The Twisted Polyakov Loop Coupling and the Search for an IR Fixed Point

PTEP (2013) 083B01 and arXiv:1307.6645[hep-lat]

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JICFuS

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Is there an IR fixed point
in SU(3) Nf=12 massless theory?

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Ishikawa, Iwasaki, Nakayama, Yoshie (phase structure, correlation fn.)

Appelquist, Fleming, Neil, M.Lin, Schaich (running coupling, mass spectrum)

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DeGrand (mass spectrum)

LatKMI (mass spectrum)

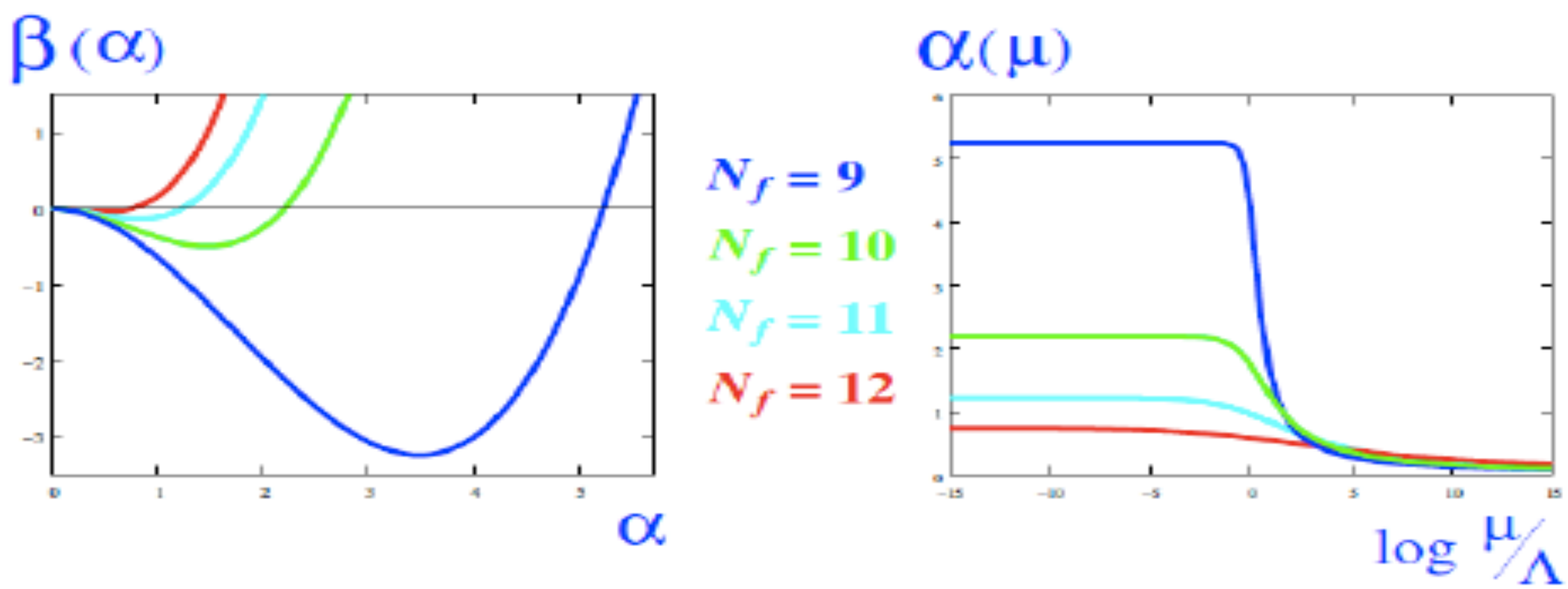
D.Lin, Ogawa, Ohki, Shintani (running coupling)

Fodor, Holland, Kuti, Nogradi, Schroeder, (running coupling, phase structure, spectrum)

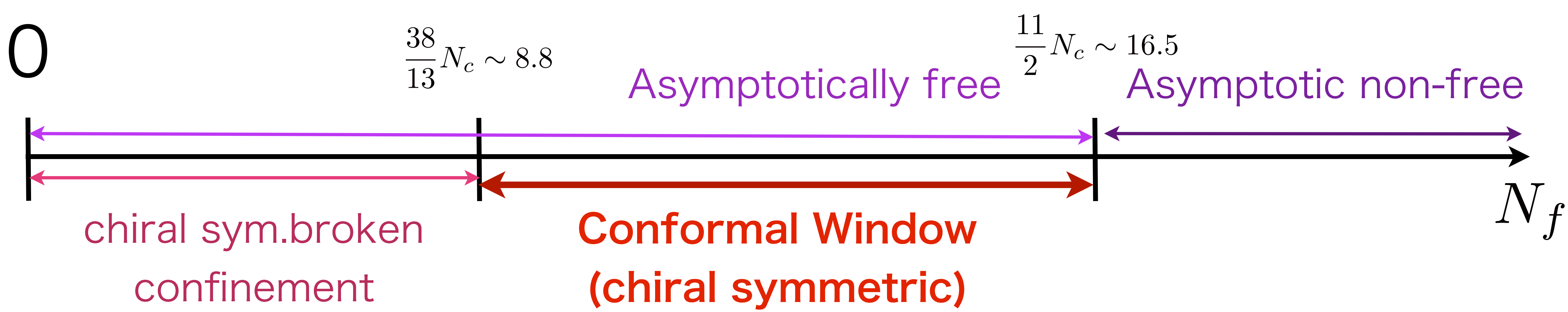
Jin and Mawhinney (phase structure)

SU(3) Nf gauge theory

Two loop analysis $\beta(\alpha) = -b\alpha^2 - c\alpha^3$



Phase structure based on two loop



Perturbative (MS bar scheme)

	2-loop	3-loop	4-loop
(alpha)	0.75	0.44	0.47
(g^2)	9.4	5.5	5.9

T.A.Ryttov and R.Shrock,
Phys.Rev.D83,056011 (2011)
20th order in Wilson loop scheme is also done by Horsley et.al.
Phys.Rev. D86 (2012) 054502

S-D eq. with large Nc

$$N_f^{cr} = 11.9$$

Exact RG

$$N_f^{cr} = 10.0^{+1.6}_{-0.7}$$

H.Gies and J.Jaeckel,
Eur.Phys.J. G46:433-438,2006

Exact RG (+ 4 fermi interaction)

$$N_f^{cr} = 11.58$$

Y.Kusafuka and H.Terao,
Phys.Rev. D84 (2011) 125006

Why are these studies contradictory?

- continuum extrapolation
- phase structure (parameter search) for each lattice setup

Methods to find interactive IR fixed point

(1) Step scaling for the renormalized coupling

Luescher, Weisz and Wolff, NPB 359 (1991) 221

(2) Hyperscaling for mass deformed theory

mass spectrum and chiral symmetry

Miransky, PRD59(1999)105003
Luty, JHEP 0904(2009)050
Del Debbio and Zwicky, PRD82(2010)014502

(3) Volume-scaling for the Dirac eigenmodes

Patella,PRD86(2012)025006
Cheng, Hasenfratz, Petropulos and Schaich, JHEP1307(2013)061

(4) Shape of the correlation fn. of mesonic operator

Ishikawa, Iwasaki, Nakayama and Yoshie, PRD87(2013)071503

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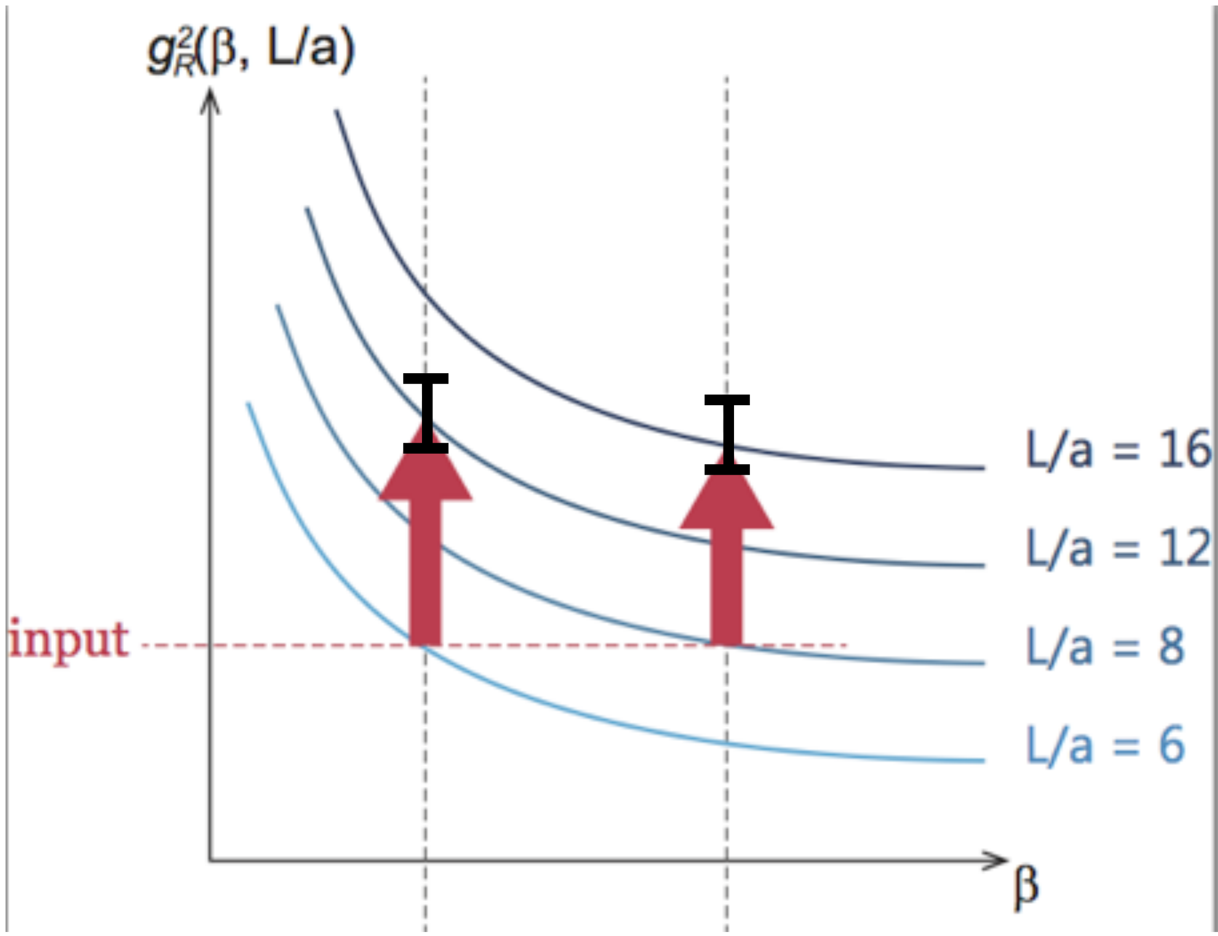
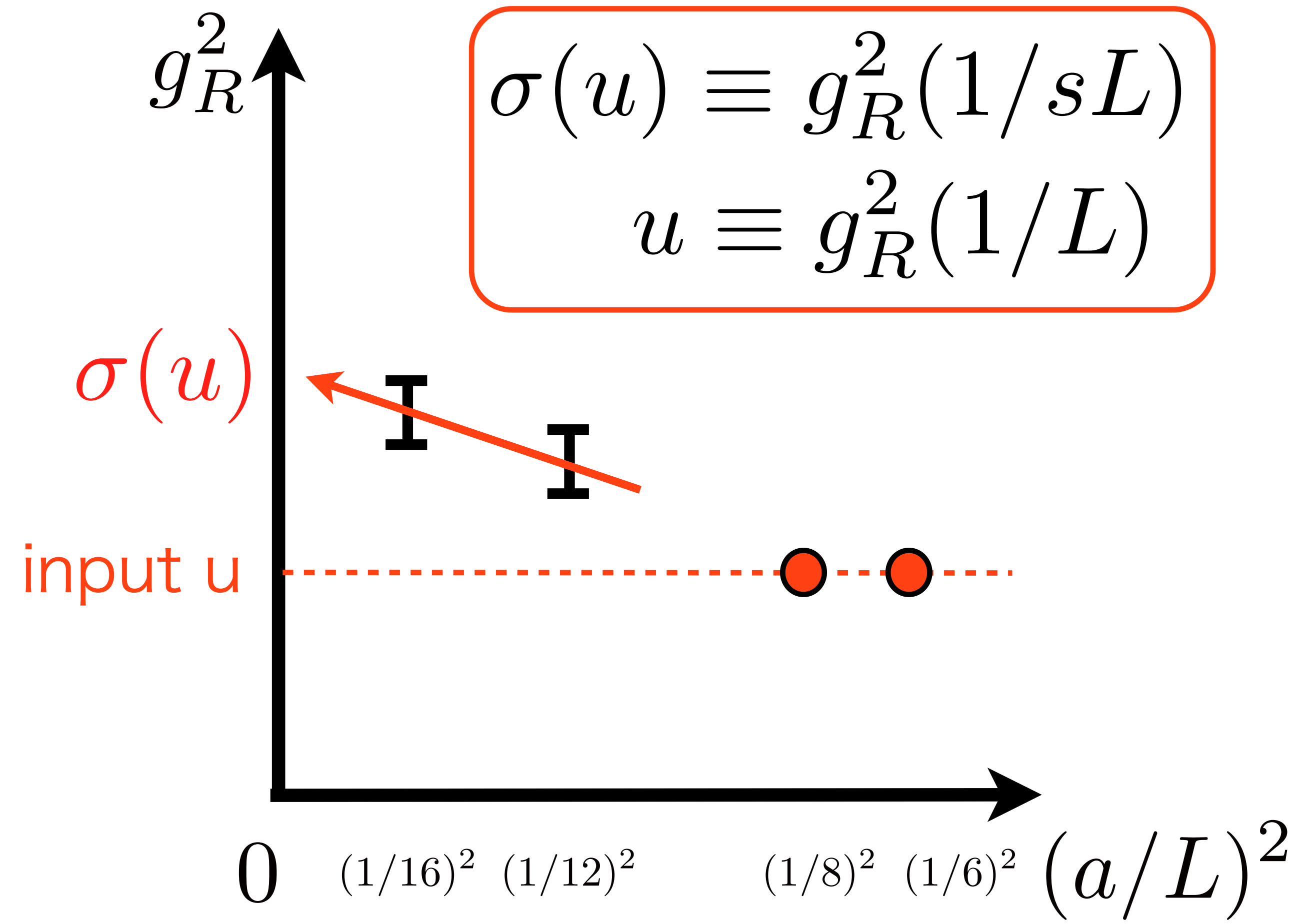
Step scaling method

- measuring the running coupling constant -

- tune beta to reproduce the input renormalized coupling
- measure the g^2 on the larger lattice with the tuned beta
- take the continuum limit





$$\sigma(u) \equiv g_R^2(1/sL)$$
$$u \equiv g_R^2(1/L)$$

We can apply this method to any
renormalization schemes on the lattice.

Several renormalization schemes and universality

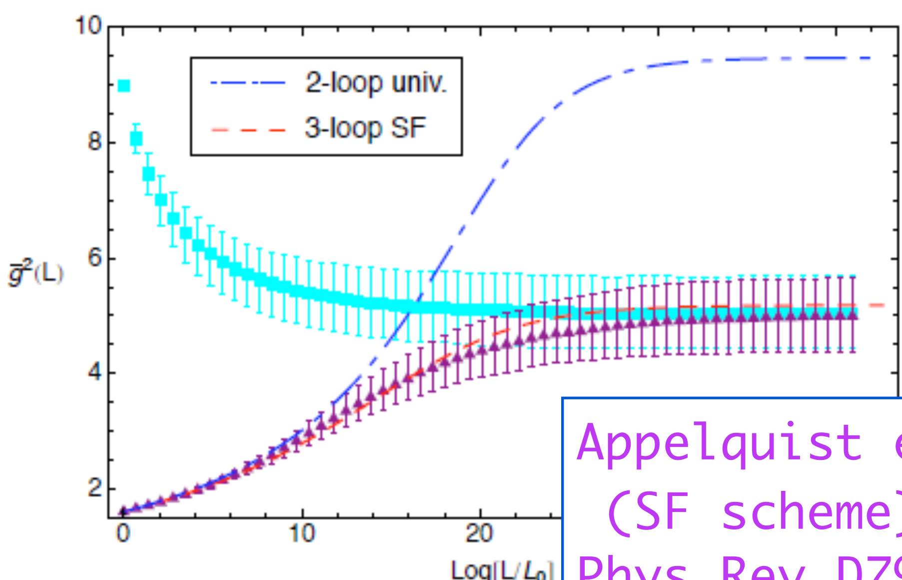
scheme transformation $$g_1 \to g_2 = f(g_1)$$

$f(g_1)$ is an analytic fn. of g_1

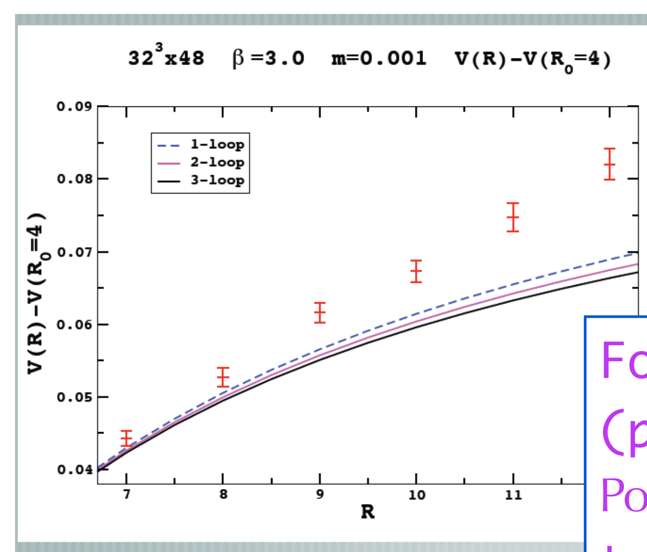
beta fn. $$\beta(g_2) = \frac{\partial f(g_1)}{\partial g_1} \beta(g_1)$$

The existence of the fixed point is scheme independent.

Recent lattice studies



Appelquist et al.
(SF scheme)
Phys.Rev.D79:076010,2009



Fodor et al.
(potential scheme)
PoS LAT2009:055,2009
talk at Lattice2010

The continuum extrapolation was not considered.

(O(a) effects depend on the renormalization scheme)

Why are these studies contradictory?

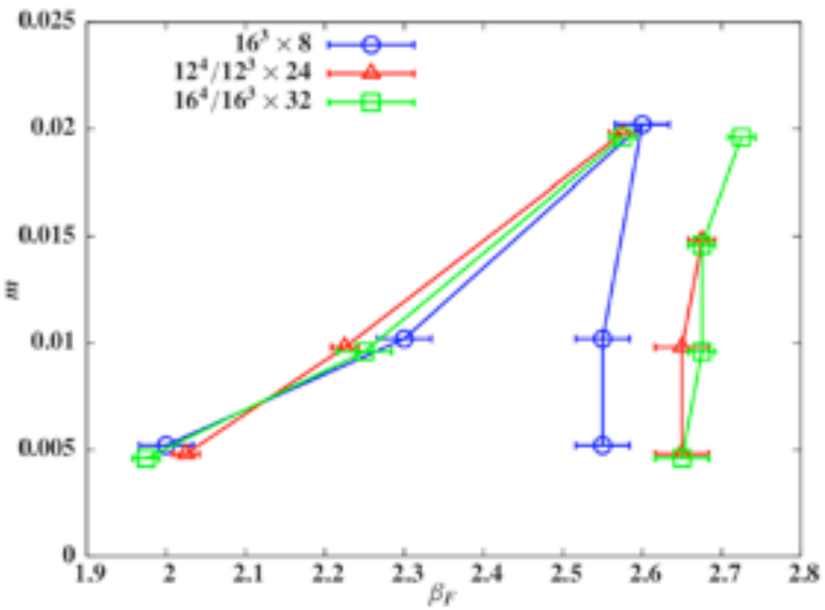
- continuum extrapolation (or infinite volume extrapolation)
- phase structure (parameter search) for each lattice

Phase structure on the lattice

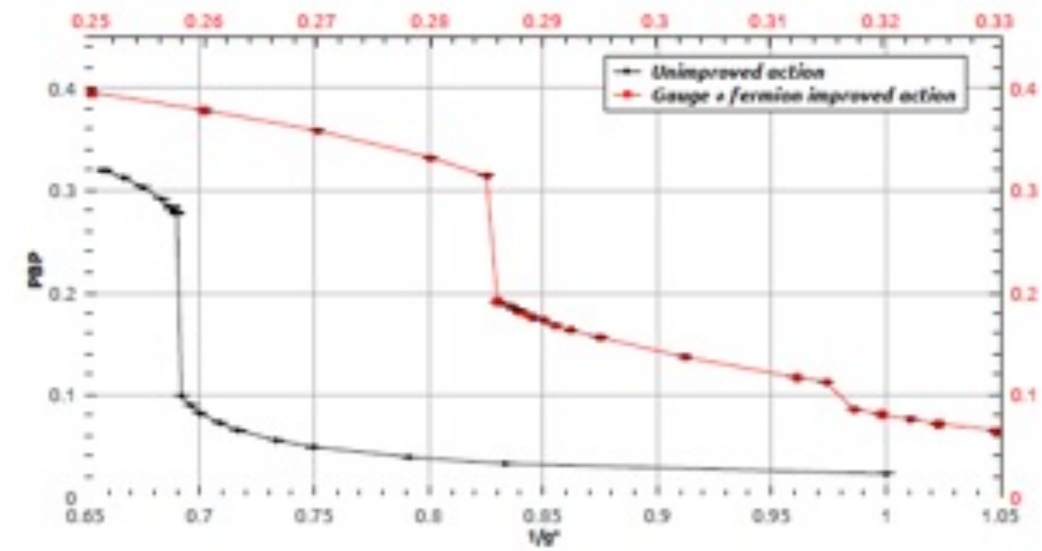
There is a bulk phase in strong coupling and near massless region.

Cheng, Hasenfratz and Schaich:
PRD85 (2012) 094509

Deuzeman, Lombardo, da Silva and Pallante:
PLB720(2013)358

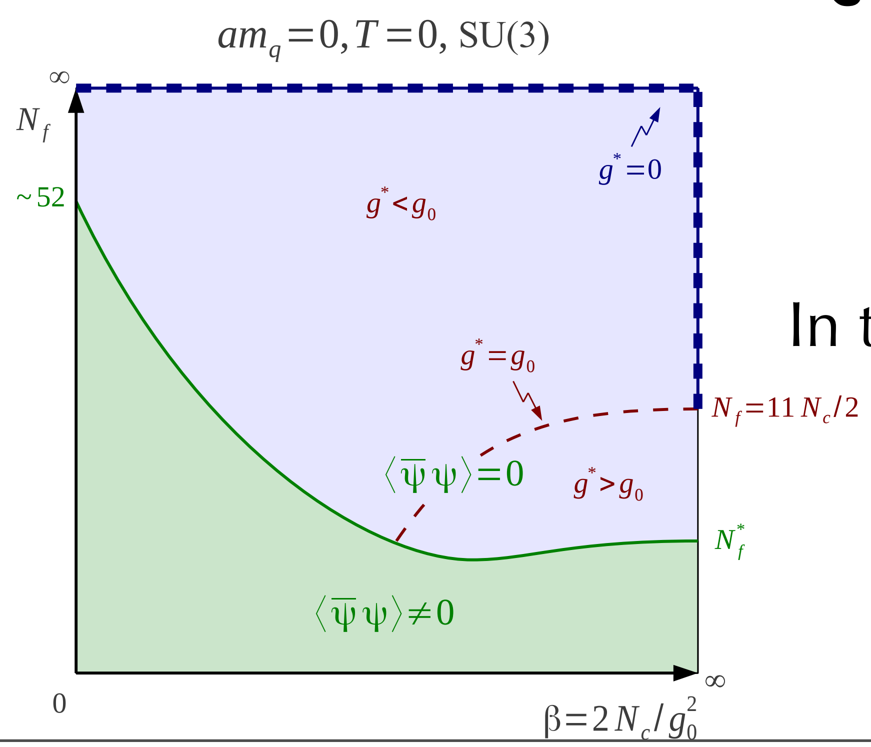


HYP smearing



Naik improvement

Conjectured phase diagram



de Forcrand, Kim and Unger:
JHEP 1302(2013)051

In the strong coupling limit, the chiral symmetry is broken Nf<52.

A careful parameter search is important for each lattice setup.

Our result

PTEP (2013) 083B01

(arXiv:1212.1353 [hep-lat])

Simulation detail

Hybrid Monte Carlo algorithm

Wilson gauge action+ naive staggered fermion

beta=4.0--100 on (L/a)^4 lattice where L/a=6,8,10,12,16,20

exact massless fermions

Twisted boundary condition for x,y directions

Link variable $U_\mu(x+\hat{\nu}L/a) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger$ $\quad \mu = x, y, z, t$, $\nu = x, y$

Fermion $\psi_\alpha^a(x+\hat{\nu}L/a) = e^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(\Omega_\nu)_{\beta\alpha}^\dagger$

Ω_ν is twist matrices (center symmetry)

$$\Omega_\nu\Omega_\nu^\dagger = \mathbb{I}, (\Omega_\nu)^3 = \mathbb{I}, \mathrm{Tr}[\Omega_\nu] = 0, \Omega_x\Omega_y = e^{i2\pi/3}\Omega_y\Omega_x$$

Twisted Polyakov loop (TPL) scheme

Examples of renormalization scheme

Schroedinger functional scheme
Wilson loop scheme
Twisted Polyakov Loop scheme
Wilson flow scheme....

} no O(a/L) error scheme

Twisted Polyakov loop (TPL) scheme
on the lattice

de Divitiis, Frezotti, Gaugnelli and Petronzio, NPB422(1994)382

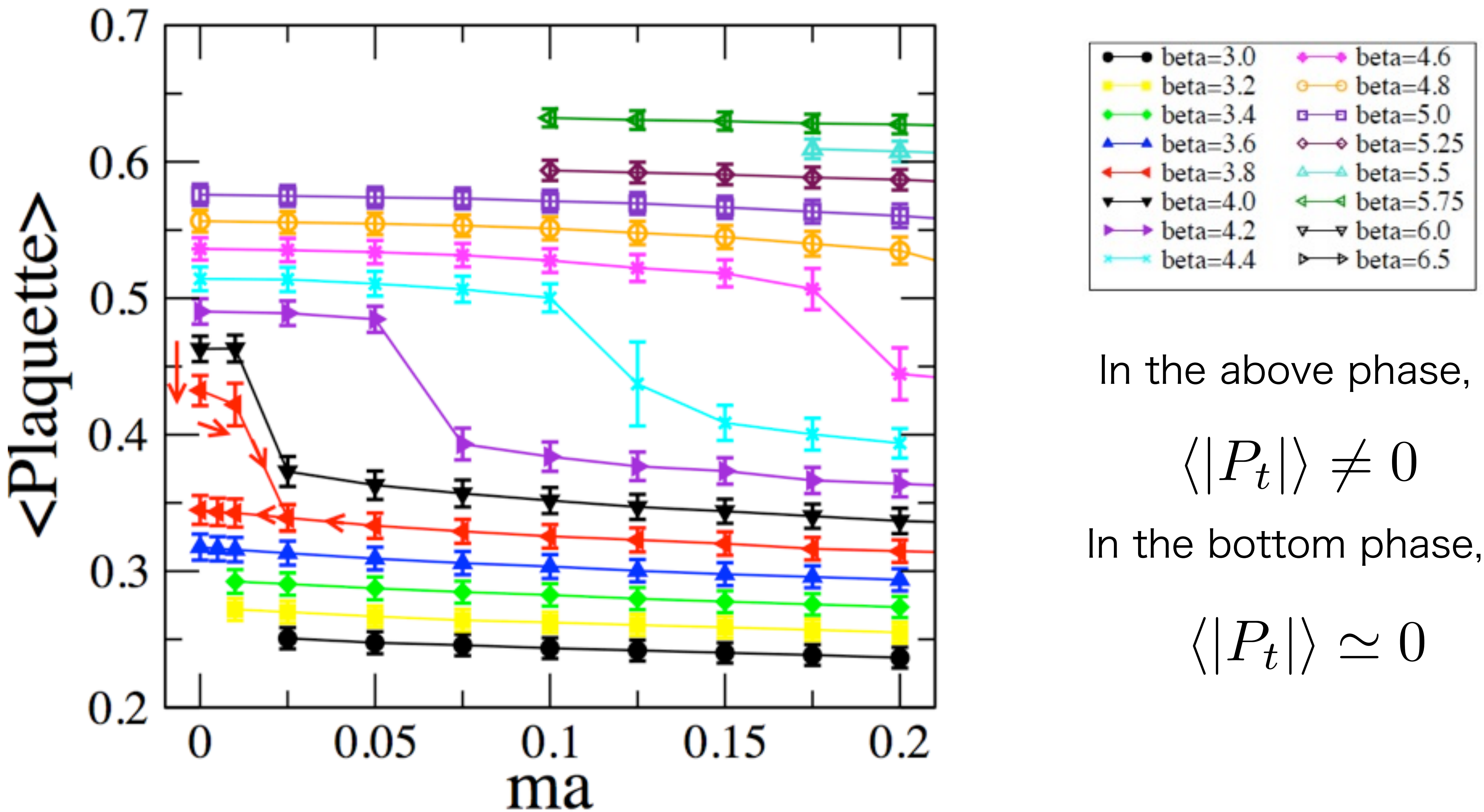
$$g^2_{\mathrm{TPL}} = \lim_{a\to 0} \frac{1}{k_{\mathrm{latt}}} \frac{\langle \sum_{y,z} P_x(y,z,L/2a) P_x(0,0,0)^\dagger \rangle}{\langle \sum_{x,y} P_z(x,y,L/2a) P_z(0,0,0)^\dagger \rangle}$$

k_{latt} is determined by the tree level value to satisfy $g^2_{\mathrm{TPL}}\big|_{\mathrm{tree}} = g_0^2$

Phase diagram in the lattice setup

In our simulation set up,

there is a bulk phase transition in small mass region.



$$(L/a)^4 = 4^4, 8^4, 12^4$$

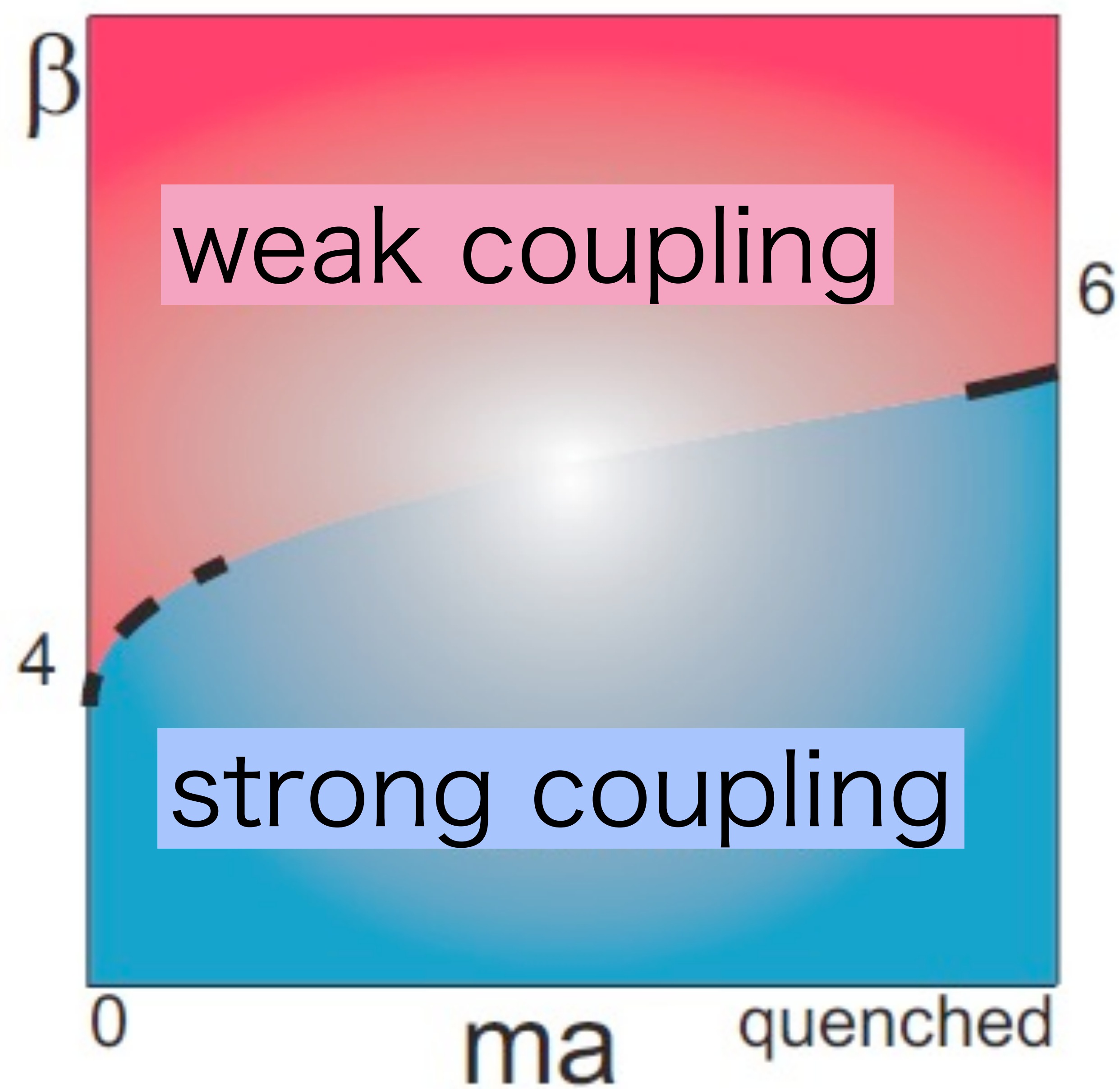
In the above phase,

$$\langle |P_t| \rangle \neq 0$$

In the bottom phase,

$$\langle |P_t| \rangle \simeq 0$$

Phase diagram for SU(3) Nf=12 naive staggered fermion with the twisted boundary condition.



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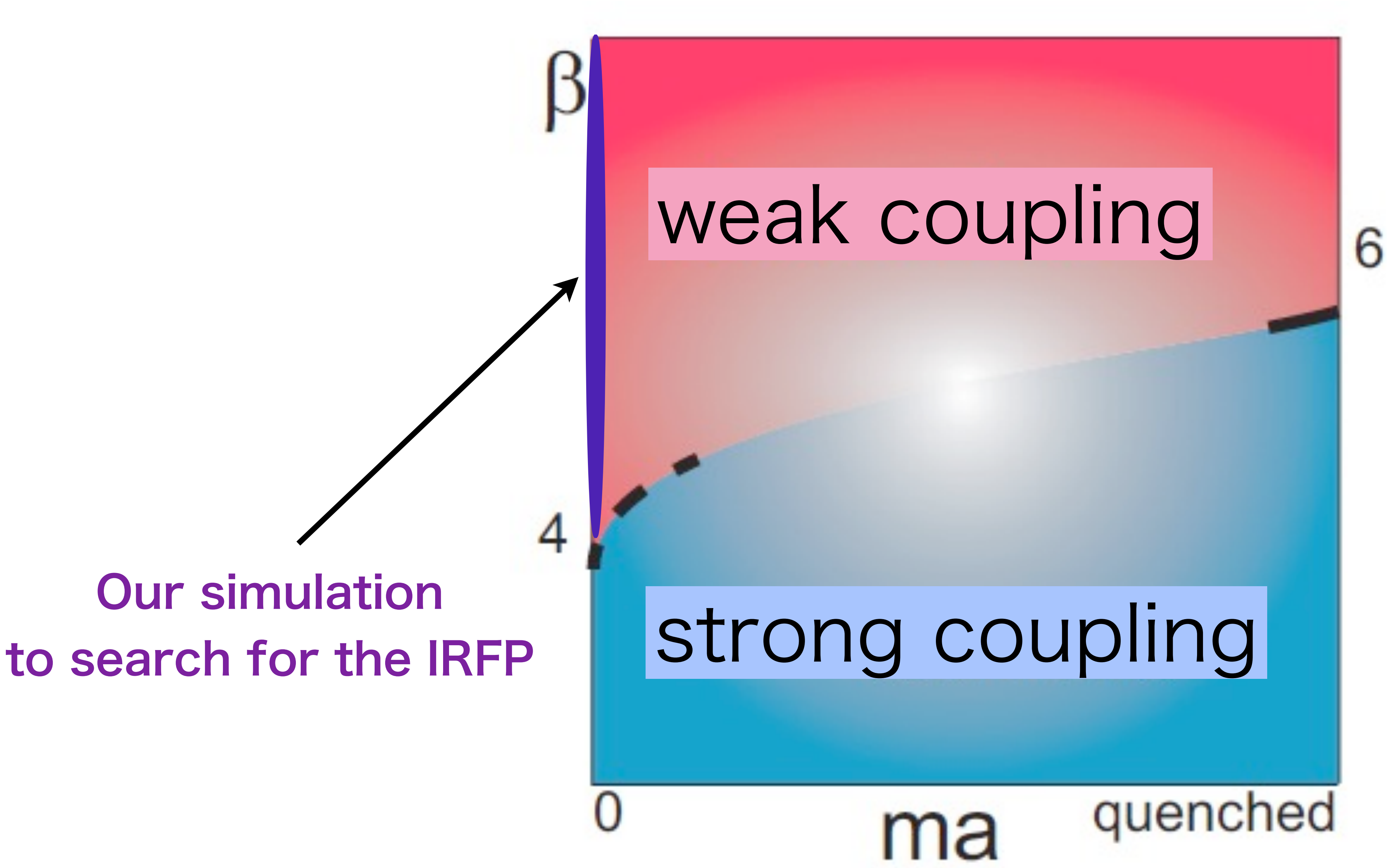


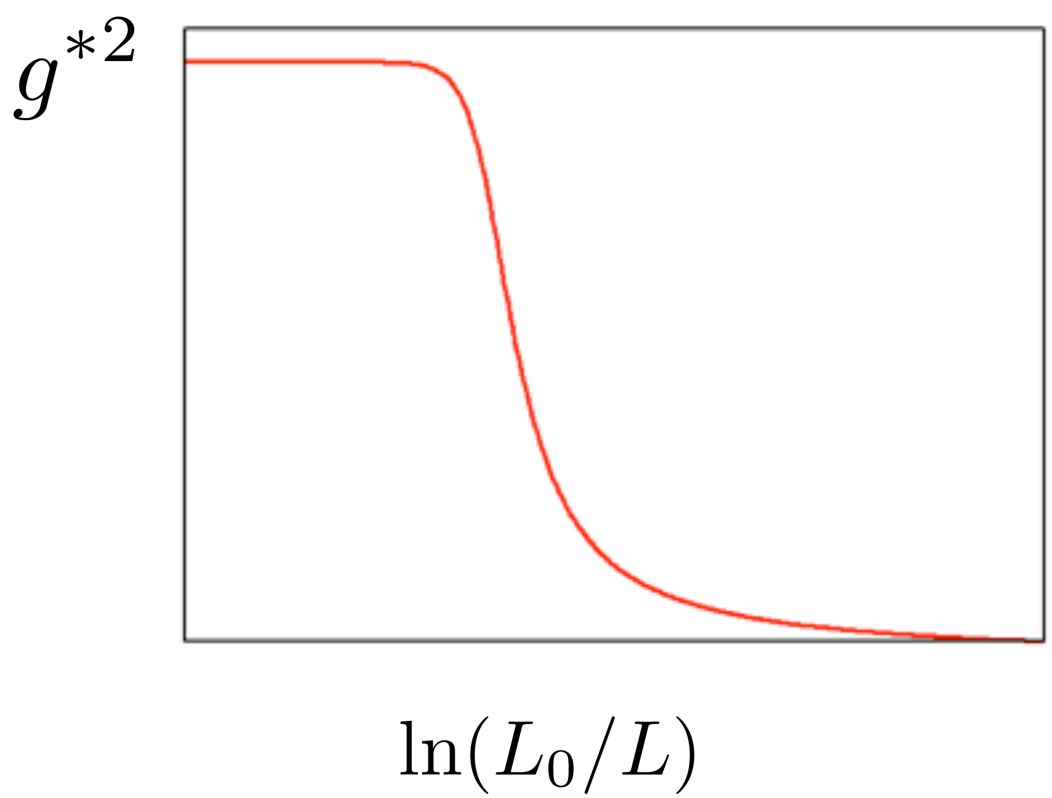
We also see that the chiral symmetry is preserved in this region.

Running coupling

Measuring the growth ratio

Obtain the growth ratio of renormalized coupling constant
to see the precise running behavior.

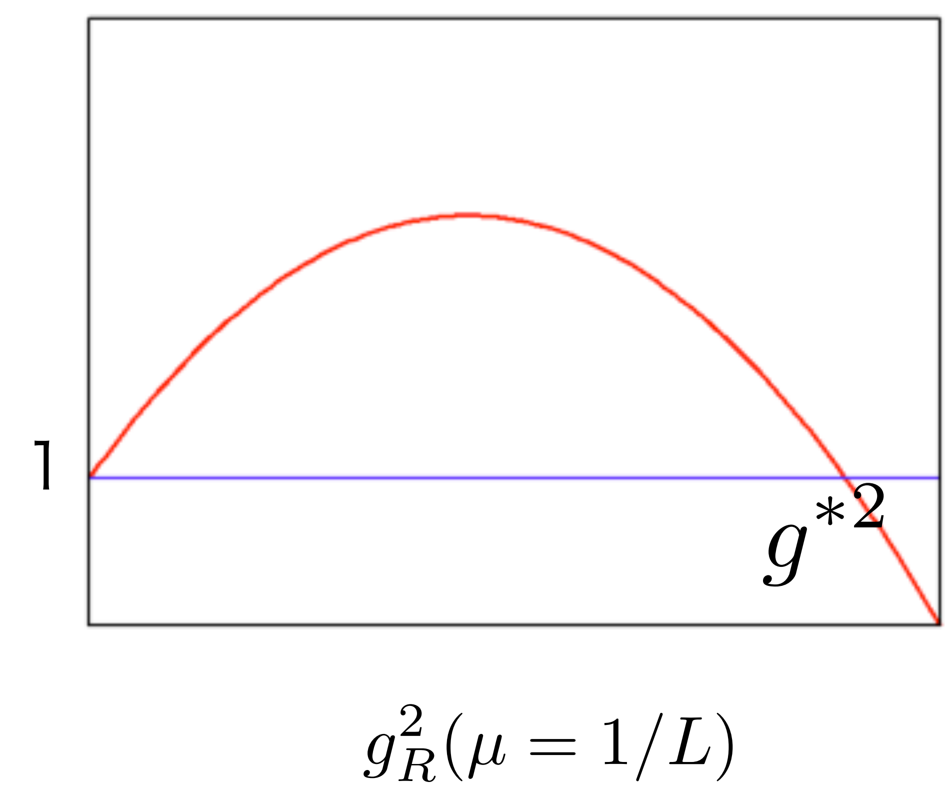
running coupling constant



systematic error is accumulated

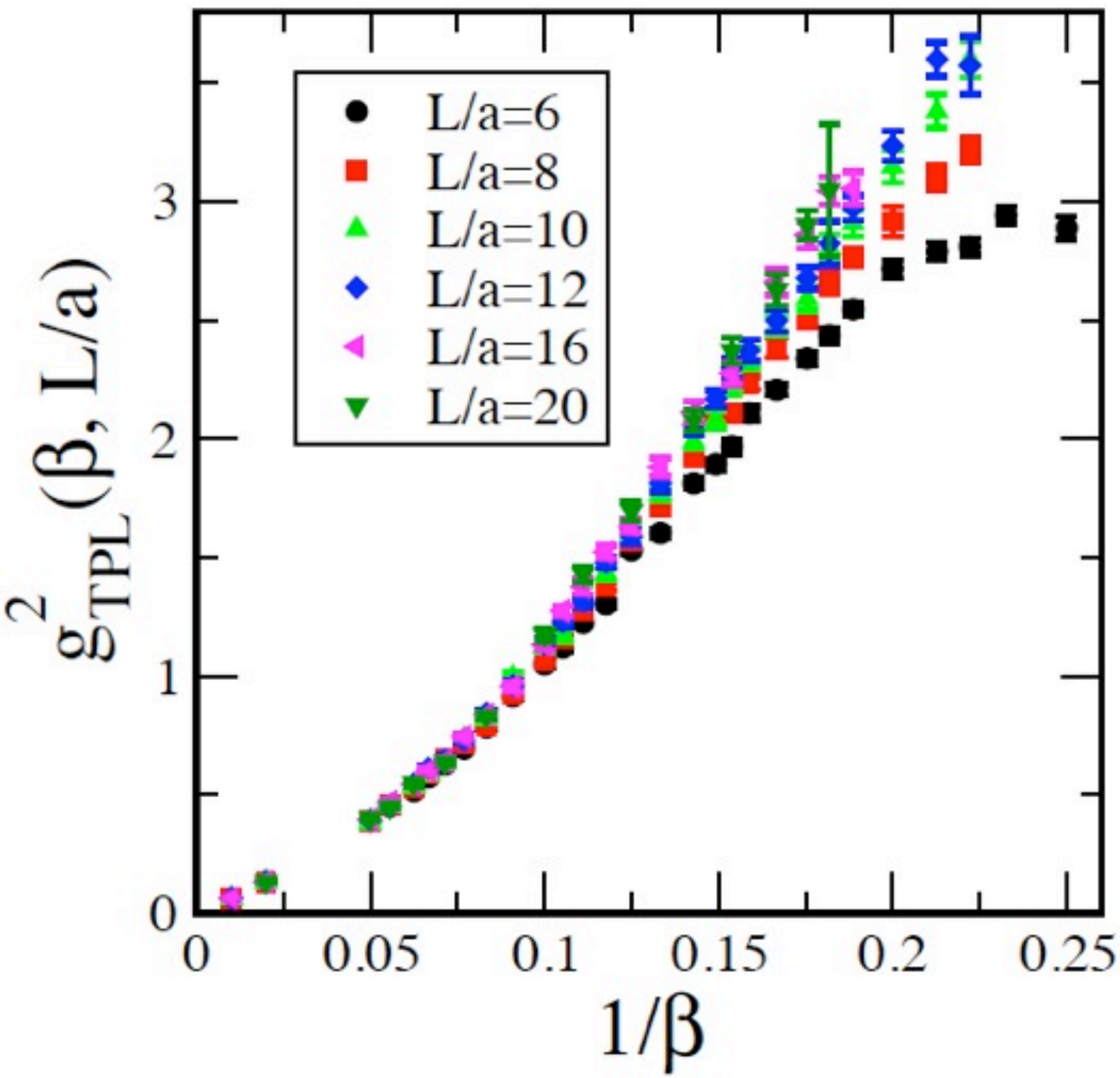
growth ratio

$$\sigma(u)/u = g_R^2(1/sL)/g_R^2(1/L)$$



systematic error is not accumulated

Raw data in TPL scheme



2-3 % statistical error.

# of Trj is 64,400- 1,892,800.

Fitting fn. for beta interpolation

$$g^2_{TPL}(\beta, L/a) = \frac{6}{\beta} + \sum_{j=1}^{N} \frac{C_j(L/a)}{\beta^{j+1}}$$

s=1.5 step scaling

L/a=6 -> L/a=9

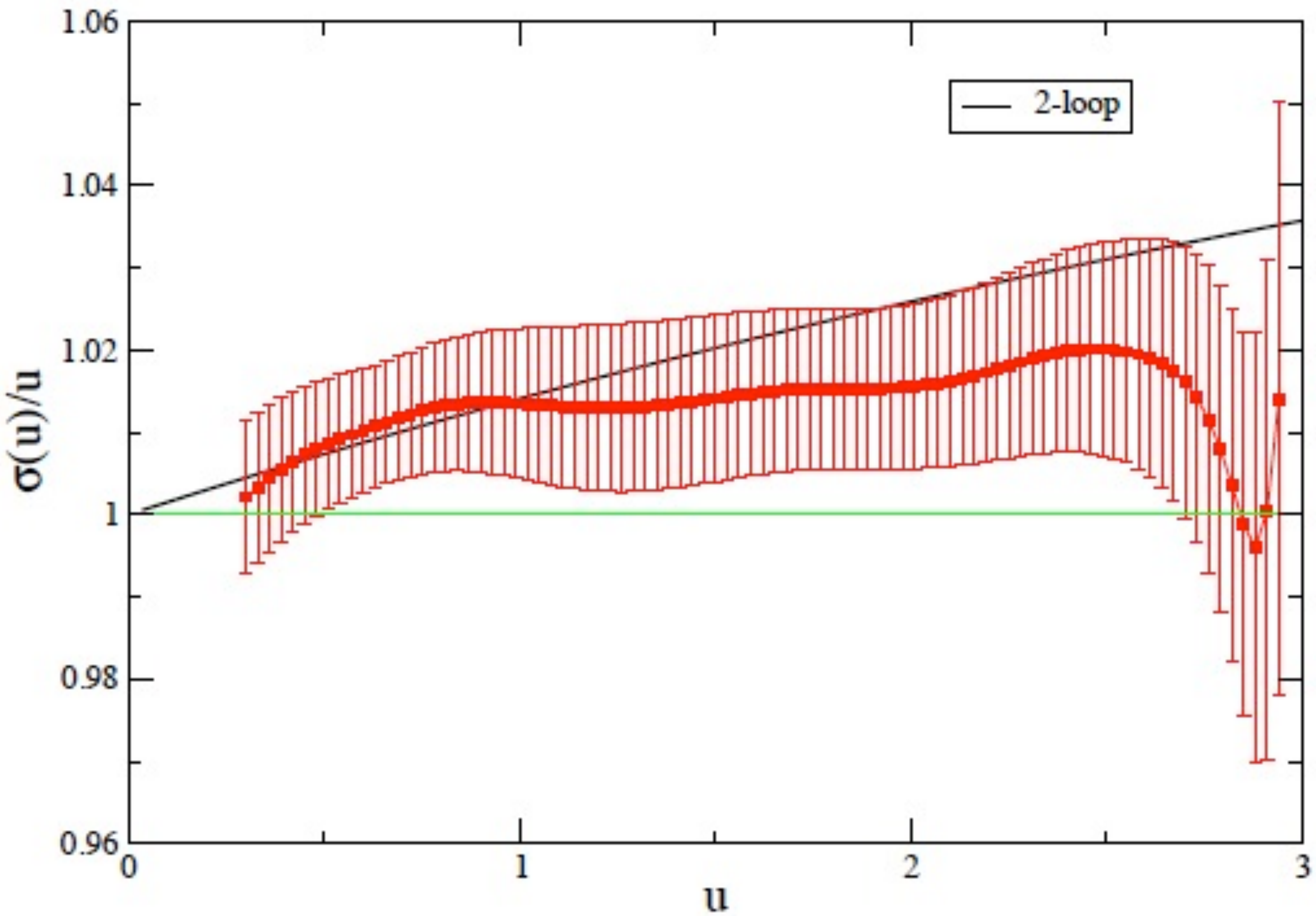
L/a=8 -> L/a=12

L/a=10 -> L/a=15

L/a=12 -> L/a=18

For L/a = 9, 15 and 18,
we estimate values of g2 for a given beta
by the linear interpolation in (a/L)2.

Growth ratio of TPL coupling (global fit analysis)



TPL coupling shows the fixed point around

$$g^{*2}_{\mathrm{TPL}} \sim 2.7$$

This is the first zero point of the beta function from the asymptotically free region.

It must be IR fixed point.

Unfortunately, the growth ratio with errorbar does not cross over the unity line.

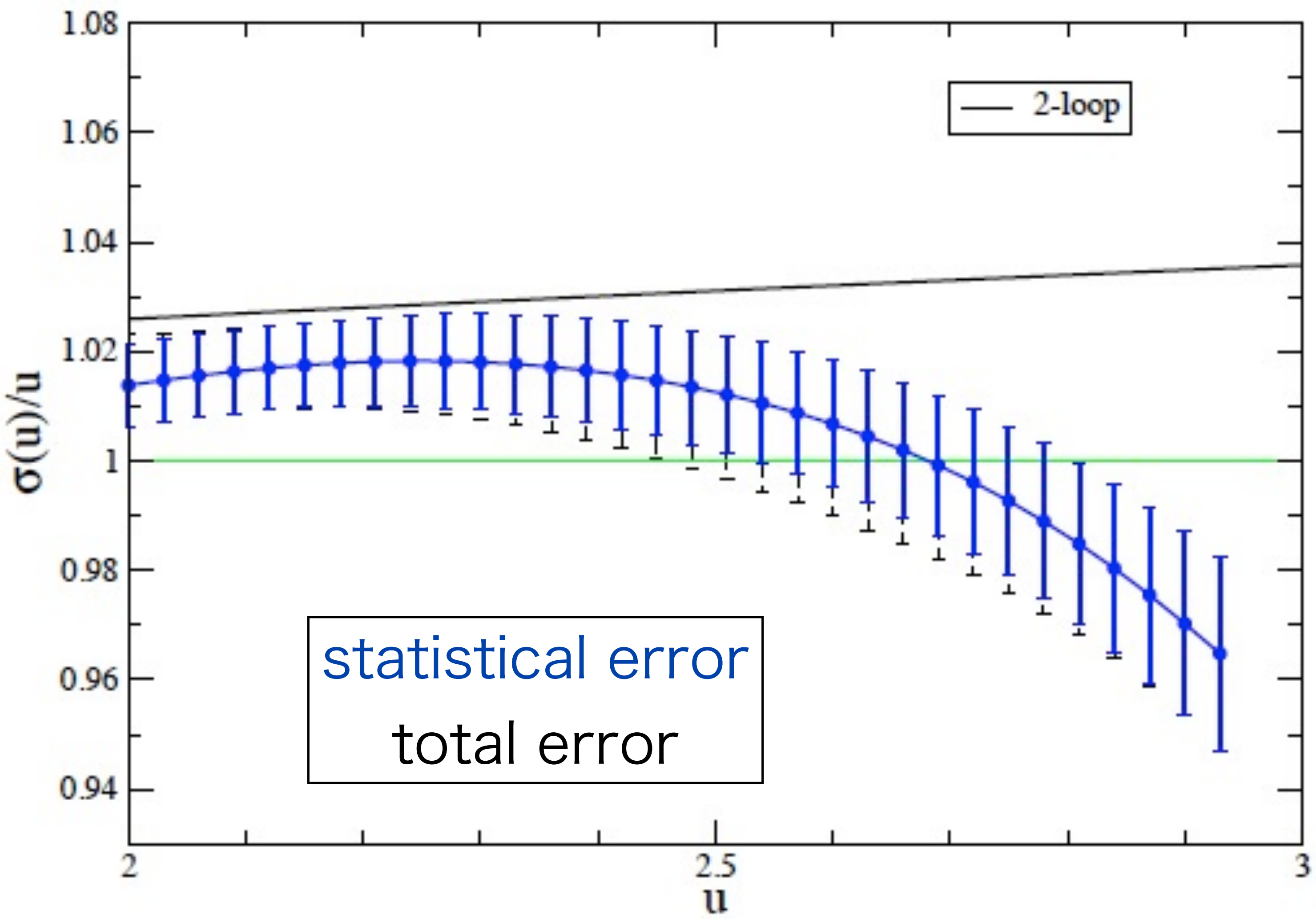
$$\sigma(u) \equiv g_R^2(1/sL)$$
$$u \equiv g_R^2(1/L)$$

Local fit analysis

Focus on the low beta region (u>2.0)

Add the data (more than 30 data points)

Growth ratio of TPL coupling (local fit analysis)



$$g^{*2}_{\mathrm{TPL}} = 2.69 \pm 0.14(\mathrm{stat.})^{+0}_{-0.16}(\mathrm{syst.})$$

The critical exponent of beta function

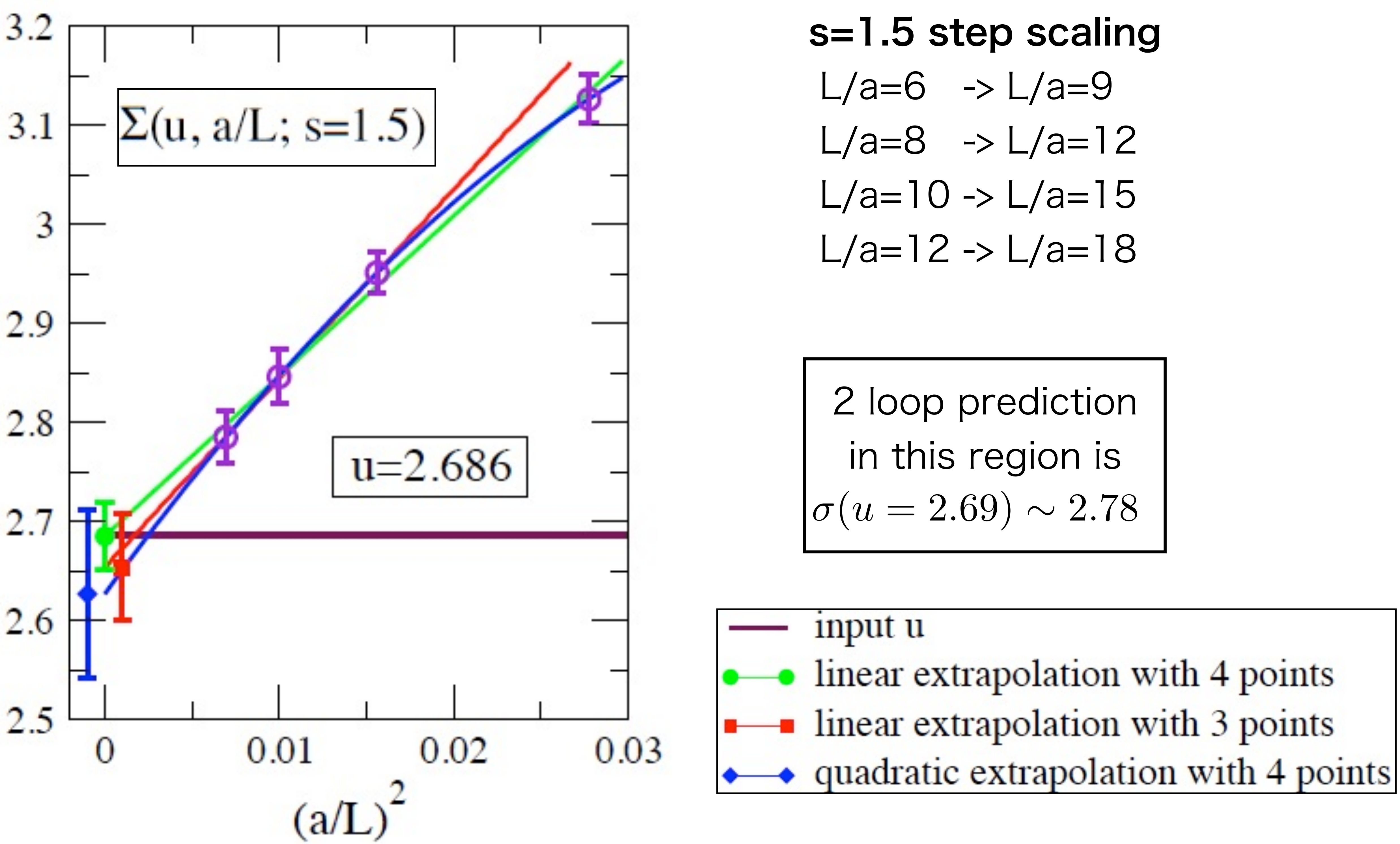
$$\beta(g^2) \sim \gamma_g^*(g^{2*} - g^2)$$

Our result

$$\gamma_g^* = 0.57^{+0.35}_{-0.31}(\mathrm{stat.})^{+0}_{-0.16}(\mathrm{syst.})$$

SF scheme	2 loop at $g^{*2} = 9.4$	4 loop (MS bar)
$\gamma_g^* = 0.13 \pm 0.03$	$\gamma_g^* = 0.36$	$\gamma_g^* = 0.28$

Continuum extrapolation



s=1.5 step scaling

L/a=6 -> L/a=9
L/a=8 -> L/a=12
L/a=10 -> L/a=15
L/a=12 -> L/a=18

2 loop prediction
in this region is
$\sigma(u = 2.69) \sim 2.78$

The systematic error is small in the strong coupling region in this scheme.

(Fit range dependence and “s” (step scaling parameter) dependence are also small.)

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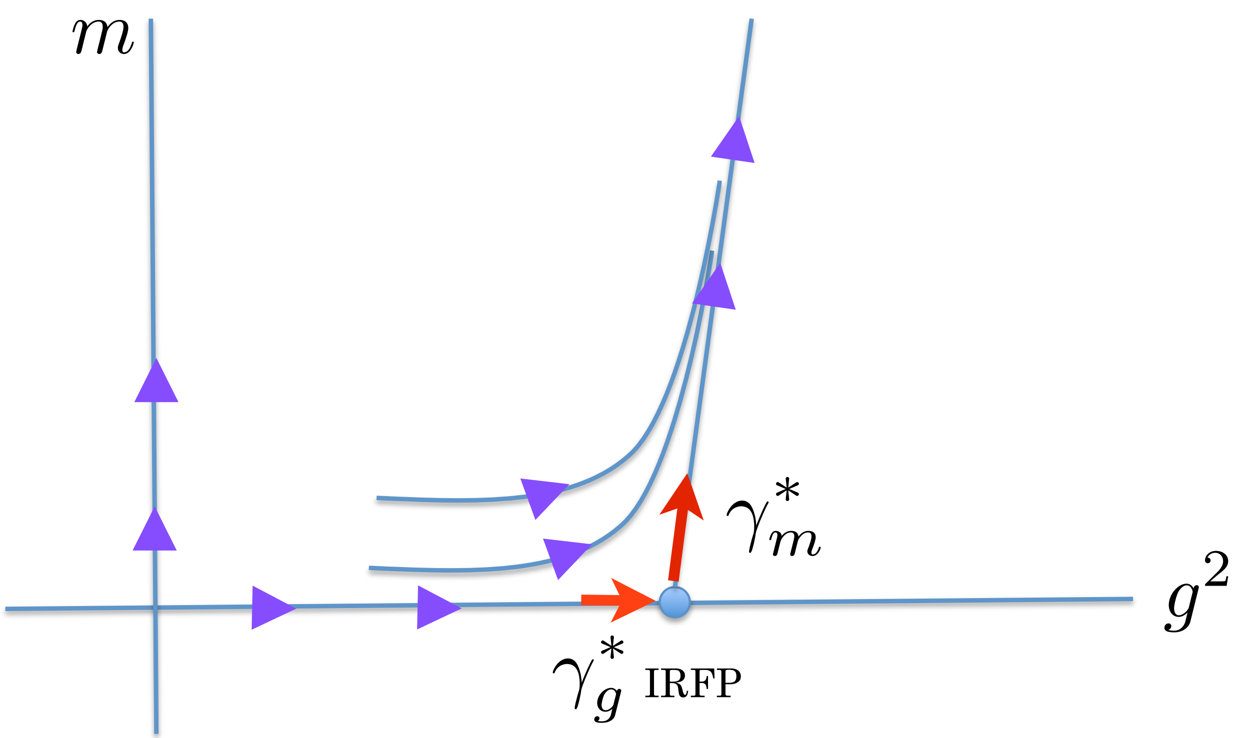
Summary



Further studies are necessary to find the universal quantities.

SF scheme : PRD79 (2009) 076010
Fodor's data: PLB703 (2011) 348-358
Fit(I): PRD84(2011) 054501
Fit(II): PRD84 (2011) 116901
LatKMI : PRD86 (2012) 054506
Cheng et.al : JHEP1307 (2013) 061
Ours : PTEP (2013)083B01
arXiv: 1307.6645

	γ_g^*	γ_m^*
2loop	0.36	0.77
4loop (MS bar)	0.28	0.25
SF scheme	0.13(3)	
Fodor's data		0.403(13) 0.35(23)
LatKMI		0.4-0.5
Cheng et. al.		0.32(3)-> 0.20
Ours	0.57(35)	$0.044^{+0.062}_{-0.040}$

Conclusion and Discussion

The IRFP exists in SU(3) Nf=12 massless theory.
Continuum extrapolation and parameter search are important.

- The phenomenological model construction for BSM using the value of mass anomalous dimension from the lattice results.

  Nf=12 theory for walking technicolor is (almost) killed by recent lattice results.
  (Minimal walking technicolor, SU(2) Nf=2 adjoint fermions, is also killed by lattice studies)

  => Change Nf? gauge group? fermion representation?

- Study on universal quantities as a conformal field theory
  (anomalous dimension, “central charge” in 4-dim)

Lattice precise data can give phenomenological and theoretical information around a nontrivial fixed point.