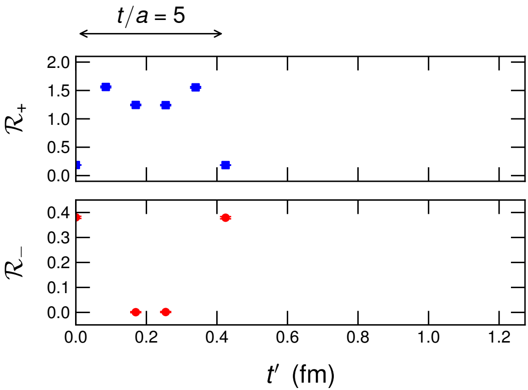
$$\mathcal{R}_\pm(|\mathbf{p}'|^2, t, t')$$

Ratios: example results

$\Lambda_Q \to \Lambda$

$|\mathbf{p}'|^2 = 4\,(2\pi/L)^2$

`F43` data set



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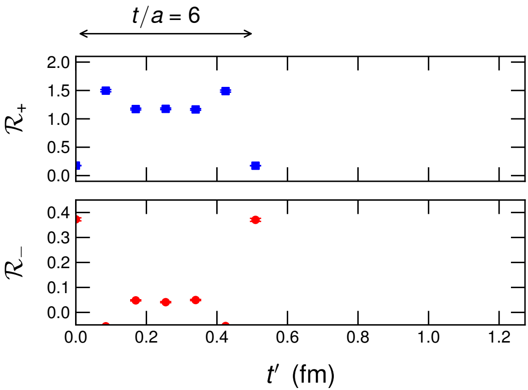

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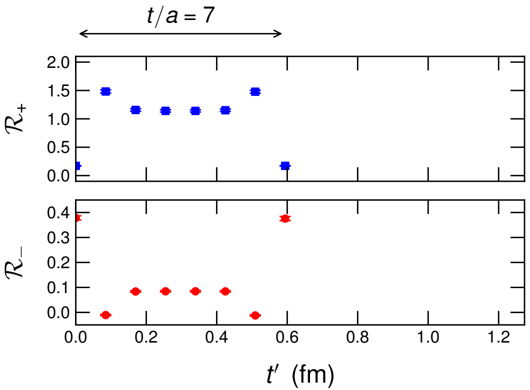

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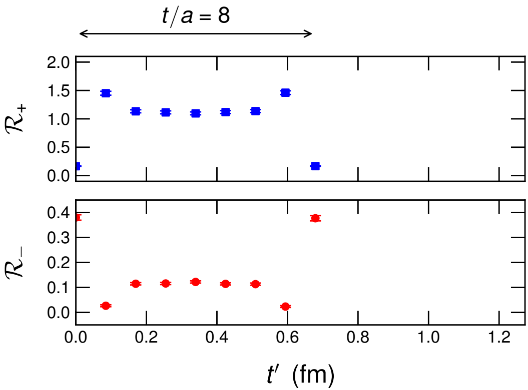

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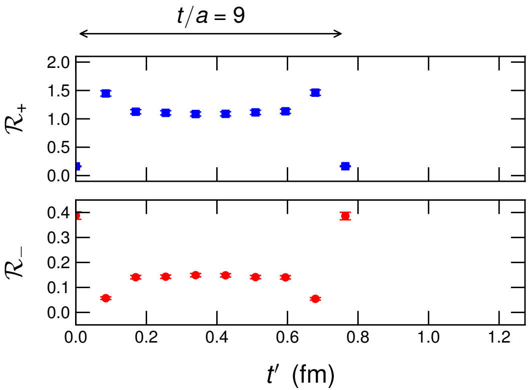
$|\mathbf{p}'|^2 = 4\,(2\pi/L)^2$

F43 data set

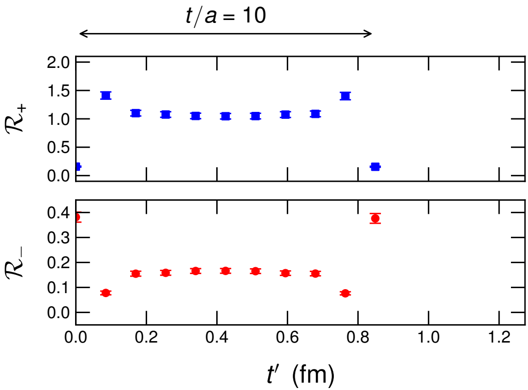


Ratios: example results

$\Lambda_Q \to \Lambda$

$|\mathbf{p}'|^2 = 4\,(2\pi/L)^2$

F43 data set

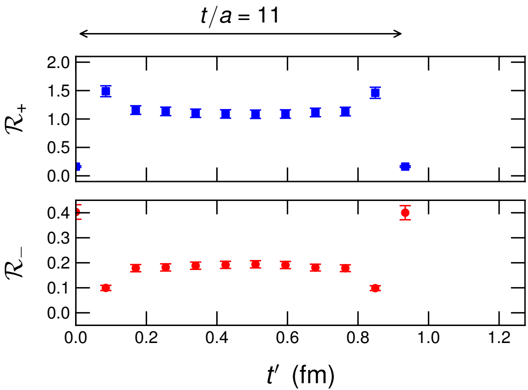


Ratios: example results

$\Lambda_Q \to \Lambda$

$|\mathbf{p}'|^2 = 4\,(2\pi/L)^2$

`F43` data set



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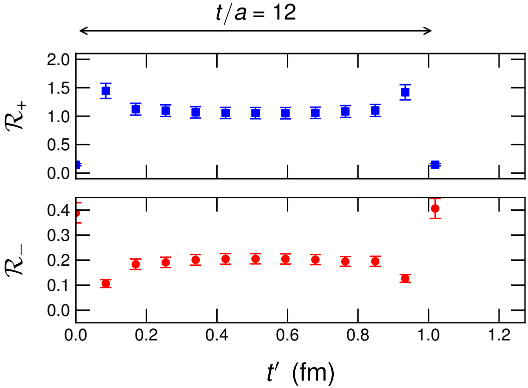

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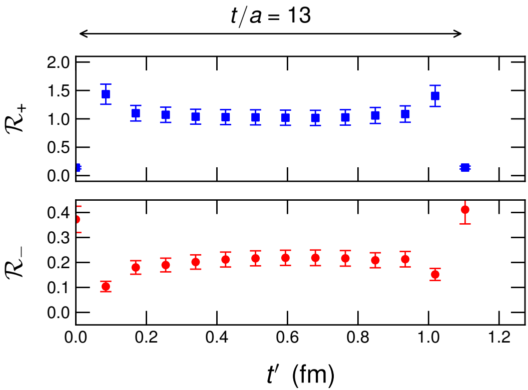

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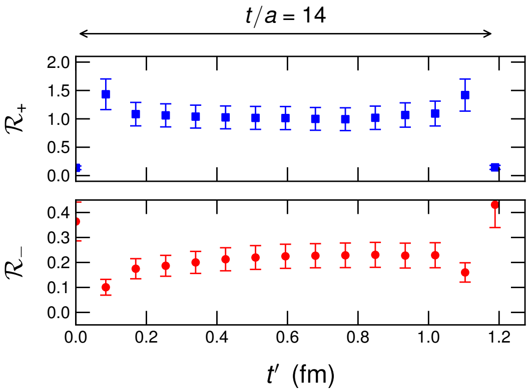
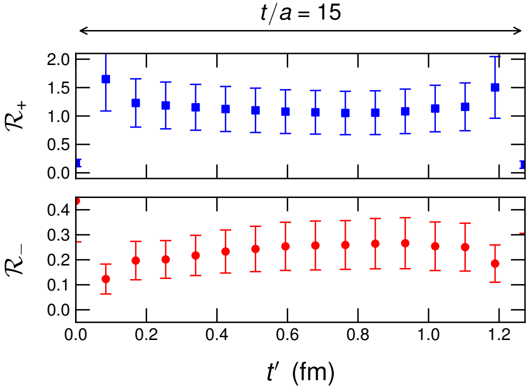


Ratios: example results

$\Lambda_Q \to \Lambda$

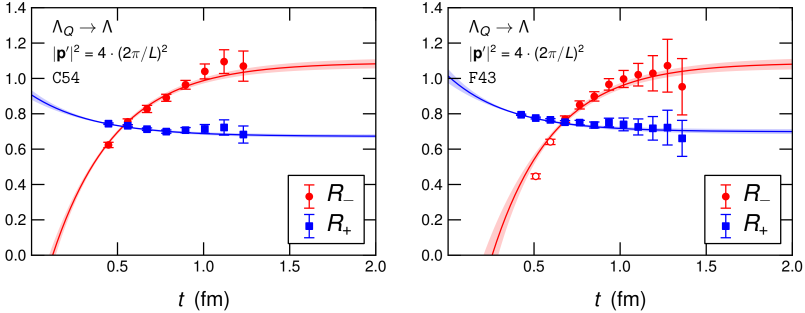
$|\mathbf{p}'|^2 = 4\,(2\pi/L)^2$

`F43` data set



Evaluate at mid-point $t' = t/2$ and compute

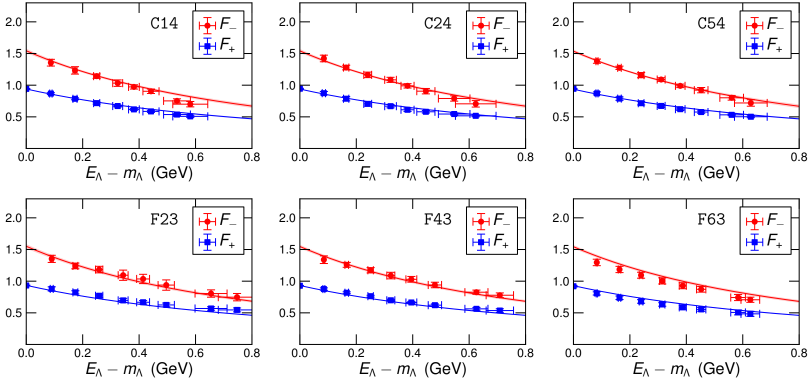
$$R_{\pm}(|\mathbf{p}'|^2, t) = \sqrt{\frac{E_\Lambda}{E_\Lambda \pm m_\Lambda} \mathcal{R}_{\pm}(|\mathbf{p}'|^2,\ t,\ t/2)} \quad \xrightarrow[t\to\infty]{} \quad F_{\pm}(E_\Lambda)$$



Fit leading excited-state contamination using

$$R_{\pm}^{i,n}(t) = F_{\pm}^{i,n} + A_{\pm}^{i,n}\ \exp[-\delta^{i,n}\ t],$$

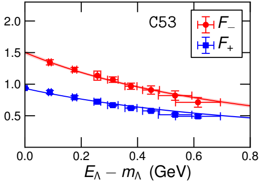
where i labels data set and n labels momentum. At each momentum, fit done simultaneously for all 7 data sets, with constraints on lattice-spacing and quark-mass dependence of $\delta^{i,n}$



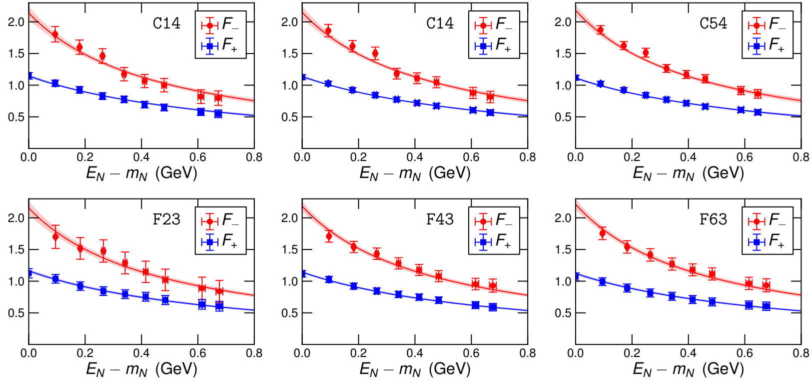
Dipole fit model:

$$F_\pm = \frac{Y_\pm}{(\widetilde{X}_\pm + E_\Lambda - m_\Lambda)^2}\left[1 + d_\pm (aE_\Lambda)^2\right],$$

$$\widetilde{X}_\pm = X_\pm + c_{l,\pm}\left[(m_\pi)^2 - (m_\pi^{\rm phys})^2\right] + c_{s,\pm}\left[(m_{\eta_s})^2 - (m_{\eta_s}^{\rm phys})^2\right]$$



$\Lambda_Q \to p$ form factors



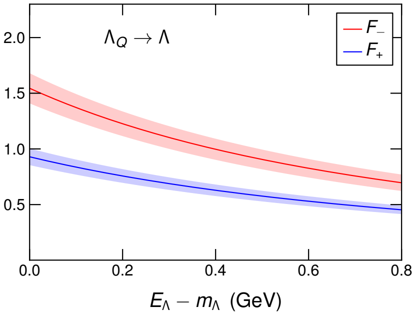
Dipole fit model:

$$F_\pm = \frac{Y_\pm}{(\widetilde{X}_\pm + E_N - m_N)^2}\left[1 + d_\pm (aE_N)^2\right],$$

$$\widetilde{X}_\pm = X_\pm + c_{l,\pm}\left[(m_\pi)^2 - (m_\pi^{\text{phys}})^2\right]$$

Note: here, E_N computed from m_N using relativistic dispersion relation

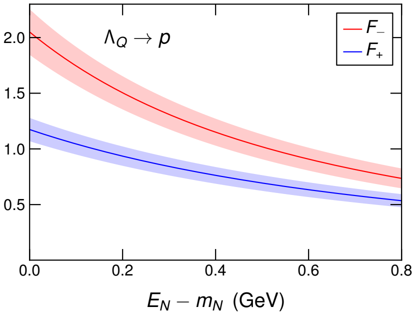
$\Lambda_Q \to \Lambda$ form factors at $m_\pi = m_\pi^{\rm phys}$, $m_{\eta_s} = m_{\eta_s}^{\rm phys}$, $a = 0$



Error bands shown here include the following estimates of systematic uncertainties:

- renormalization: ∼6%
- finite volume: ∼3%
- chiral extrapolation: ∼3%
- continuum extrapolation: ∼2%

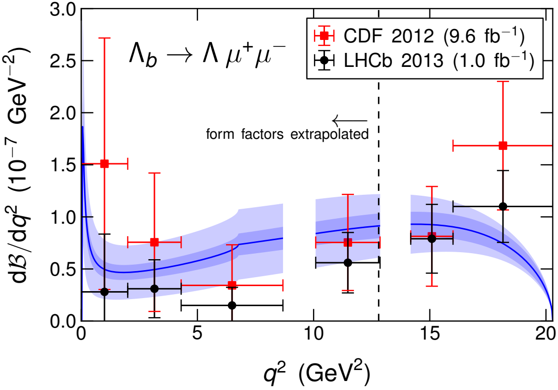
$\Lambda_Q \to p$ form factors at $m_\pi = m_\pi^{\rm phys}$, $m_{\eta_s} = m_{\eta_s}^{\rm phys}$, $a = 0$



Error bands shown here include the following estimates of systematic uncertainties:

- renormalization: $\sim$6%
- finite volume: $\sim$3%
- chiral extrapolation: $\sim$3%
- continuum extrapolation: $\sim$2%

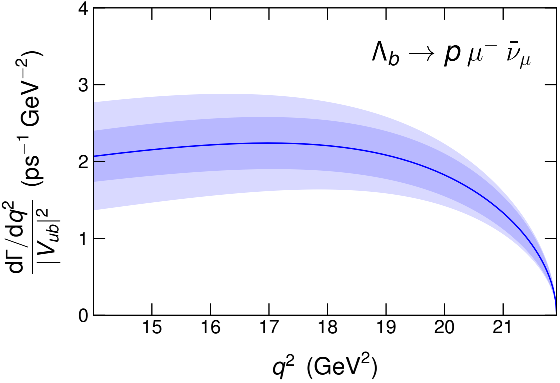
$\Lambda_b \to \Lambda\mu^+\mu^-$ differential branching fraction



[CDF data: Public Note 10726, LHCb data: arXiv:1306.2577]

- Inner error band from uncertainty in static form factors
- Outer error band includes $\dfrac{\sqrt{|\mathbf{p}'|^2 + \Lambda_{QCD}^2}}{m_b}$ uncertainty from static approximation

$\Lambda_b \to p\,\mu^- \bar{\nu}_\mu$ differential decay rate



- Only the kinematic range where we have lattice data is shown here.
  Integrated rate:

$$\frac{1}{|V_{ub}|^2}\int_{14\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to p\,\mu^-\bar{\nu}_\mu)}{dq^2}\,dq^2 \;=\; 15.3 \pm 2.4 \pm 3.4\ \ \mathrm{ps}^{-1}$$

  $\to |V_{ub}|$ extraction with theory uncertainty $\sim 15\%$ (dominated by static approximation), which is already smaller than the exclusive-inclusive discrepancy

Lattice calculation with relativistic b quarks

Work in progress. All results are very preliminary.

Credit: Oak Ridge National Laboratory

Relativistic form factors

To eliminate the $1/m_b$ systematic uncertainty, replace static heavy-quark action by "relativistic" heavy-quark action and perform direct calculation of the relativistic form factors

$$
\begin{aligned}
\langle X|\,\overline{q}\,\gamma^\mu\,b\,|\Lambda_b\rangle &= \overline{u}_X\left[f_1^V\,\gamma^\mu - f_2^V\,i\sigma^{\mu\nu}q_\nu/m_{\Lambda_b} + f_3^V\,q^\mu/m_{\Lambda_b}\right]u_{\Lambda_b},\\
\langle X|\,\overline{q}\,\gamma^\mu\gamma_5\,b\,|\Lambda_b\rangle &= \overline{u}_X\left[f_1^A\,\gamma^\mu - f_2^A\,i\sigma^{\mu\nu}q_\nu/m_{\Lambda_b} + f_3^A\,q^\mu/m_{\Lambda_b}\right]\gamma_5\,u_{\Lambda_b},\\
\langle X|\,\overline{q}\,i\sigma^{\mu\nu}q_\nu\,b\,|\Lambda_b\rangle &= \overline{u}_X\left[f_1^{TV}\,(\gamma^\mu q^2 - q^\mu\not{q})/m_{\Lambda_b} - f_2^{TV}\,i\sigma^{\mu\nu}q_\nu\right]u_{\Lambda_b},\\
\langle X|\,\overline{q}\,i\sigma^{\mu\nu}q_\nu\,\gamma_5\,b\,|\Lambda_b\rangle &= \overline{u}_X\left[f_1^{TA}\,(\gamma^\mu q^2 - q^\mu\not{q})/m_{\Lambda_b} - f_2^{TA}\,i\sigma^{\mu\nu}q_\nu\right]\gamma_5\,u_{\Lambda_b}
\end{aligned}
$$

Actions

- Reuse the existing domain-wall propagators for light and strange quarks
- For the b quarks, adopt the three-parameter RHQ action used by RBC/UKQCD [Aoki et al., arXiv:1206.2554]

$$S = a^4 \sum_x \bar{Q} \left(m_0 + \gamma_0 \nabla_0 + \zeta \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} - \frac{a}{2} \nabla_0^2 - \frac{a}{2} \zeta \boldsymbol{\nabla}^2 + \frac{a}{4} c_P \, i \sigma^{\mu\nu} F_{\mu\nu} \right) Q$$

See the following talks for more details:

Oliver Witzel on f_B, f_{B_s} (Tue 2:00)
Taichi Kawanai on $\bar{B} \to \pi \ell^- \bar{\nu}_\ell$ (Tue, 2:20)
Ben Samways on $g_{B^* B \pi}$ (Tue, 4:40)

- in addition to $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$, also compute $\Lambda_b \to \Lambda_c$

[$\Lambda_b \to \Lambda_c$ first studied by Bowler et al. (UKQCD), hep-lat/9709028; Gottlieb and Tamhankar, hep-lat/0301022]

Mostly nonperturbative renormalization

Heavy-light current written as follows: [El-Khadra, Kronfeld, Mackenzie, Ryan, Simone, hep-ph/0101023]

$$J_\Gamma = \sqrt{Z_V^{QQ}\, Z_V^{qq}}\ \rho_\Gamma \left[\bar{q}\,\Gamma\,Q + \mathcal{O}(a)\right]$$

where

■ = nonperturbative

Determination of Z_V^{QQ}: see talk by Taichi Kawanai (Tue, 2:20). Thanks for providing preliminary results!

■ = one loop

Computed using **PhySyHCAl** framework (`http://physyhcal.lhnr.de`) by Christoph Lehner [arXiv:1211.4013]

See talk by Christoph Lehner (Tue, 2:40). Thanks for providing preliminary results and for discussions!

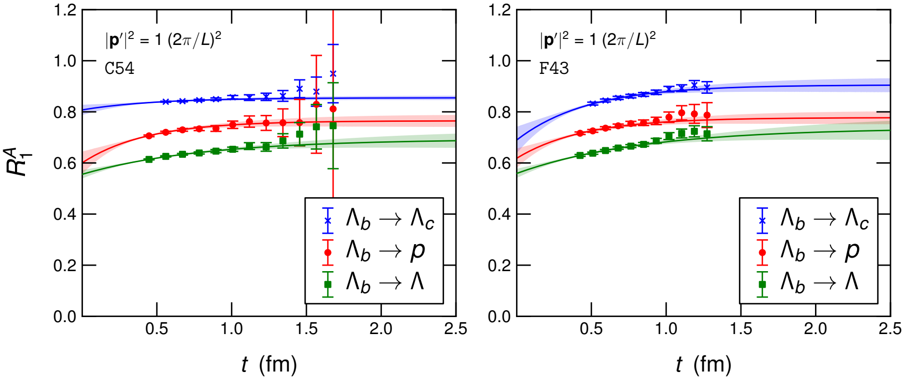
Relativistic form factors from ratios

- Similar method as used in static calculation (current insertion at mid-point $t' = t/2$, then extrapolate to $t = \infty$).
- Have constructed combinations of ratios that directly give the 8 form factors:

$$
\begin{aligned}
R_1^V(t) &\xrightarrow[t\to\infty]{} f_1^V, \\
R_2^V(t) &\xrightarrow[t\to\infty]{} f_2^V, \\
R_1^A(t) &\xrightarrow[t\to\infty]{} f_1^A, \\
R_2^A(t) &\xrightarrow[t\to\infty]{} f_2^A, \\
R_1^{TV}(t) &\xrightarrow[t\to\infty]{} f_1^{TV}, \\
R_2^{TV}(t) &\xrightarrow[t\to\infty]{} f_2^{TV}, \\
R_1^{TA}(t) &\xrightarrow[t\to\infty]{} f_1^{TA}, \\
R_2^{TA}(t) &\xrightarrow[t\to\infty]{} f_2^{TA}
\end{aligned}
$$

(f_3^V and f_3^A do not contribute to decay rates in massless lepton approximation)

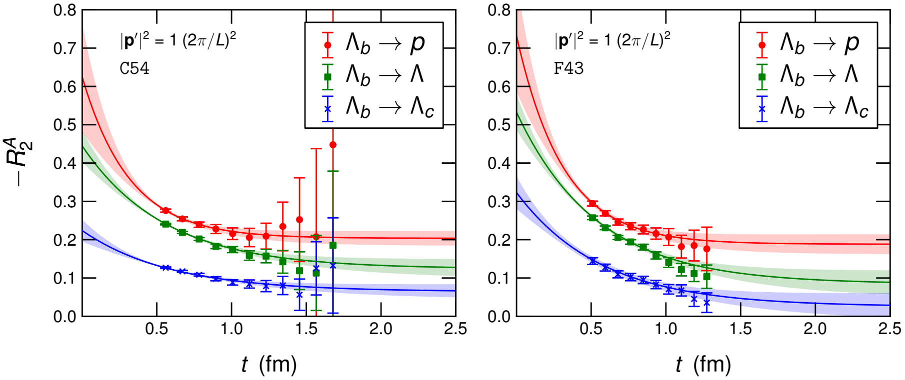
Example results: $R_1^A(t)$



Caveats:

- for $\Lambda_b \to \Lambda_c$, Z_V^{cc} computed only at tree level, ρ factors missing
- $\mathcal{O}(a)$ improvement not yet included

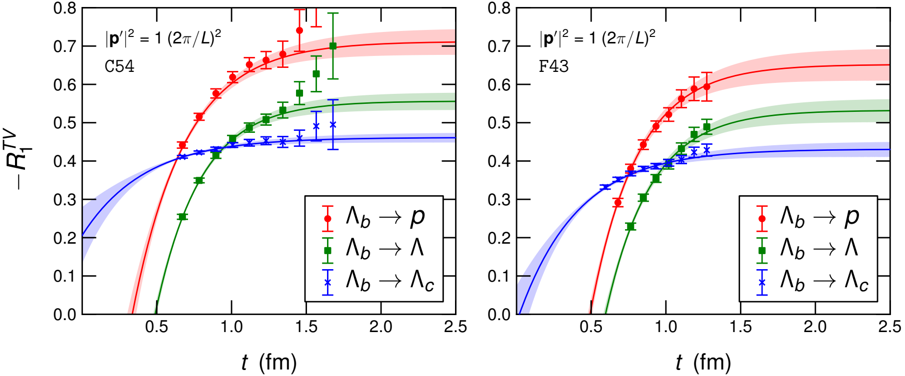
Example results: $-R_2^A(t)$



Caveats:

- for $\Lambda_b \to \Lambda_c$, Z_V^{cc} computed only at tree level, ρ factors missing
- $\mathcal{O}(a)$ improvement not yet included

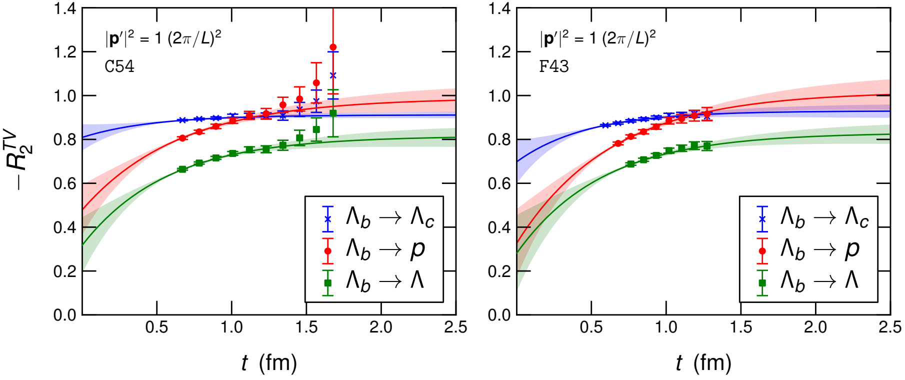
Example results: $-R_1^{TV}(t)$



Caveats:

- for $\Lambda_b \to \Lambda_c$, Z_V^{cc} computed only at tree level, ρ factors missing
- $\mathcal{O}(a)$ improvement not yet included
- ρ factors for tensor currents missing

Example results: $-R_2^{TV}(t)$



- for $\Lambda_b \to \Lambda_c$, Z_V^{cc} computed only at tree level, ρ factors missing
- $\mathcal{O}(a)$ improvement not yet included
- ρ factors for tensor currents missing

Conclusions / Outlook

- Lattice calculation of heavy→light baryon form factors is less expensive than lattice calculation of light→light baryon form factors (sequential propagators for heavy quark only)

- Substantial excited-state contamination successfully removed through wide range of source-sink separations and $t \to \infty$ extrapolation

- Plan to update predictions of decay rates using relativistic form factors, also predict angular distributions

- Plan to calculate relativistic form factors on new $48^3 \times 96$ RBC/UKQCD Möbius DW ensemble at physical pion mass using all-mode-averaging
  [Blum, Izubuchi, Shintani, arXiv:1208.4349]

- $\Lambda_b \to \Lambda \mu^+ \mu^-$: theory uncertainty will be dominated by long-distance contributions ($B \to K^{(*)} \mu^+ \mu^-$ has the same problem). LHCb measurement is statistics limited; higher precision results and angular analysis forthcoming
  [Watson, QFTHEP 2013]

- $\Lambda_b \to p \mu^- \bar{\nu}$ and $\Lambda_b \to \Lambda_c \mu^- \bar{\nu}$: theory predictions (in low-recoil region) will be very precise. Awaiting experimental data from LHCb [Egede and Sutcliffe, 2013]