I am very honored to receive this award

Kenneth G. Wilson
8th June 1936 - 15th June 2013

I would have liked to meet him

I feel a sense of responsibility

Acknowledgements

I would like to thank those who have had particular influence on my physics education and development

Martin Savage, Steve Sharpe

Will Detmold, David Lin, Brian Tiburzi

Paulo Bedaque, Kostas Orginos

Maarten Golterman, Wick Haxton

all of my collaborators and friends

Acknowledgements

I would like to especially acknowledge all those who have made contributions to Lattice Field Theory as significant or more so than my own, who were not eligible simply because of the rules of consideration - particularly the other "young" researchers as deserving as myself

"For significant contributions to our understanding of baryons using lattice QCD and effective field theory"

"For significant contributions to our understanding of baryons using lattice QCD and effective field theory"

Effective Field Theory (EFT) teaches us how things should be...

I grew up learning effective field theory

Lattice QCD teaches us how things are...

in my postdoc youth, I learned some lattice QCD

“For significant contributions to our understanding of baryons using lattice QCD and effective field theory”

Lattice QCD provides numerical answers to specific questions

EFT provides a framework to understand these numbers in a broader context, and provides a quantitative connection with many other questions

"For significant contributions to our understanding of baryons using lattice QCD and effective field theory"

Chiral Perturbation Theory (χPT), the low-energy EFT of QCD, has been needed to extrapolate results from lattice QCD calculations to the real world physical quark mass, infinite volume, (continuum limit)

Lattice QCD calculations are now performed close to the real world: Lattice QCD can now be used to significantly improve our understanding of χPT

What separates Chiral Perturbation Theory from a simple Taylor expansion?

The chiral expansion informs you approximately the range of validity of the theory (EFT)

χPT is described by universal coefficients which describe many observables

χPT predicts *chiral logarithms* or rather non-analytic dependence upon the light-quark masses

Evidence for these chiral logarithms is deemed essential for finding the chiral regime

Chiral Expansion: all hadrons have a mass

$$M_H = M_{H,0} + \alpha_H m_l + \ldots$$

except the pion!

$$m_\pi^2 = -2m_l \frac{\langle 0|\bar{q}_l q_l|0\rangle}{f^2} + \ldots$$

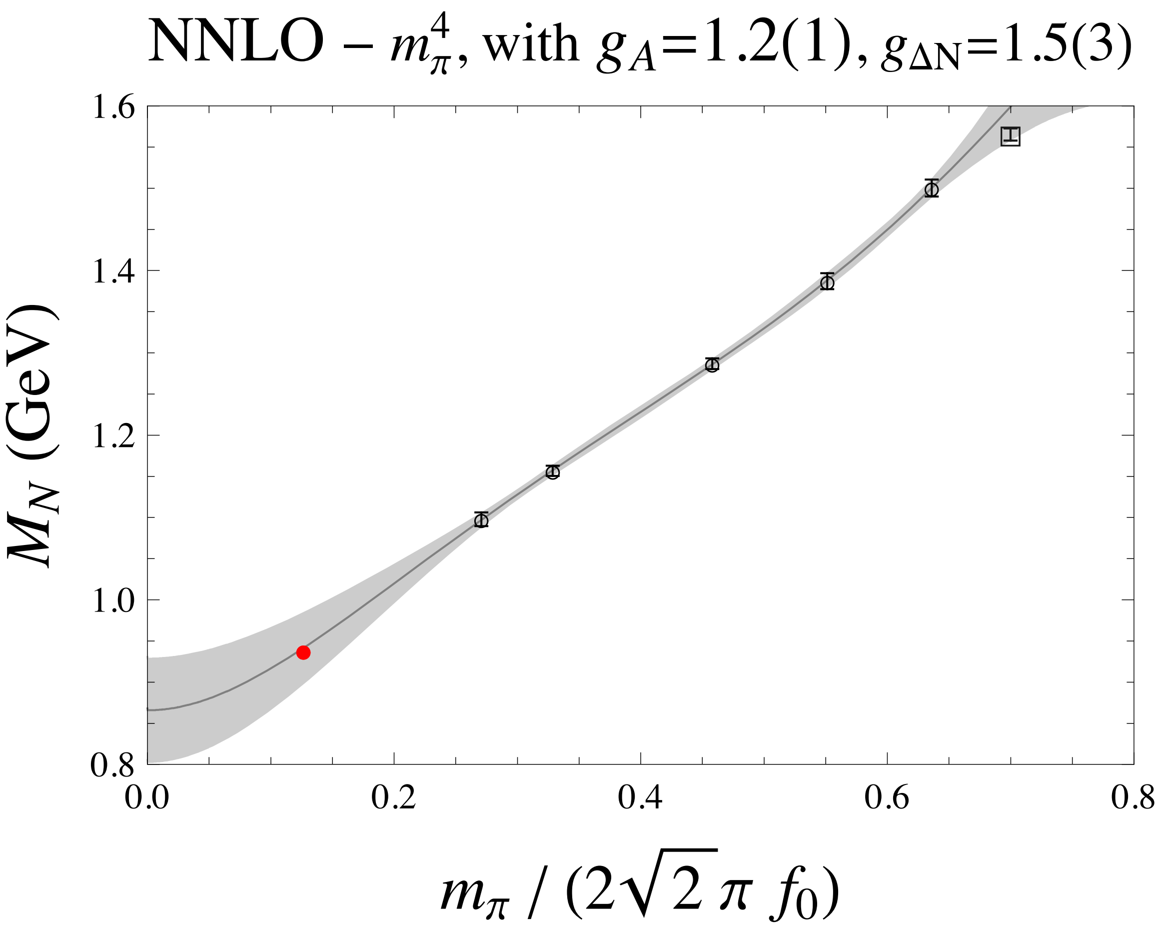
eg. the nucleon

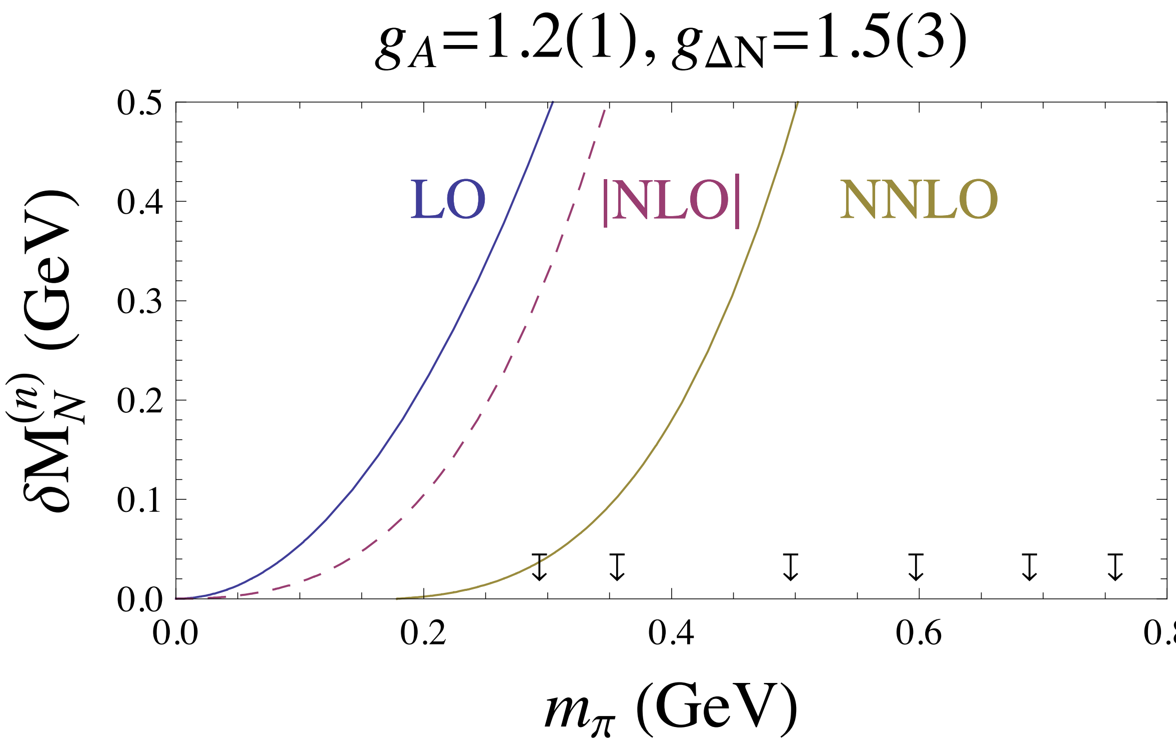
$$M_N = M_{N,0} + \alpha_N m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \ldots$$

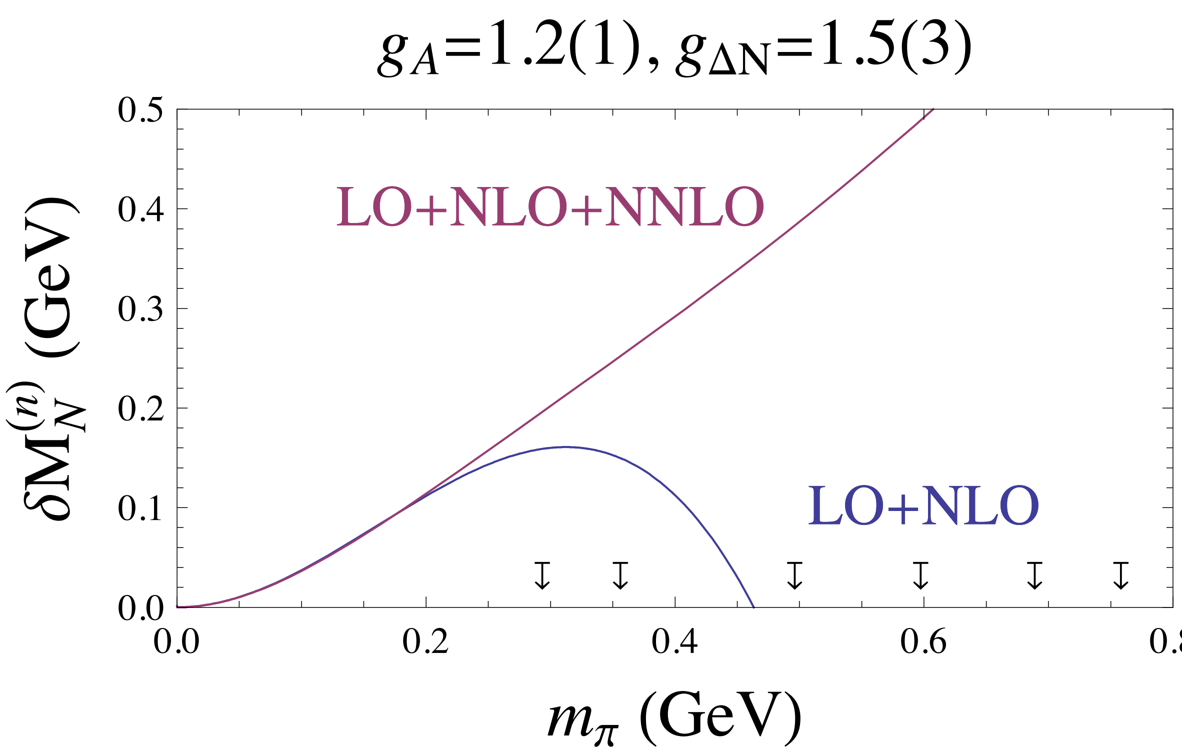
$$(\Delta = M_\Delta - M_N)$$

Can we observe this non-analytic light quark mass dependence of the nucleon mass?

Baryons in lattice QCD   Light quark mass dependence of M_N







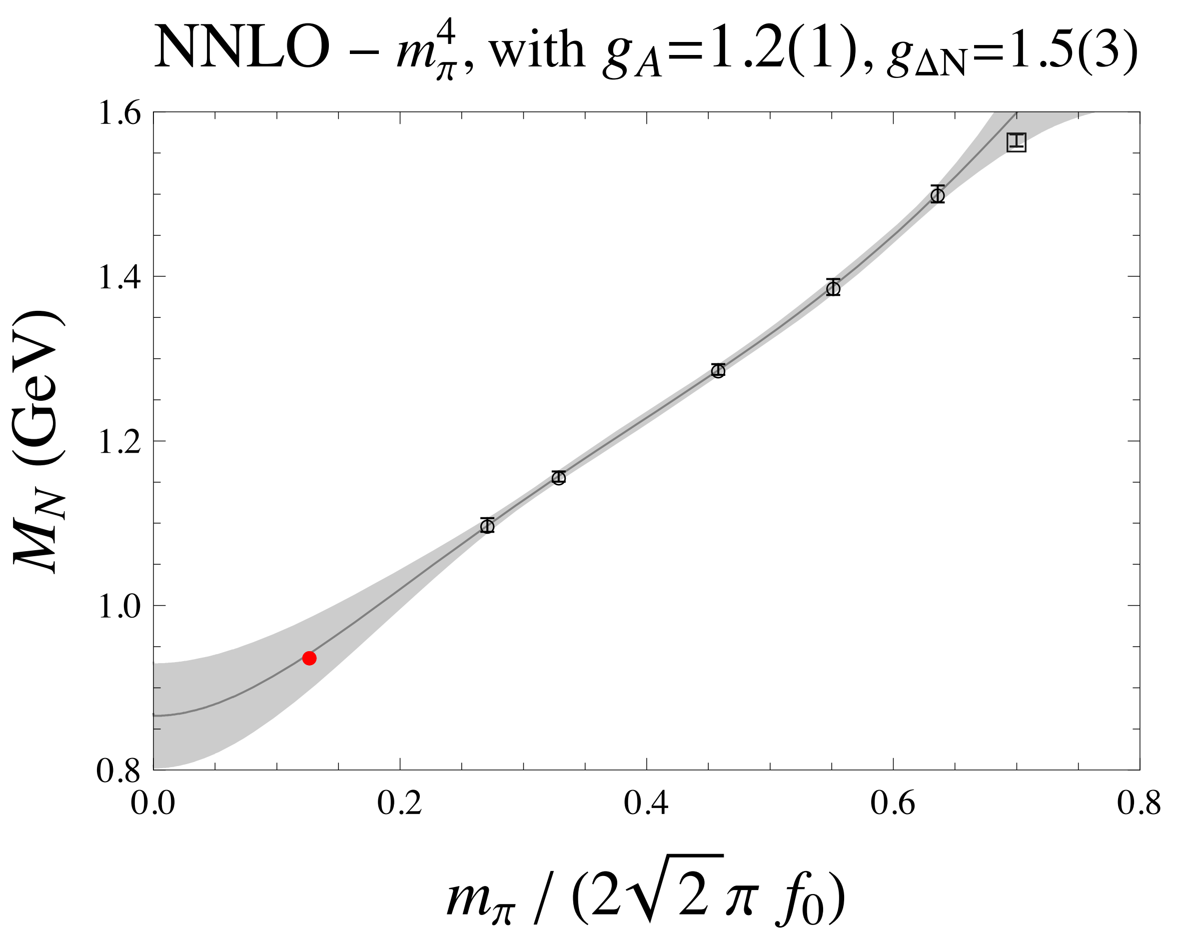
NNLO Heavy Baryon Fit

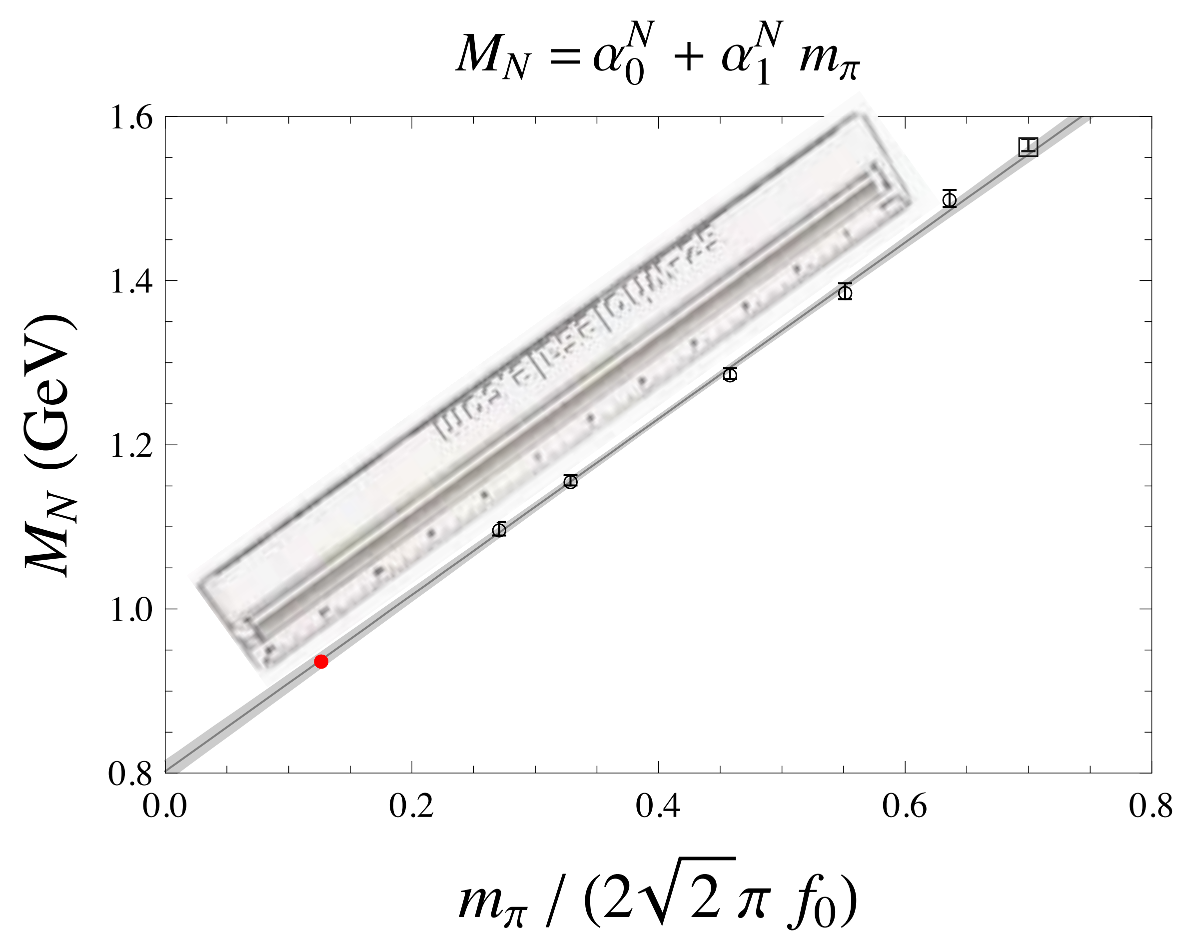
$$M_N = 941 \pm 42 \pm 17 \text{ MeV}$$

statistical   varying inputs

LHP Collaboration arXiv:0806.4549

Baryons in lattice QCD   Light quark mass dependence of M_N





NNLO Heavy Baryon Fit

$$M_N = 941 \pm 42 \pm 17 \text{ MeV}$$

Ruler Approximation

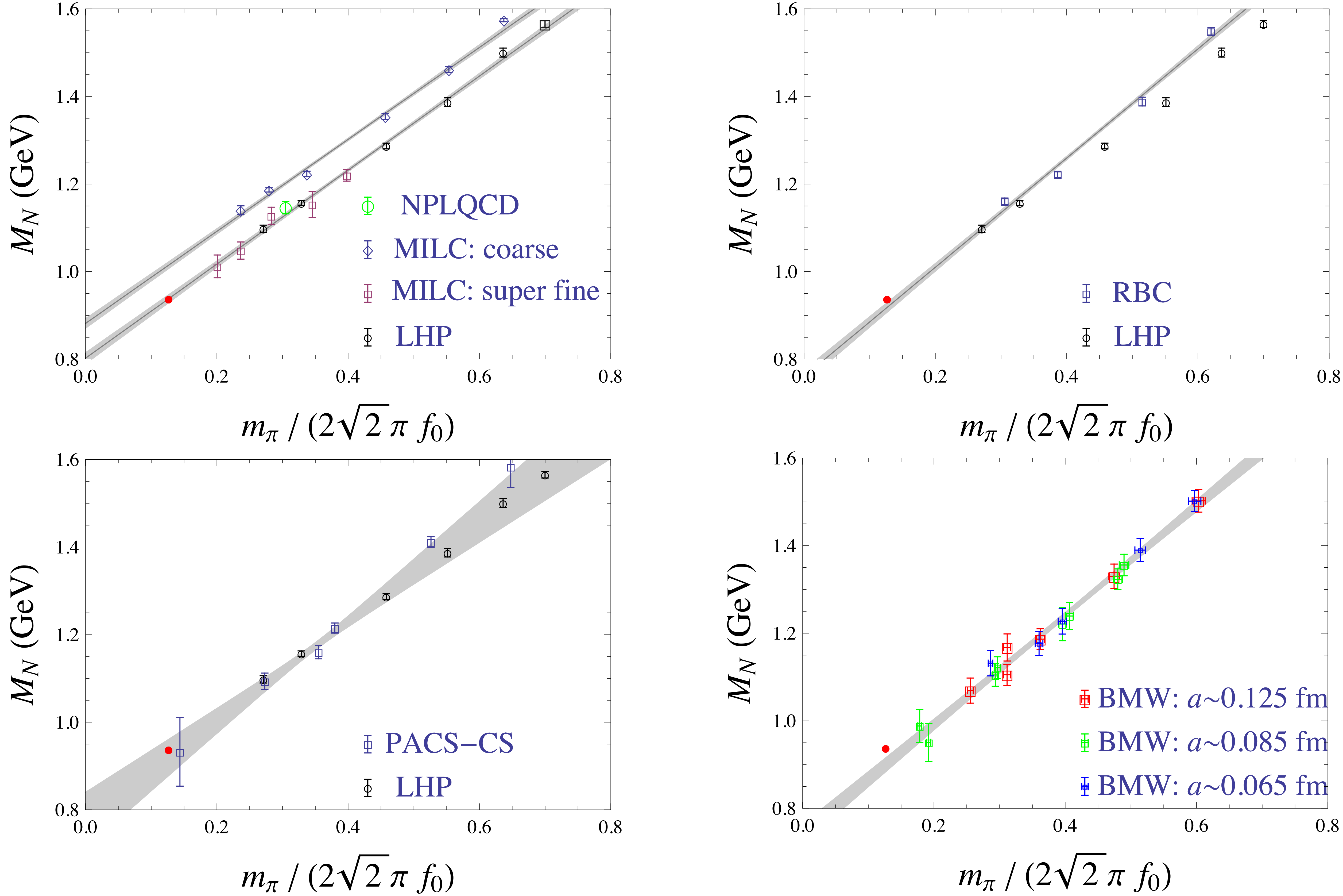
$$M_N = \alpha_0^N + \alpha_1^N m_\pi$$
$$= 938 \pm 9 \text{ MeV}$$

I am not advocating this as a good model for QCD!

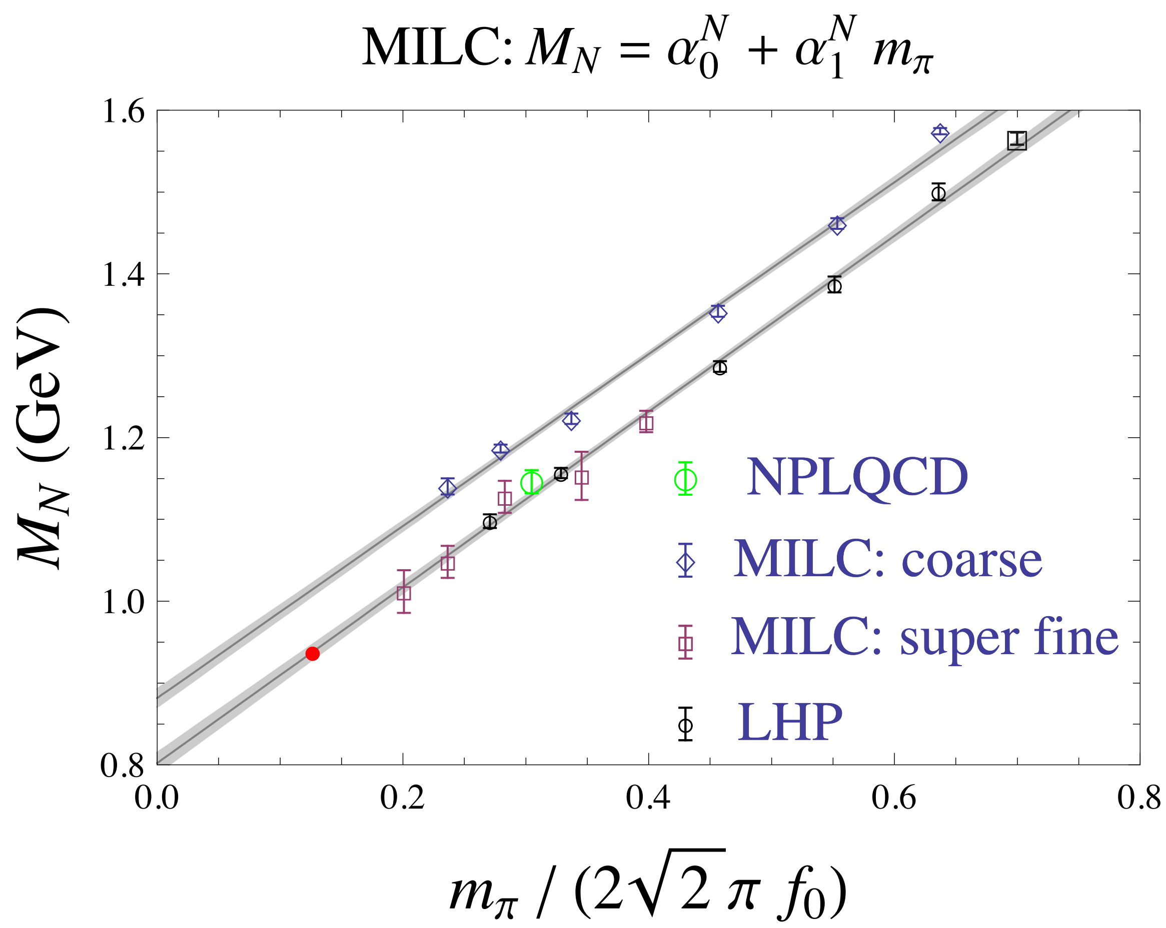
LHP Collaboration arXiv:0806.4549

Baryons in lattice QCD   Light quark mass dependence of M_N

MILC: $M_N = \alpha_0^N + \alpha_1^N m_\pi$ Latt 2008, arXiv:0810.0663 $M_N = \alpha_0^N + \alpha_1^N m_\pi$



Baryons in lattice QCD   Light quark mass dependence of M_N



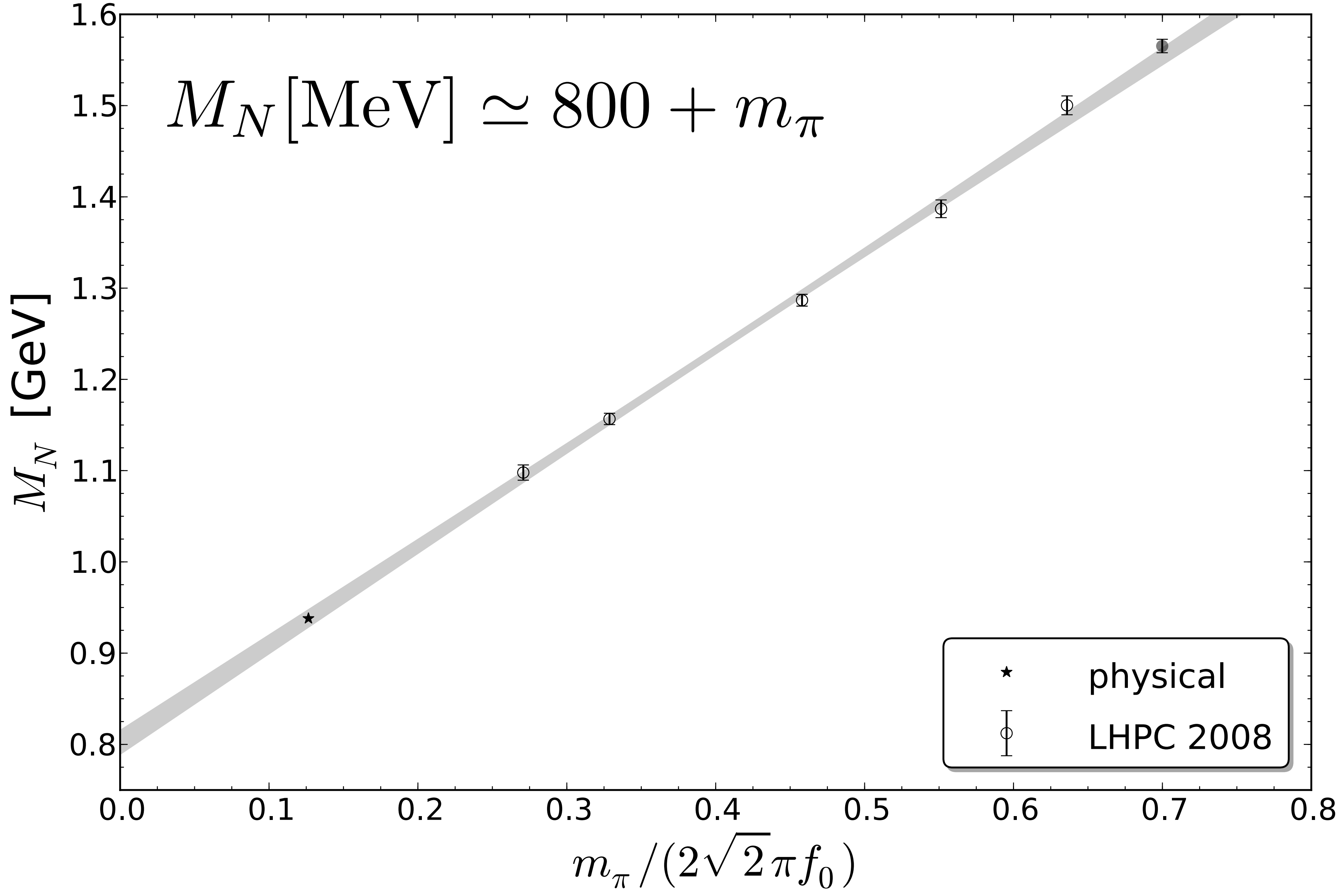
What does this teach us?

For these pion masses, there is a strong cancelation between LO, NLO and NNLO $\chi\mathrm{PT}$ contributions

perhaps should have been expected given poor convergence (but just not a straight line!!!)

Chiral Dynamics 2012
arXiv:1304.6341

What is the status now (2012)?

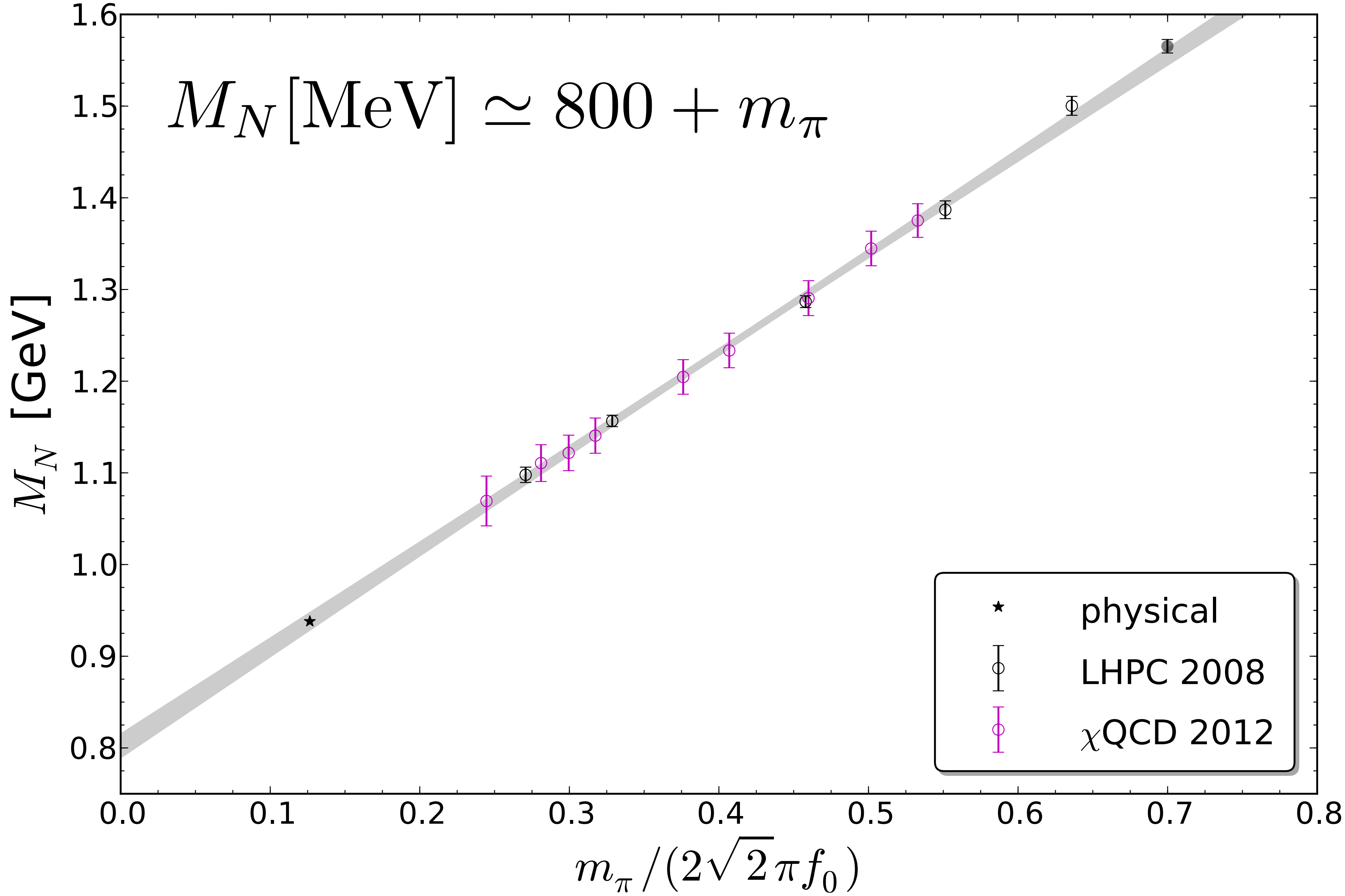


Physical point NOT included in fit

Light quark mass dependence of M_N

Chiral Dynamics 2012
arXiv:1304.6341

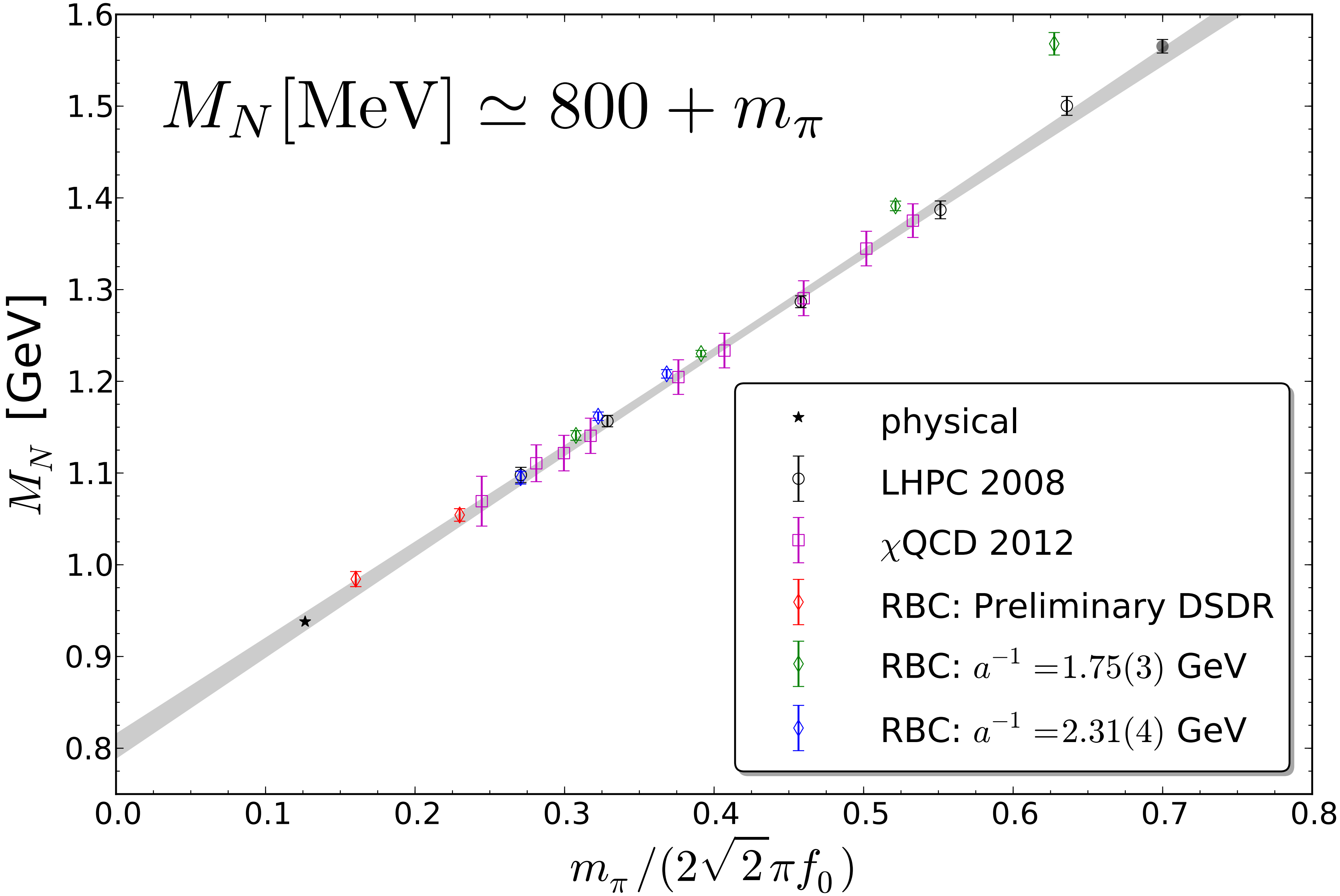
What is the status now (2012)?



χQCD Collaboration uses Overlap Valence fermions on Domain-Wall (RBC-UKQCD) sea fermions

Chiral Dynamics 2012
arXiv:1304.6341

What is the status now (2012)?

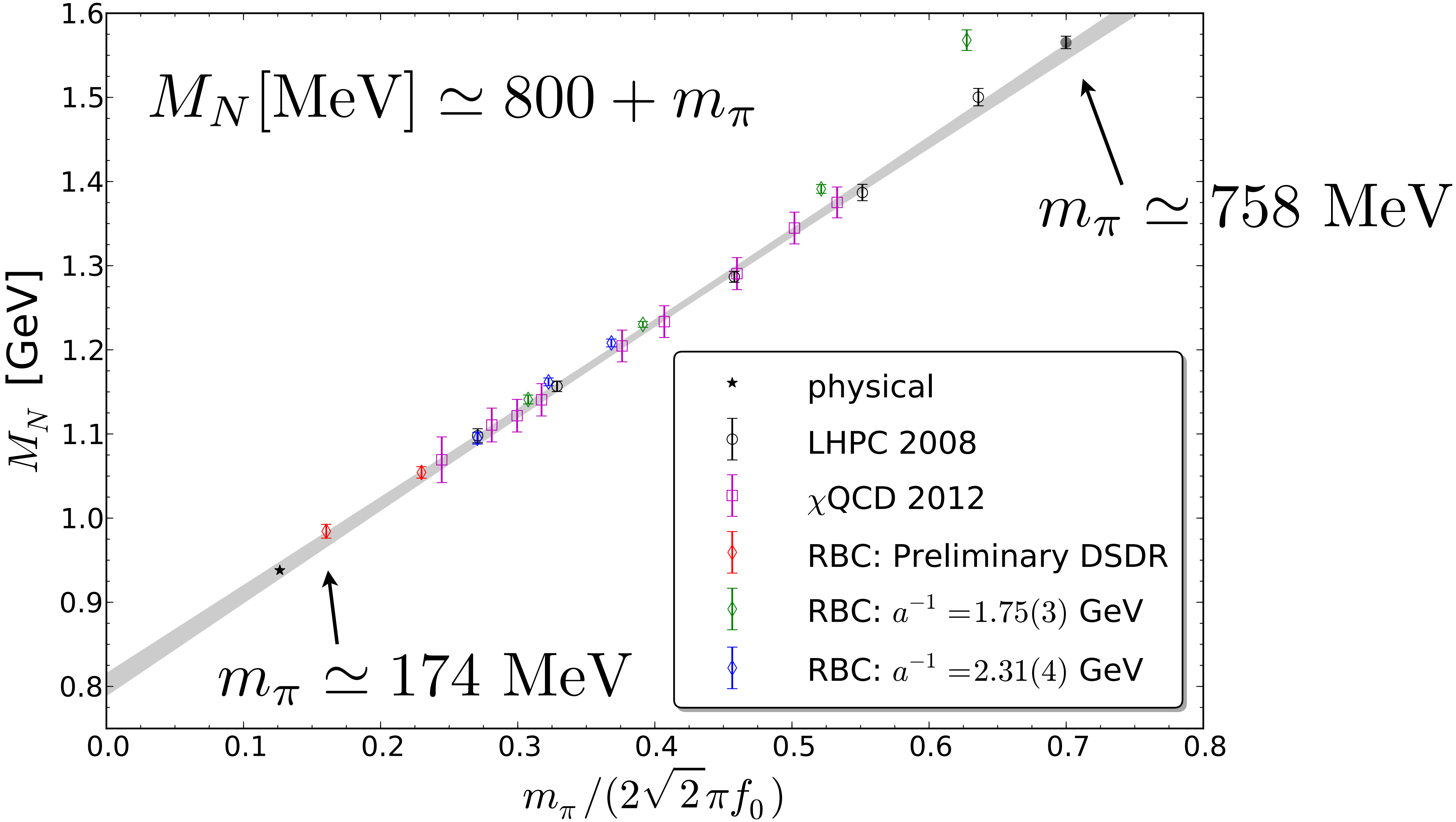


RBC-UKQCD Collaboration uses Domain-Wall valence and sea fermions

Light quark mass dependence of M_N

Chiral Dynamics 2012
arXiv:1304.6341

What is the status now (2012)?



Taking this seriously yields

$$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$$

I am not advocating this as a good model for QCD!

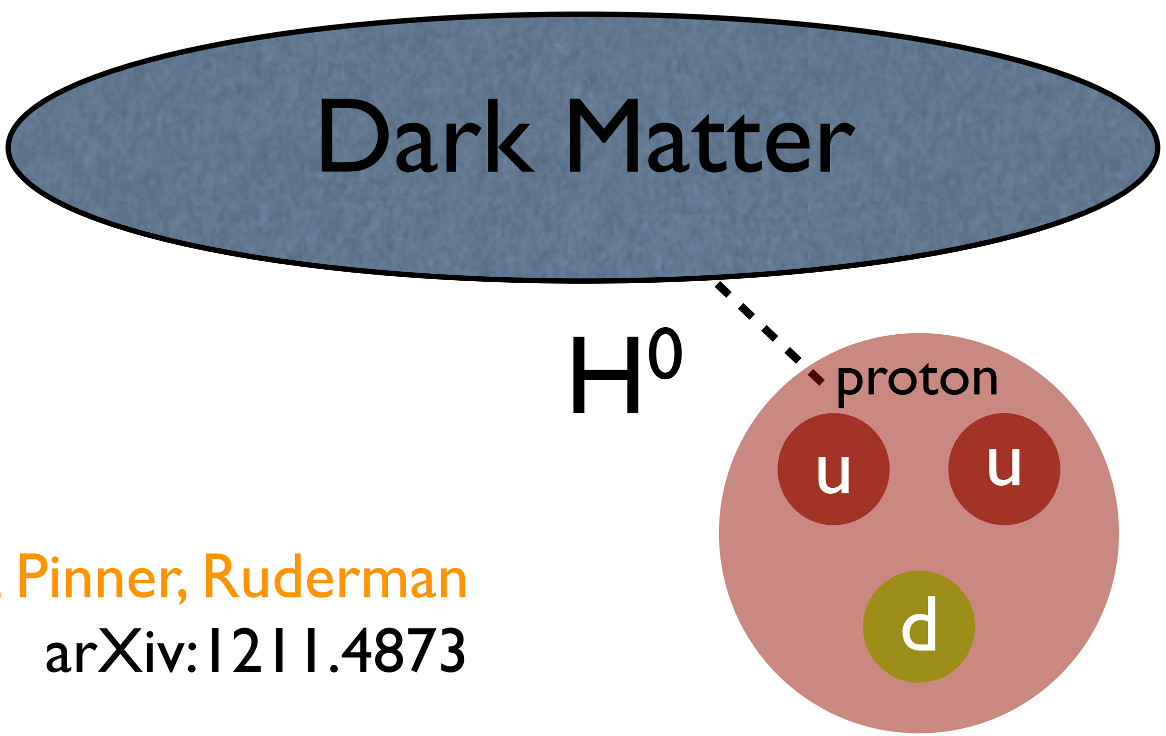
This is not merely an academic exercise:

The light-quark mass dependence of the nucleon allows us to determine the scalar quark matrix elements in the nucleon

$$m_q\langle N|\bar{q}q|N\rangle = m_q\frac{\partial}{\partial m_q}m_N(m_q)$$

which in turn dictate the strength of potential dark-matter nucleus scattering cross sections

$$\sigma \propto |f|^2 \qquad f = \frac{2}{9} + \frac{7}{9}\sum_{q=u,d,s} f_q$$



$$f_q \equiv \frac{\langle N|m_q\bar{q}q|N\rangle}{m_N}$$

see eg. Cheung, Hall, Pinner, Ruderman
arXiv:1211.4873

I would like to share some new preliminary work

Isospin Breaking: M_n - M_p

- Nature: $M_n - M_p = 1.29333217(42)\ \mathrm{MeV}$

CODATA
PDG (2012)

- Standard Model has two sources of isospin breaking

$$\hat{Q} = \frac{1}{6}\mathbb{1} + \frac{1}{2}\tau_3 \qquad m_q = \hat{m}\mathbb{1} - \delta\tau_3$$

- Given only electro-static forces, one would predict

$$M_p > M_n$$

- The contribution from $m_d - m_u$ is comparable in size but opposite in sign

Isospin Breaking: M_n - M_p

- M_n - M_p plays an extremely significant role in the evolution of the universe as we know it

Initial conditions for Big Bang Nucleosynthesis (BBN)

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

- The neutron lifetime is highly sensitive to the value of this mass splitting

$$\frac{1}{\tau_n} = \frac{(G_F \cos\theta_C)^2}{2\pi^3} m_e^5 (1 + 3g_A^2)\, f\left(\frac{M_n - M_p}{m_e}\right)$$

Point Nucleons $f(a) \simeq \frac{1}{15}\left(2a^4 - 9a^2 - 8\right)\sqrt{a^2 - 1} + a \ln\left(a + \sqrt{a^2 - 1}\right)$

Griffiths "Introduction to Elementary Particles"

10% change in $M_n - M_p$ corresponds to ~100% change neutron lifetime

Isospin Breaking: $M_n - M_p$

- What is Big Bang Nucleosynthesis?

  Describes our understanding of the evolution of the early universe from a time approximately one second after the Big Bang to approximately 15 minutes after the Big Bang.

  At this time, the only relevant degrees of freedom in the universe are protons, neutrons, electrons and photons

  A chain of coupled nuclear reactions produces the primordial abundance of light nuclei H, D, ^{3}He, ^{4}He, ^{7}Li

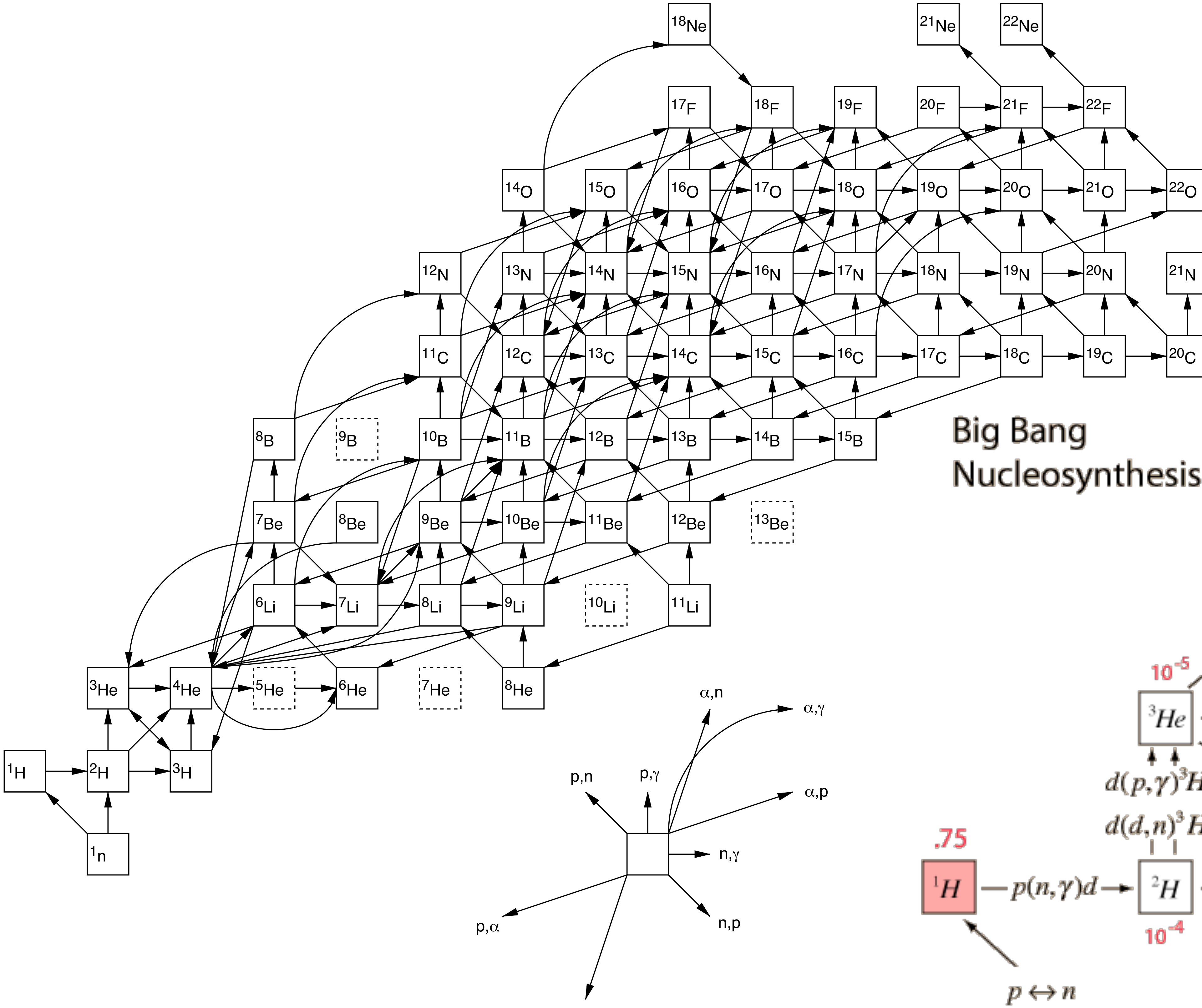
  Given the measured nuclear reactions, the only input/output to our understanding of BBN is the primordial ratio of baryons to photons

$$\eta \equiv \frac{X_N}{X_\gamma}$$

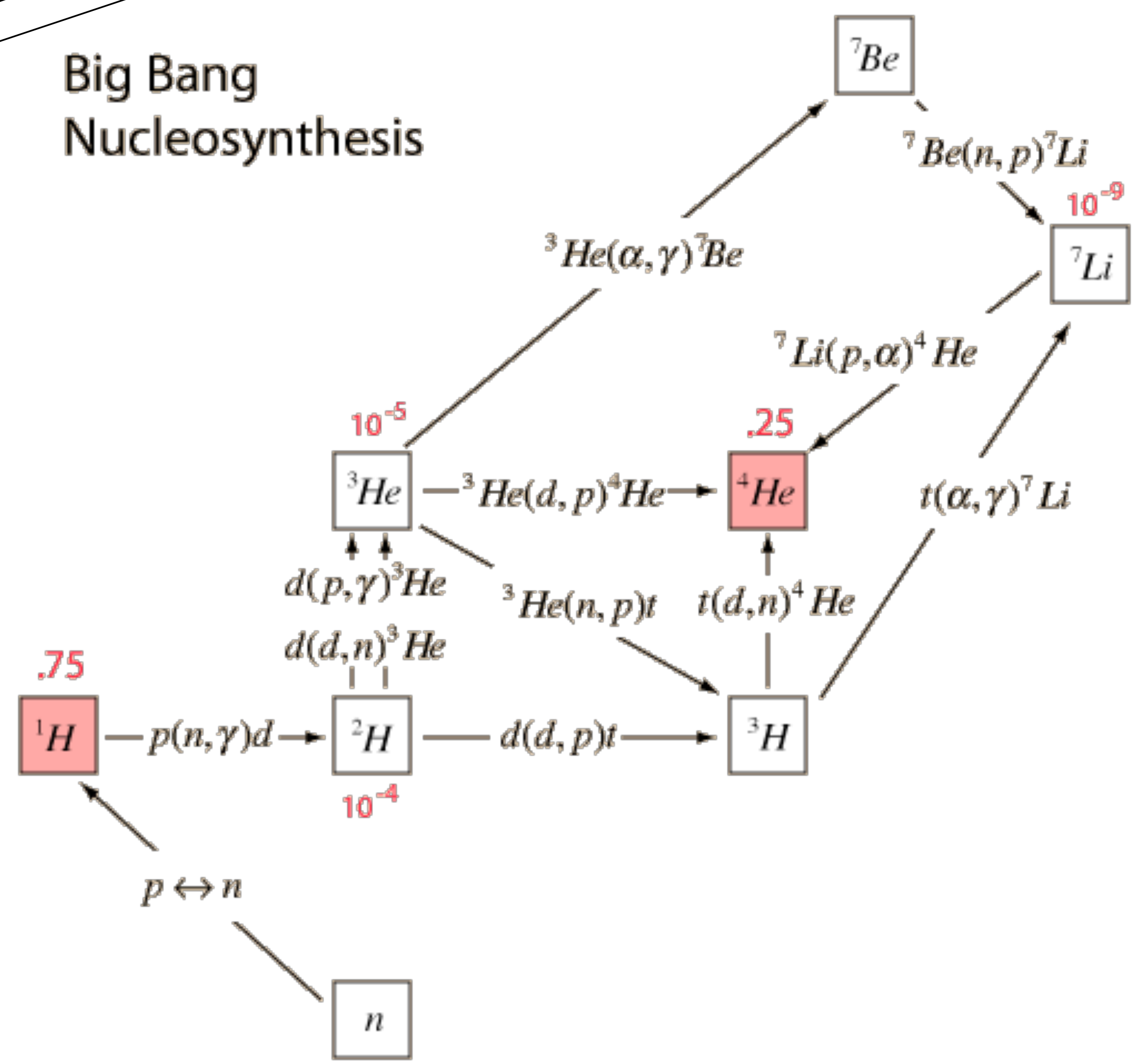
a good review

"The violent Universe: the Big Bang"
Keith A. Olive
arXiv:1005.3955

Isospin Breaking: M_n - M_p



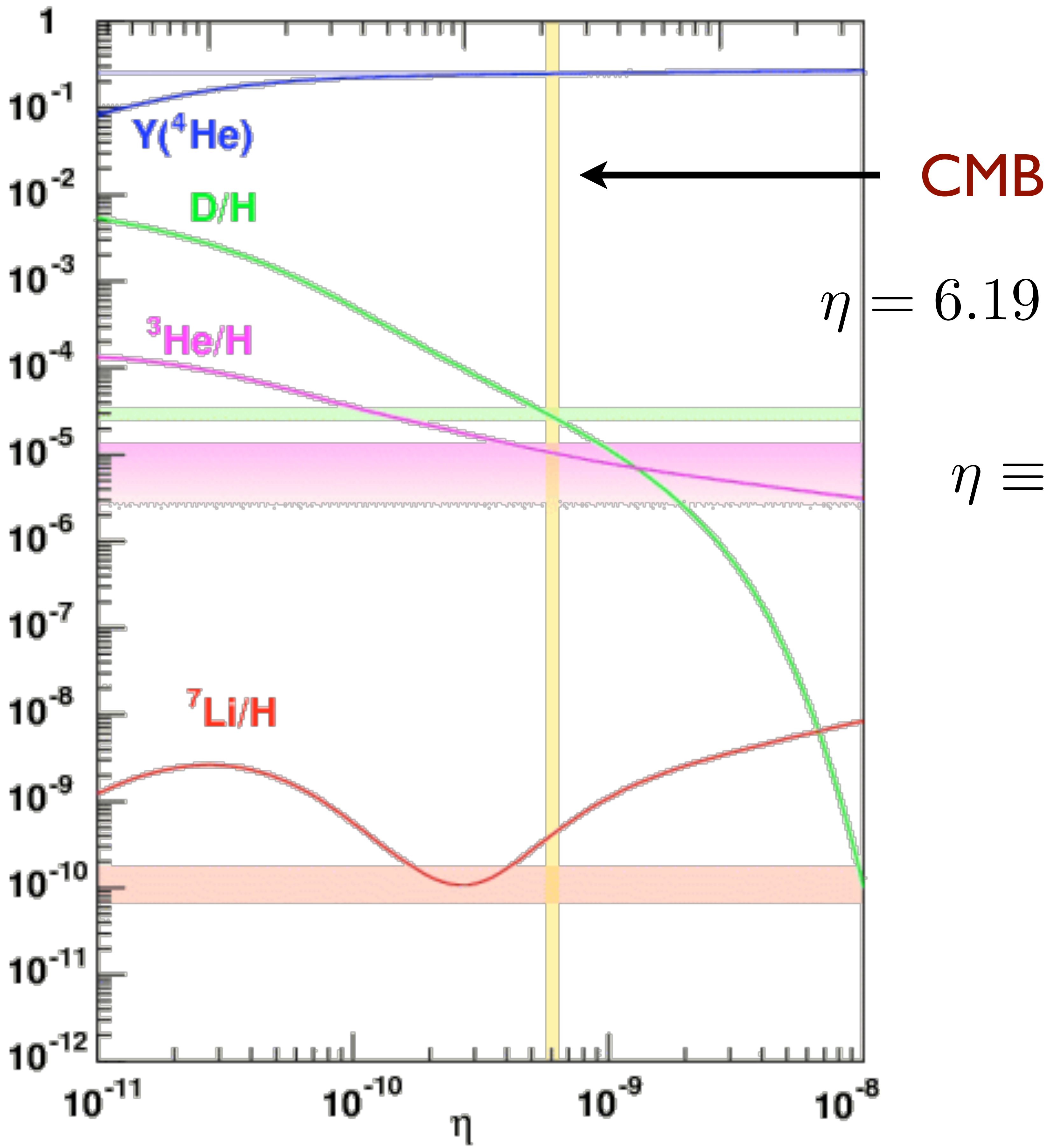
Big Bang Nucleosynthesis



Isospin Breaking: M_n - M_p

Primordial Universe (Mass Fraction)

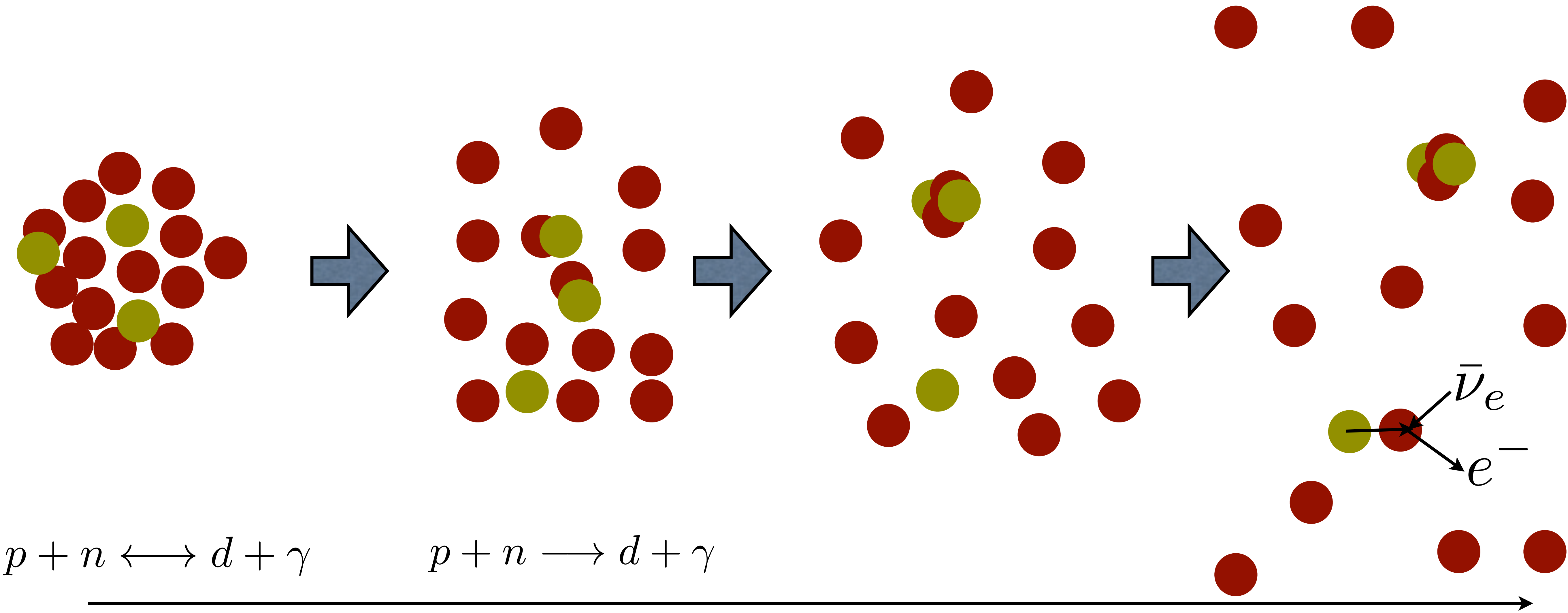
~75% H
~25% ^{4}He



$$\eta = 6.19(15) \times 10^{-10}$$

$$\eta \equiv \frac{X_N}{X_\gamma}$$

Isospin Breaking: $M_n - M_p$ Big Bang Nucleosynthesis



$t \sim 1$ sec	$t \sim 3$ min	$t \sim 3^+$ min	$t \sim 15$min
$T \sim 1$ MeV	$T \sim 0.1$ MeV	$T \sim 0.1^-$ MeV	$T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

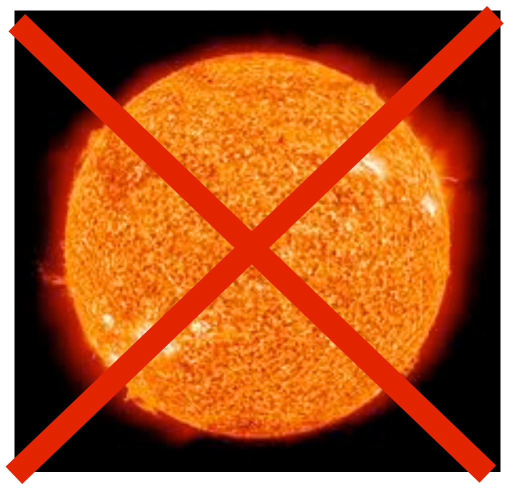
$$B_d$$

deuterium binding energy

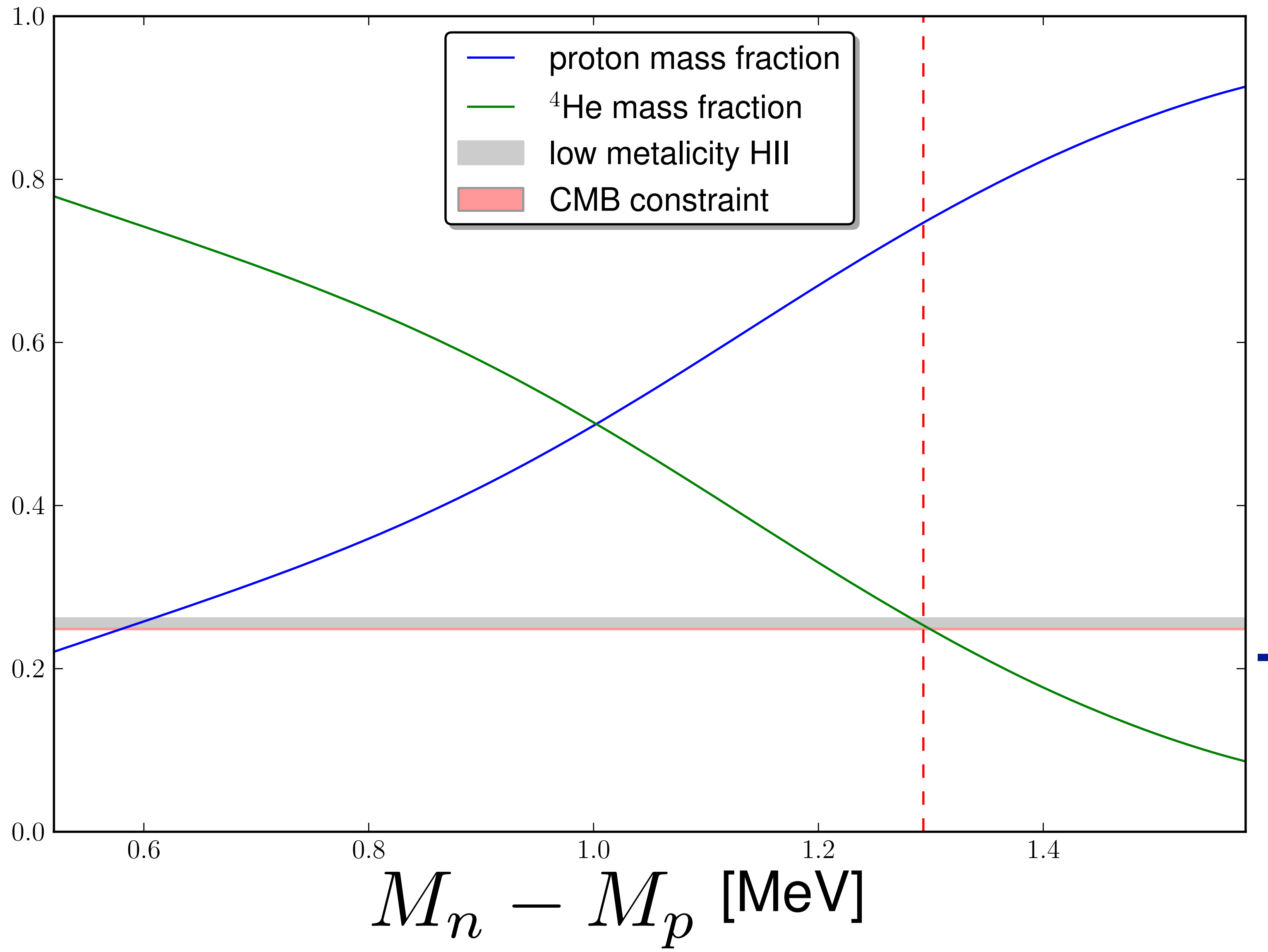
$$\tau_n$$

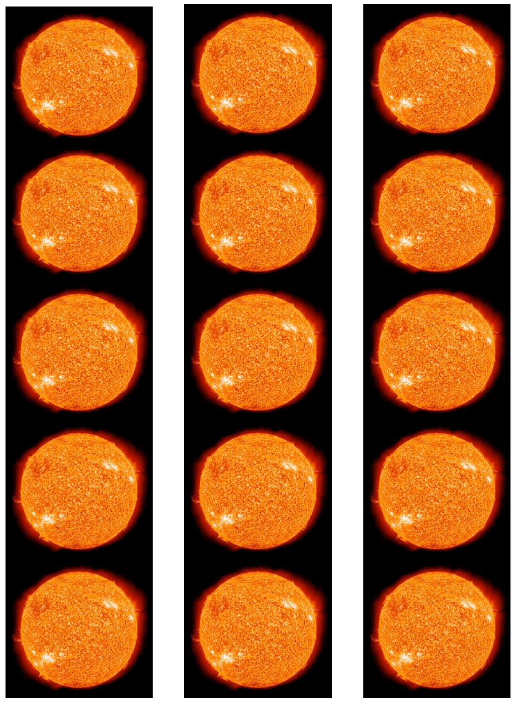
neutron lifetime

Isospin Breaking: M_n - M_p



No Sun!





Too many suns?

Isospin Breaking: M_n - M_p What do we know?

- We would like to understand the Neutron-Proton mass splitting from first principles

- $M_n - M_p = \delta M^{\gamma} + \delta M^{m_d - m_u}$ Separation only valid at LO in isospin breaking

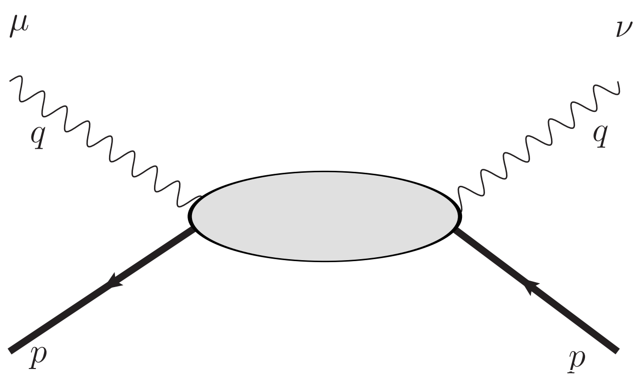
- $\delta M^{m_d - m_u}$ Well understood from lattice QCD

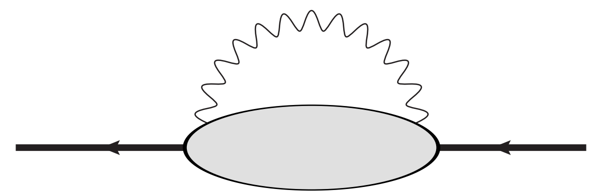
  δM^{γ} Disparate scales relevant for QCD and QED make this a very challenging problem to solve with LQCD: large systematic uncertainties

- Alternative means to determine δM^{γ}
  Cottingham Formulation

Isospin Breaking: M_n - M_p What do we know?

Cottingham Formulation





Cini, Ferrari, Gato PRL 2 (1959)
Cottingham Annals Phys 25 (1963)
Gasser, Leutwyler Nucl. Phys. B94 (1975)
Collins Nucl. Phys. B149 (1979)
Gasser, Leutwyler Phys. Rept 87 (1982)
AWL, C.Carlson, G.Miller PRL 108 (2012)
AWL, C.Carlson, G.Miller PoS LATT (2012)

$$\delta M^\gamma = \frac{i}{2M}\frac{e^2/4\pi}{(2\pi)^3}\int_R d^4q \frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

$$T_{\mu\nu} = \frac{i}{2}\sum_\sigma \int d^4\xi\ e^{iq\cdot\xi}\langle p\sigma|T\{J_\mu(\xi)J_\nu(0)\}|p\sigma\rangle$$

After some manipulations, renormalization and a subtracted dispersion integral

AWL, C.Carlson, G.Miller PRL 108 (2012)

$$\delta M^\gamma_{p-n}[\text{MeV}] = 0.83(03) - \frac{3\beta_M^{p-n}}{8\pi}\int_0^{\Lambda_0^2} dQ^2 Q^2 f(Q^2) \qquad \lim_{Q^2\to\infty} f(Q^2) \propto \frac{1}{Q^4}$$

$$\beta_M^{p-n} = -1.0 \pm 1.0\times 10^{-4}\ \text{fm}^3$$

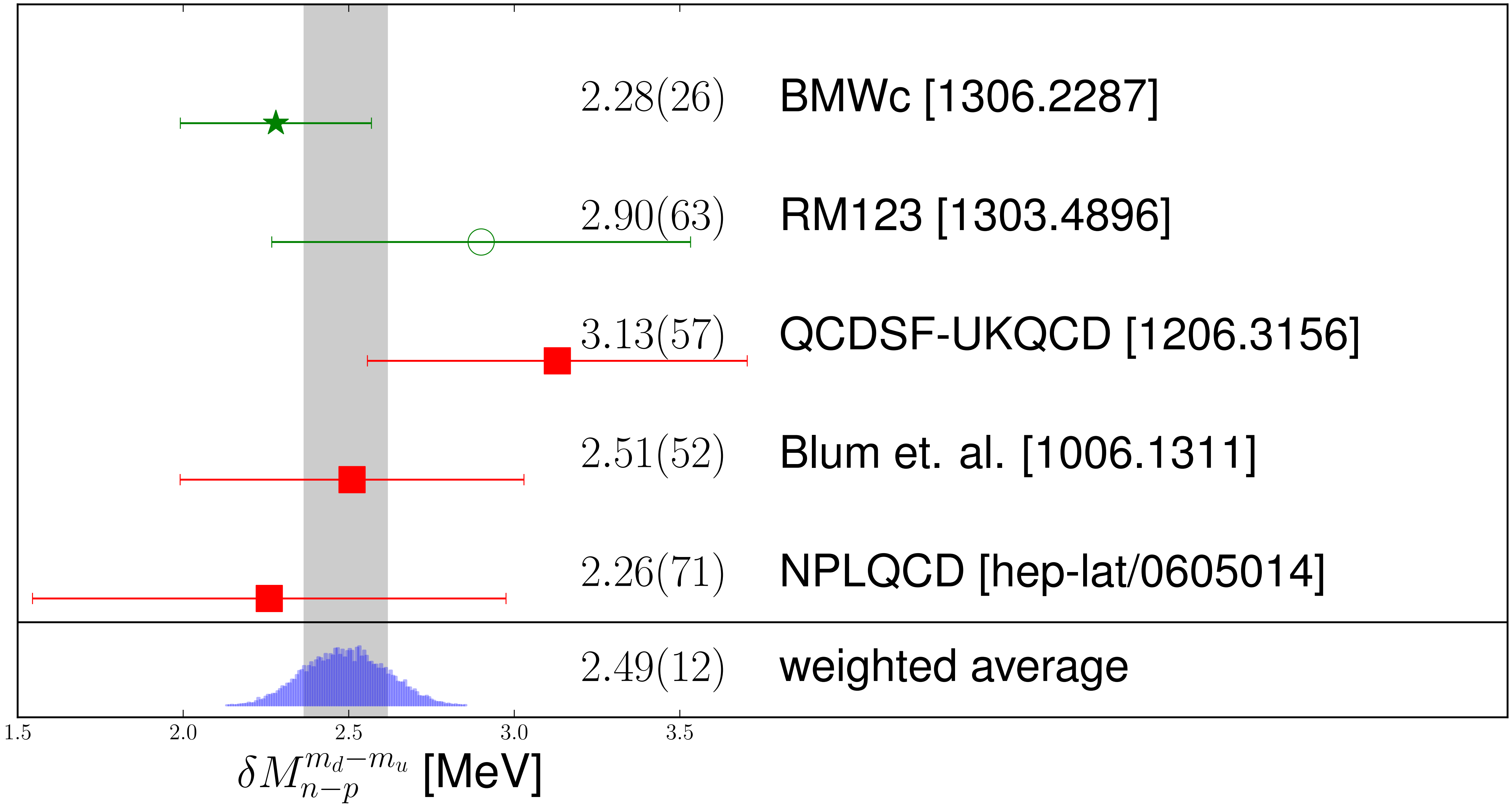
H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

Magnetic polarizability

$$f(Q^2) = \left(\frac{1}{1+Q^2/m_0^2}\right)^2 \quad \Rightarrow \quad \delta M^\gamma_{p-n}[\text{MeV}] = 1.40(.03)(.47)$$

Isospin Breaking: M_n - M_p What do we know?

- $\delta M_{n-p}^{m_d-m_u} = 2.49(12)\ \mathrm{MeV}$



$$\delta M_{p-n}^{\gamma} = M_p - M_n - \delta M_{p-n}^{m_d-m_u} = 1.20(12)\ \mathrm{MeV}$$

[AWL, C.Carlson, G.Miller PRL 108 (2012) $1.40(03)(47)\ \mathrm{MeV}$]

Strong Isospin Breaking: m_d - m_u

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d - m_u} = 2.49(12)\ \mathrm{MeV}$$

lattice average

B. Tiburzi, AWL	Nucl. Phys. A764 (2006)
Beane, Orginos, Savage	Nucl. Phys. B768 (2007)
AWL	arXiv:0904.2404
Blum, Izubuchi, etal	Phys. Rev. D82 (2010)
AWL	PoS Lattice2010 (2010)
de Divitiis etal	JHEP 1204 (2012)
Horsley etal	Phys. Rev. D86 (2012)
de Divitiis etal	Phys. Rev. D87 (2013)
Borsanyi etal	arXiv:1306.2287

But in lattice calculations $m_u = m_d = m_l$?
(except latest)

Strong Isospin Breaking: $m_d - m_u$

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

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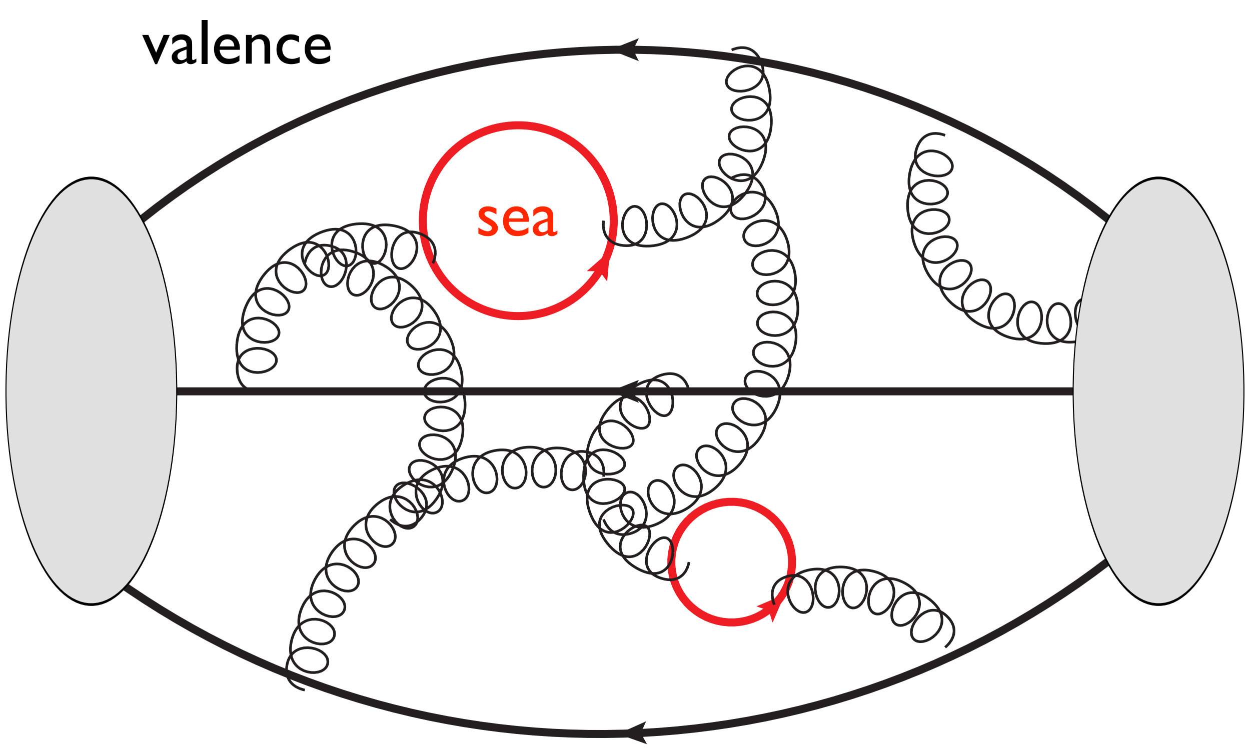

$$m_{u,d}^{valence} \neq m_l^{sea}$$

"partially quenched" lattice QCD trick that works on the computer but introduces error which must be corrected

Strong Isospin Breaking: m_d - m_u

strong isospin breaking correction

$$\delta M_{n-p}^{m_d-m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d-m_u} = 2.49(12)\ \mathrm{MeV}$$

lattice average

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can we improve this method?

of course!

“Symmetric breaking of isospin symmetry” AWL arXiv:0904.2404

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

$$\mathcal{Z}_{u,d} = \int DU_\mu \, \mathrm{Det}(D + m_l - \delta\tau_3) \, e^{-S[U_\mu]}$$

$$= \int DU_\mu \, \mathrm{Det}(D + m_l) \, \det\left(1 - \frac{\delta^2}{(D + m_l)^2}\right) \, e^{-S[U_\mu]}$$

Isospin symmetric quantities: error $\mathcal{O}(\delta^2)$

Isospin violating quantities: error $\mathcal{O}(\delta^3)$

see also

de Divitiis etal JHEP 1204 (2012)

de Divitiis etal Phys. Rev. D87 (2013)

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Pion Chiral Lagrangian

$$\mathcal{L} = \frac{f^2}{8} \mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right) - \frac{f^2}{8} \mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right) - \frac{l_1}{4}\left[\mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right)\right]^2 - \frac{l_2}{4} \mathrm{tr}\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right) \mathrm{tr}\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right)$$
$$- \frac{l_3 + l_4}{16}\left[\mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right)\right]^2 + \frac{l_4}{8} \mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right) \mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right) - \frac{l_7}{16}\left[\mathrm{tr}\left(\chi'^\dagger \Sigma - \Sigma^\dagger \chi'\right)\right]^2$$

$$m_{\pi^\pm}^2 = 2Bm_l \left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right) + \frac{4m_\pi^2}{f_\pi^2} l_4^r(\mu)\right\} - \frac{\Delta_{PQ}^4}{2(4\pi f_\pi)^2}$$

$$m_{\pi^0}^2 = m_{\pi^\pm}^2 + \frac{16B^2\delta^2}{f_\pi^2} l_7$$

$$\Delta_{PQ}^2 = 2B\delta$$

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Can also construct the partially quenched baryon chiral Lagrangian

$$M_p = M_0 - \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$M_n = M_0 + \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$M_n - M_p = \alpha(m_d - m_u) + \mathcal{O}(\delta^2, m_\pi^2 \delta)$$

$$(2\delta = m_d - m_u)$$

Problematic terms exactly drop out of expansion for mass difference!
This only works for this symmetric choice of partial quenching

Strong Isospin Breaking: m_d - m_u PRELIMINARY

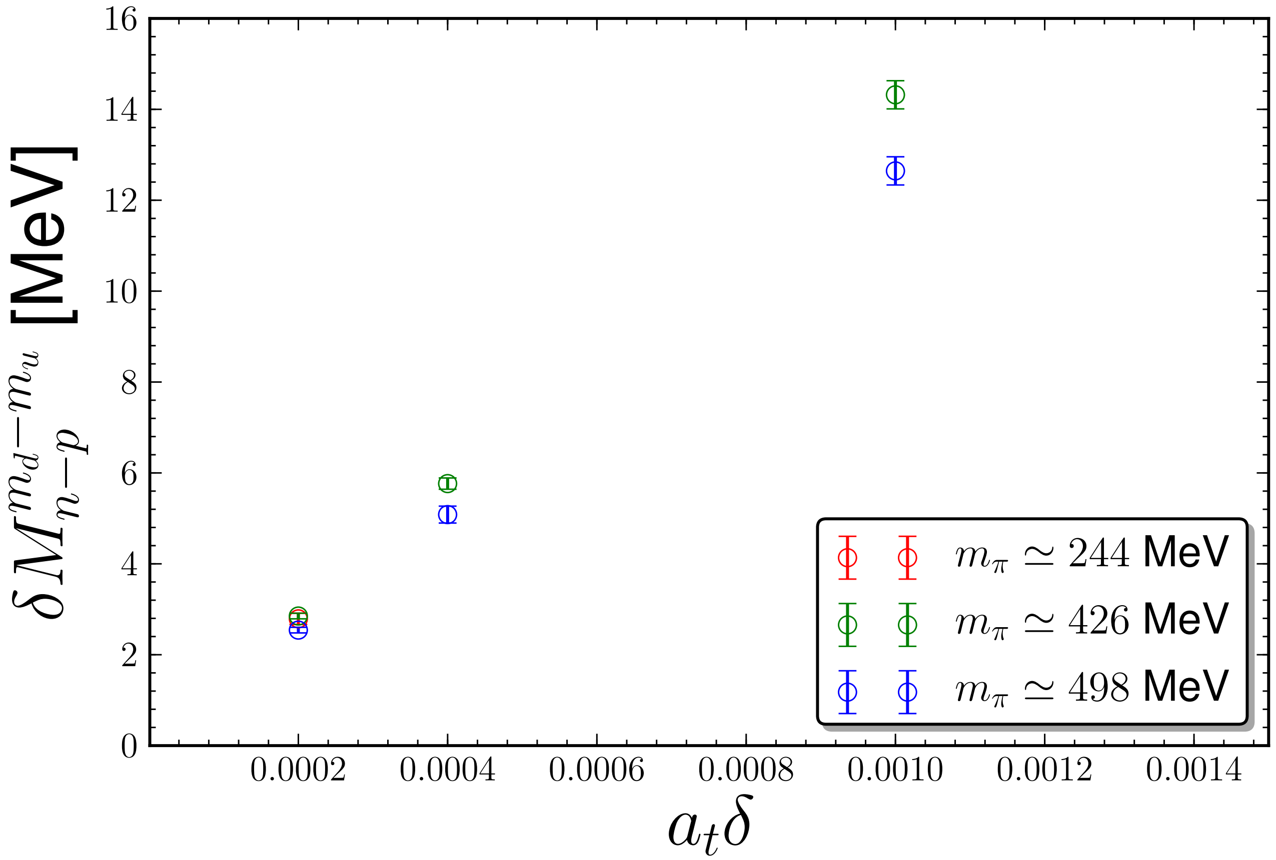
lattice QCD calculation performed using the Spectrum Collaboration anisotropic clover-Wilson gauge ensembles (developed @JLAB)

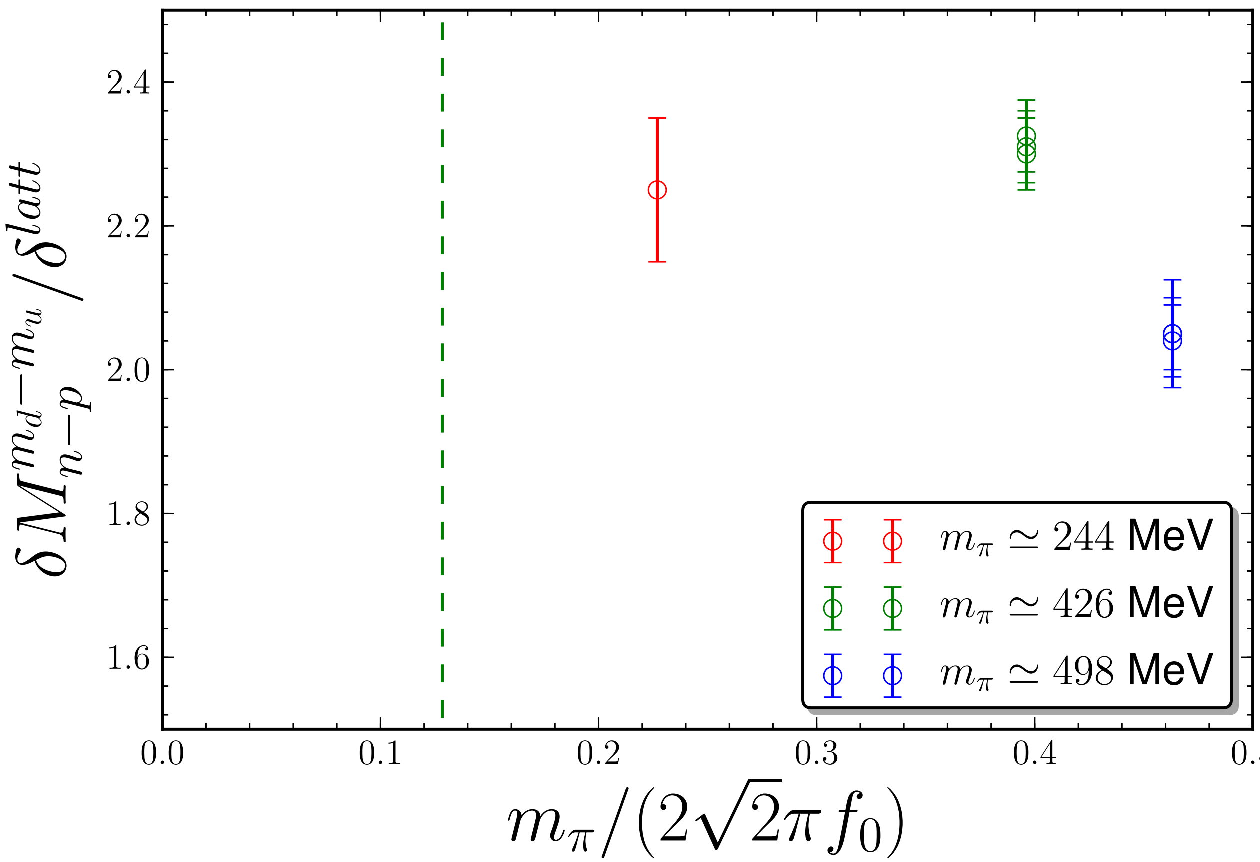
ensemble				m_π	m_K	$a_t\delta$ $[N_{cfg} \times N_{src}]$			
L	T	$a_t m_l$	$a_t m_s$	[MeV]	[MeV]	0.0002	0.0004	0.0010	0.0020
16	128	-0.0830	-0.0743	500	647	207×16	207×16	207×16	207×16
16	128	-0.0840	-0.0743	426	608	166×25	166×25	166×25	166×50
20	128	-0.0840	-0.0743	426	608	120×25	–	–	–
24	128	-0.0840	-0.0743	426	608	97×25	–	193×25	–
32	256	-0.0840	-0.0743	426	608	291×10	291×10	291×10	–
24	128	-0.0860	-0.0743	244	520	118×26	–	–	–
32	256	-0.0860	-0.0743	244	520	842×11	–	–	–

M_Ω scale setting

C.Aubin, W.Detmold,
Emanuele Mereghetti,
K.Orginos, S.Syritsyn,
B.Tiburzi,
AWL

Strong Isospin Breaking: m_d - m_u PRELIMINARY

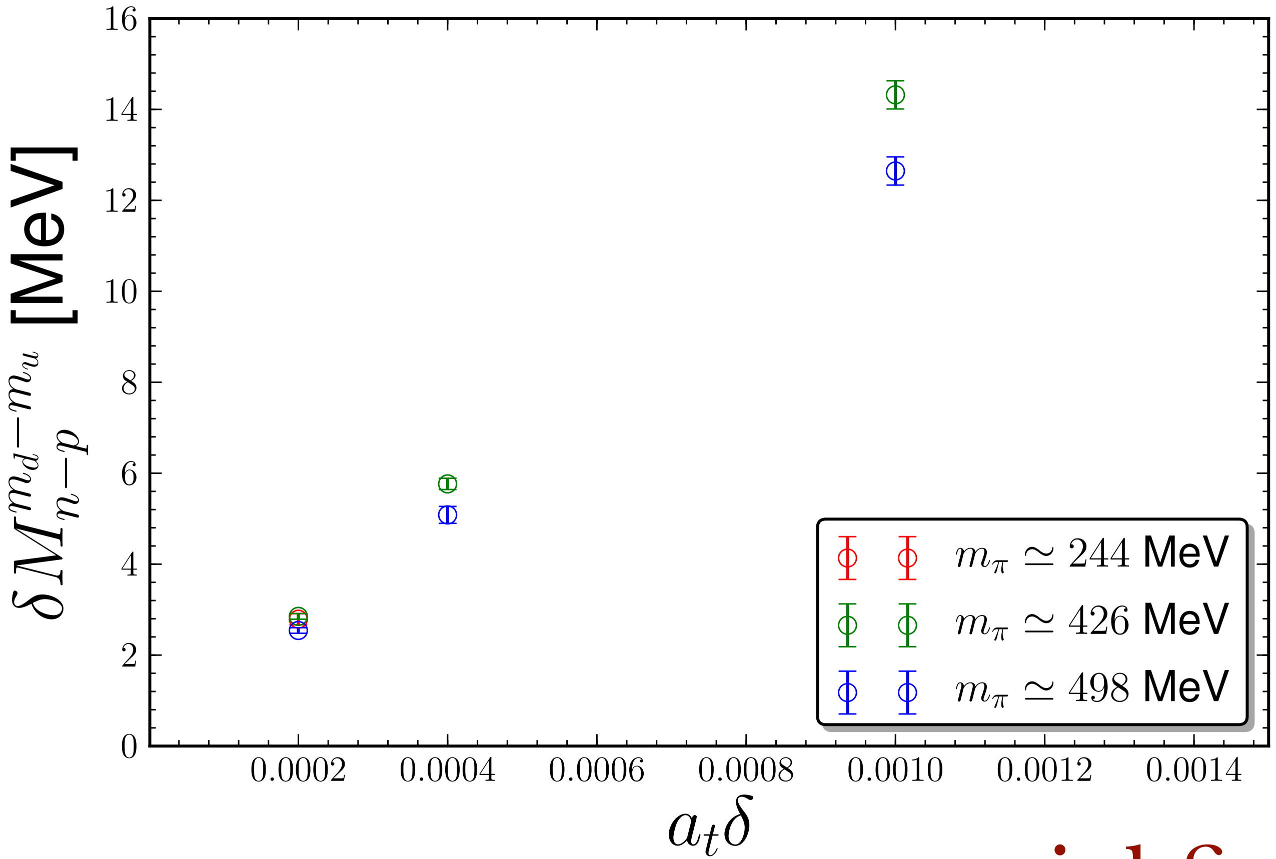


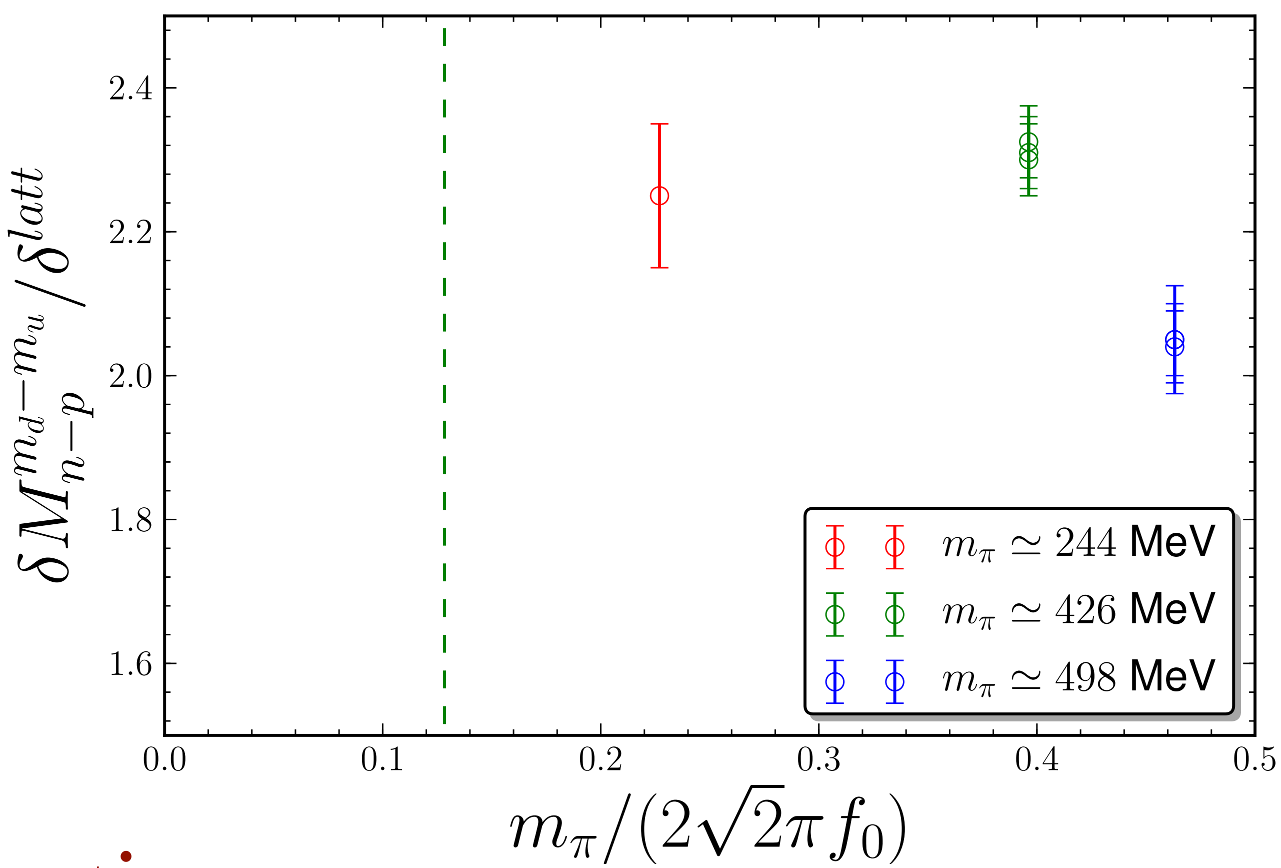


slope depends slightly on pion mass

no evidence for deviations from linear δ dependence

Strong Isospin Breaking: $m_d - m_u$ PRELIMINARY





trial fit functions

polynomial in m_π^2

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha + \beta\frac{m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\chi^2/dof = 13/5 = 2.6$$

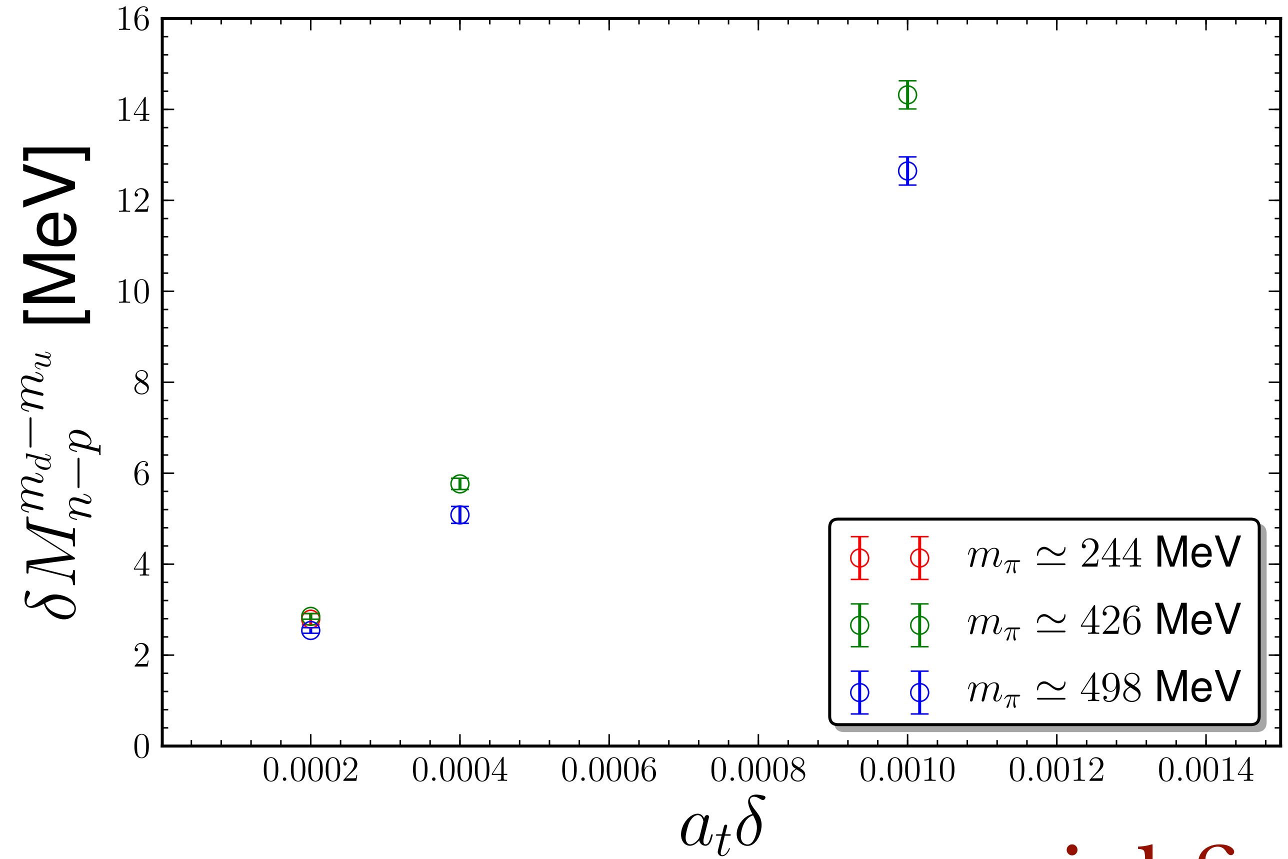
NNLO χPT

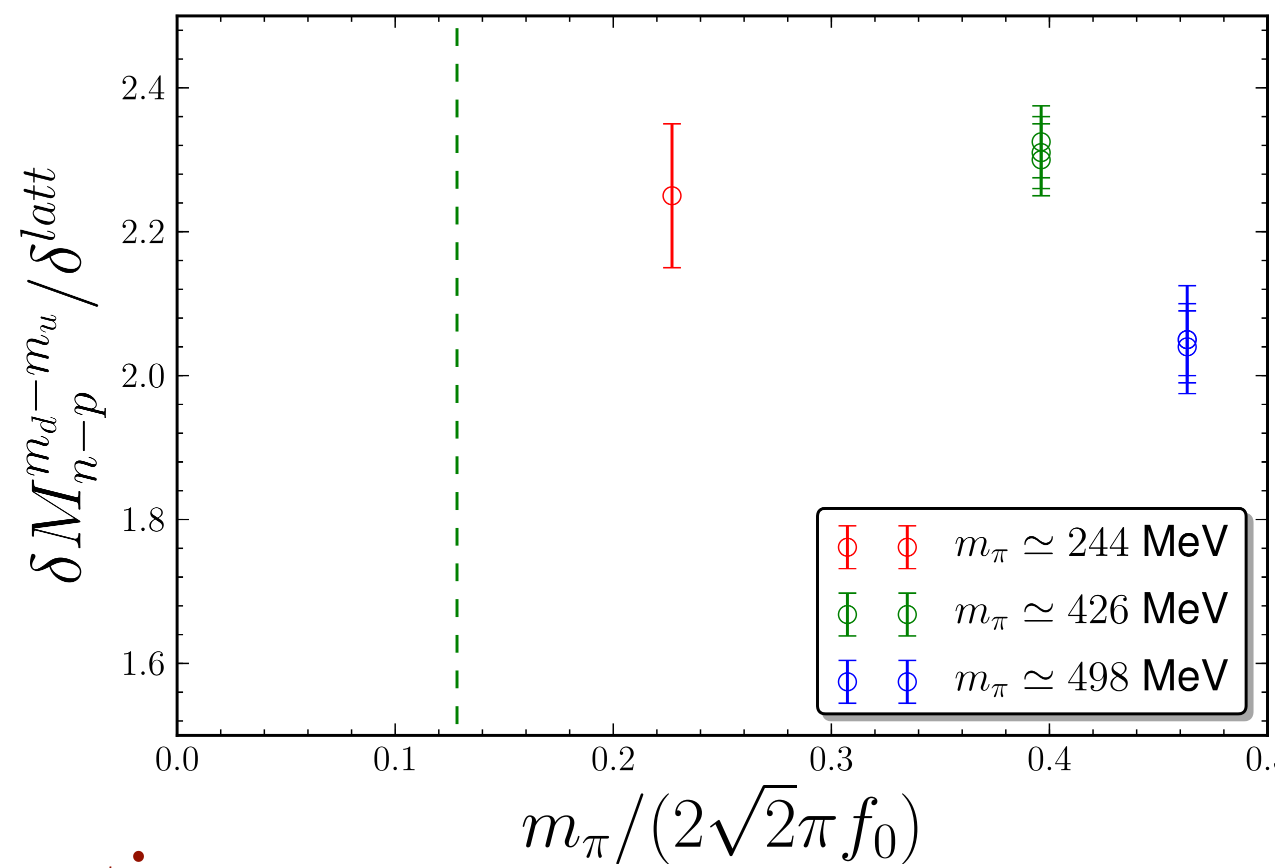
$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$(g_A = 1.27, f_\pi = 130 \text{ MeV})$

$$\chi^2/dof = 1.66/5 = 0.33$$

Strong Isospin Breaking: $m_d - m_u$ PRELIMINARY





trial fit functions

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$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha + \beta\frac{m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

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NNLO χPT

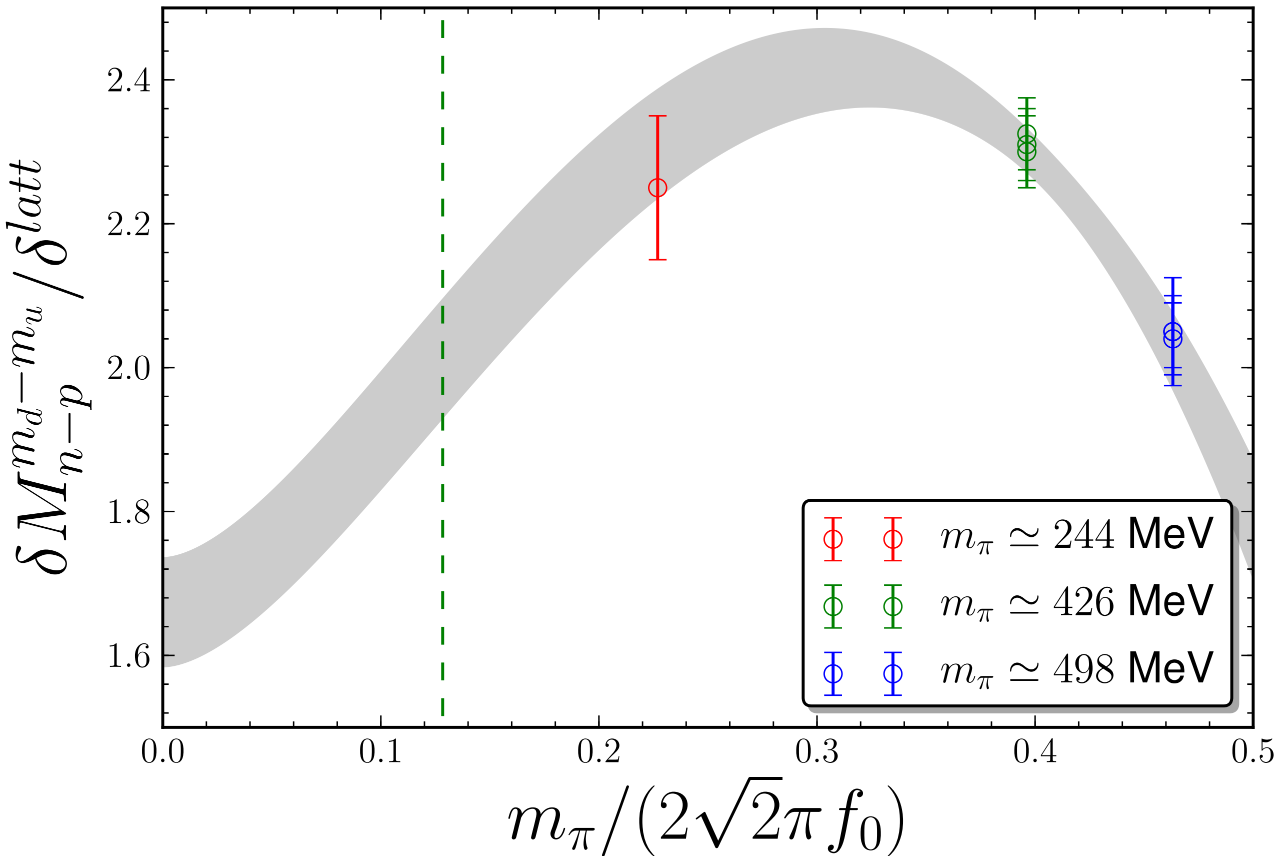
$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

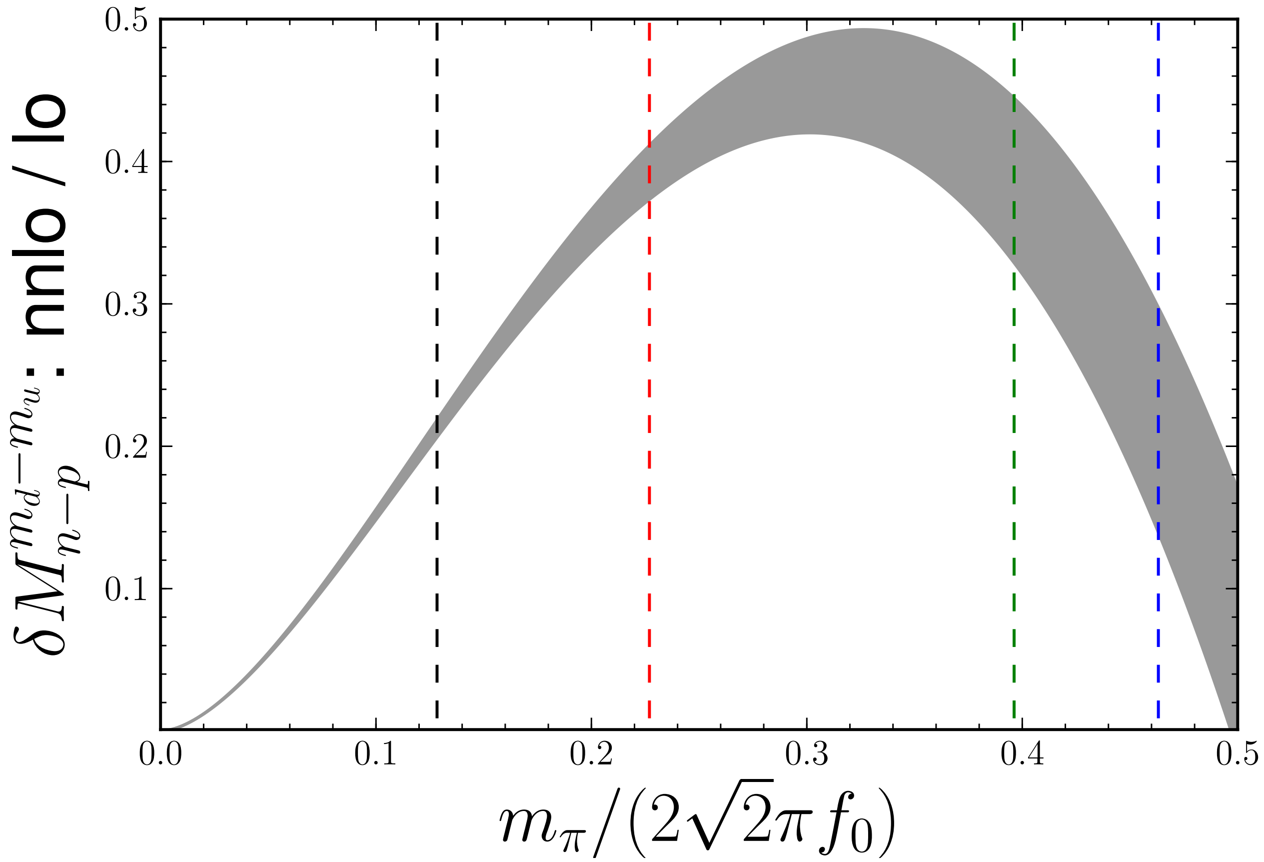
$(f_\pi = 130 \text{ MeV})$

$$\chi^2/dof = 1.34/4 = 0.33$$

→ $g_A = 1.50(.29)$

Strong Isospin Breaking: $m_d - m_u$ PRELIMINARY





NNLO χPT

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

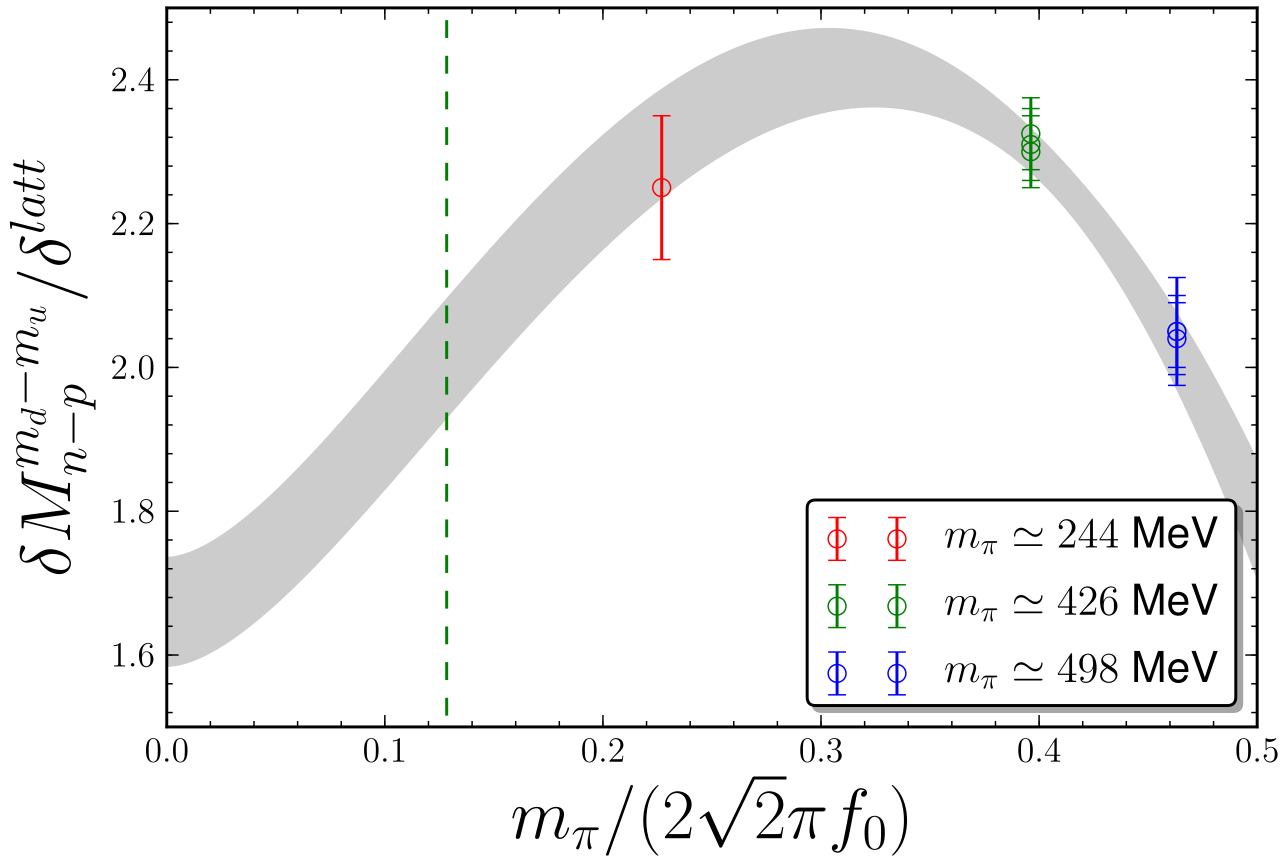
$(g_A = 1.27, f_\pi = 130 \text{ MeV})$

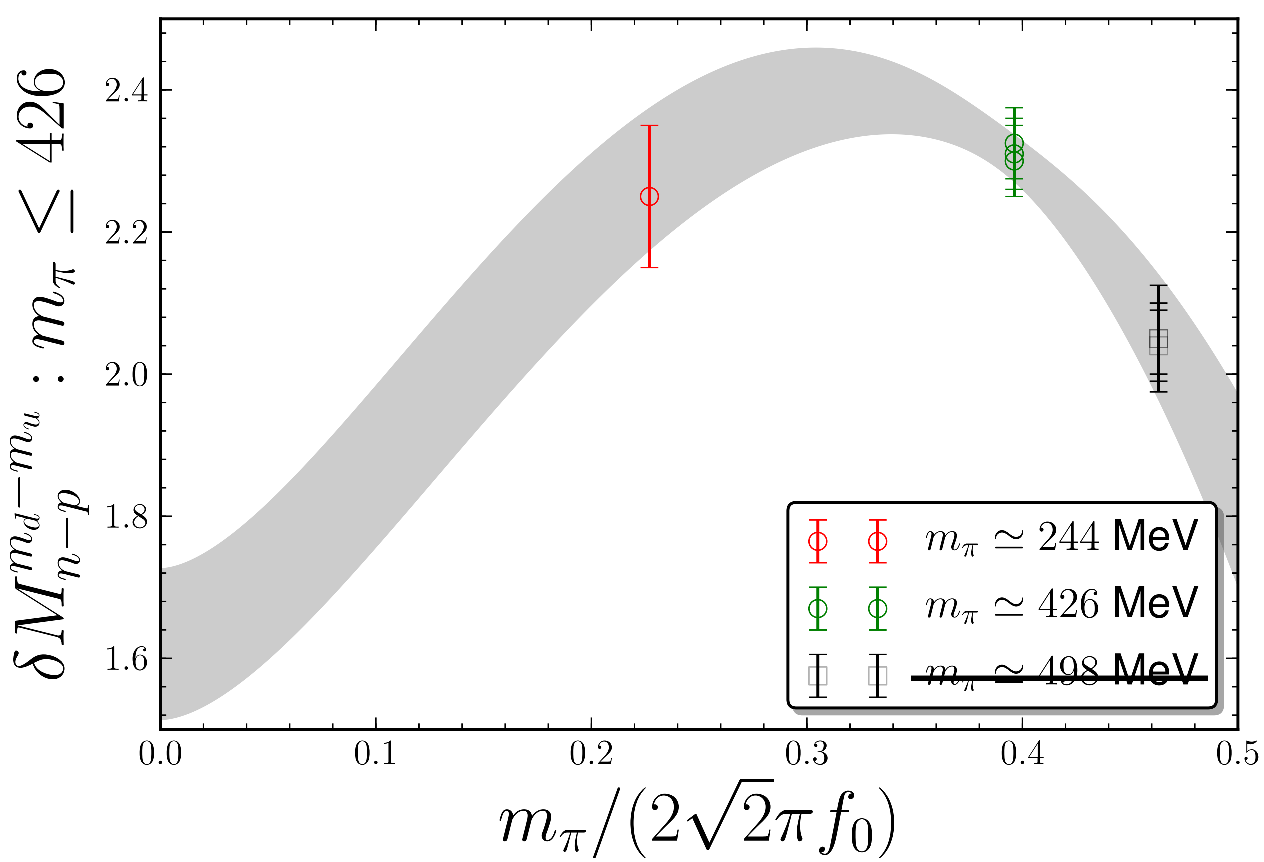
$$\chi^2/dof = 1.66/5 = 0.33$$

ratio of NNLO to LO correction

C.Aubin, W.Detmold,
Emanuele Mereghetti,
K.Orginos, S.Syritsyn,
B.Tiburzi,
AWL

Strong Isospin Breaking: $m_d - m_u$ PRELIMINARY





NNLO χPT

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$(g_A = 1.27, f_\pi = 130 \text{ MeV})$

$$\chi^2/dof = 1.66/5 = 0.33$$

exclude heavy mass point

this is striking evidence of a chiral logarithm

C.Aubin, W.Detmold, Emanuele Mereghetti, K.Orginos, S.Syritsyn, B.Tiburzi, AWL

Big Bang Nucleosynthesis and $M_n - M_p$

$$M_n - M_p = \delta M^{\gamma}_{n-p} + \delta M^{m_d - m_u}_{n-p}$$
$$= -178(04)(64)\ \text{MeV} \times \alpha_{f.s.} + 1.01(5)(9) \times (m_d - m_u)$$

(lattice average)

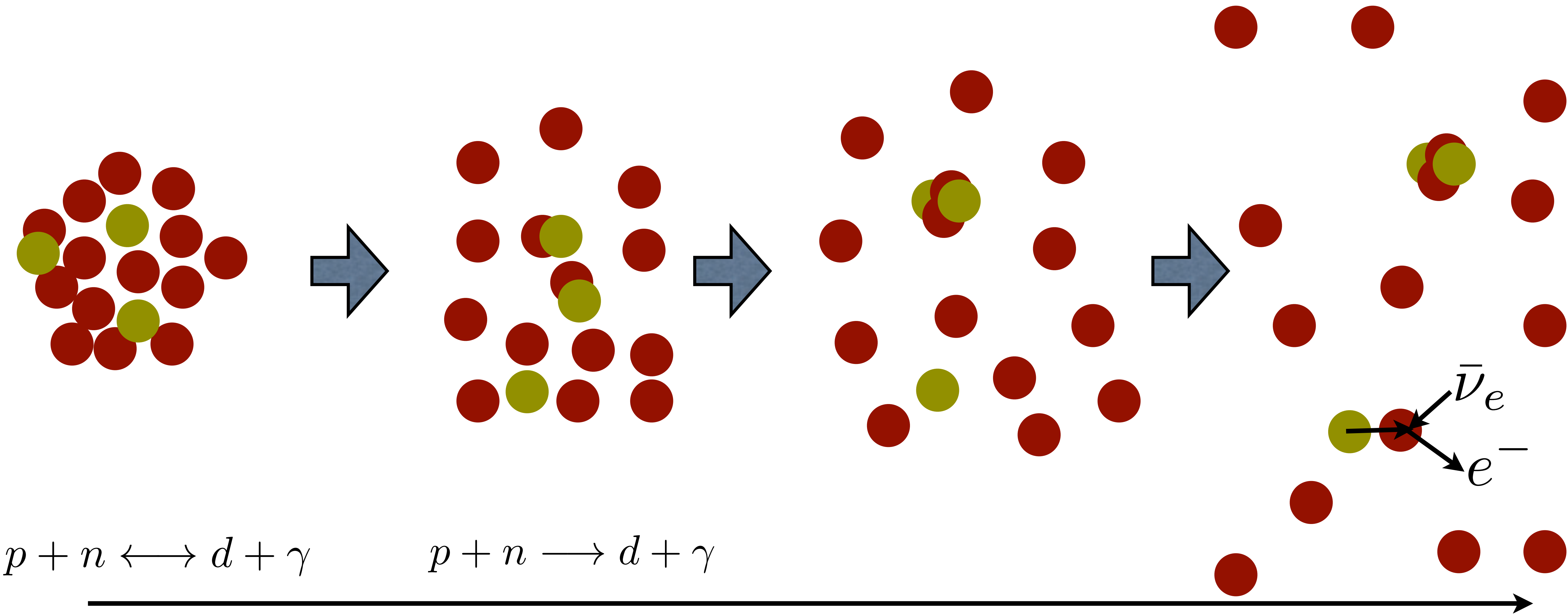
my value *hopefully* more precise

Big Bang Nucleosynthesis highly constrains variation of $M_n - M_p$ and hence variation of fundamental constants

considering $\alpha_{f.s.}$ and $m_d - m_u$ simultaneously relaxes constraints (not yet simultaneously considered)

for now - freeze electromagnetic coupling and just look at effects of quark mass splitting

Big Bang Nucleosynthesis and $M_n - M_p$



$t \sim 1$ sec	$t \sim 3$ min	$t \sim 3^+$ min	$t \sim 15$min
$T \sim 1$ MeV	$T \sim 0.1$ MeV	$T \sim 0.1^-$ MeV	$T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

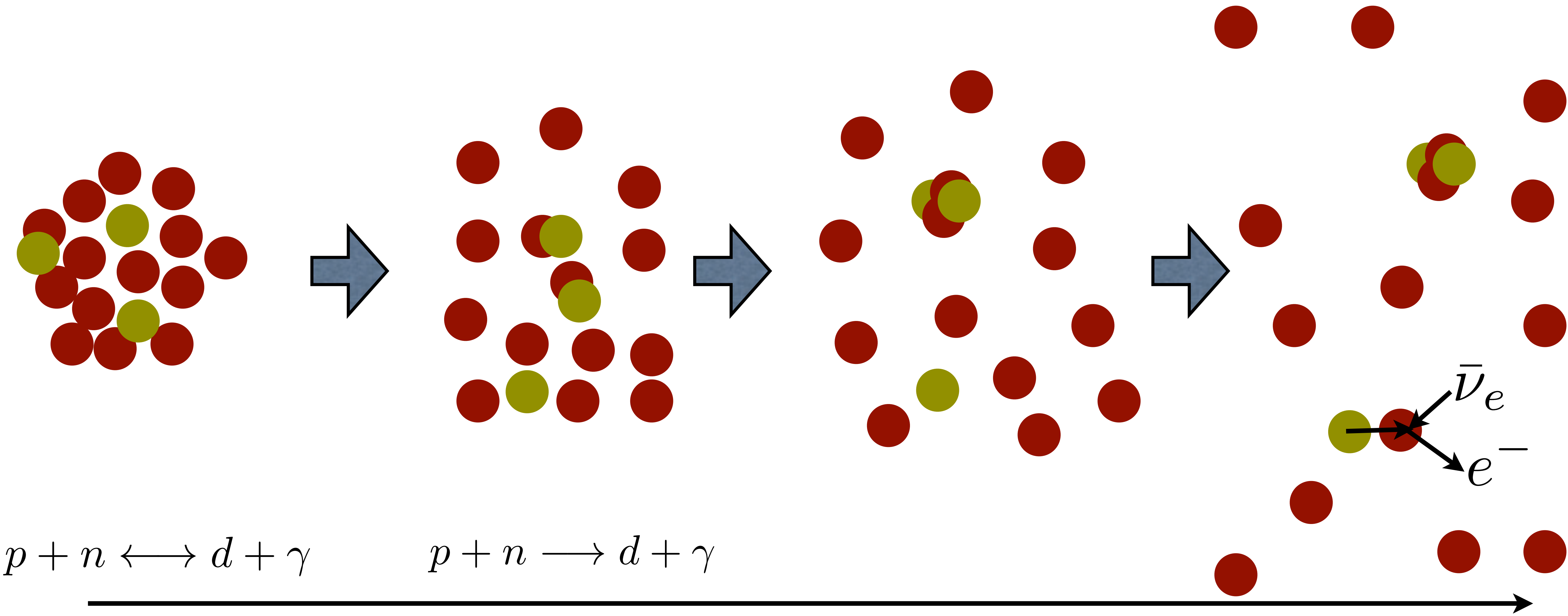
B_d

deuterium binding energy

τ_n

neutron lifetime

Big Bang Nucleosynthesis and $M_n - M_p$



$t \sim 1$ sec	$t \sim 3$ min	$t \sim 3^+$ min	$t \sim 15$min
$T \sim 1$ MeV	$T \sim 0.1$ MeV	$T \sim 0.1^-$ MeV	$T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

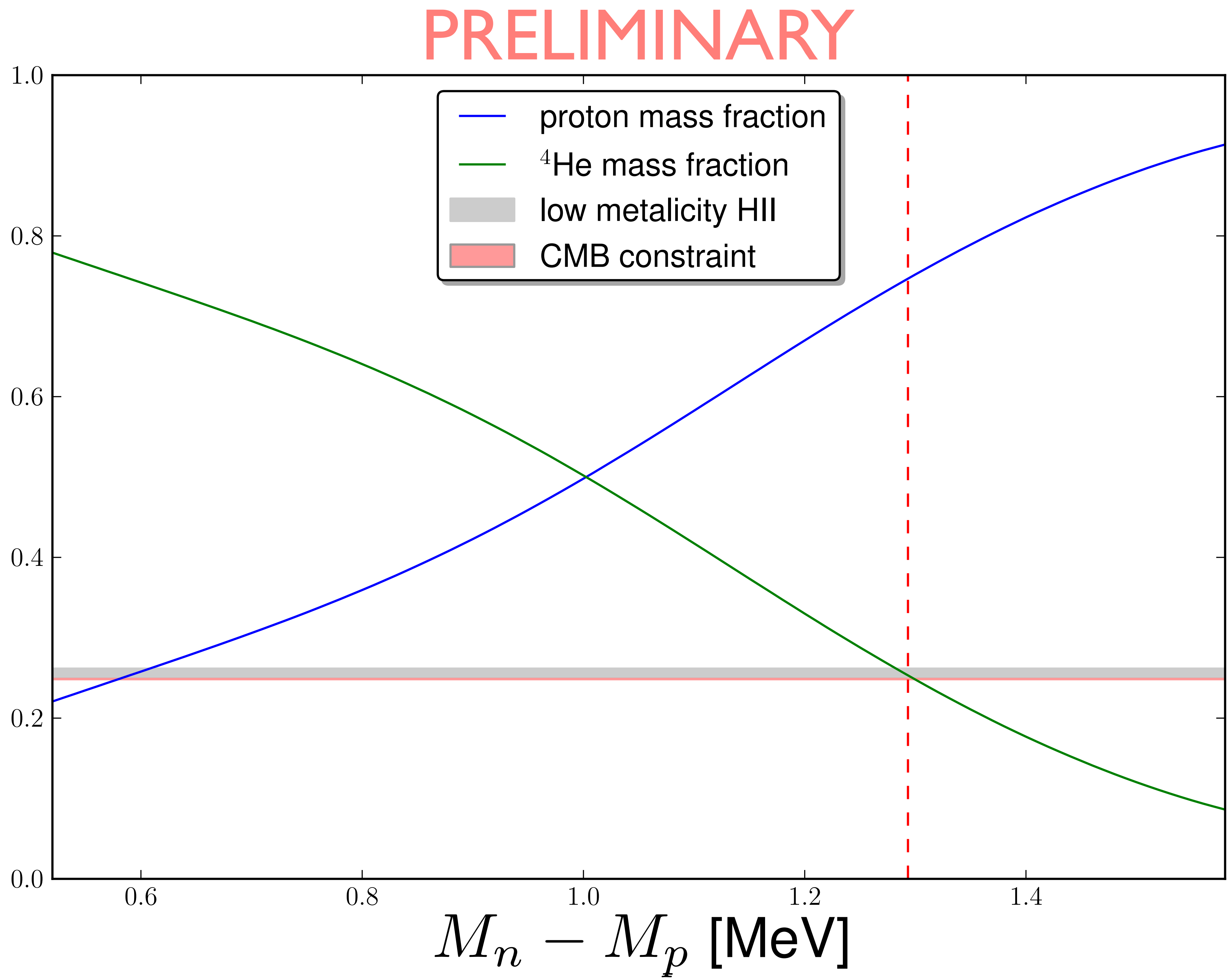
focus on leading isospin breaking

τ_n

neutron lifetime

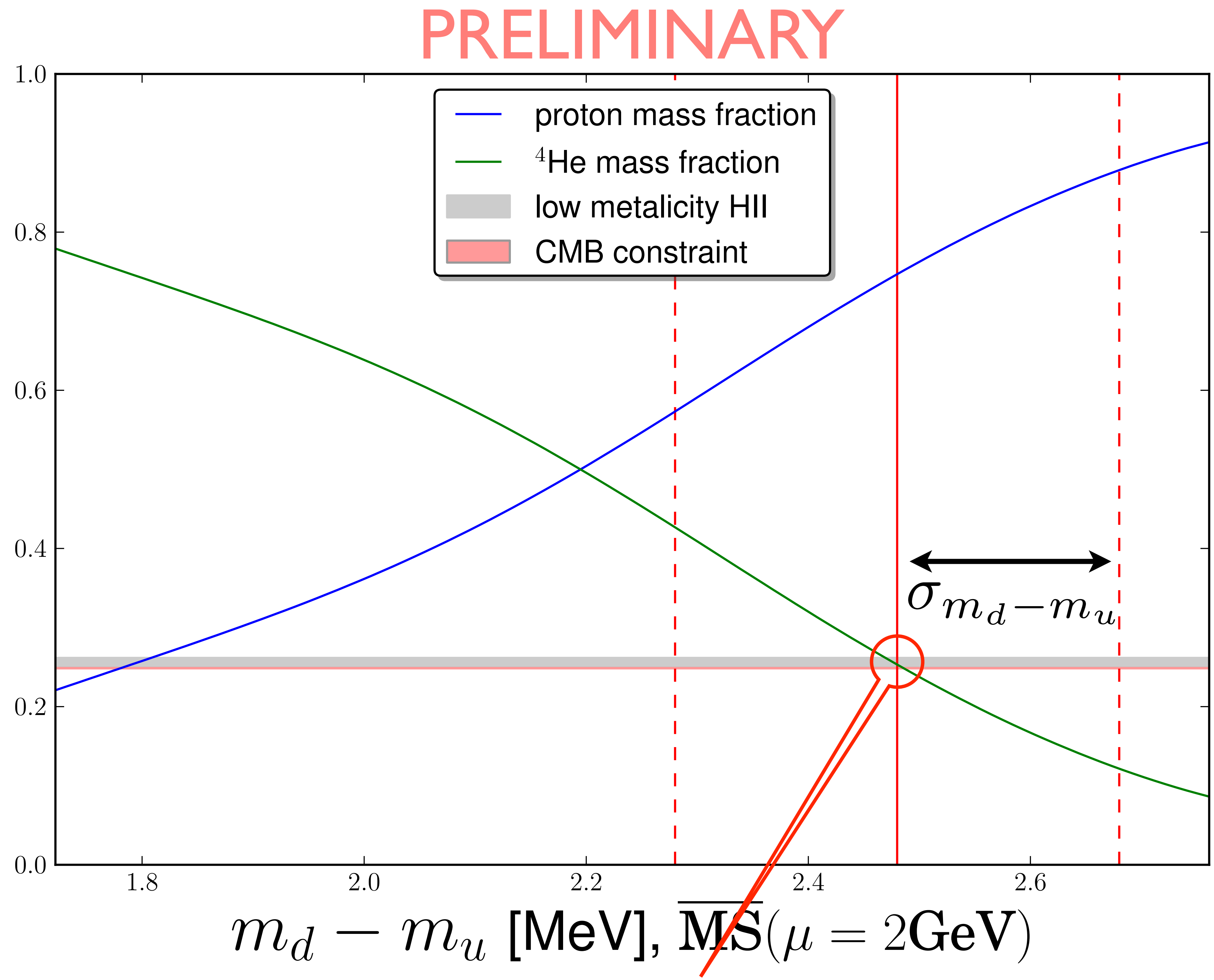
Big Bang Nucleosynthesis and $M_n - M_p$

P. Banerjee, T. Luu,
S. Syritsyn AWL



Big Bang Nucleosynthesis and $M_n - M_p$

P. Banerjee, T. Luu, S. Syritsyn AWL



Lattice QCD

A precise determination of α + BBN can constrain $m_d - m_u$

$\delta M_{n-p}^{m_d - m_u} \equiv \alpha(m_d - m_u)$ *connect the quarks with the cosmos*

Thank You

Nuclear Physics Review

André Walker-Loud

Berkeley Lab → The College of William & Mary + Jefferson Lab

LATTICE 13

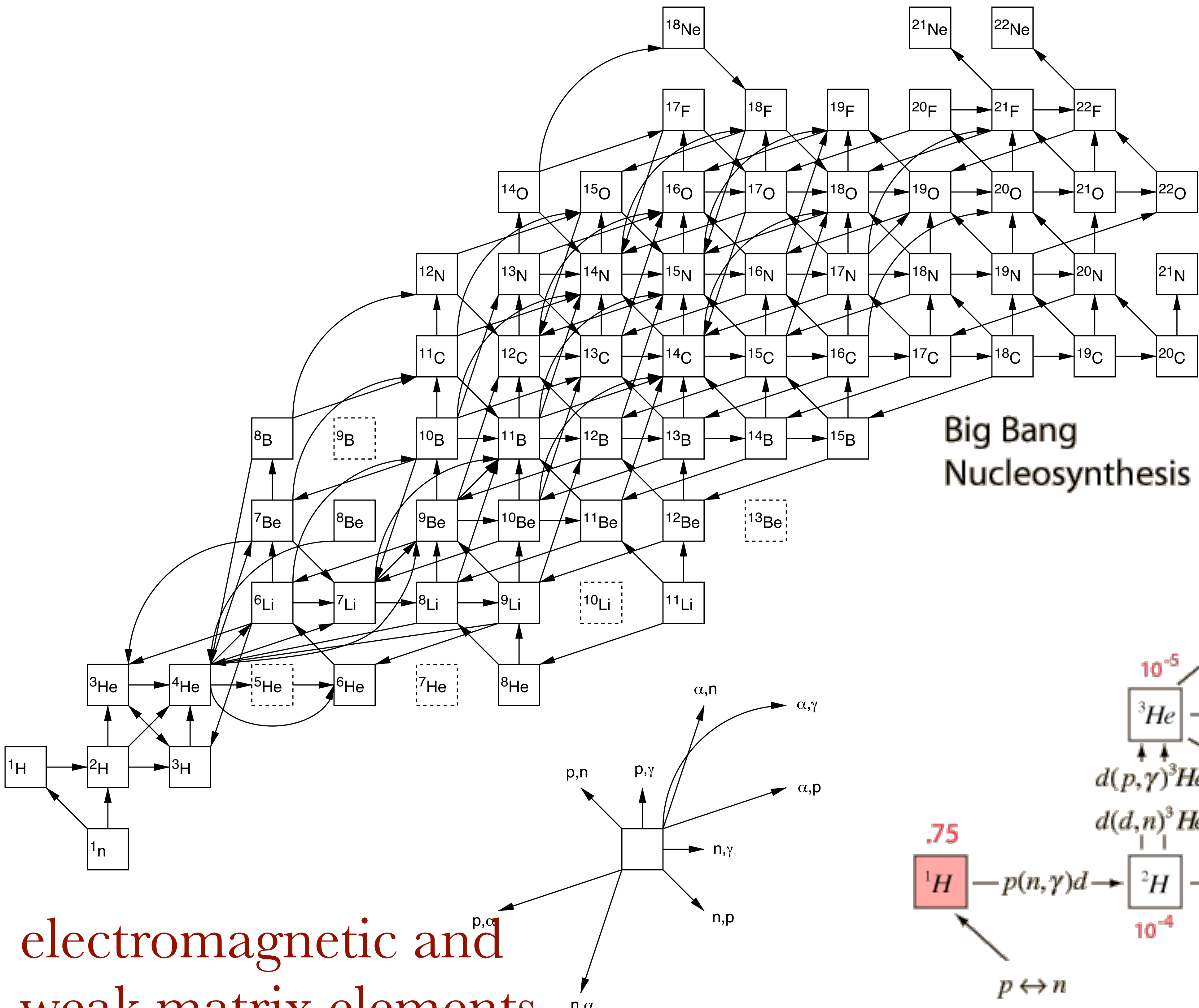
JULY 29 – AUGUST 03 2013

MAINZ, GERMANY

Outline

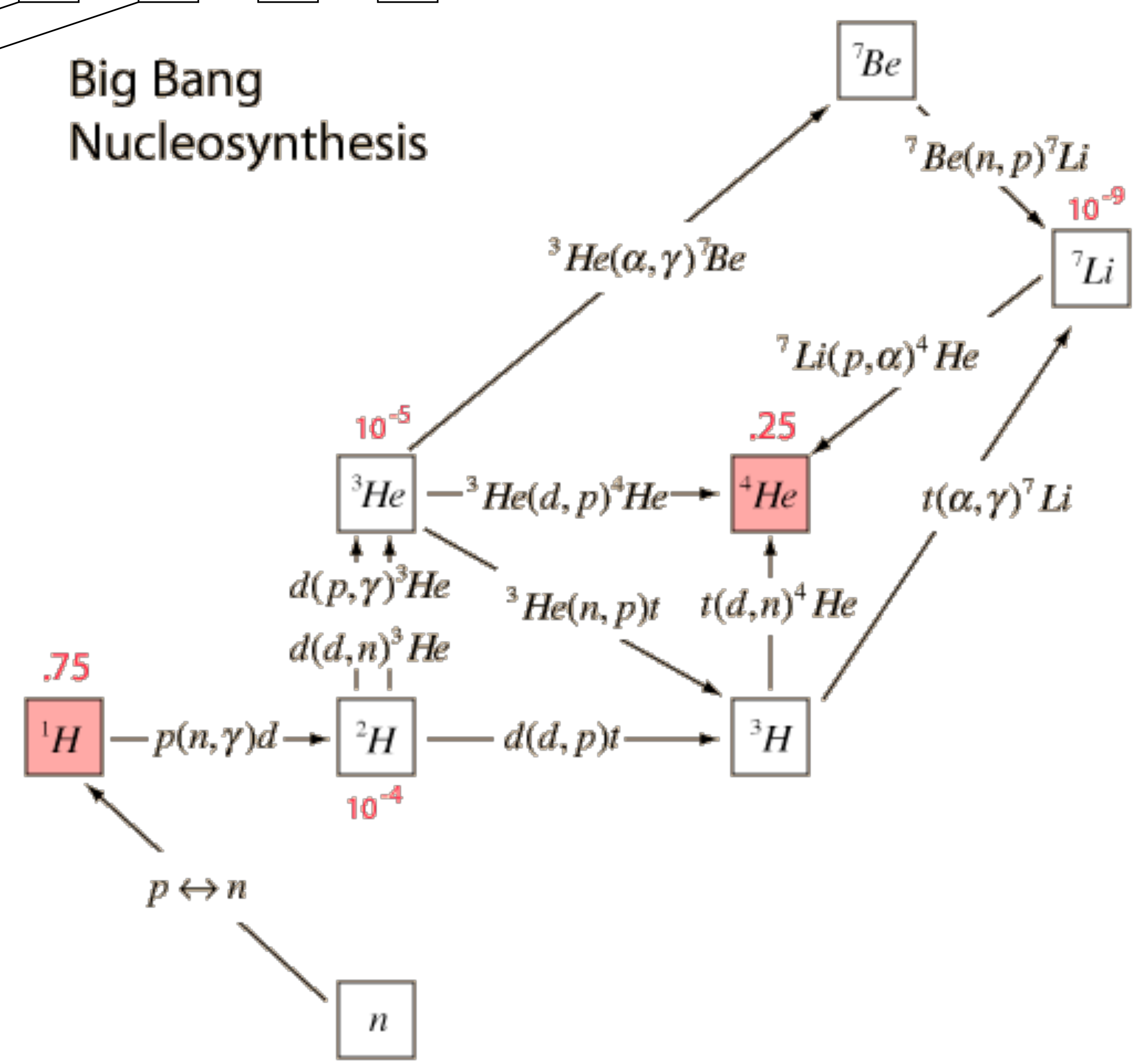
- Introduction
- Methods and Results
- Challenges and Progress
- Current and Future Developments
- Conclusions

Introduction



QCD plays a significant, if not dominant role in all these reactions

Big Bang Nucleosynthesis

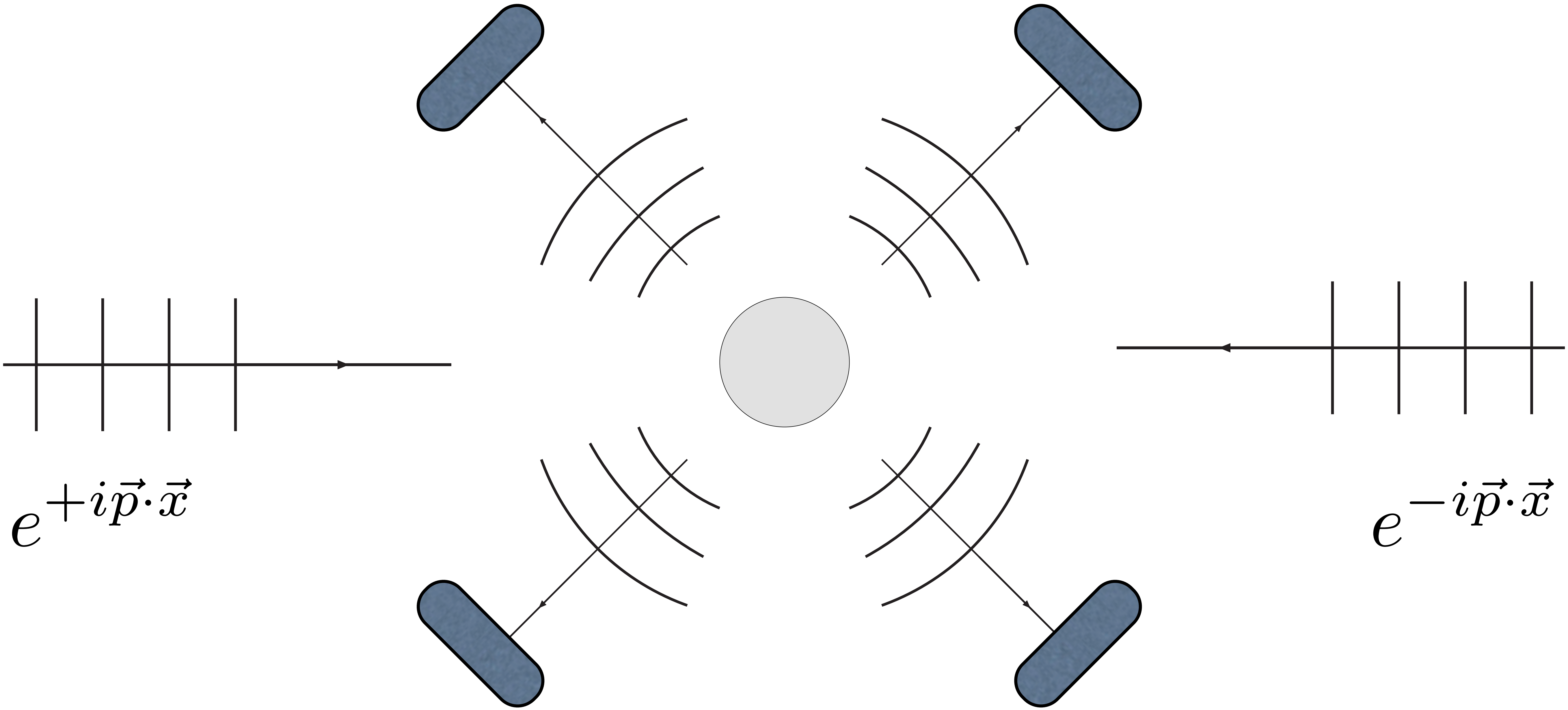


electromagnetic and weak matrix elements needed also

Methods and Results

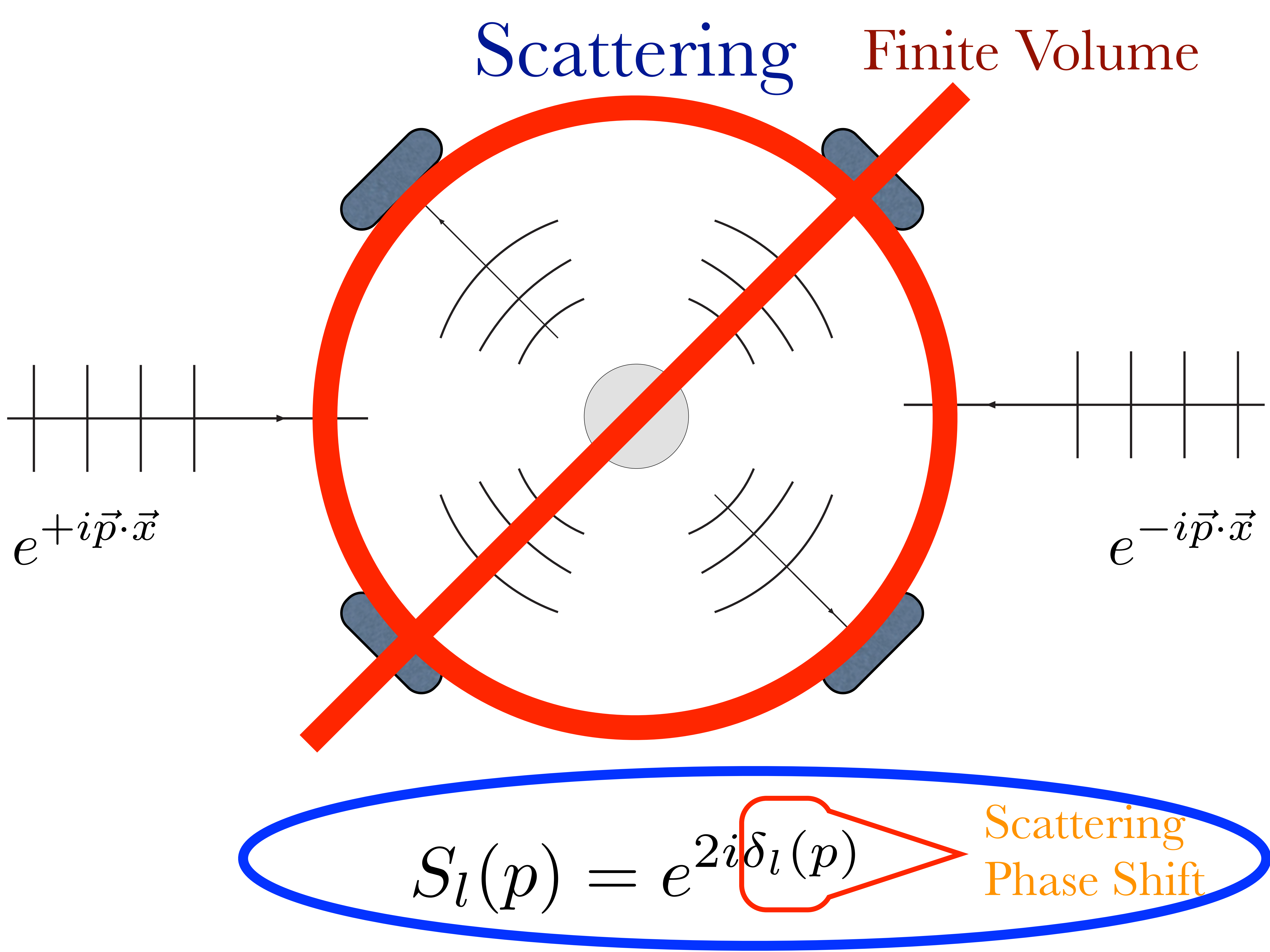
- Lüscher Method
- HALQCD 1
- HALQCD 2
- Results

Scattering Infinite Volume



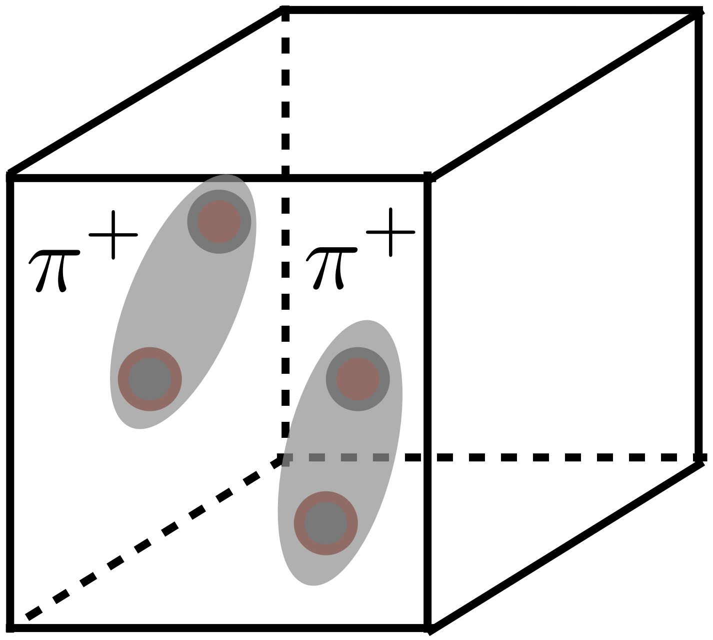
$$S_l(p) = e^{2i\delta_l(p)}$$

Scattering Phase Shift



Scattering
Finite Volume
$e^{+i\vec{p}\cdot\vec{x}}$
$e^{-i\vec{p}\cdot\vec{x}}$
$S_l(p) = e^{2i\delta_l(p)}$
Scattering
Phase Shift

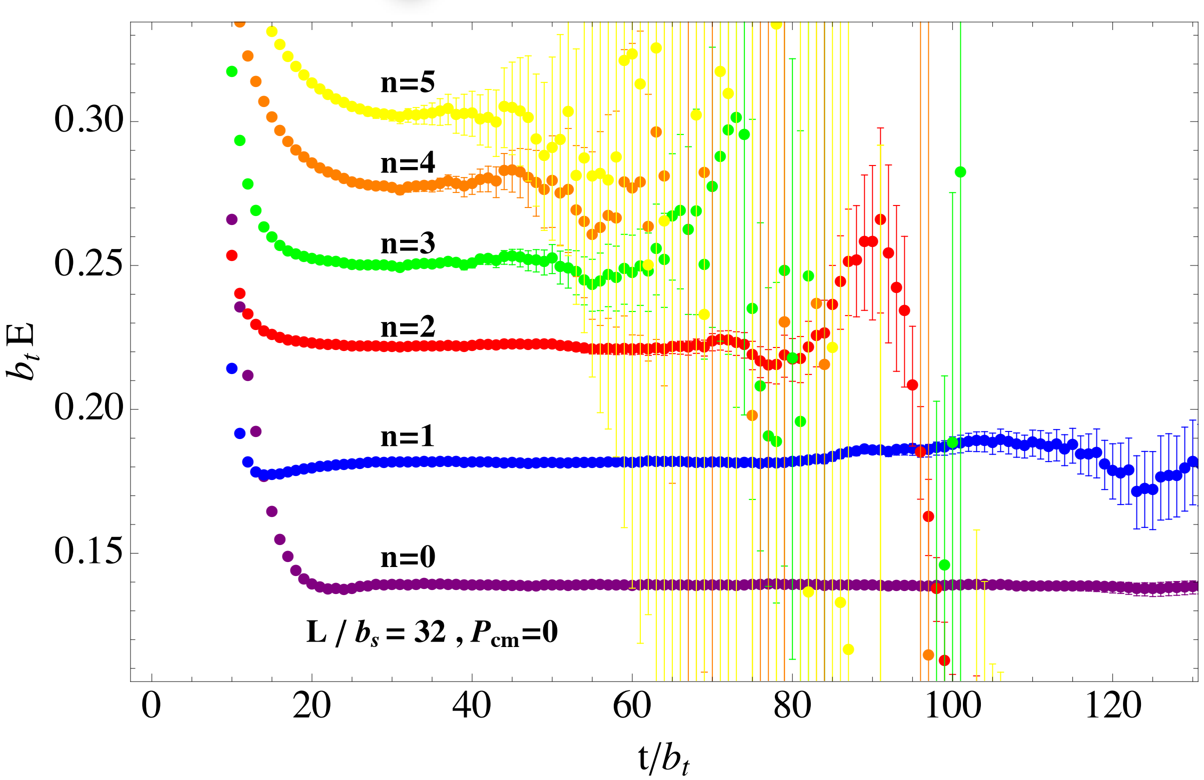
lattice QCD calculations performed in finite volume
infinite volume scattering phase shifts
(a la Lüscher)



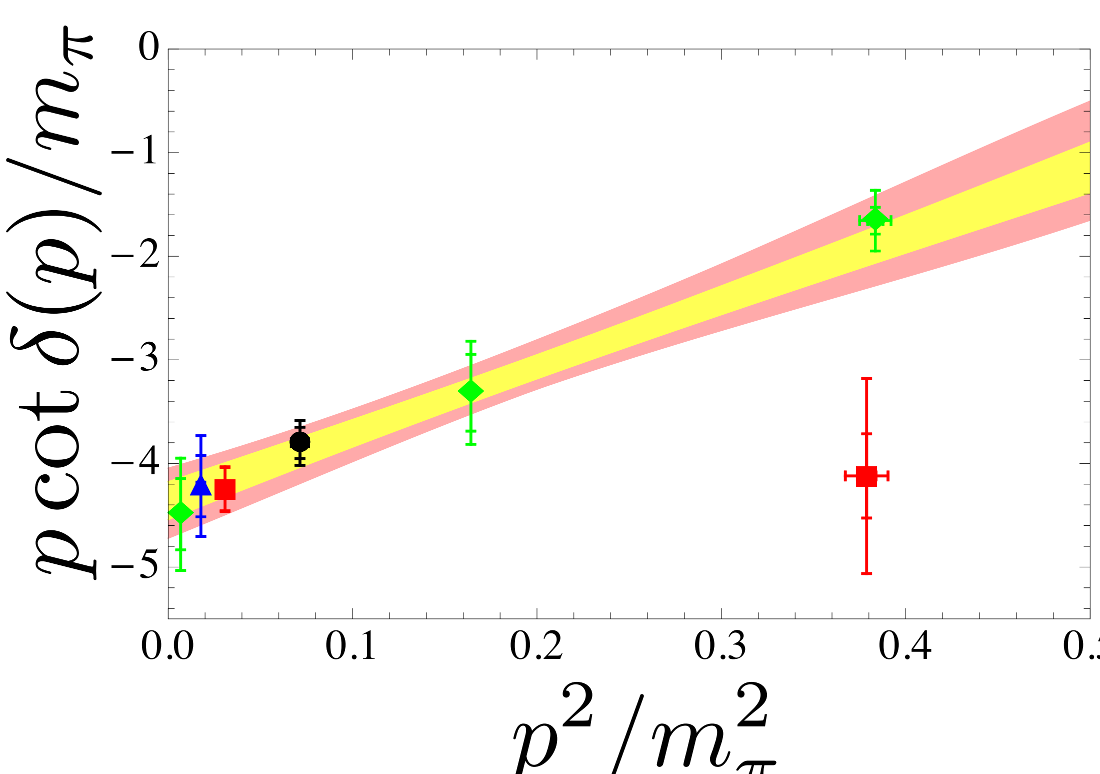
$$\underline{E} = 2\sqrt{m^2 + \underline{p}^2}$$ (two identical particles)

$$\underline{p \cot \delta(p)} = \frac{1}{\pi L} \sum_{|\vec{n}|<\Lambda} \frac{1}{|\vec{n}|^2 - \frac{p^2 L^2}{4\pi^2}} - 4\pi\Lambda$$

(includes bound states)



NPLQCD PRD 85 (2012) 034505



Methods and Results

Assumptions/Approximations/Challenges

- I have ignored partial wave mixing so far. The formalism for including these corrections is well established.

M. Lüscher, Commun. Math. Phys. 105 (1986)
M. Lüscher, Nucl. Phys. B 354 (1991)
K. Rummukainen and S.A. Gotlieb, Nucl.Phys.B 450 (1995)
C.H. Kim, C.T.Sachrajda and S.R.Sharpe, Nucl.Phys.B 727 (2005)
T.Luu and M.J.Savage, PRD 83 (2011)
M.T.Hansen and S.R.Sharpe, PRD 86 (2012)
R.Briceño and Z.Davoudi, arXiv:1204.1110
R.Briceño, Z.Davoudi and T.Luu, arXiv:1305.4903
R.Briceño, Z.Davoudi, T.Luu and M.J.Savage, in preparation

HalQCD Method 1

Solve for a potential with the (Schwinger)(Gell-Mann-Low) Bethe-Salpeter (Nambu) wave-function

$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0\right]\psi_{\mathbf{p}}(\mathbf{r}) = \int d^3r' U(\mathbf{r},\mathbf{r}')\psi_{\mathbf{p}}(\mathbf{r}') \qquad H_0 = -\frac{\nabla_{\mathbf{r}}^2}{2\mu}$$

$$\mu = M/2$$

In the absence of interactions $H_0\psi_{\mathbf{p}}(\mathbf{r}) = \frac{\mathbf{p}^2}{2\mu}\psi_{\mathbf{p}}(\mathbf{r})$

A choice of Bethe-Salpeter wave-function is

$$\psi_{\mathbf{p}}(\mathbf{r}) = \frac{1}{V}\sum_{\mathbf{x}}\langle 0|N(\mathbf{x}+\frac{\mathbf{r}}{2})N(\mathbf{x}-\frac{\mathbf{r}}{2})|N(\mathbf{p})N(\mathbf{p})\rangle_{in}$$

HalQCD Method 1

Consider the two-particle correlation function

$$
\begin{aligned}
C_{NN}(\mathbf{r},t) &= \sum_{\mathbf{x}} \langle 0|N(\mathbf{x}+\frac{\mathbf{r}}{2},t)N(\mathbf{x}-\frac{\mathbf{r}}{2},t)N^{\dagger}(\mathbf{x_0},0)N^{\dagger}(\mathbf{x_0},0)|0\rangle \\
&= \sum_{n}\sum_{\mathbf{x}} e^{-E_n t}\langle 0|N(\mathbf{x}+\frac{\mathbf{r}}{2},0)N(\mathbf{x}-\frac{\mathbf{r}}{2},0)|n\rangle\langle n|N^{\dagger}(\mathbf{x_0},0)N^{\dagger}(\mathbf{x_0},0)|0\rangle \\
&= \sum_{n} e^{-E_n t}\psi_n(\mathbf{r})A_n^{\dagger}
\end{aligned}
$$

$$
\lim_{t\to\infty} C_{NN}(\mathbf{r},t) = e^{-E_0 t}\psi_0(\mathbf{r})A_0^{\dagger}
$$

NOTE: two-particle correlation function used in standard Lüscher method is

$$
C_{NN}(\mathbf{P},t) = \sum_{\mathbf{r}} e^{i\mathbf{P}\cdot\mathbf{r}} C_{NN}(\mathbf{r},t)
$$

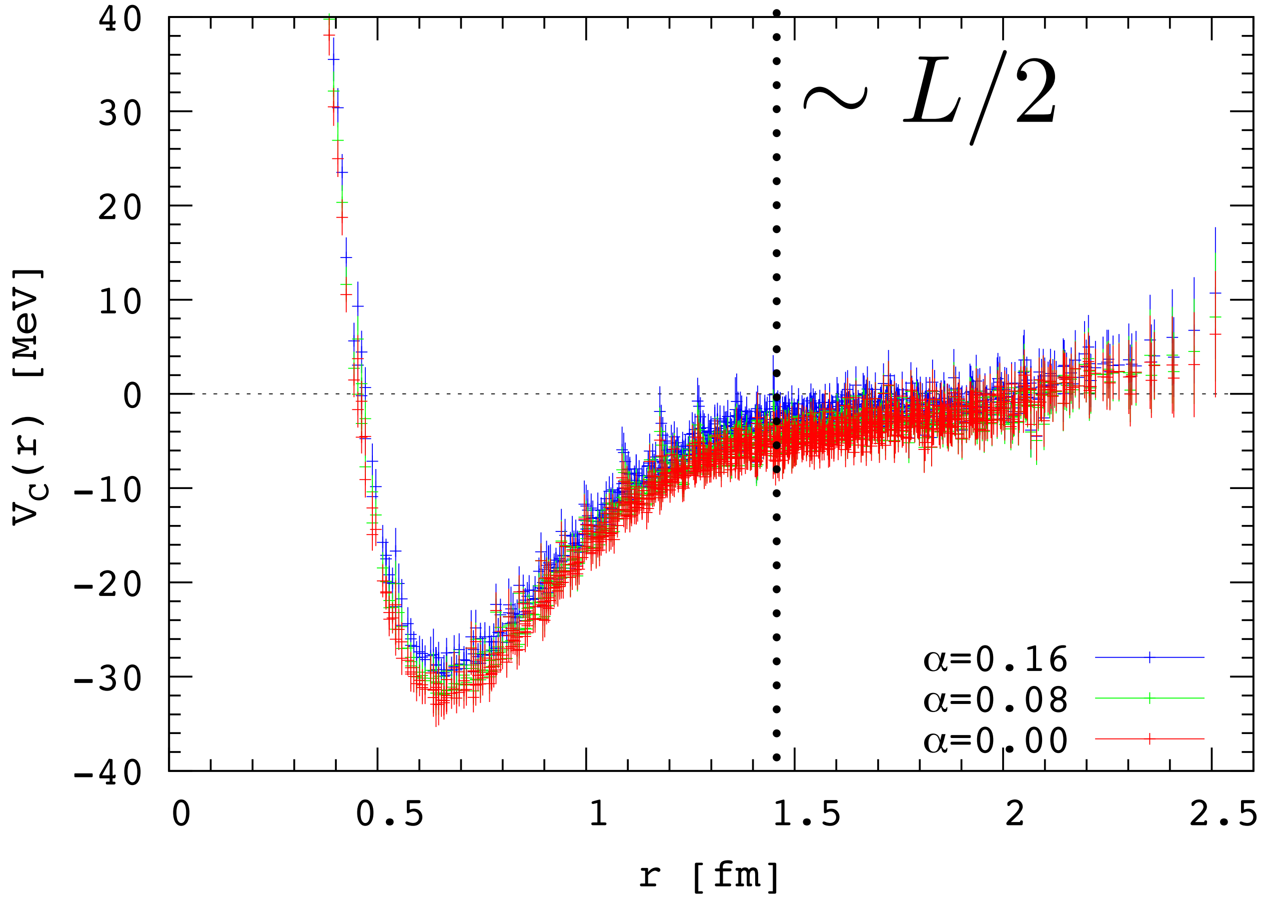
Methods and Results

HalQCD Method 1

$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0\right]\psi_{\mathbf{p}}(\mathbf{r}) = \int d^3r' U(\mathbf{r},\mathbf{r}')\psi_{\mathbf{p}}(\mathbf{r}')$$

Approximate potential $U(\mathbf{r},\mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$

HalQCD PLB 712 (2012)



$$V_C(\mathbf{r}) \simeq \frac{\mathbf{p}^2}{2\mu} + \lim_{t\to\infty}\frac{1}{2\mu}\frac{\nabla_{\mathbf{r}}^2 C_{NN}(\mathbf{r},t)}{C_{NN}(\mathbf{r},t)}$$

$$= \frac{\mathbf{p}^2}{2\mu} + \frac{1}{2\mu}\frac{\nabla_{\mathbf{r}}^2(e^{-E_0 t}\psi(\mathbf{r})A_0^\dagger)}{e^{-E_0 t}\psi(\mathbf{r})A_0^\dagger}$$

$$= \frac{\mathbf{p}^2}{2\mu} + \frac{1}{2\mu}\frac{\nabla_{\mathbf{r}}^2\psi(\mathbf{r})}{\psi(\mathbf{r})}$$

Methods and Results

Assumptions/Approximations/Challenges

- The gradient expansion for the potential is difficult to systematically quantify

$$U(\mathbf{r},\mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \mathcal{O}(\nabla^2_{\mathbf{r}}/\Lambda^2)$$

$$\Lambda = \begin{cases} \Delta E^* & \text{excitation energy to inelastic state} \\ \Lambda_{QCD} & \text{typical QCD scale} \\ \cdots & \end{cases}$$

- Periodic images of potential must be accounted for (HALQCD does include image potentials)

Methods and Results

HalQCD Method 2

Develop a "time-dependent" Schrödinger-like equation to avoid ground state saturation

$$\left[\frac{1}{4M}\partial_t^2 - \partial_t - H_0\right] R(\mathbf{r},t) = \int d^3r' U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) \qquad R(\mathbf{r},t) = \frac{C_{NN}(\mathbf{r},t)}{(C_N(t))^2}$$

Take t large enough that only the ground state contributes to $C_N(t)$

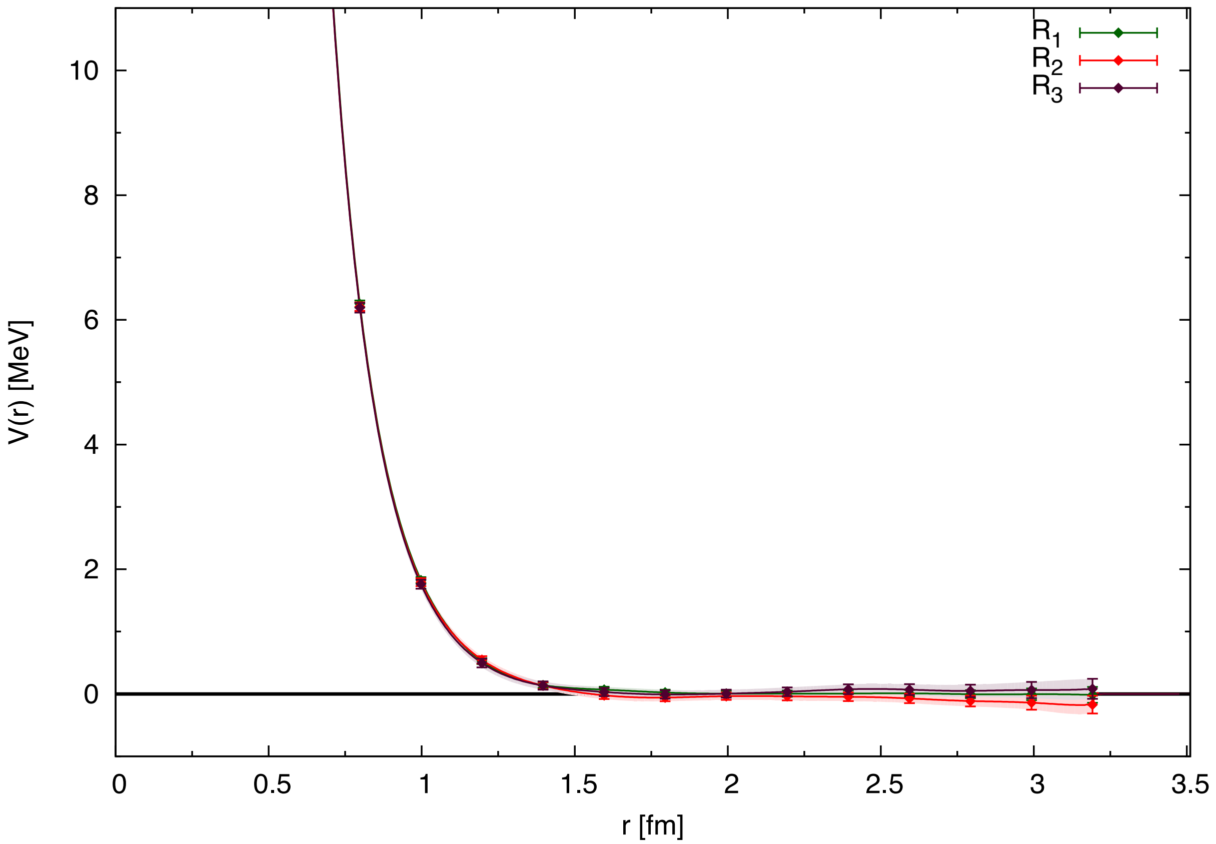
Assume only elastic states contribute to $C_{NN}(\mathbf{r},t)$

$$U(\mathbf{r},\mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$

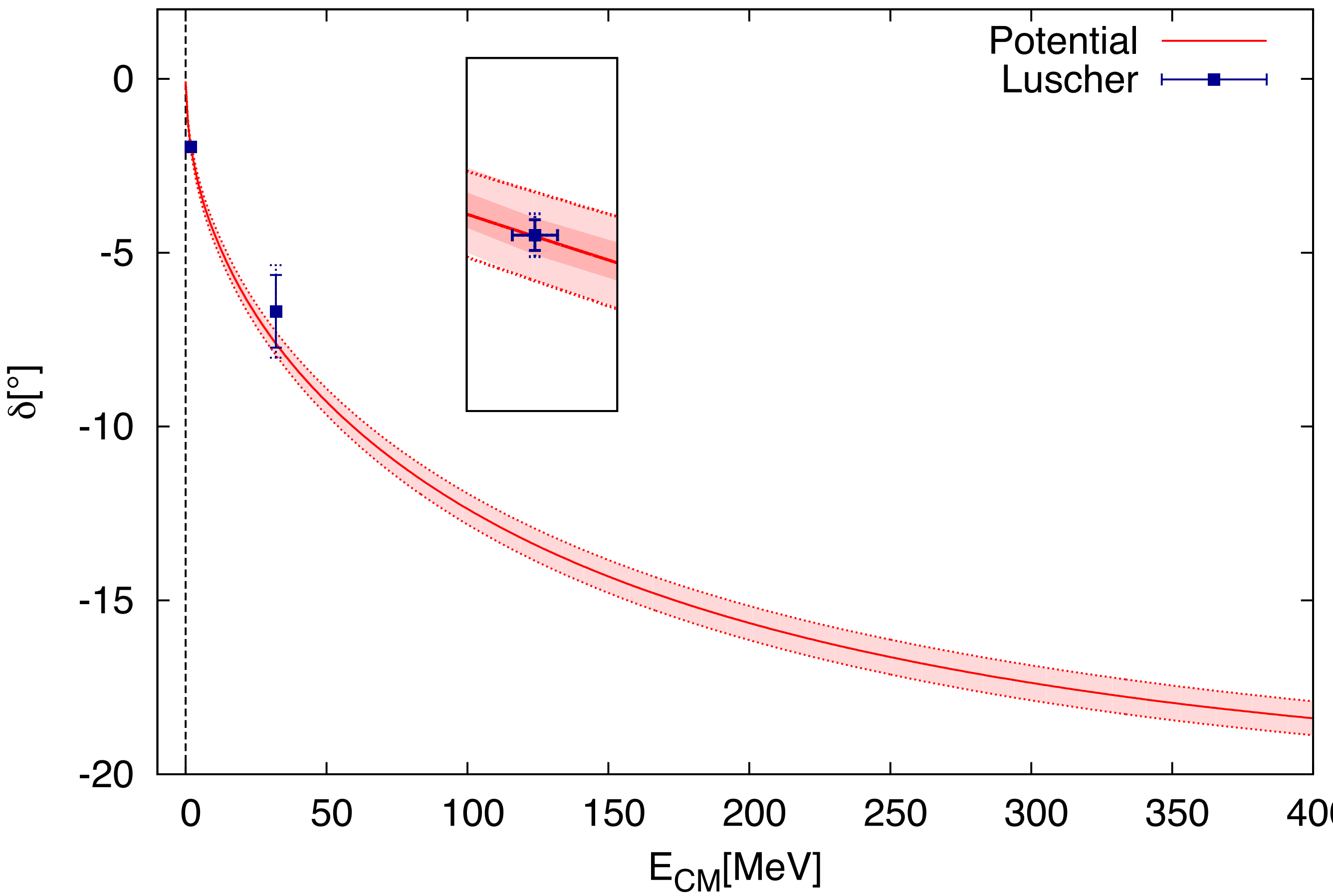
$$V_C(\mathbf{r}) \simeq \frac{1}{M}\frac{\nabla_{\mathbf{r}}^2 R(\mathbf{r},t)}{R(\mathbf{r},t)} - \frac{\partial_t R(\mathbf{r},t)}{R(\mathbf{r},t)} + \frac{1}{4M}\frac{\partial_t^2 R(\mathbf{r},t)}{R(\mathbf{r},t)}$$

Methods and Results

HalQCD Method 2



I=2 ππ

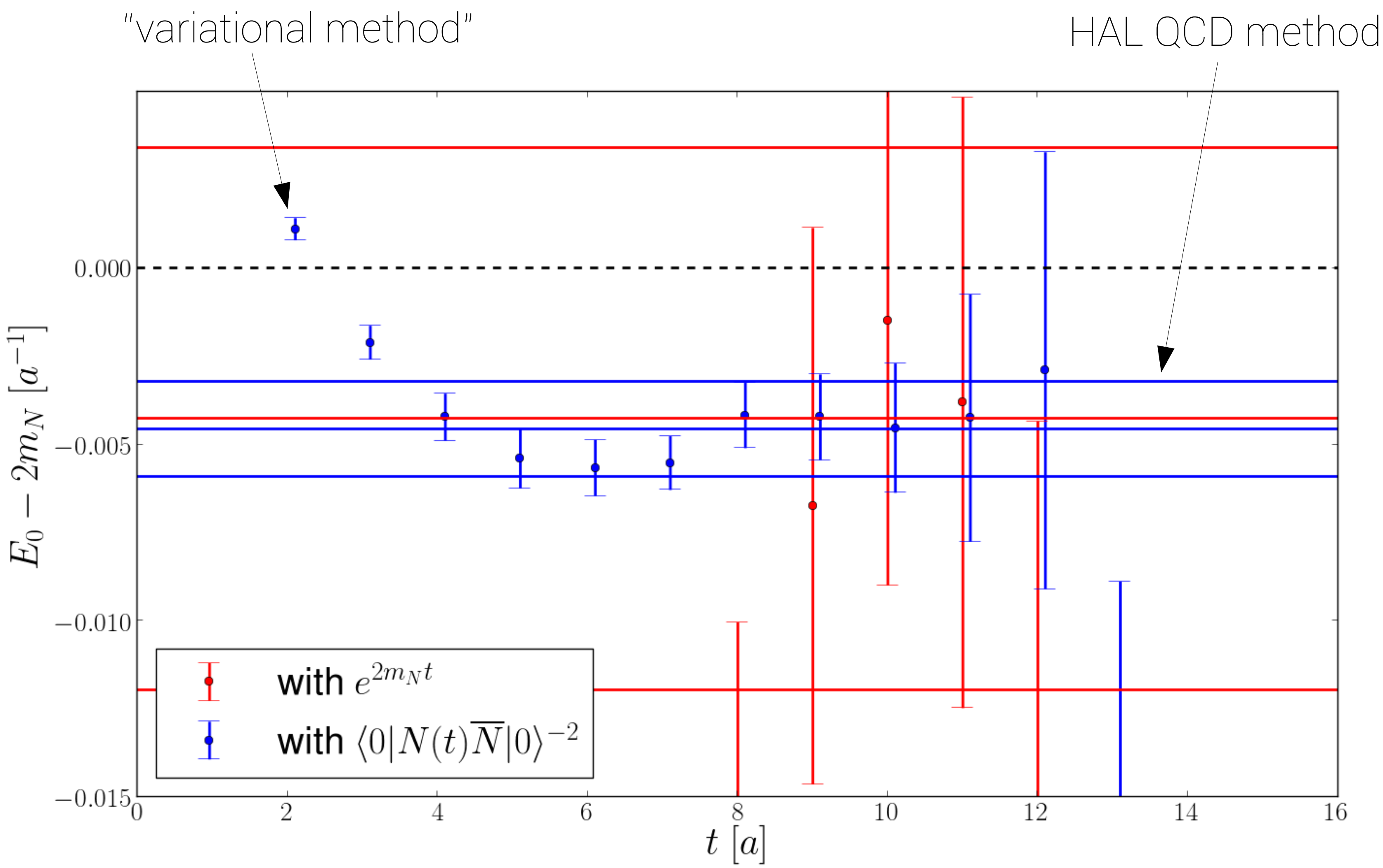


HalQCD arXiv:1305.4462
TALK: T. Kurth, Tues 3G 15:20

HalQCD method provides "interpolation" between allowed Lüscher eigen-values

HalQCD Method 2 NN

NN: energy shift



BAND: HalQCD method

Numerical Points: Ratio of correlation functions

$$R(\mathbf{r},t) = \frac{C_{NN}(\mathbf{r},t)}{e^{-2m_N t}}$$

$$R(\mathbf{r},t) = \frac{C_{NN}(\mathbf{r},t)}{(C_N(t))^2}$$

Normalization by 2-pt function: better signal but no control on time dependence.

HalQCD: TALK: B. Charron, Tues 3G 15:00

Methods and Results

Assumptions/Approximations/Challenges

- The two-body correlation function is free from contamination from inelastic states. It is challenging to demonstrate

$$C_{NN}(\mathbf{r}, t) \equiv \sum_{n \in \mathrm{elastic}} e^{-E_n t} Z_n A_n^\dagger$$

Otherwise, an unquantifiable systematic is introduced

Methods and Results

Results

NPLQCD	2006
HALQCD	2006
...	
T. Yamazaki, K. Ishikawa, Y. Kurumashi, A. Ukawa	2010
...	
R. Briceno, Z. Davoudi	2012
J. Gunther, B. Toth, L. Varnhorst	2013
A. Francis, C. Miao, T.D. Rae, H. Wittig	2013

many others working on very similar topics

Methods and Results

2010: First dynamical lattice calculation of bound two-baryon system (still at unphysical pion masses), the H-dibaryon
NPLQCD PRL 106 (2011) 162001
HALQCD PRL 106 (2011) 162002

the H-dibaryon $|H\rangle = -\sqrt{\frac{1}{8}}|\Lambda\Lambda\rangle + \sqrt{\frac{3}{8}}|\Sigma\Sigma\rangle + \sqrt{\frac{4}{8}}|N\Xi\rangle$ has the quantum numbers of a flavor singlet and S=-2, proposed by R.L. Jaffe (PRL 38 1977), the most symmetric two-baryon state, as a relatively deeply bound di-baryon

To date, there is no experimental evidence of a bound h-dibaryon

Nevertheless, this was exciting as it was the beginning of the era of lattice QCD calculations of bound multi-baryons

Methods and Results

2010: First dynamical lattice calculation of bound two-baryon system (still at unphysical pion masses), the H-dibaryon
NPLQCD PRL 106 (2011) 162001
HALQCD PRL 106 (2011) 162002

Calculating a negatively shifted energy in finite volume is not sufficient to demonstrate a bound vs. scattering state.

$$\Delta E = 2\sqrt{M^2 + k^2} - 2M \qquad \kappa^2 = -k^2$$

scattering state $$\Delta E = -\frac{4\pi a}{ML^3}\left[1 + \mathcal{O}\left(\frac{a}{L}\right)\right]$$

bound state $$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L}\right) + \cdots \qquad \gamma = \sqrt{M_\Lambda^\infty B_H^\infty}$$

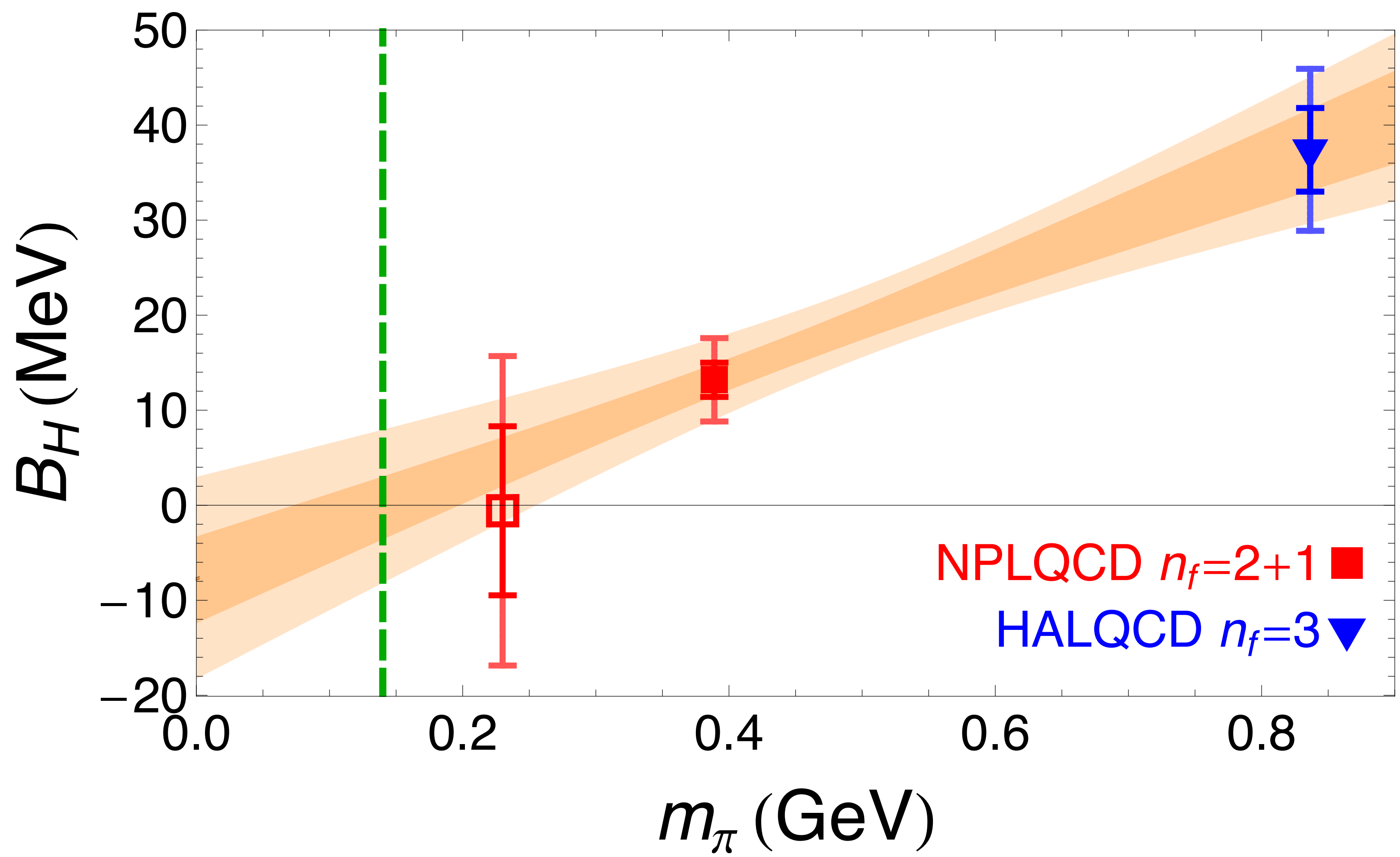
One needs to check the volume dependence either by varying the box size (or boosting the system to non-zero P_{com})

Methods and Results

2010: First dynamical lattice calculation of bound two-baryon system (still at unphysical pion masses), the H-dibaryon

NPLQCD PRL 106 (2011) 162001

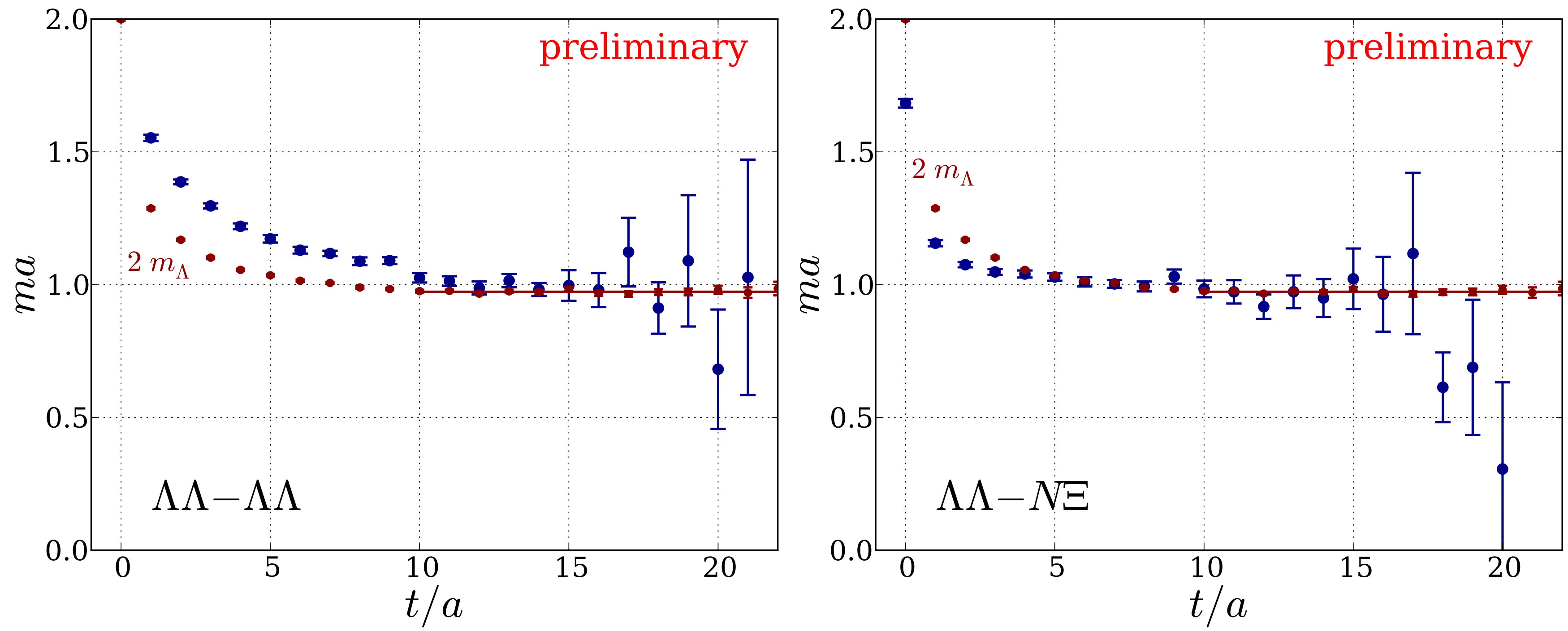
HALQCD PRL 106 (2011) 162002



crude estimate of binding at physical pion mass (QCD only)

Methods and Results

Two-baryon correlators with $m_s^{quench.} \simeq m_s^{phys}$ and $m_\pi^{sea} = 460$MeV



a/fm	$T \times L^3$	L/fm	m_π/MeV	$m_\pi L$	n_{conf}	n_{src}	n_{meas}
0.063	64×32^3	2.02	460	4.7	900	8	7200

This conference - A. Francis, C. Miao, T. Rae, H. Wittig. (CLS Mainz)

They are increasing statistics and implementing the full 3x3 matrix of correlation functions which will allow for a variational approach

Methods and Results

NN Interactions

$^1S_0$

di-neutron

$a \simeq -24$ fm

$$\left(\lim_{p \to 0} p \cot \delta(p) = -\frac{1}{a} \right)$$

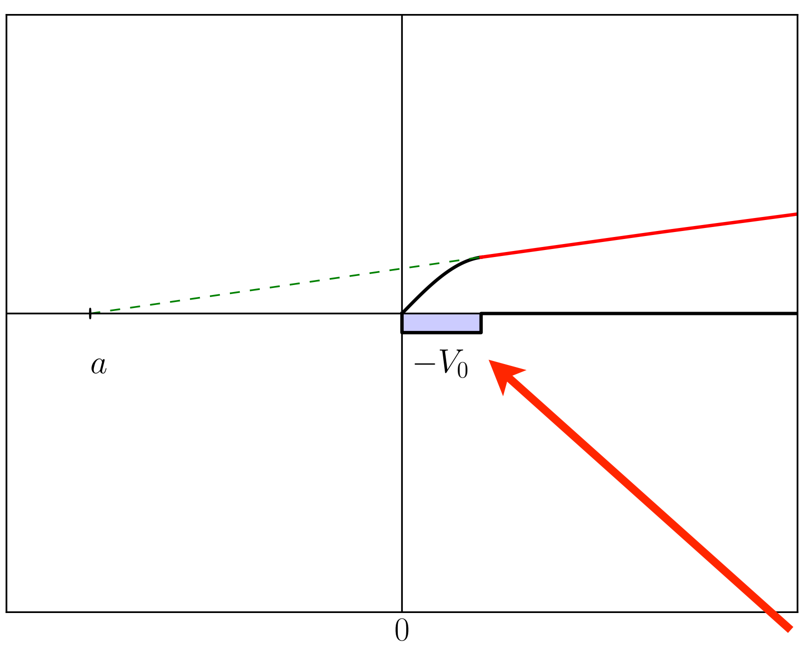
$^3S_1$

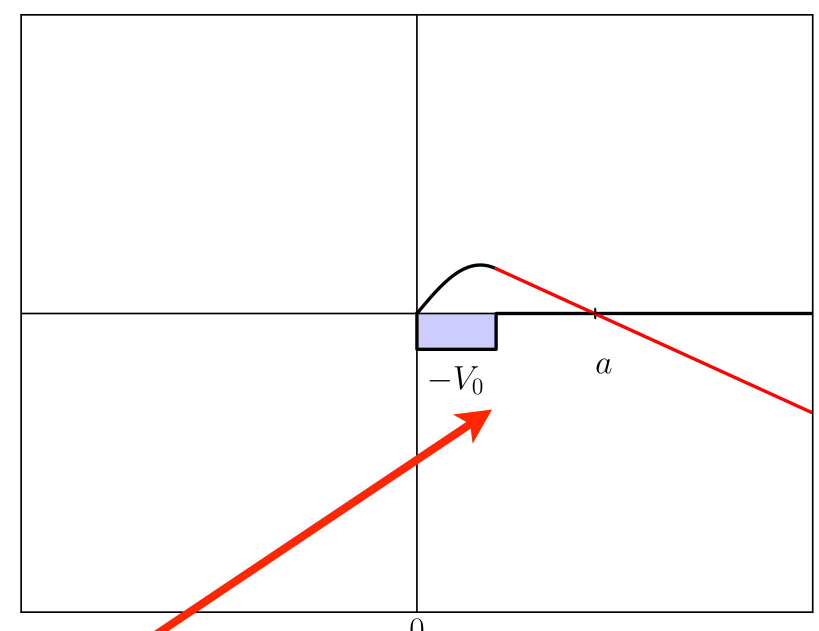
deuteron

$B_d = 2.2245(2)$ MeV

$a \simeq 5.5$ fm

threshold scattering is finely tuned



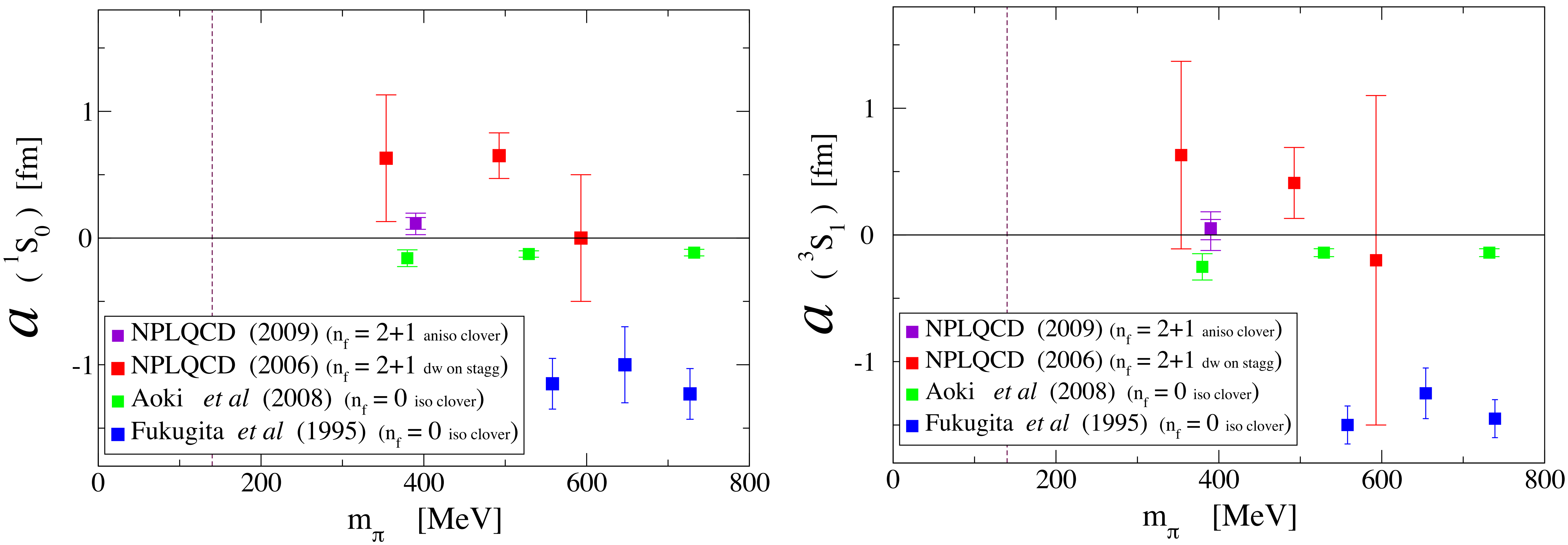


$$R_{NN} \sim \frac{1}{m_\pi} \sim 1.4 \text{ fm}$$

Methods and Results

NN Interactions

early calculations indicate the large scattering lengths relax for larger pion masses



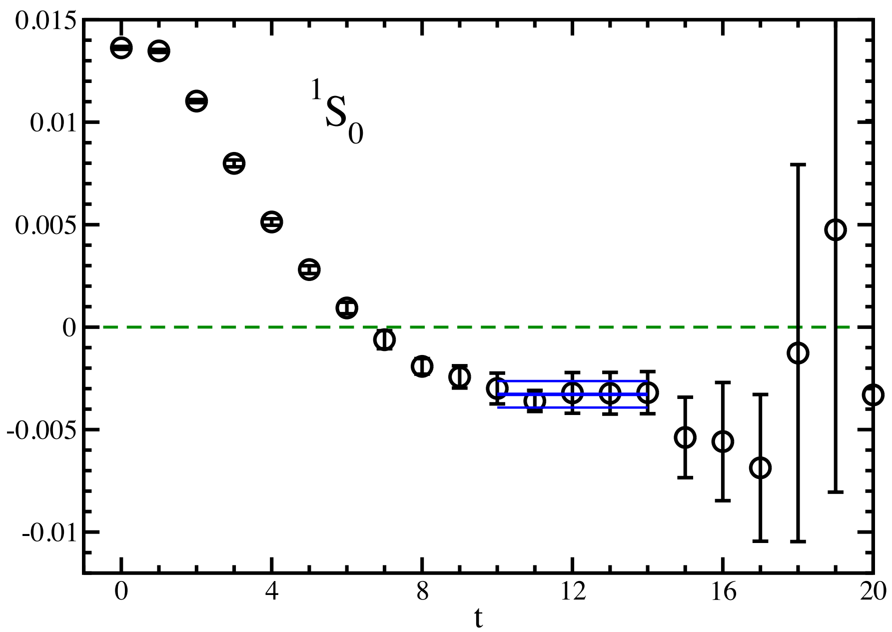
NPLQCD PRD 81 (2010)

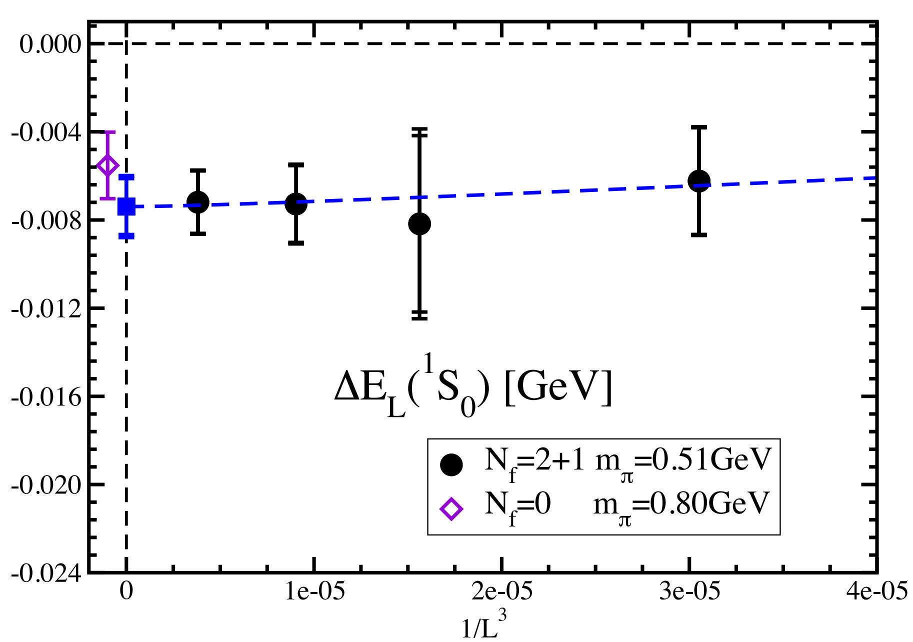
Methods and Results

NN Interactions $^1S_0$

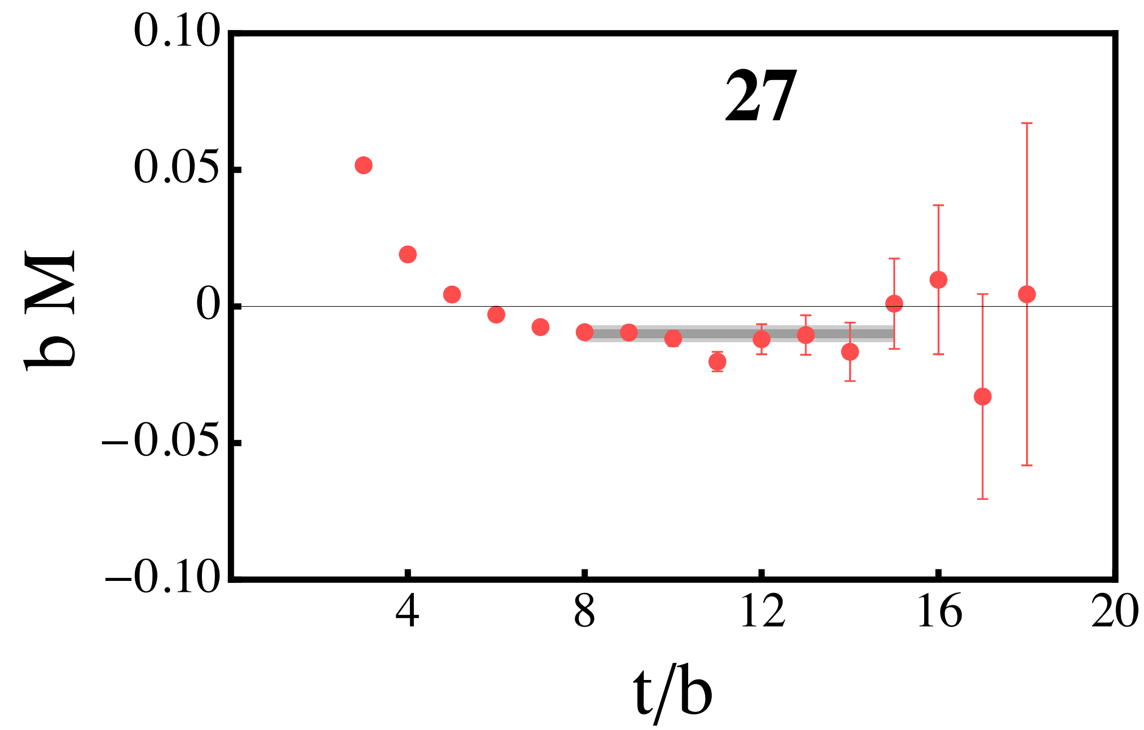
more recent calculations, with higher statistics, have indicated the di-neutron even becomes bound

Yamazaki et al., PRD 86 (2012)





NPLQCD PRD 87 (2013)

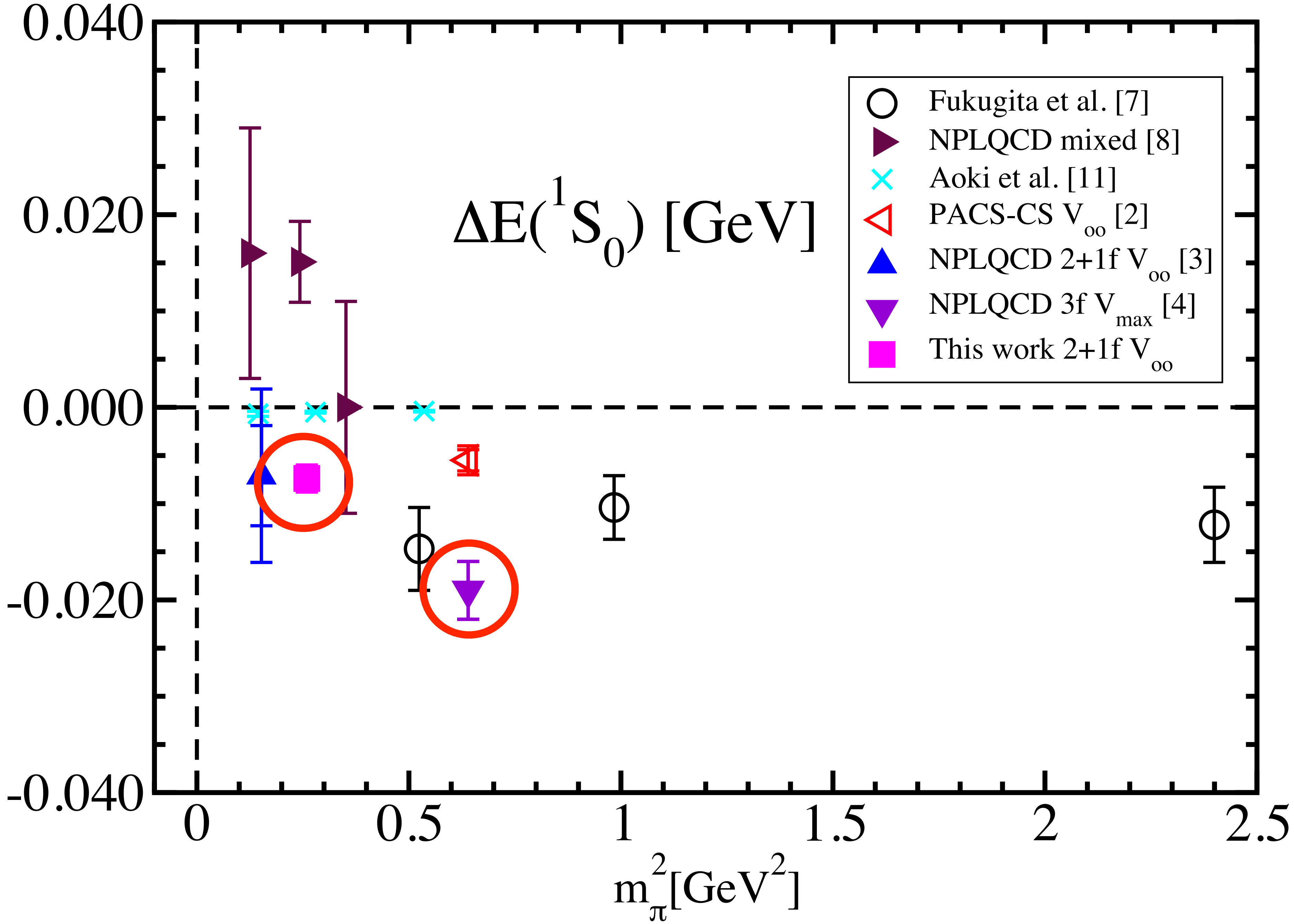


both calculations clearly find a bound di-neutron

Methods and Results

NN Interactions $^1S_0$

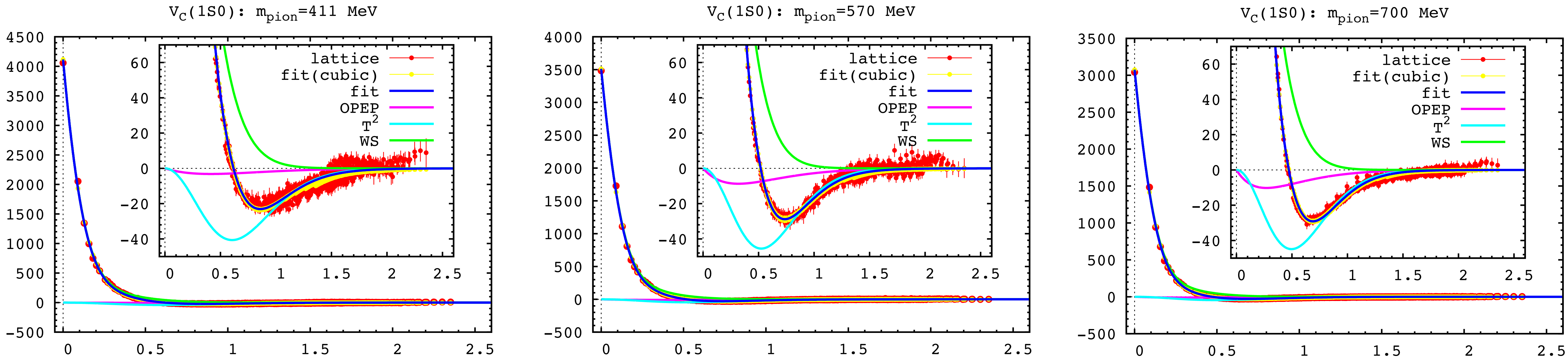
more recent calculations, with higher statistics, have indicated the di-neutron even becomes bound

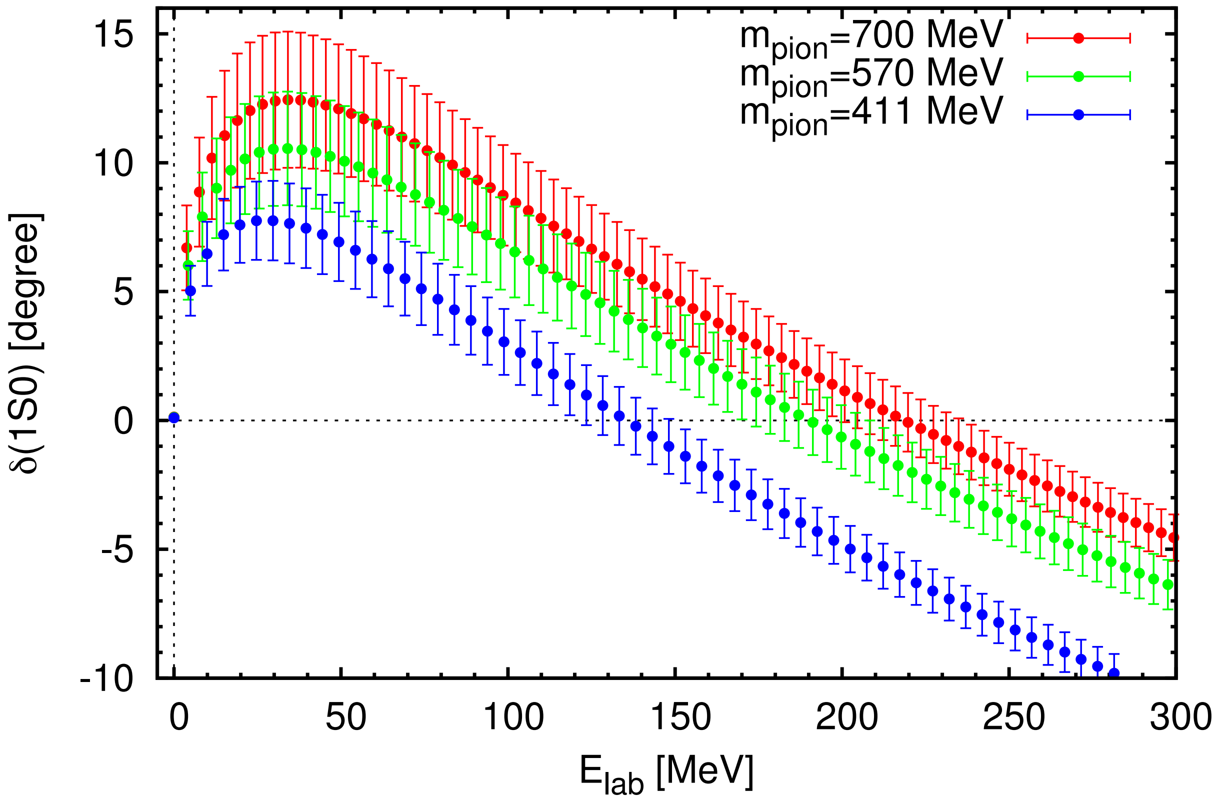


Methods and Results

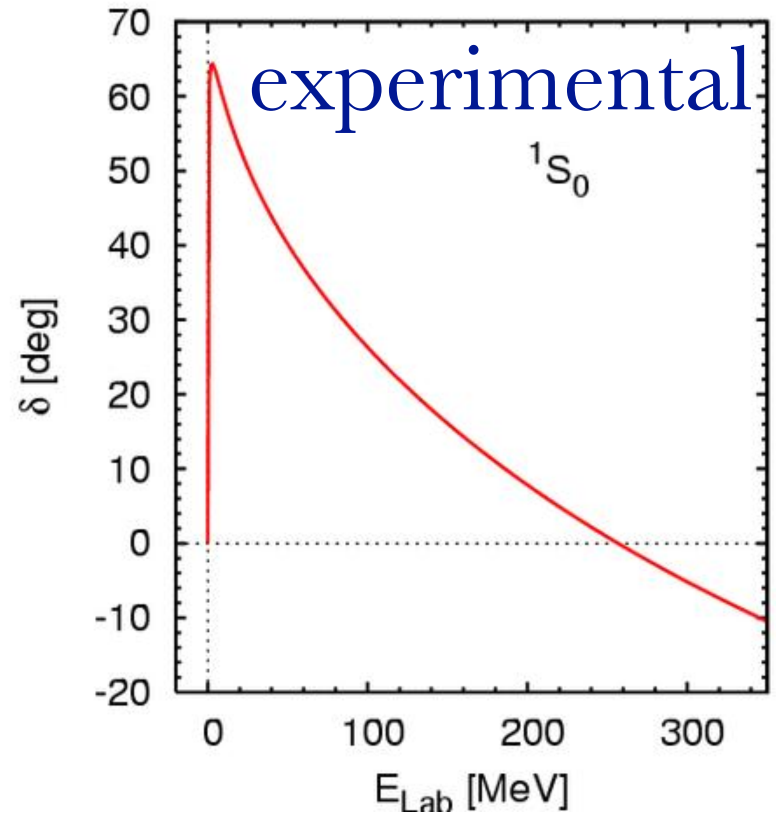
NN Interactions $^1S_0$

contrast with results from the HALQCD method





(Thanks to HALQCD)

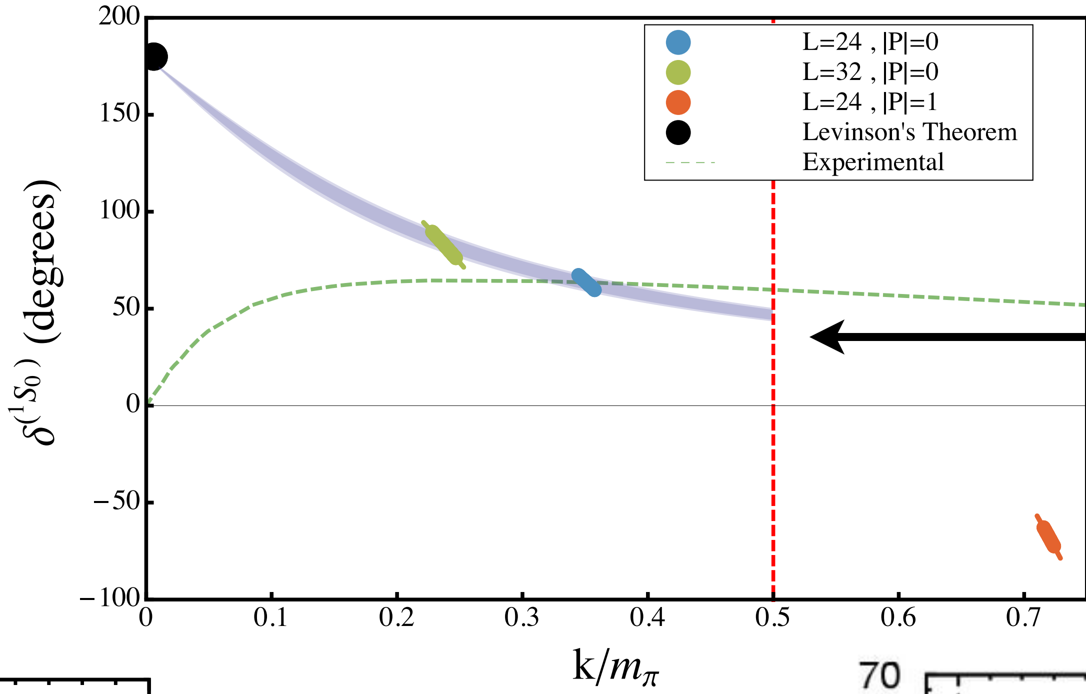


Methods and Results

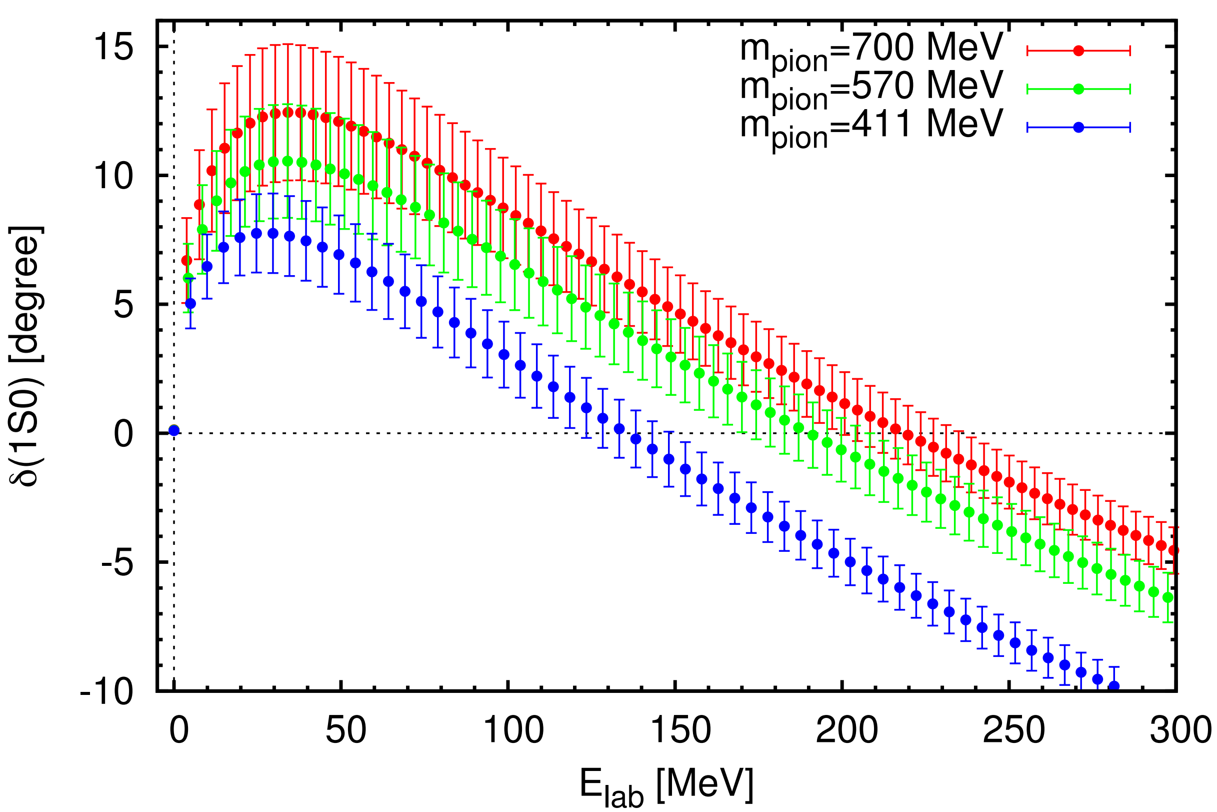
NN Interactions $^1S_0$

contrast with results from the HALQCD method

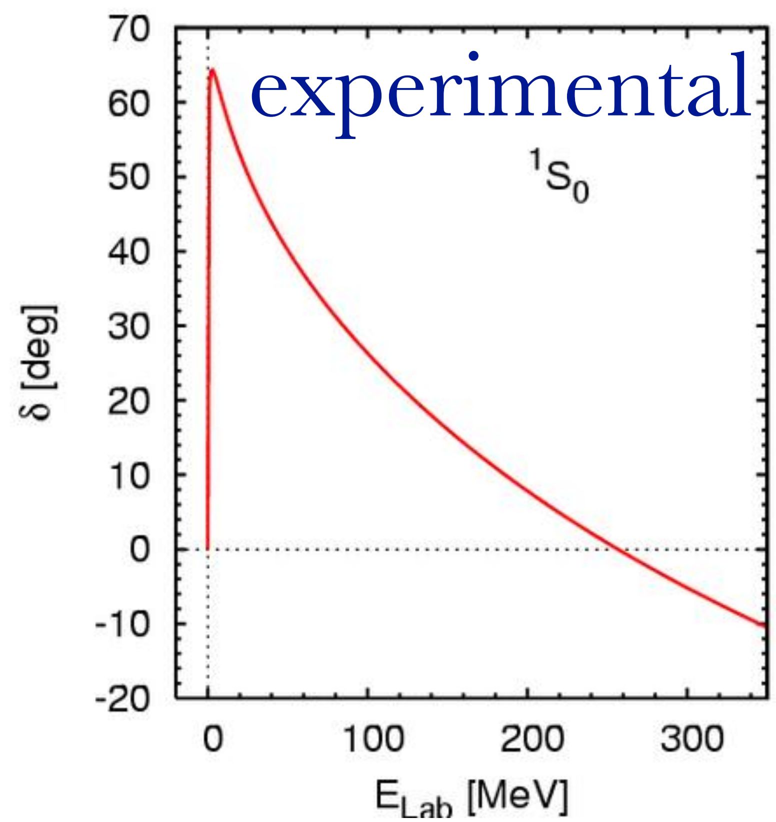
NPLQCD arXiv:1301.5790
to be published in PRC (2013)



t-channel cut



(Thanks to HALQCD)



Methods and Results

NN Interactions $^1S_0$

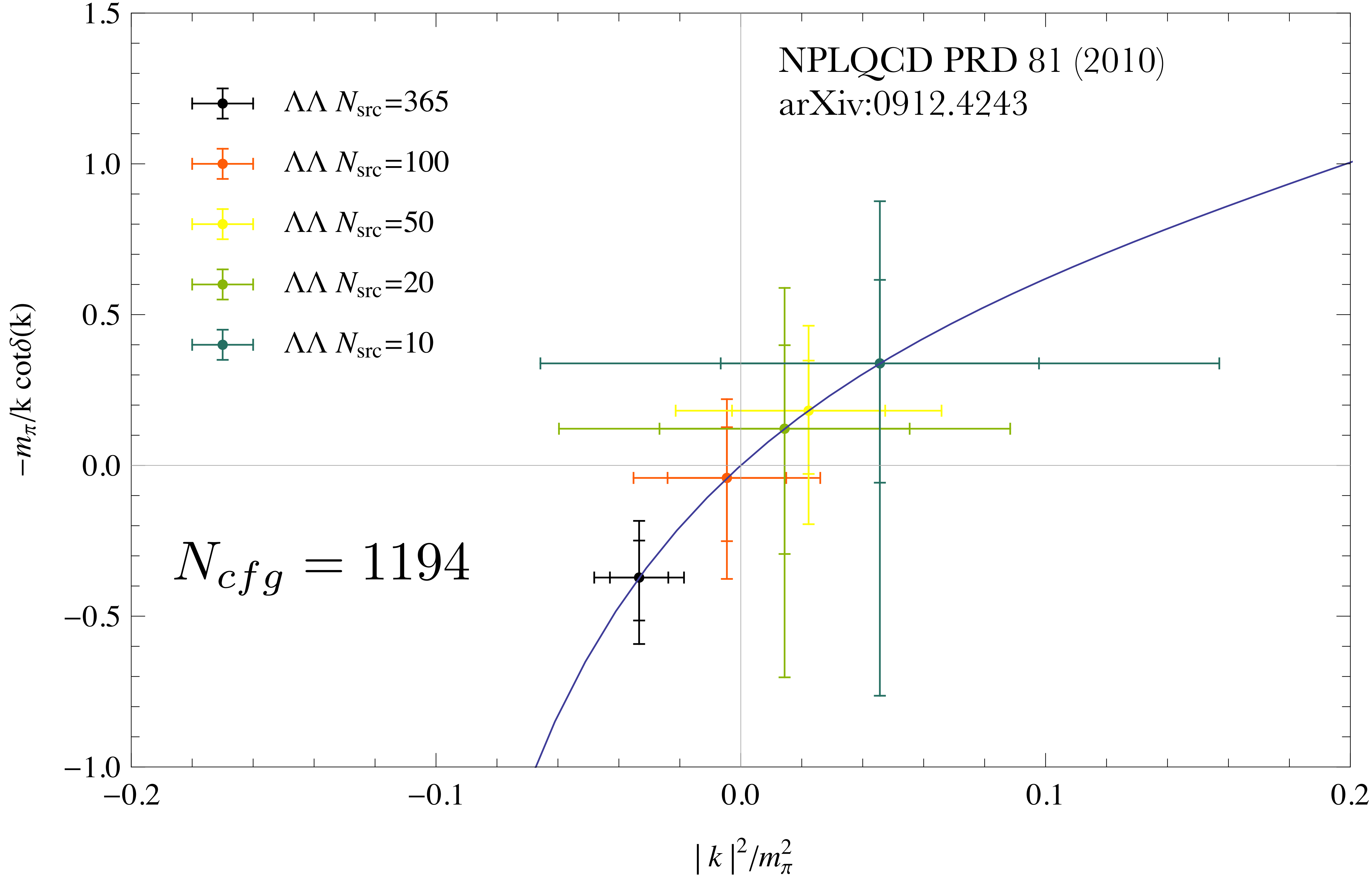
Heavy pion mass: $(m_\pi \gtrsim 390 \text{ MeV})$

- NPLQCD finds a bound state
- Yamazaki et.al. find a bound state
- HALQCD does not find a bound state

my speculation: HALQCD does not have enough statistics to resolve the long-range potential, which contributes significantly to the low-energy phase shift

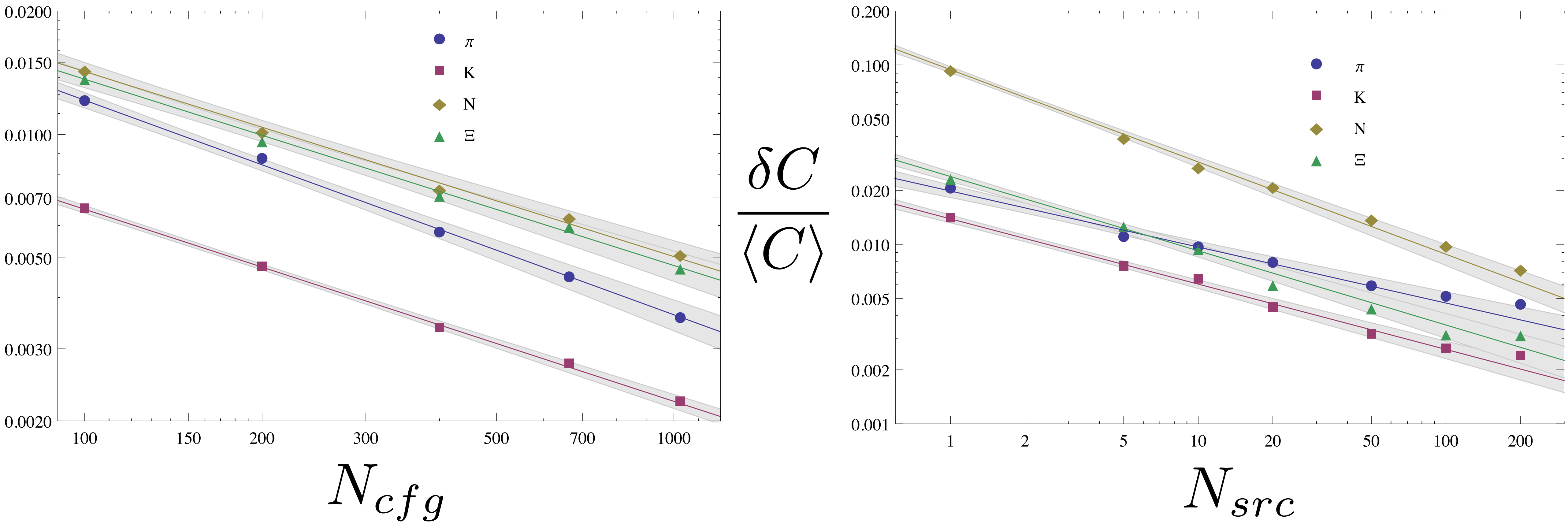
Methods and Results

NN Interactions $^1S_0$



Methods and Results

NN Interactions $^1S_0$



$$\frac{\delta C}{\langle C \rangle} = A(N_{cfg})^b$$

$$\frac{\delta C}{\langle C \rangle} = A(N_{src})^b$$

$$b^N_{cfg} = -0.45(2)$$

$$b^N_{src} = -0.51(9)$$

NPLQCD PRD 79 (2009)
arXiv:0903.2990

Methods and Results

NPLQCD, PRD 87 (2013)

Light Nuclei

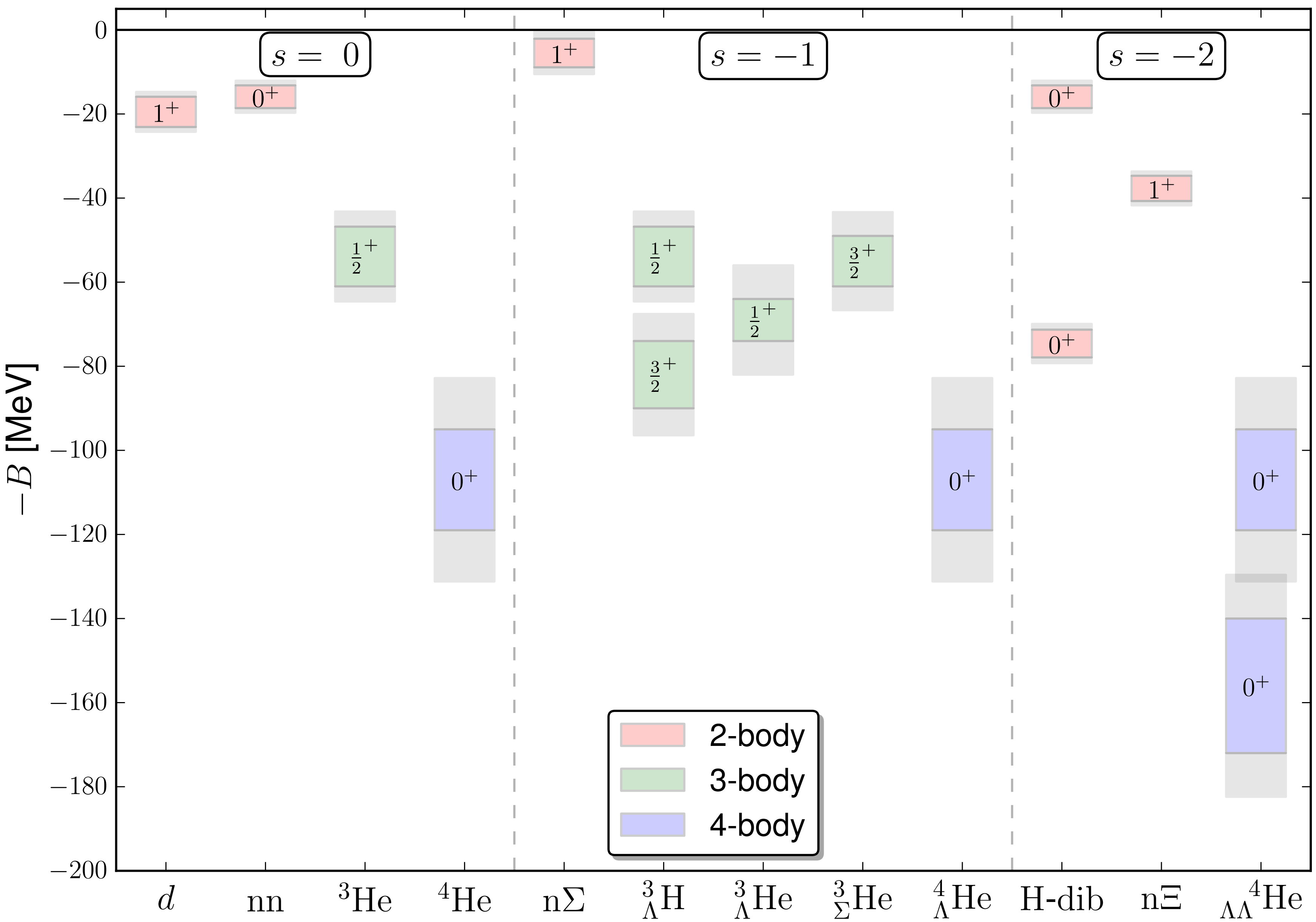
$N_f = 3$

clover-Wilson

$a \sim 0.145$ fm

$m_{\pi,K} = 807$ MeV

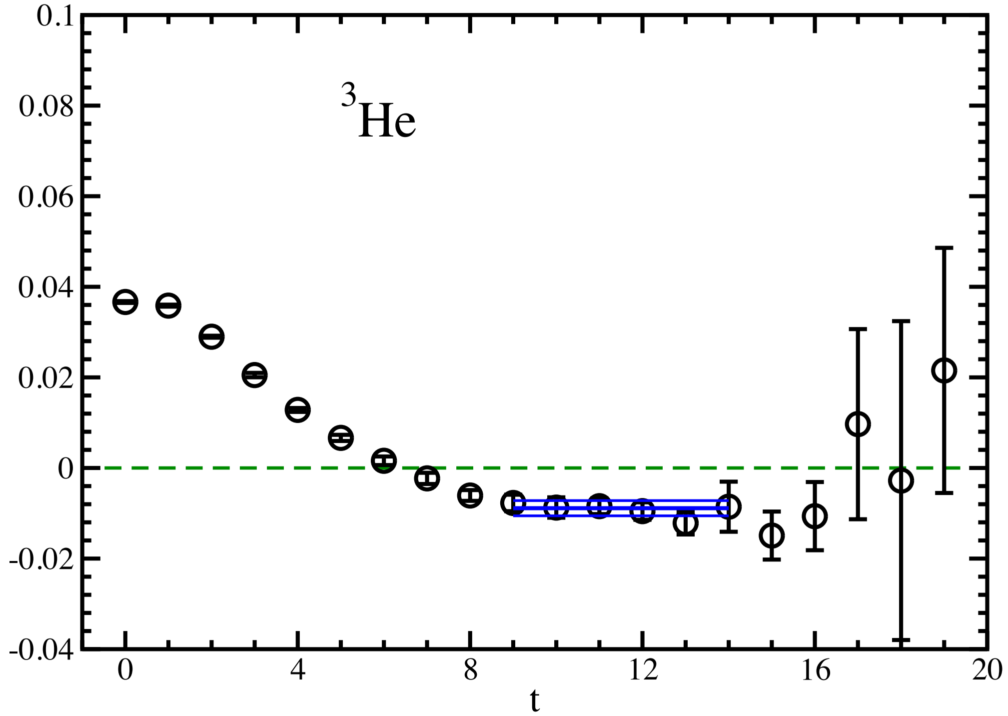
$$V = \begin{cases} 24^3 \times 48 \\ 32^3 \times 48 \\ 48^3 \times 64 \end{cases}$$

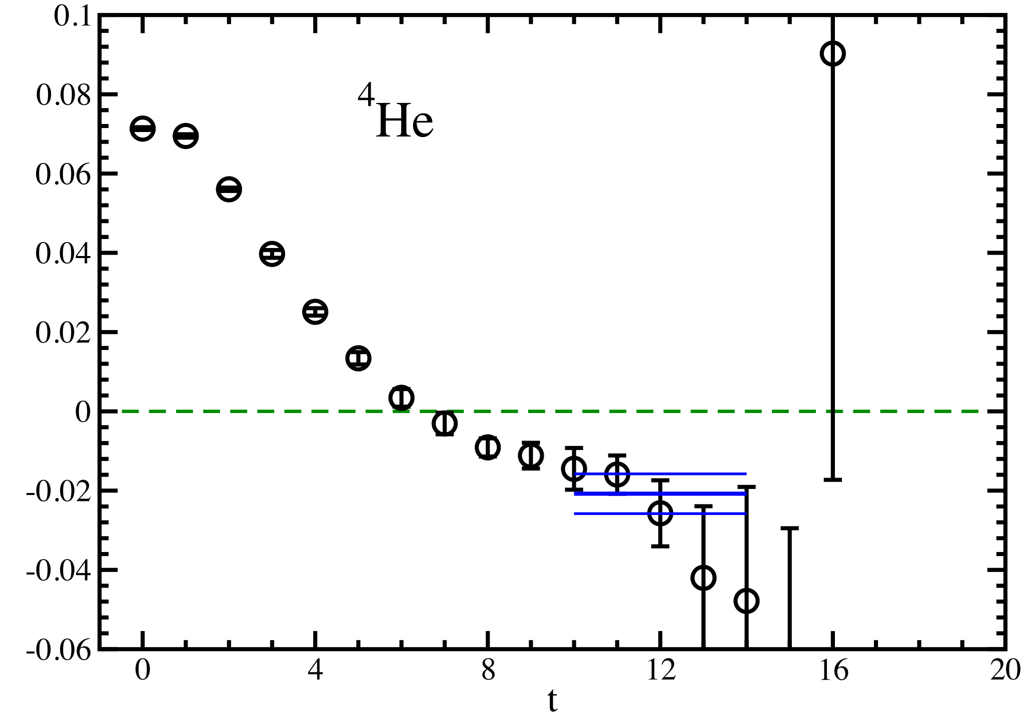


Methods and Results

Yamazaki et al., PRD 86 (2012)

Light Nuclei

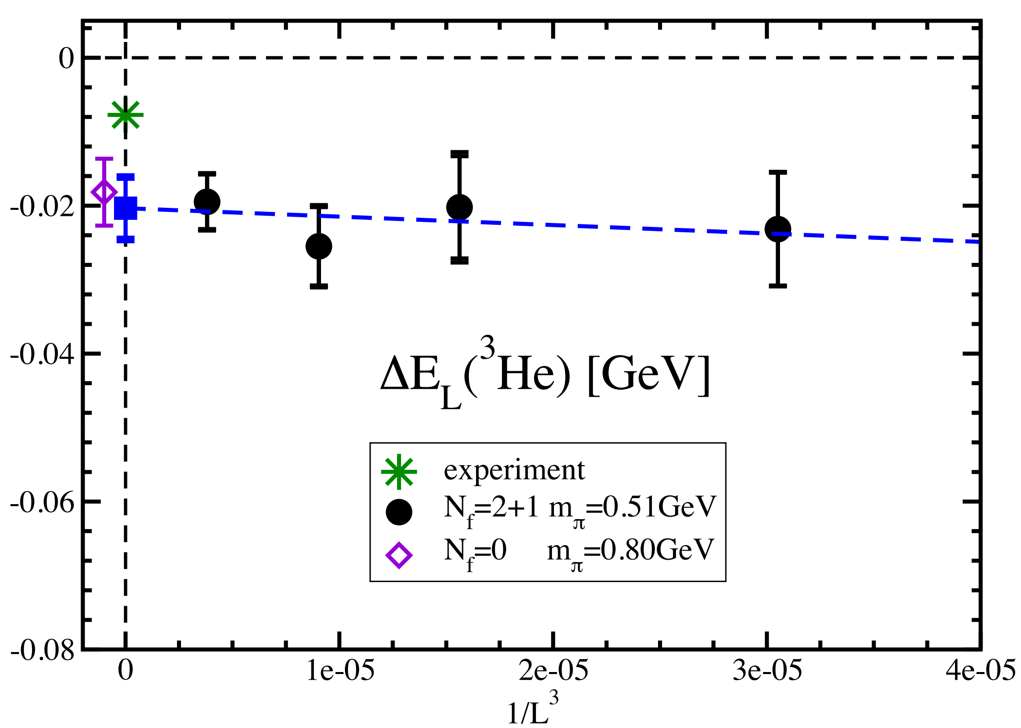


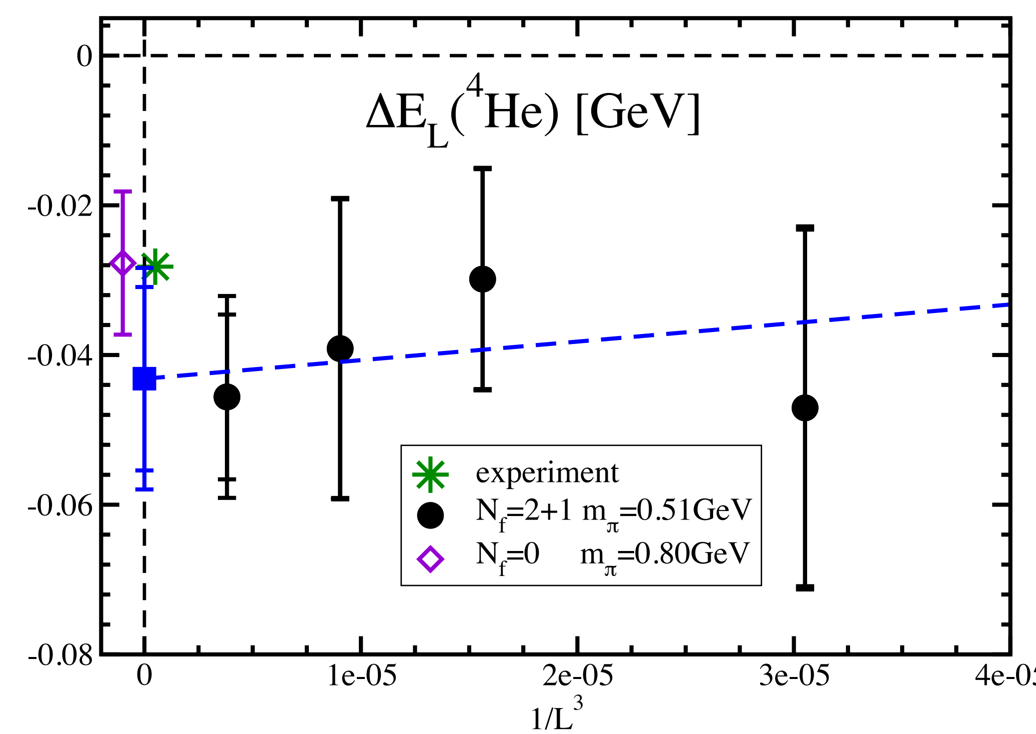


$m_\pi \simeq 510$ MeV

$N_f = 2+1$

PACS-CS parameters





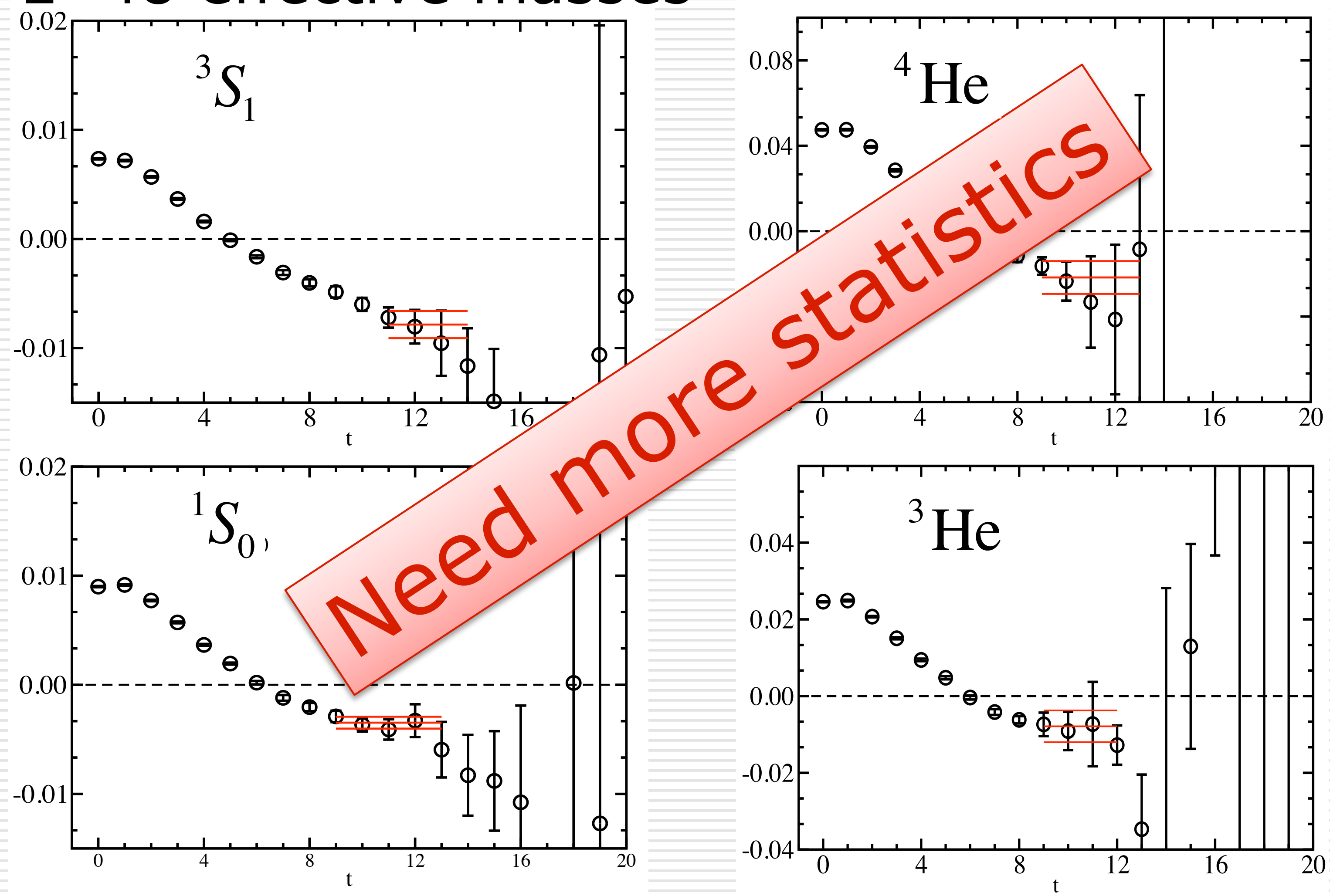
Methods and Results

A.Ukawa: 1G 15:20

Light Nuclei

Results at m_π=0.30GeV (preliminary)

- L=48 effective masses



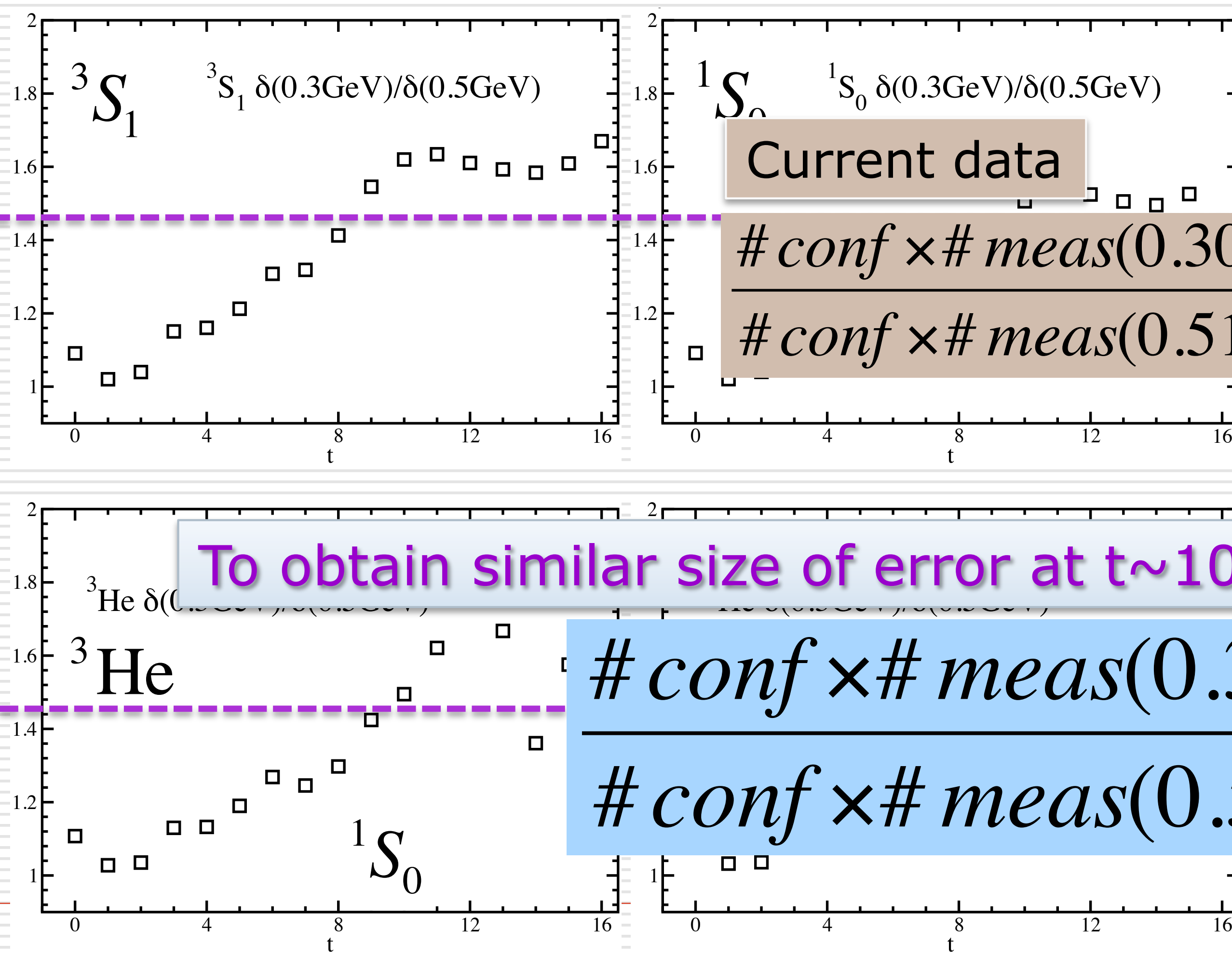
A.Ukawa: 1G 15:20

Light Nuclei

Results at m_π=0.30GeV (preliminary) (II)

- Comparison of relative errors for m_π=0.30GeV and m_π=0.51GeV (L=48)

$$\delta_H(t) = \frac{\delta C_H(t)}{C_H(t)}$$



Current data

$$\frac{\#conf \times \#meas(0.30GeV)}{\#conf \times \#meas(0.51GeV)} \approx 8$$

statistics

To obtain similar size of error at t~10, we need

$$\frac{\#conf \times \#meas(0.30GeV)}{\#conf \times \#meas(0.51GeV)} \approx 16$$

Challenges and Progress

- Contractions
- Finite Volume Dependence and Boosted Systems
- Coupled Channels and Inelastic Thresholds

Challenges and Progress

Performing the quark-level Wick contractions to form the nuclear correlation functions remains one of the most challenging and computationally demanding aspects of these calculations. Naively, the contractions scale as $N_{contr} = N_u!\, N_d!\, N_s!$

$$N_{contr}^{^4He} = 6! \times 6! = 518,400$$

Challenges and Progress

However, there is a high amount of symmetry in various nuclei: one can reduce the contractions from 518400 → 1107 distinct ways of contracting the quark lines for ^{4}He

Yamzaki, Kuramashi, Ukawa PRD 81 (2010)

There has been significant development in recursive contraction algorithms

T. Doi and M. Endres, Comp. Phys. Comm. 184 (2013)
W. Detmold and K. Orginos, PRD 87 (2013)
J. Günther, B.C. Toth and L.Varnhorst PRD 87 (2013)

J. Günther, Mon. 1G 14:20
L. Varnhorst, Mon. 1G 14:40

even with these improvements, the contractions remain a significant computational challenge

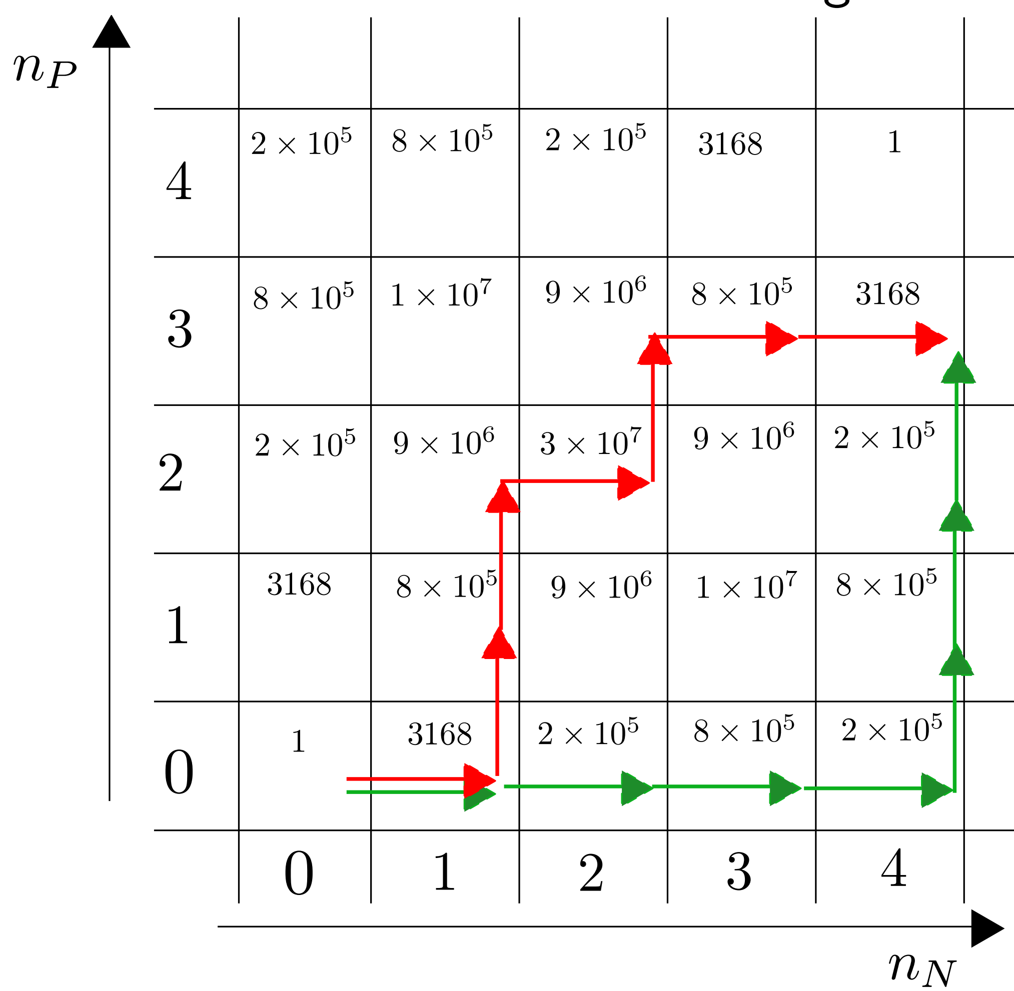
Challenges and Progress

J. Günther, Mon. 1G 14:20
L. Varnhorst, Mon. 1G 14:40

J. Günther, B.C. Toth and L.Varnhorst PRD 87 (2013)

Recursive relations for nuclei

The actual effort to calculate a given F_{-} depends on the chosen path.



The red path is less efficient then the green one.

In general path which add first all baryons of one type and then the baryons of the other type are advantageous.

Challenges and Progress

J. Günther, Mon. 1G 14:20
L. Varnhorst, Mon. 1G 14:40

J. Günther, B.C. Toth and L.Varnhorst PRD 87 (2013)

Comparison with naïve method

Relativistic Operators, 1 Quark source:

	N_P	N_N	No. of op.	Naïve No. of op.	η
^{3}He	2	1	19241280	5.5×10^{11}	2.9×10^{4}
^{4}He	2	2	531321120	5.7×10^{16}	1.1×10^{8}
^{6}Li	3	3	2905079520	4.9×10^{27}	1.7×10^{18}
^{7}Li	3	4	404946240	3.0×10^{33}	7.5×10^{24}
(^{8}Be)	4	4	448496928	2.8×10^{39}	6.2×10^{30}

FV dependence and boosted systems

Challenges and Progress

- The deuteron is coupled in partial waves

- mostly S-wave, with small D-wave admixture

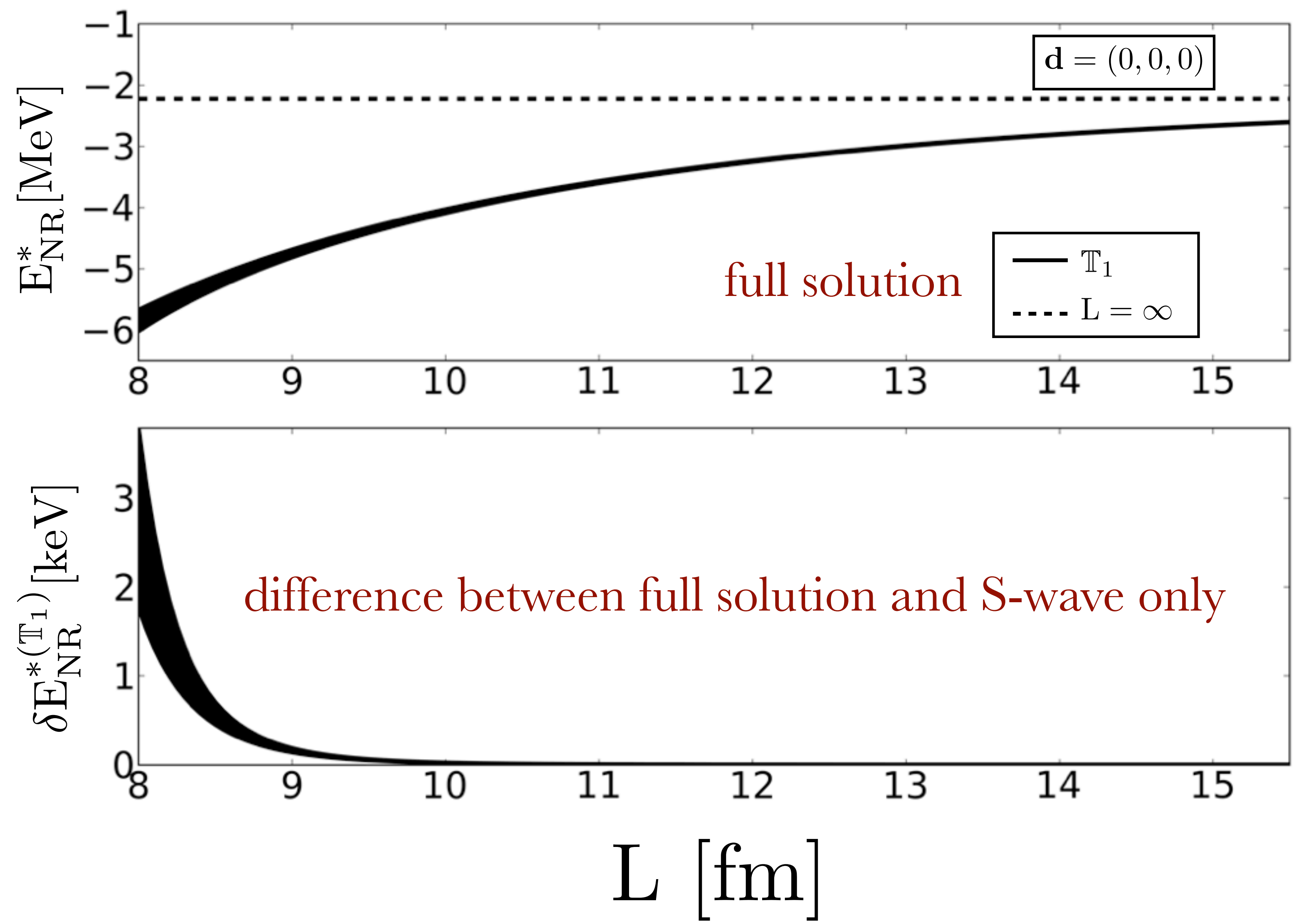
- one way to parameterize S-matrix is "barred" representation

$$S_{2\to 2} = \begin{pmatrix} e^{2i\delta_1}\cos 2\bar{\epsilon} & ie^{i(\delta_1+\delta_2)}\sin 2\bar{\epsilon} \\ ie^{i(\delta_1+\delta_2)}\sin 2\bar{\epsilon} & e^{2i\delta_2}\cos 2\bar{\epsilon} \end{pmatrix}$$

- three pieces of information, $\delta_1, \delta_2, \bar{\epsilon}$ completely parameterize the S-matrix

Challenges and Progress

Distortion of the deuteron binding energy in finite volume



Motivated people to consider boosted multi-baryon systems

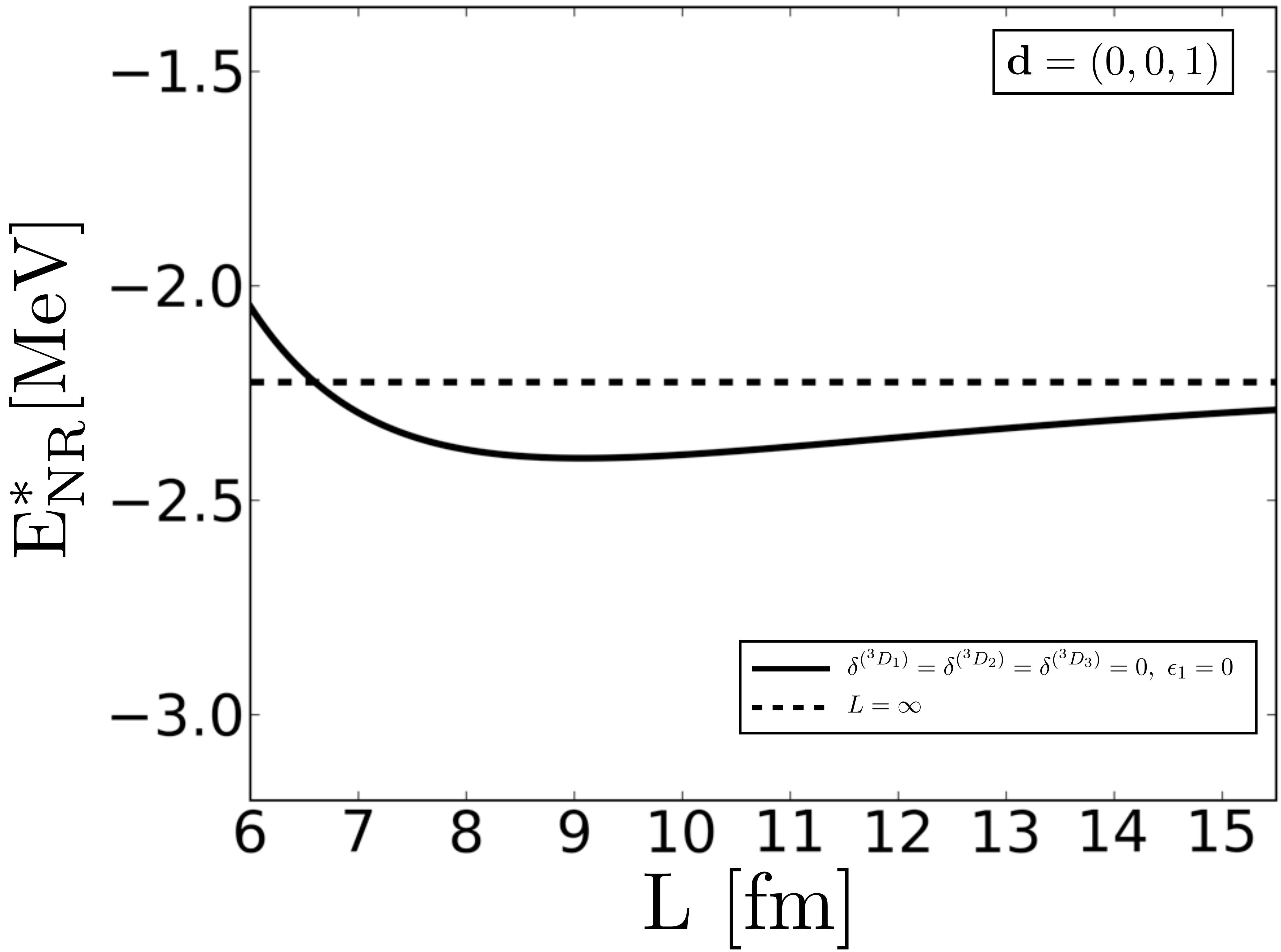
Z. Davoudi and M.J. Savage PRD 84 (2011)

R. Briceño, Z. Davoudi, T. Luu and M.J. Savage, in preparation

Courtesy of Raúl Briceño in collaboration with Z. Davoudi, T. Luu and M.J. Savage

Challenges and Progress

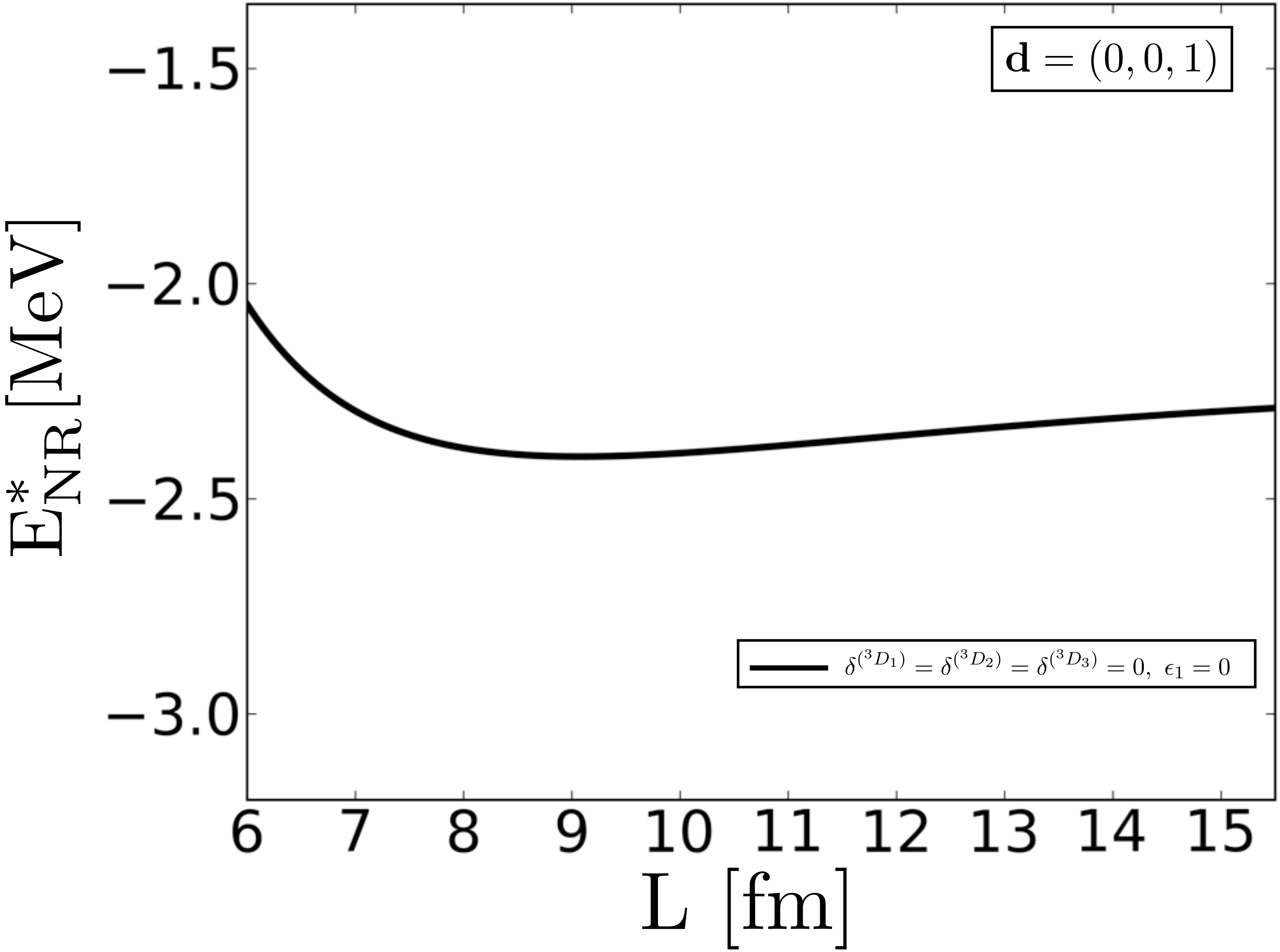
Distortion of the deuteron binding energy in finite volume



Courtesy of Raúl Briceño in collaboration with Z. Davoudi, T. Luu and M.J. Savage

Challenges and Progress

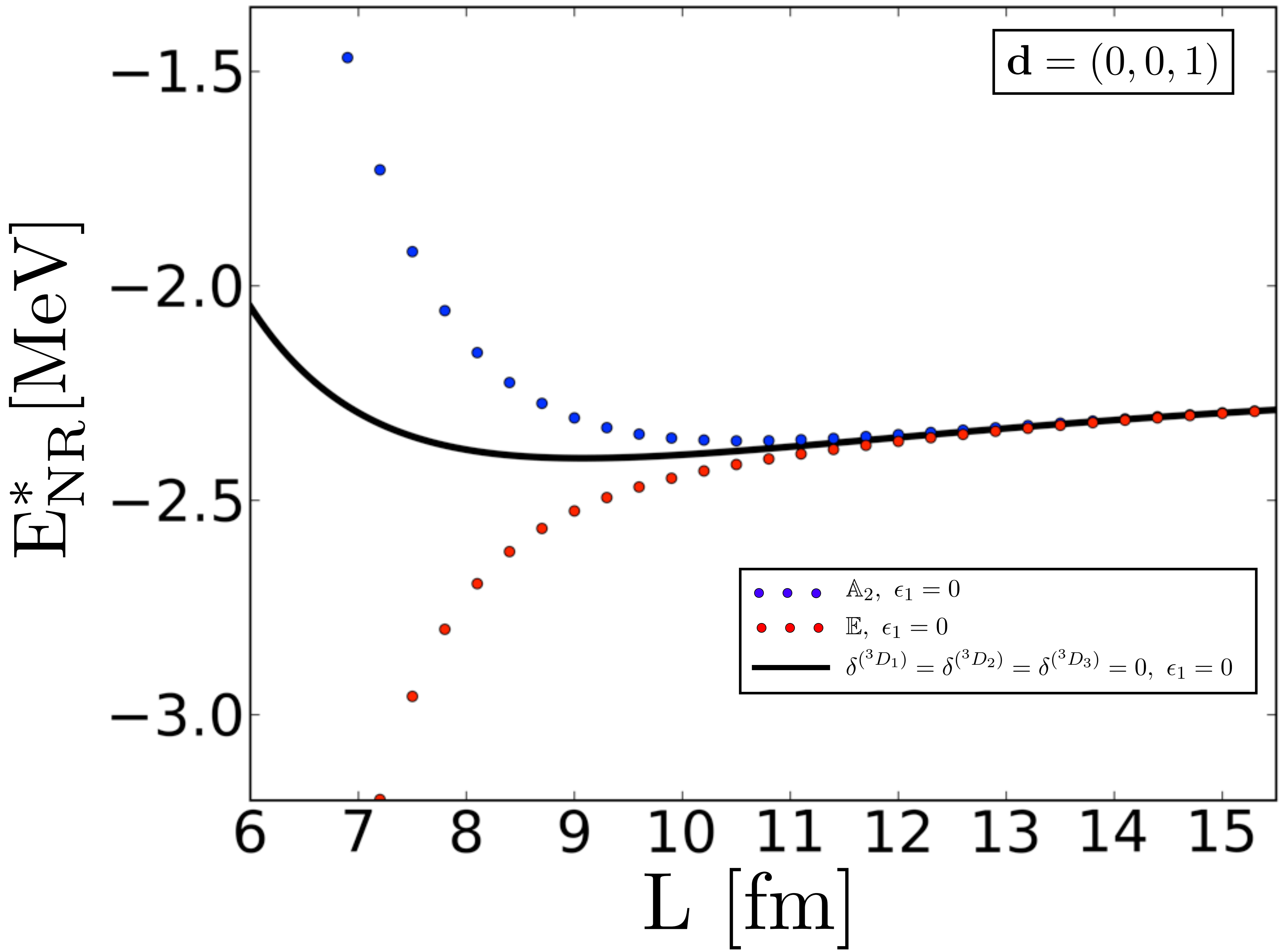
Distortion of the deuteron binding energy in finite volume



Courtesy of Raúl Briceño in collaboration with Z. Davoudi, T. Luu and M.J. Savage

Challenges and Progress

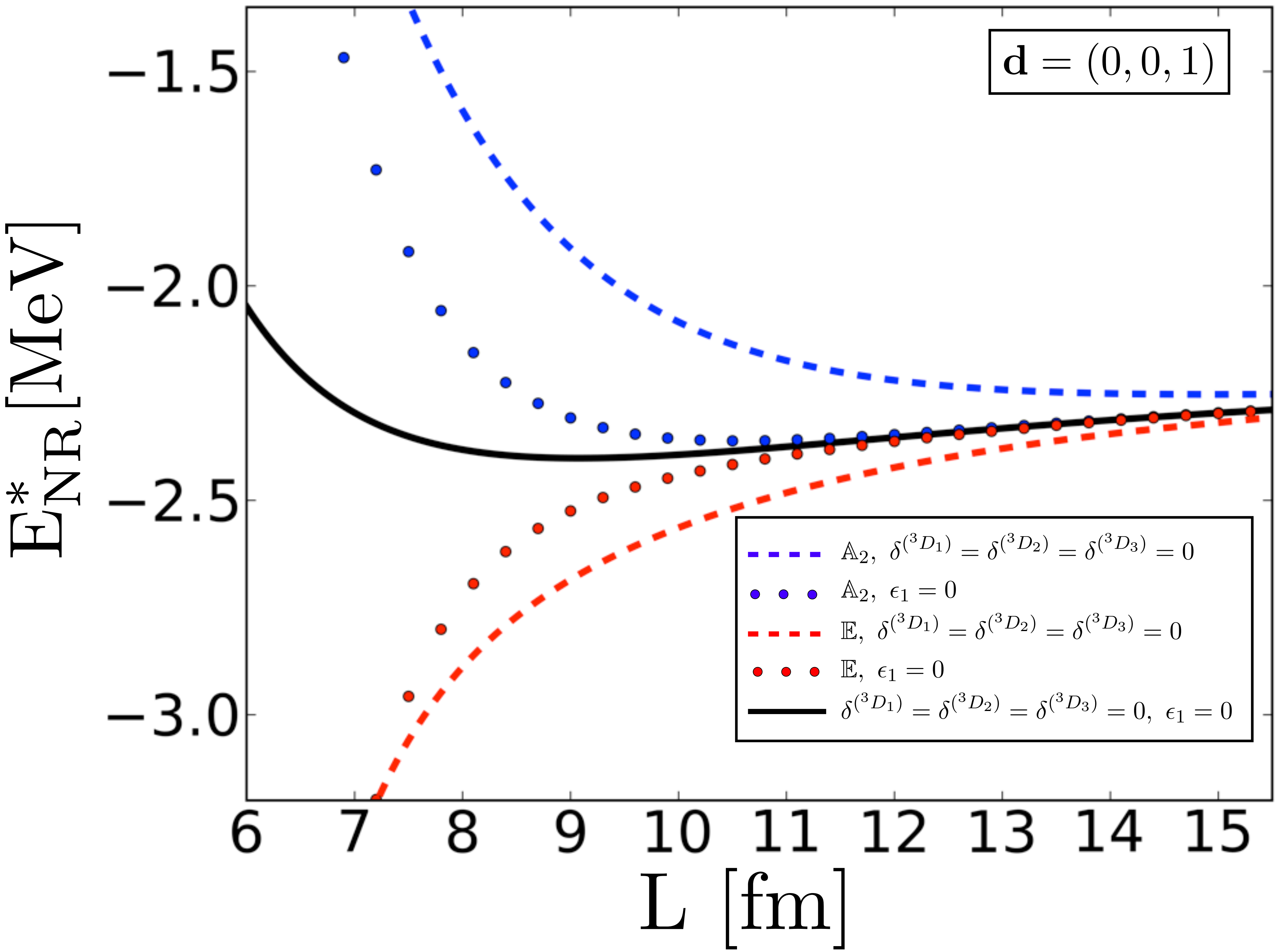
Distortion of the deuteron binding energy in finite volume



Courtesy of Raúl Briceño in collaboration with Z. Davoudi, T. Luu and M.J. Savage

Challenges and Progress

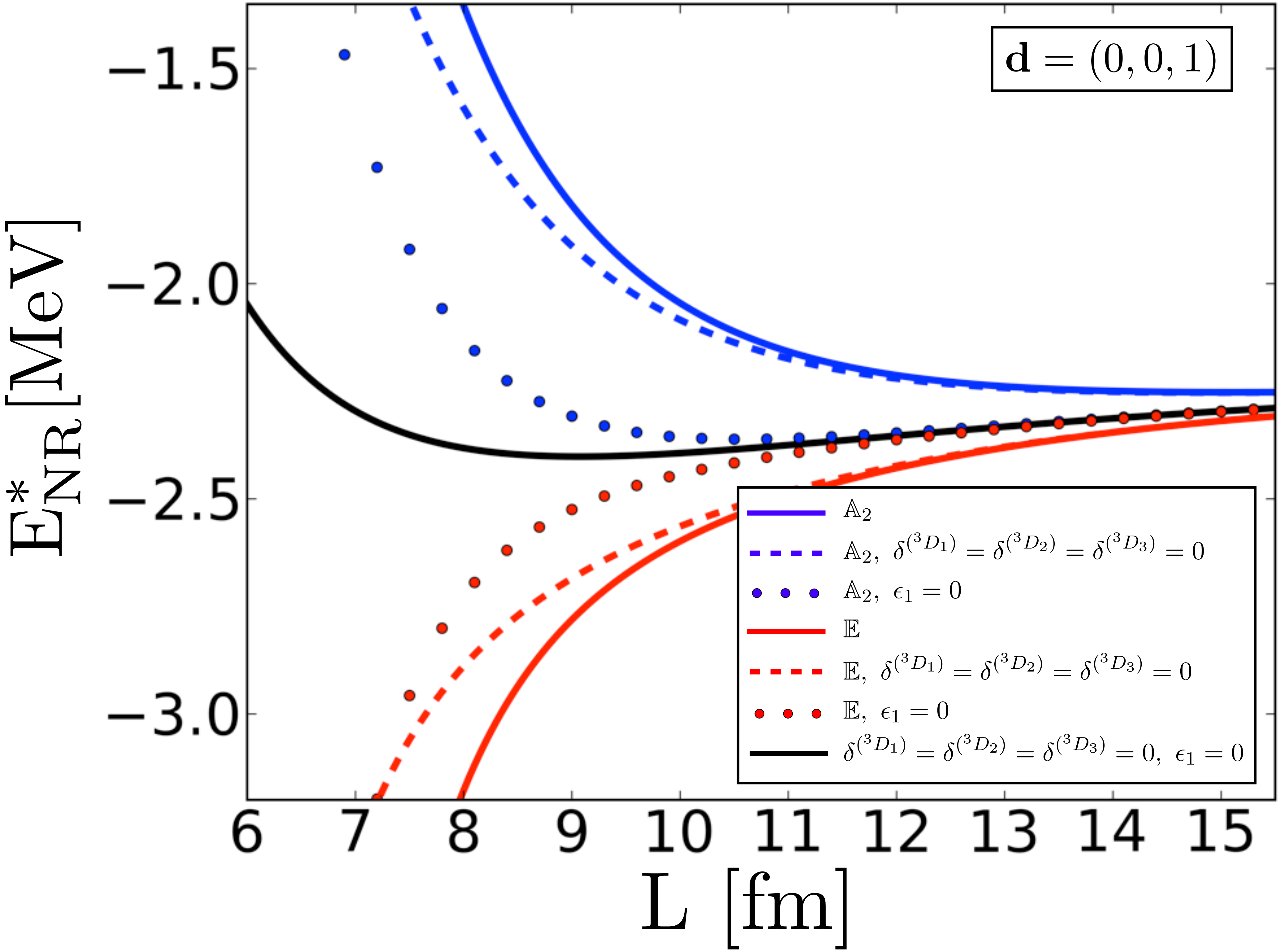
Distortion of the deuteron binding energy in finite volume



Courtesy of Raúl Briceño in collaboration with Z. Davoudi, T. Luu and M.J. Savage

Challenges and Progress

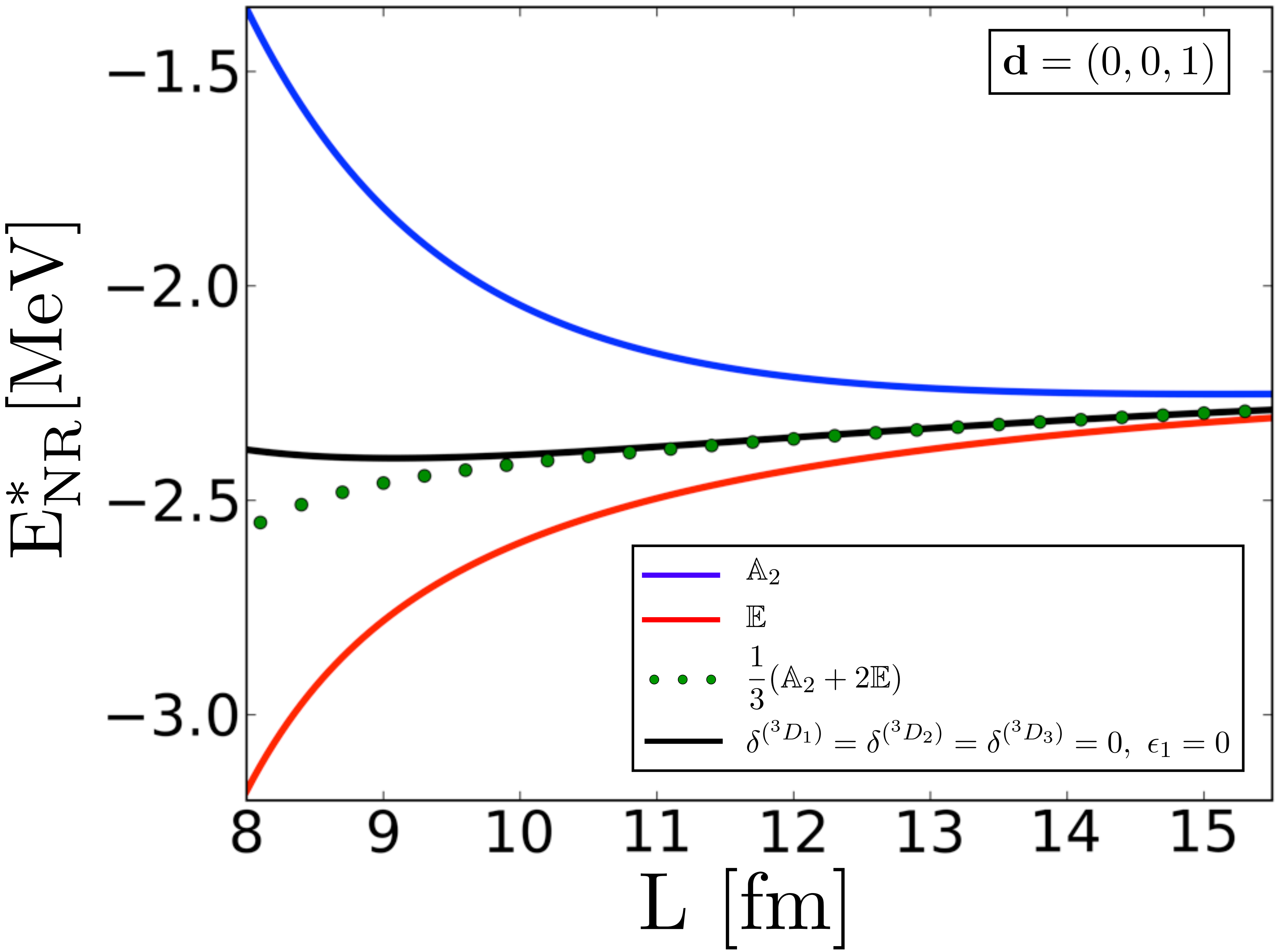
Distortion of the deuteron binding energy in finite volume



Courtesy of Raúl Briceño in collaboration with Z. Davoudi, T. Luu and M.J. Savage

Challenges and Progress

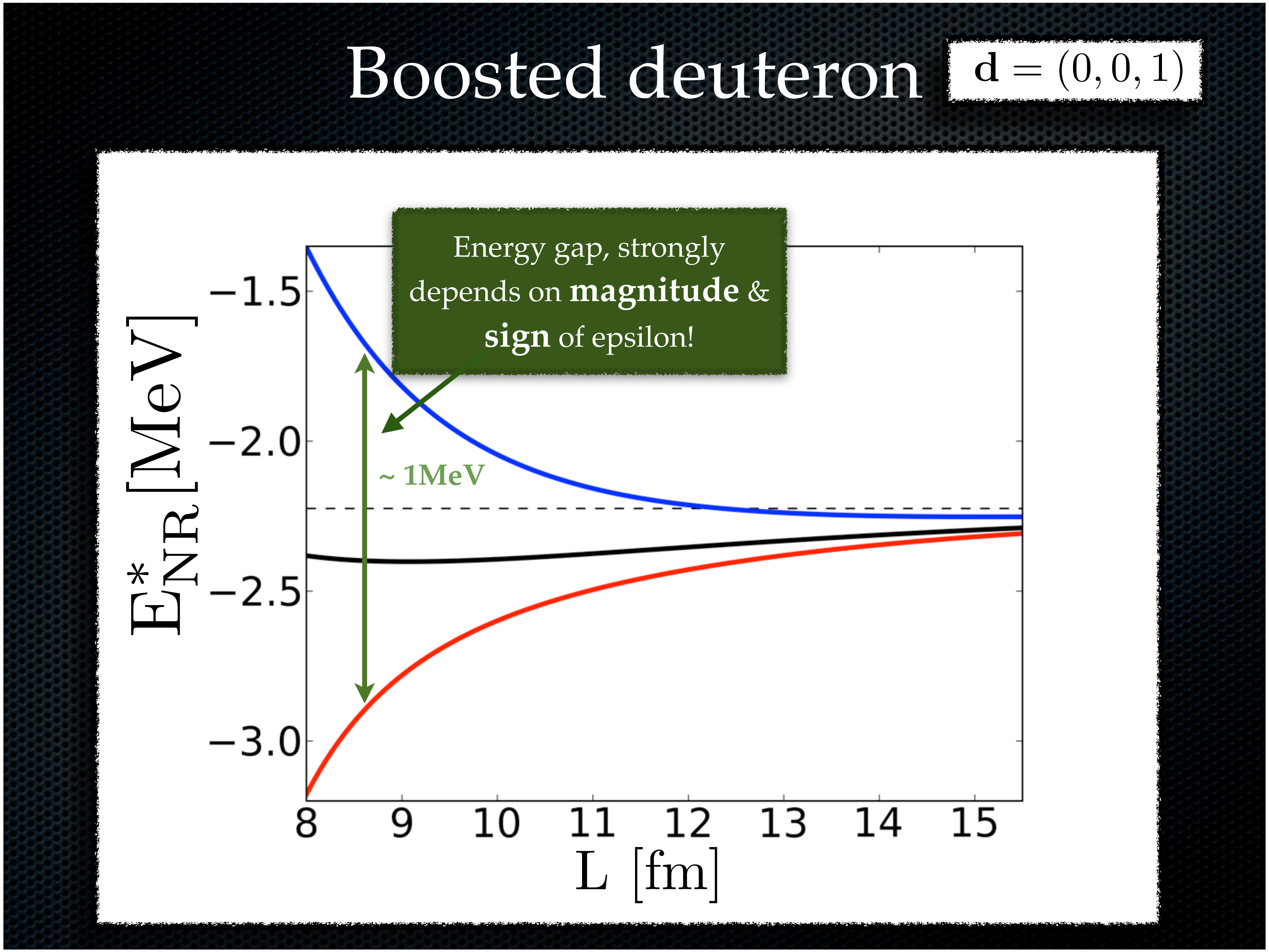
Distortion of the deuteron binding energy in finite volume



Courtesy of Raul Briceno in collaboration with Z. Davoudi, T. Luu and M.J. Savage

Challenges and Progress

Raúl Briceño Tues. 3G 15:40



Challenges and Progress

Coupled Channels and Inelastic States

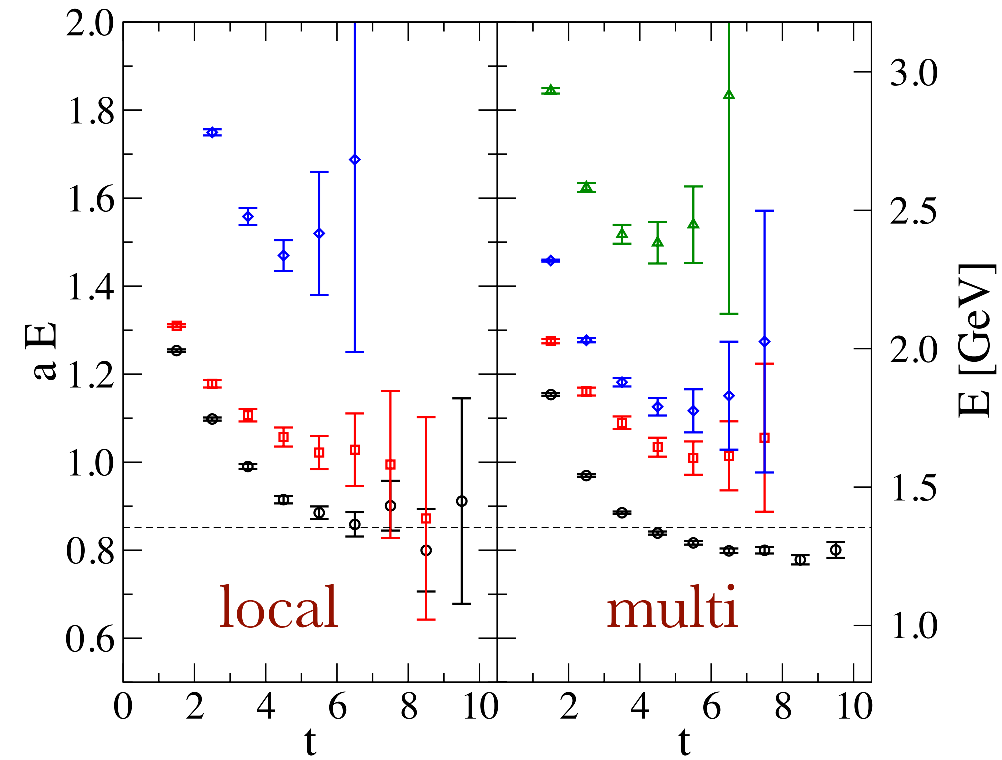
- For detailed discussion, see plenary talk by Michael Döring, Sat. 9:45

Challenges and Progress

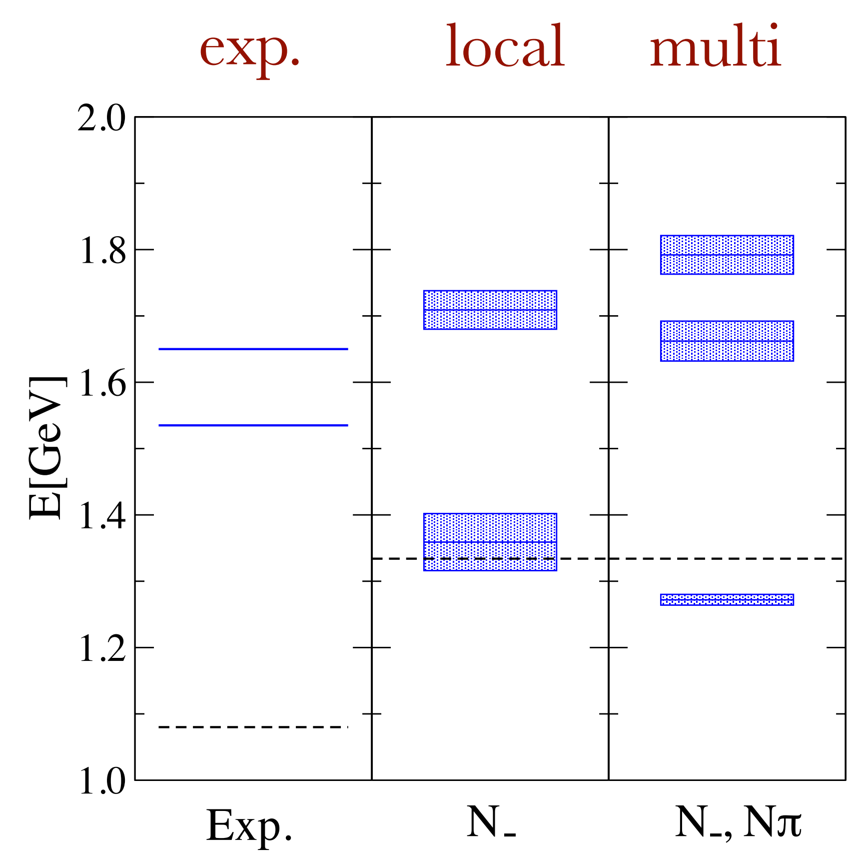
Coupled Channels and Inelastic States

- Calculations of NN interactions with near physical pion masses and large volumes (8-10 fm) requires an understanding of coupled channels and use of multiple operators

$$NN \rightarrow NN\pi$$



- without including operators which couple to all relevant states - the spectrum is not determined correctly



C.Lang and V.Verduci PRD 87 (2013)

V.Verduci, Fri. 10C 17:50

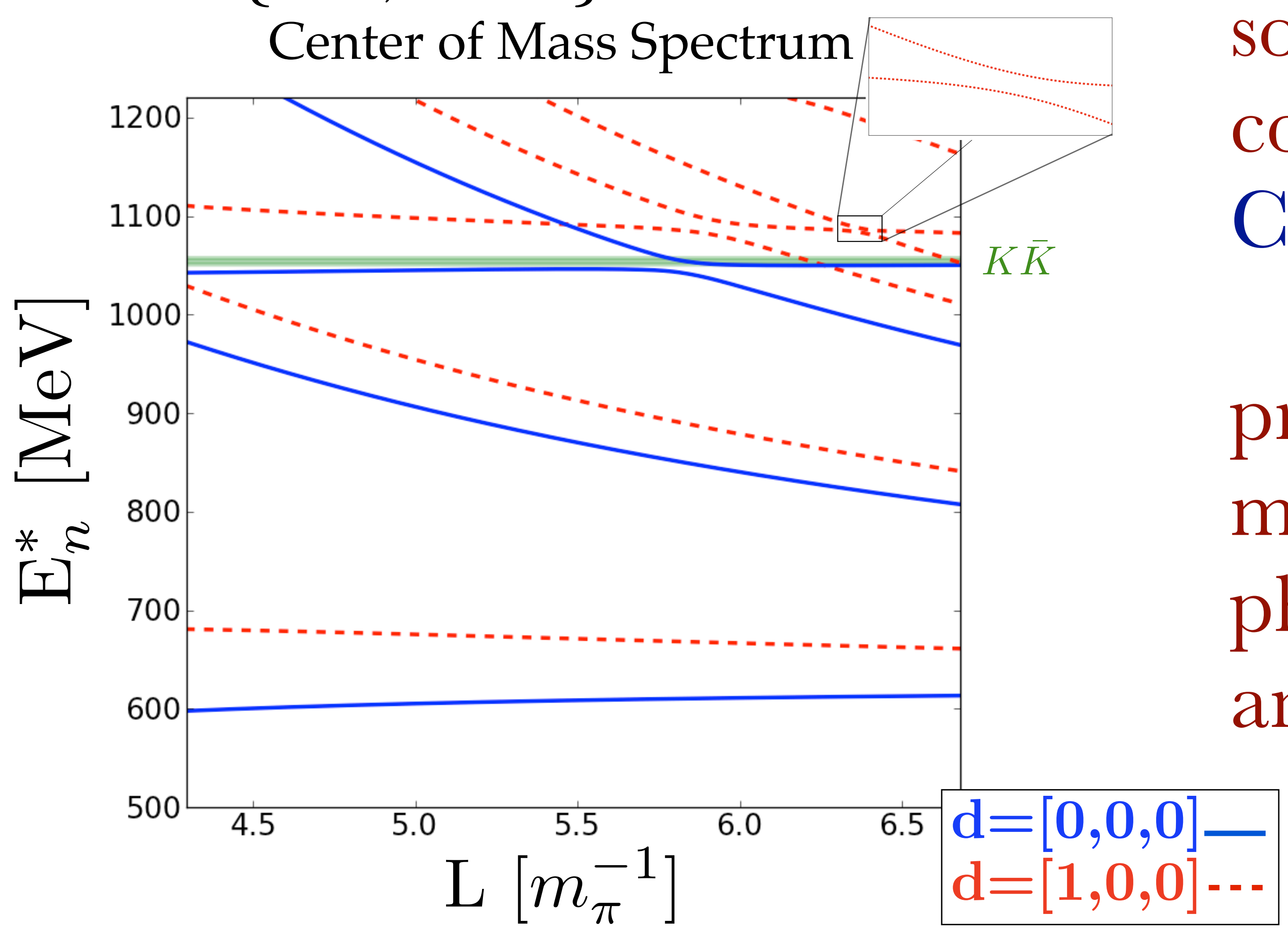
Challenges and Progress

Coupled Channels and Inelastic States

- The HALQCD potential method may be very useful

two coupled channels

$I = 0 \; \{\pi\pi, KK\} \;\; 4m_\pi < 2m_K$



for two channels, 3 pieces of information required to solve the quantization condition - at the same COM energy

practically impossible - need means to interpolate the phase shifts and mixing angle to different E*

Challenges and Progress

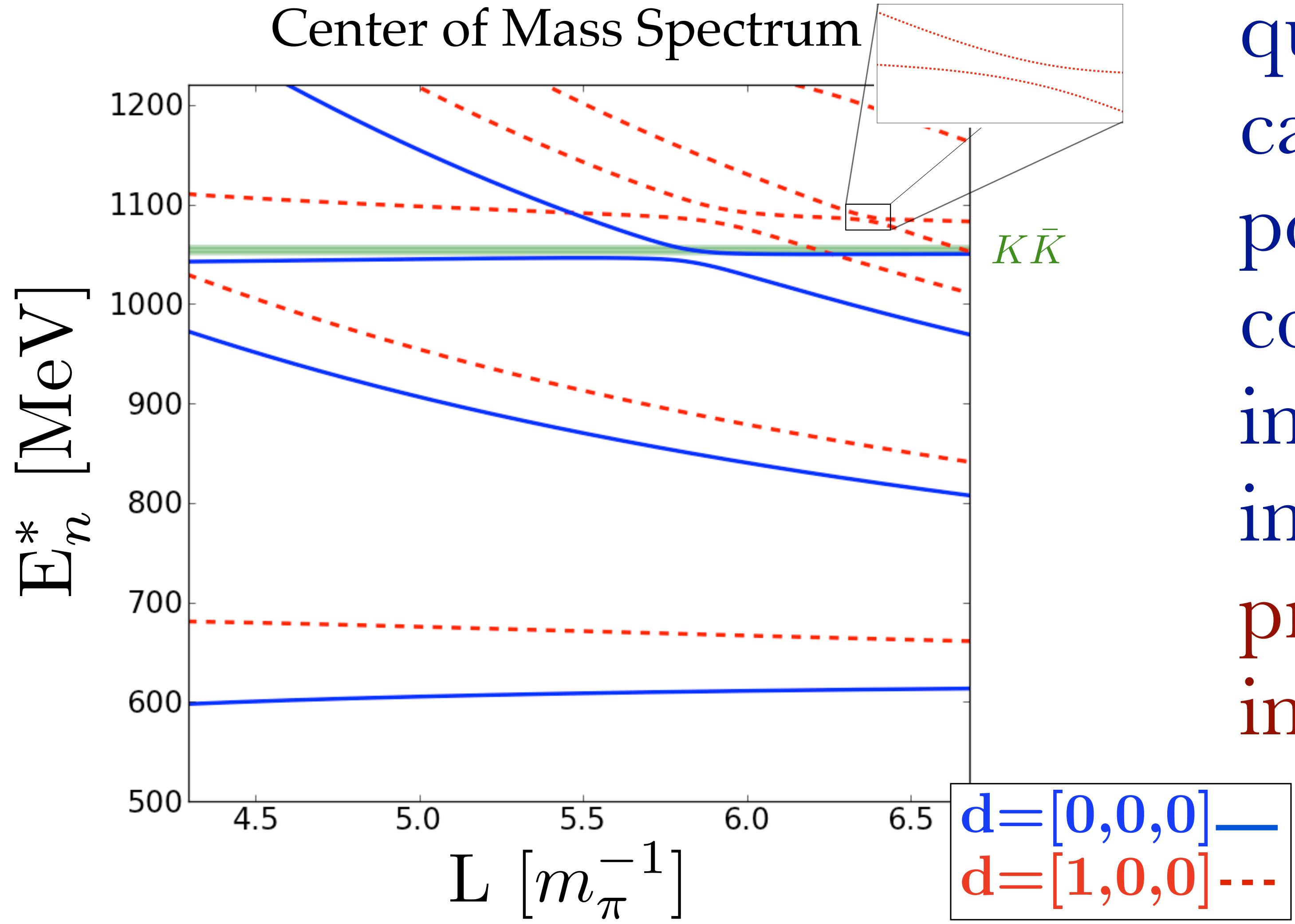
Coupled Channels and Inelastic States

- The HALQCD potential method may be very useful

two coupled channels

$I = 0 \; \{\pi\pi, KK\} \;\; 4m_\pi < 2m_K$

Center of Mass Spectrum



given the symmetry between hadronic and/or quark level operators, one can easily construct potentials with relative couplings which are interpolating field independent - the potentials provide the needed interpolating functions

Current and Future Developments

- 3 particles in a Box
- Other examples

Current and Future Developments

M. Hansen, Tues. 3G 14:00
S. Sharpe, Tues. 3G 14:20

3 identical, spinless particles in a box

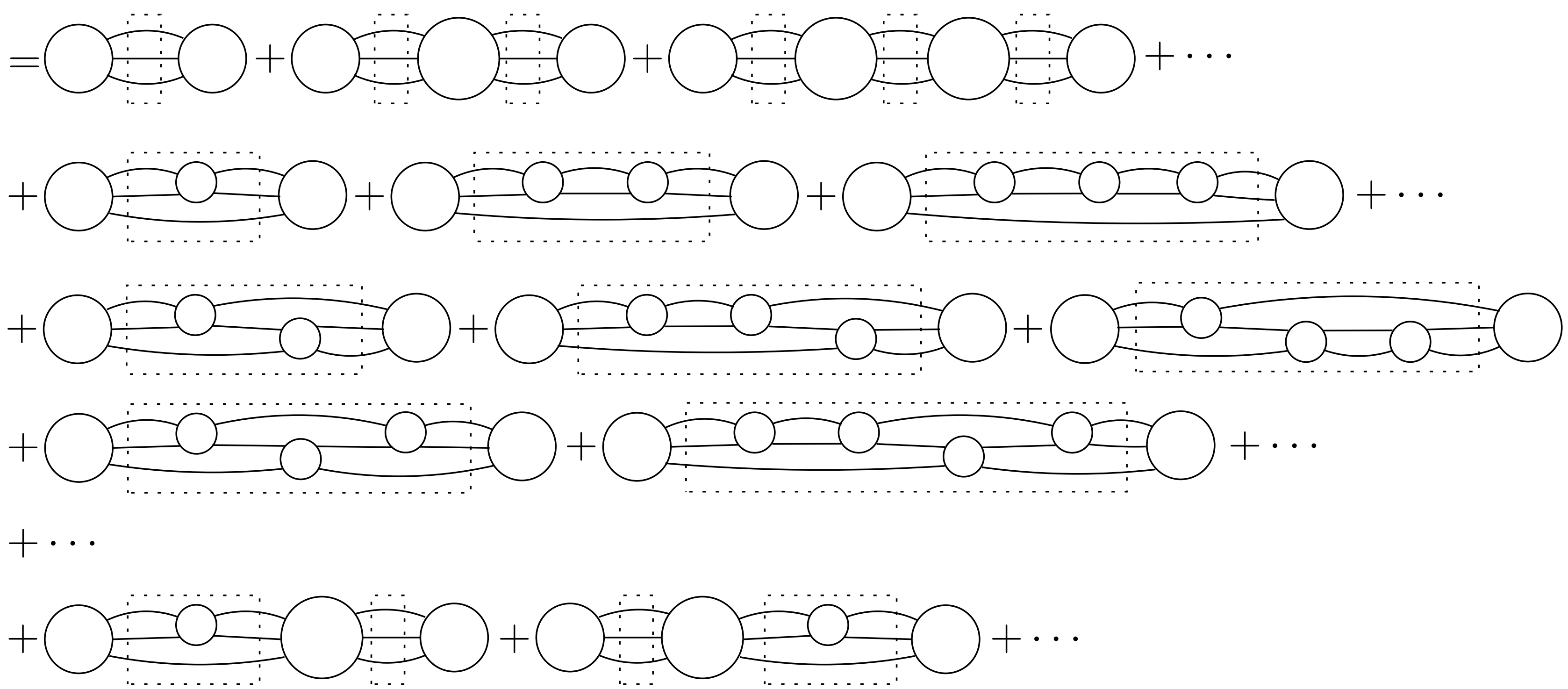
(see also Briceño and Davoudi arXiv:1212.3398)

We give a **relativistic**, **model-independent** relation between finite-volume spectrum and S-matrix for three identical particles.

Our result, valid in a moving frame, is reached by summing all power-law finite-volume corrections to a three-to-three finite-volume correlator.

Restricting energies and assuming a $\mathbb{Z}_2$ symmetry, we reduce the correlator to the following skeleton expansion

$C_L(E, \vec{P}) =$



Current and Future Developments

M. Hansen, Tues. 3G 14:00
S. Sharpe, Tues. 3G 14:20

3 identical, spinless particles in a box

Our final result is a condition for C_L to diverge. It has the form

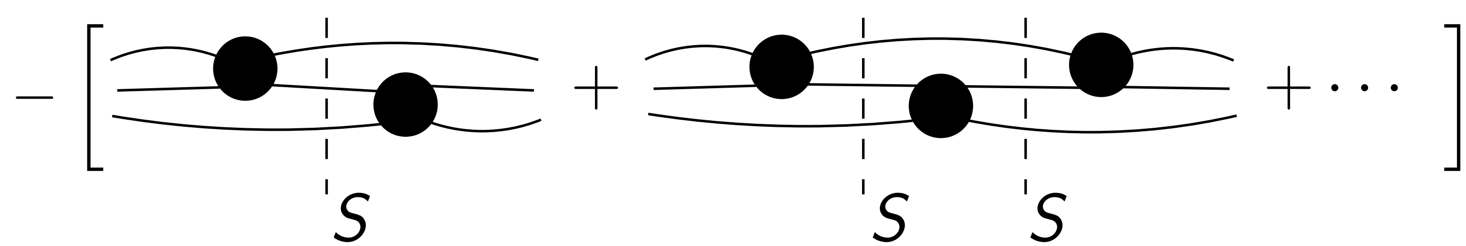
$$\Delta_{\{i\mathcal{M}_{2\to 3}, i\mathcal{M}_{df,3\to 3}\}}(E, \vec{P}, L) = 0\,.$$

Here Δ is a function of $E, \vec{P}, L$ with the property that solutions $E_1, E_2, \cdots$ at fixed $\vec{P}, L$ give the finite-volume spectrum.

The functional form of Δ is governed by the two-to-two scattering amplitude $i\mathcal{M}_{2\to 2}$ as well as a "divergence-free three-to-three scattering amplitude" $i\mathcal{M}_{df,3\to 3}$.

$i\mathcal{M}_{df,3\to 3}$ is defined by removing divergent terms from $i\mathcal{M}_{3\to 3}$

$i\mathcal{M}_{df,3\to 3} \equiv i\mathcal{M}_{3\to 3}$



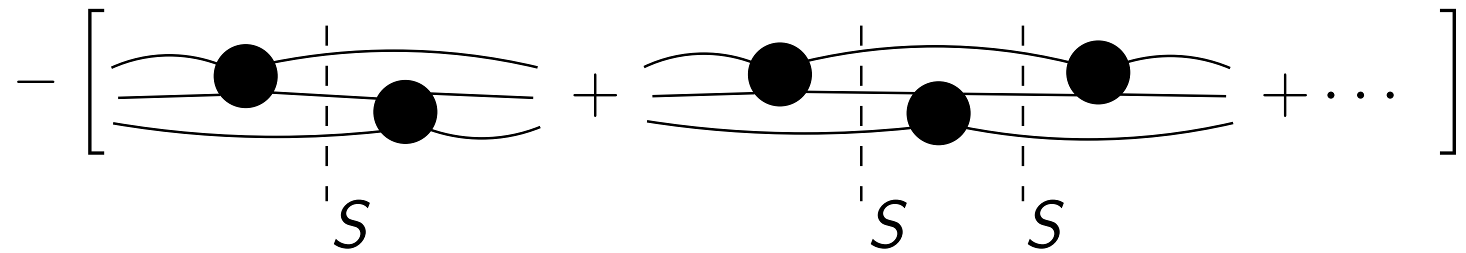
Here the filled circles represent on-shell $i\mathcal{M}_{2\to 2}$. The cuts represent factors with the same singularities as the full diagrams.

Current and Future Developments

M. Hansen, Tues. 3G 14:00
S. Sharpe, Tues. 3G 14:20

3 identical, spinless particles in a box

$$i\mathcal{M}_{df,3\to3} \equiv i\mathcal{M}_{3\to3}$$



Three comments:

(1) $i\mathcal{M}_{df,3\to3}$ is well motivated, since $i\mathcal{M}_{3\to3}$ diverges for certain momenta above threshold (nothing to do with bound states).[1]

(2) Unlike $i\mathcal{M}_{3\to3}$, $i\mathcal{M}_{df,3\to3}$ can be decomposed in harmonics and truncating gives a finite number of unknowns.[2] If $i\mathcal{M}_{2\to2}$ is determined separately, one can in principle **extract** $i\mathcal{M}_{df,3\to3}$ **from the spectrum.**

(3) After extraction one can add back subtracted terms (only depend on $i\mathcal{M}_{2\to2}$) **to determine** $i\mathcal{M}_{3\to3}$.

---

[1] Potapov & Taylor. *PRA 16-6* (1977).
[2] Rubin, e. *PR 146-4* (1966).

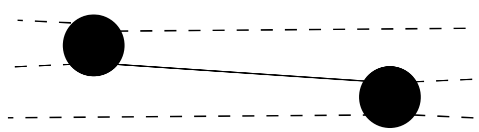
M. Hansen, Tues. 3G 14:00
S. Sharpe, Tues. 3G 14:20

3 identical, spinless particles in a box

Three-to-three amplitude singularity

Next natural step would be to decompose unit-vectors in spherical harmonics. This is invalid due to singularities in amplitude.

$i\mathcal{M}_{3\to 3}$ always has physical singularities above threshold which have nothing to do with bound states. For example consider



$$\equiv i\mathcal{M}_{\mathrm{on,off}} \frac{i}{(E-\omega_p-\omega_k)^2-\omega_{pk}^2} i\mathcal{M}_{\mathrm{off,on}} \,,$$

where $\omega_{pk} \equiv \sqrt{(\vec{P}-\vec{p}-\vec{k})^2+m^2}$.

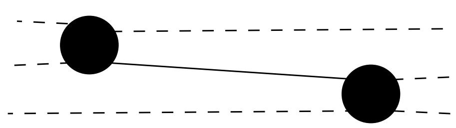
Here filled circles represent two-to-two scattering amplitudes and subscripts indicate whether in- and out-state is on- or off-shell.

Current and Future Developments

M. Hansen, Tues. 3G 14:00
S. Sharpe, Tues. 3G 14:20

3 identical, spinless particles in a box

Three-to-three amplitude singularity



$$\equiv i\mathcal{M}_{\text{on,off}}\Delta(P-p-k)i\mathcal{M}_{\text{off,on}} \, .$$

As a result of the singularity, harmonic decomposition is not convergent.

this has been known for a while Potapov & Taylor PRA 16 (1977)
and is in contradiction with statements made in (HALQCD)
"Asymptotic behavior of Nambu-Bethe-Salpeter wave functions for multi-particles in quantum field theories" arXiv:1303.2210

without directional dependence
no singularity from above

$$\begin{aligned} T([\boldsymbol{q}^A]_n, [\boldsymbol{q}^B]_n) &\equiv T(\boldsymbol{Q}_A, \boldsymbol{Q}_B) \\ &= \sum_{[L],[K]} T_{[L][K]}(Q_A, Q_B) Y_{[L]}(\Omega_{\boldsymbol{Q}_A}) \overline{Y_{[K]}(\Omega_{\boldsymbol{Q}_B})} \end{aligned} \quad (37)$$

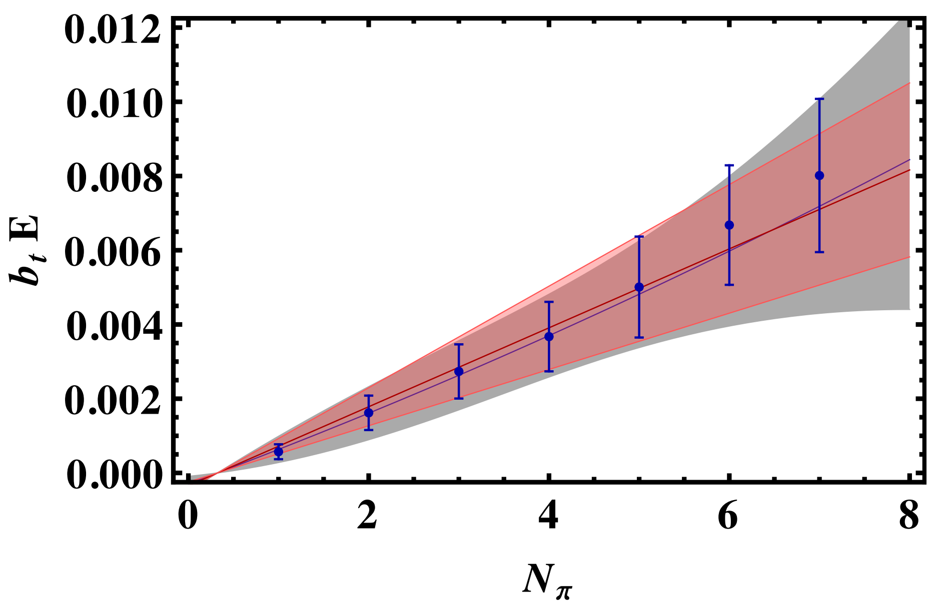
$$Q_{A,B} = |\mathbf{Q}_{A,B}|$$

Current and Future Developments

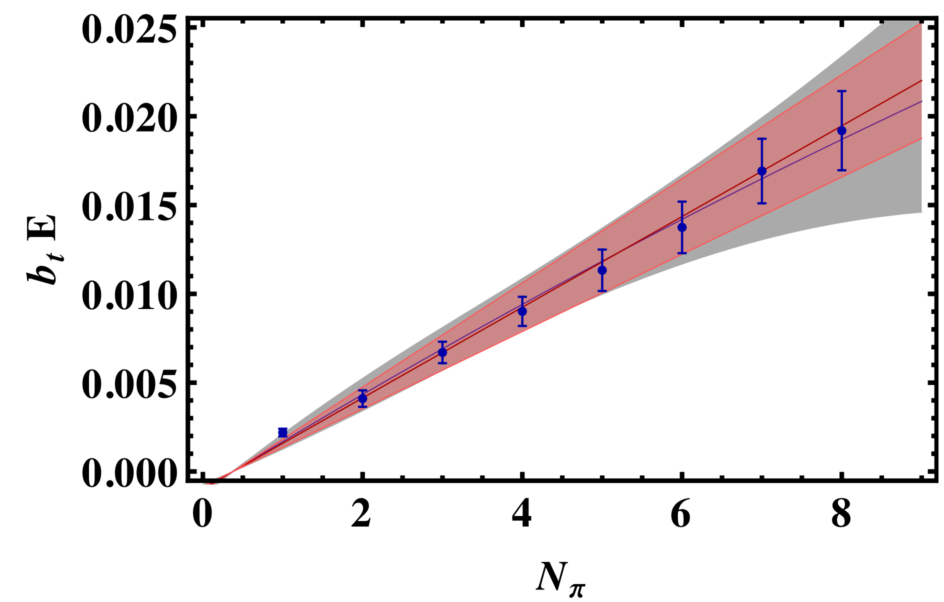
A. Nicholson: Thursday 7G 15:40
(work with W. Detmold)

Computing modifications to baryons in a sea of pions or kaons

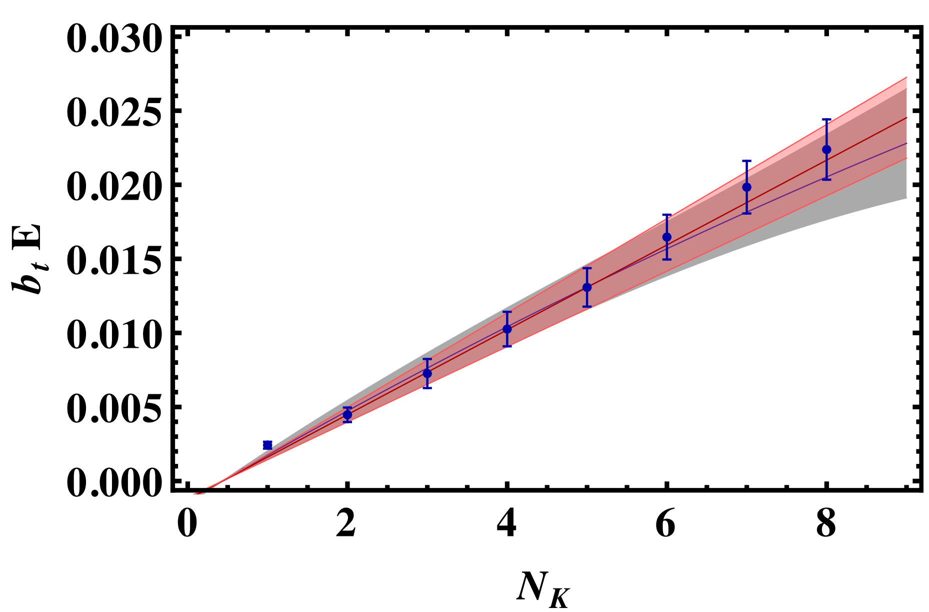
Ξ^0, π^+



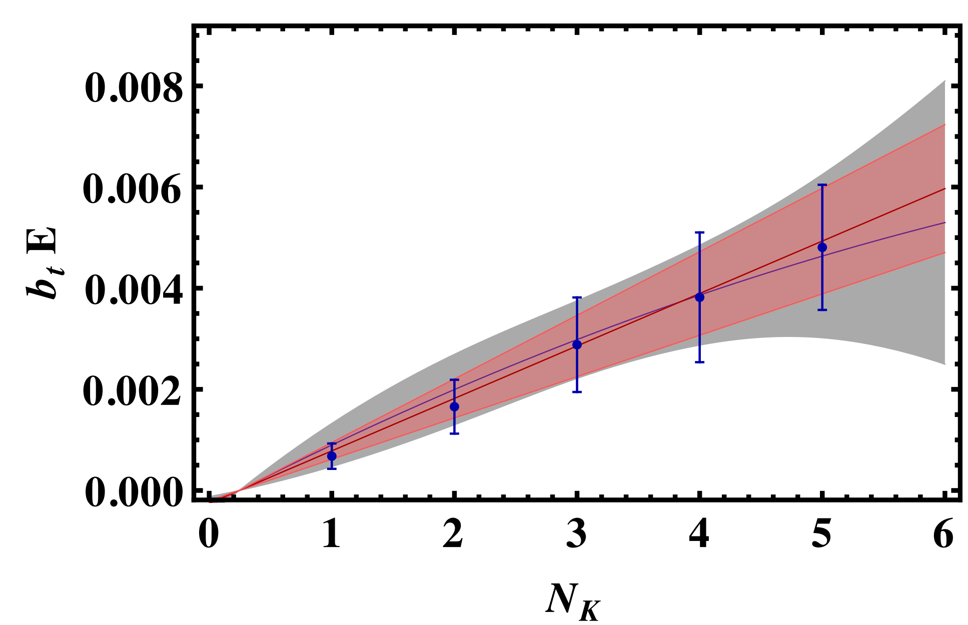
Σ^+, π^+



p,K^+



n,K^+

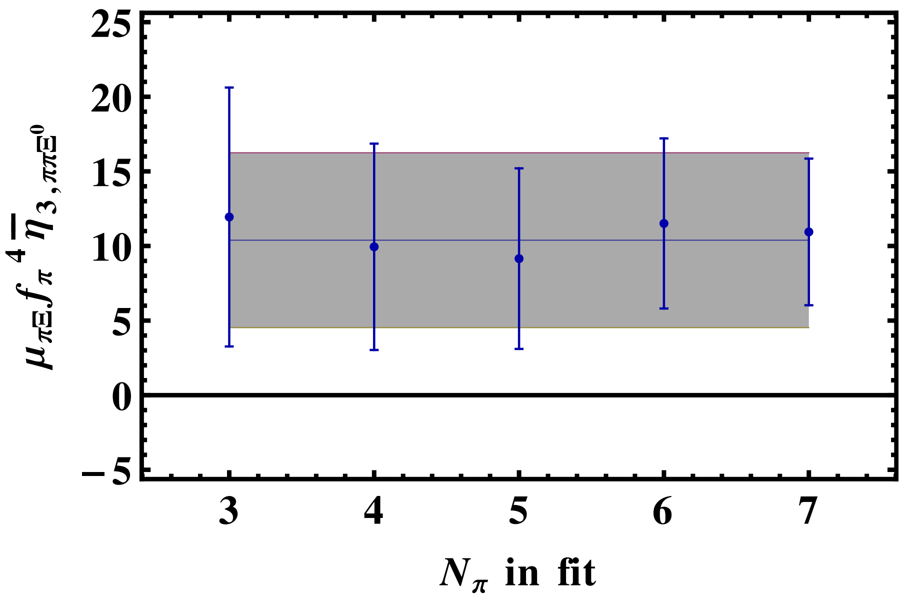


Current and Future Developments

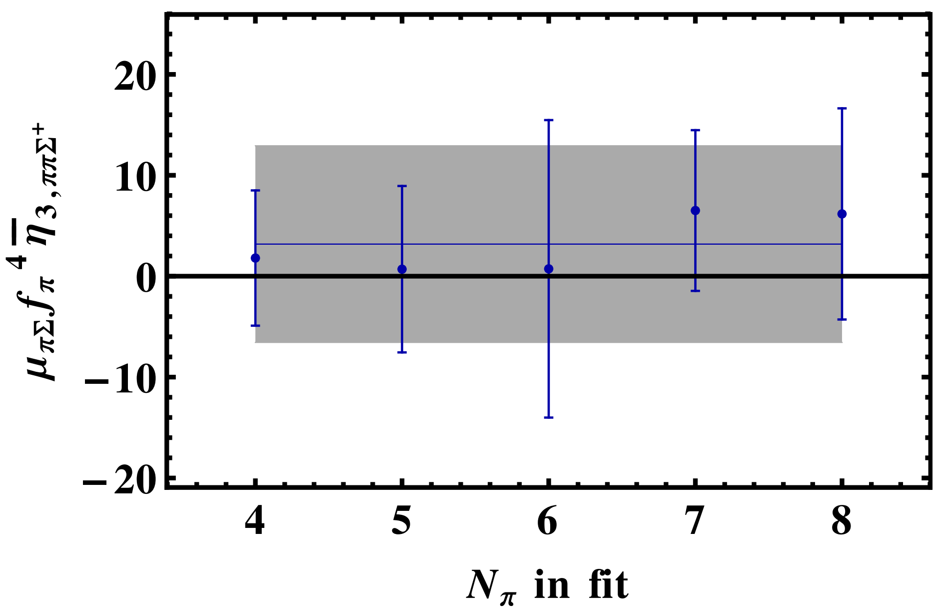
A. Nicholson: Thursday 7G 15:40
(work with W. Detmold)

able to determined the 3-body interactions

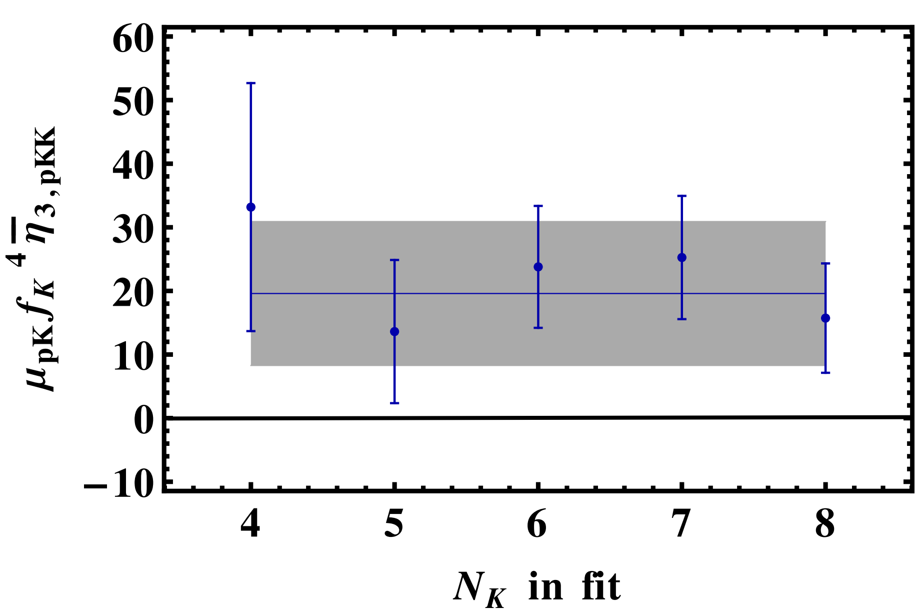
Ξ^0, π^+



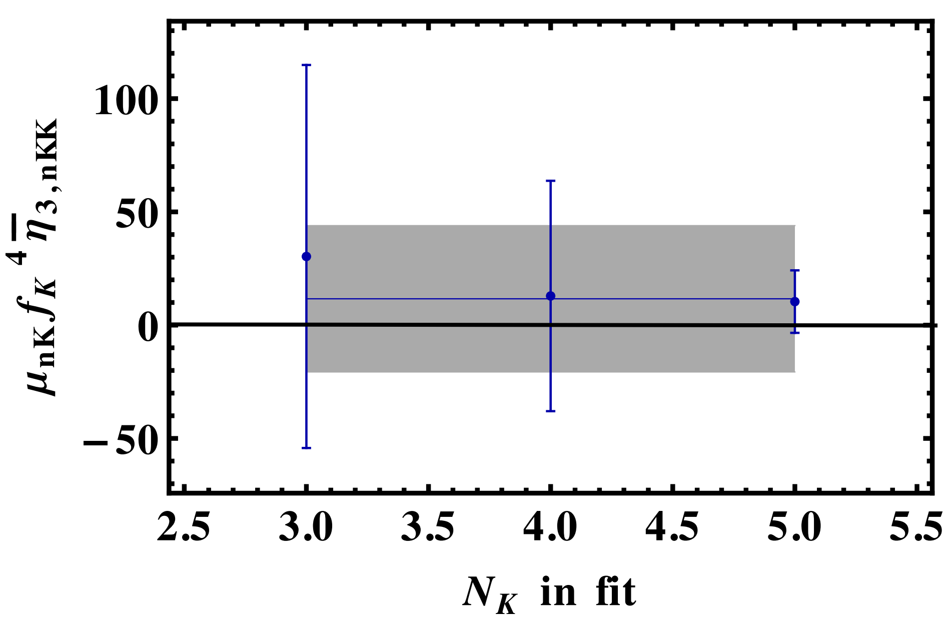
Σ^+, π^+



p,K+



n,K+

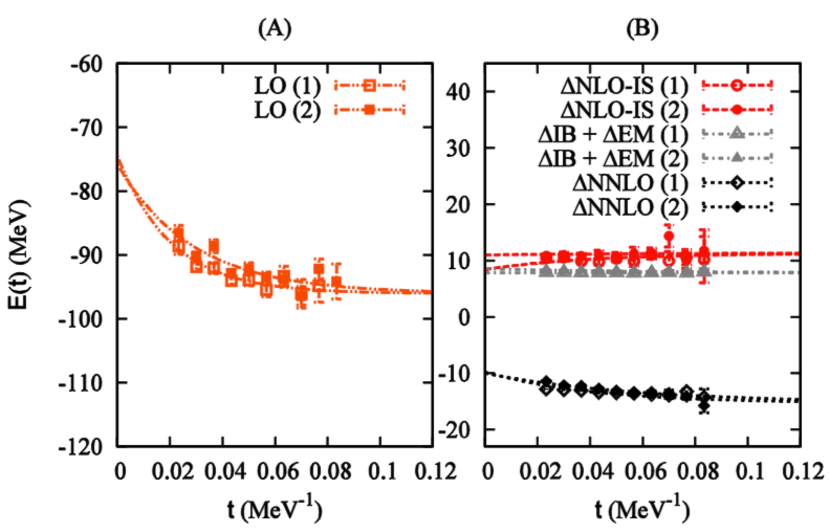


Current and Future Developments

T. Lähde: Monday 1G 15:40

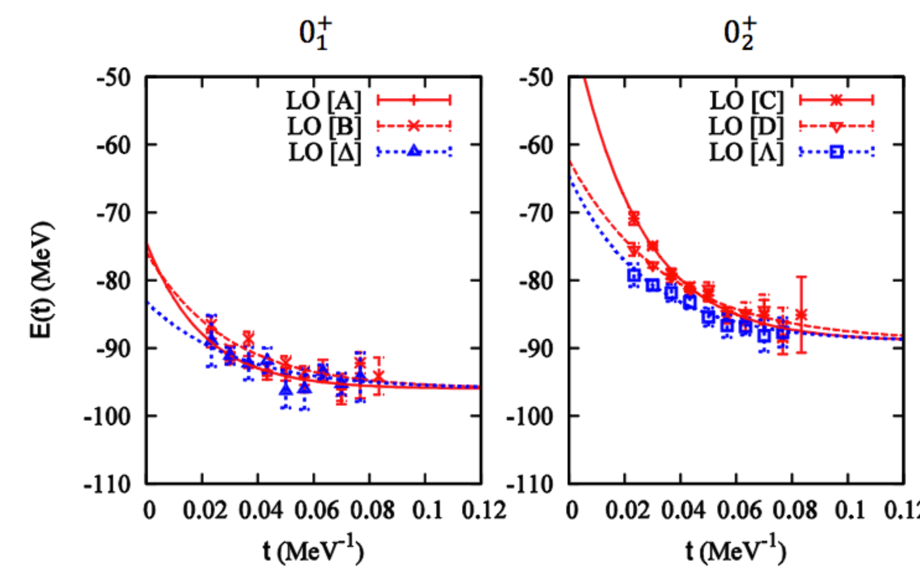
NN EFT on the Lattice

LECs of the EFT are fixed by experiment in light nuclei. The Lattice EFT is then used to compute states in ^{12}C

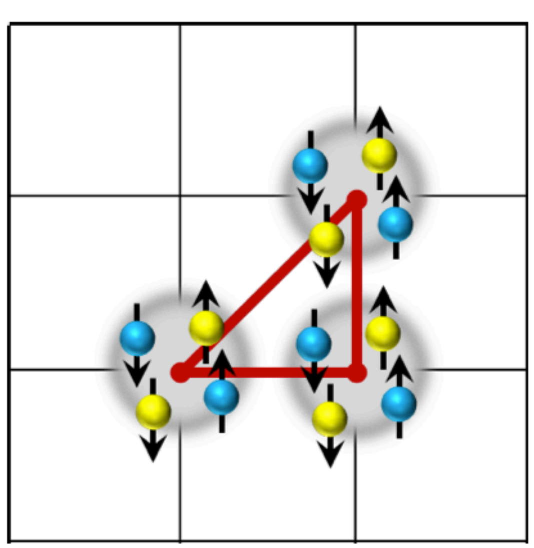


Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 (2012) 252501

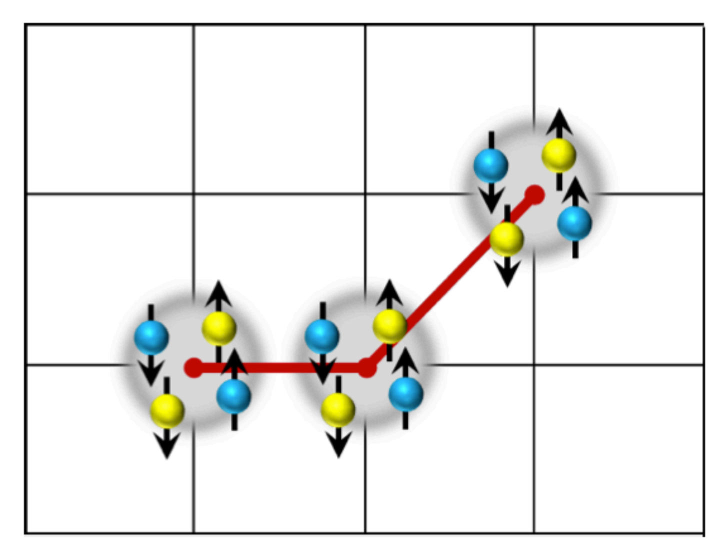
desire to match these LECs directly to QCD by comparing with Lattice QCD



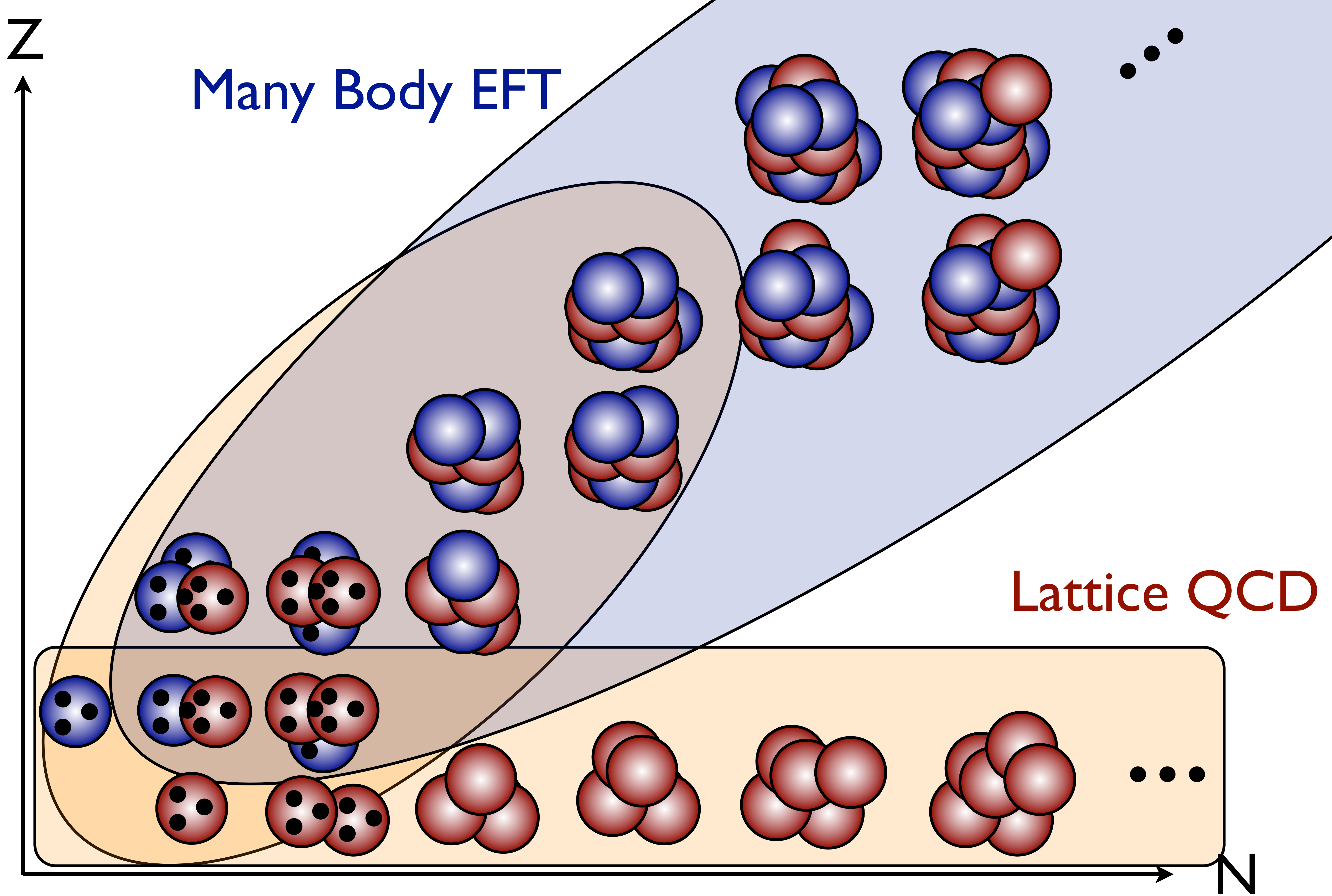
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)



12 rotational orientations



24 rotational orientations



Determine 2, 3, 4 body forces directly from QCD
match onto many body effective field theory

Current and Future Developments

Many other related talks - I list the ones yet to come

W. Kamleh Thur. 7G 14:00	Exploring the Roper Resonance in lattice QCD
K-F Liu Thur. 7G 14:20	The Roper Puzzle
A. Nicholson Thur. 7G 15:40	Baryon Properties in meson mediums from lattice QCD
S. Prelovsek Thur. 8G 17:30	Charmonium-like states from scattering on the lattice
M. Yamada Fri. 10C 16:30	Omega-Omega interaction on the Lattice
K. Sasaki Fri. 10C 16:50	LQCD studies of multi-strange baryon-baryon interactions
N. Ishii Fri. 10C 17:10	The anti-symmetric LS potential in flavor SU(3) limit
K. Murano Fri. 10C 17:30	Quark mass dependence of LS force in parity-odd NN
V. Verduci Fri. 10C 17:50	Pion-nucleon scattering in lattice QCD
S. Cohen Fri. 10C 18:10	Looking for a Quarkonium-Nucleus Bound State on the Lattice
P. Rakow Fri. 10C 18:30	The Hadronic Decays of Decuplet Baryons

Conclusions

- Nuclear Physics is beginning a Renaissance with lattice QCD
- Very open field with room/need for new ideas
- Exciting to see more people getting involved, especially so many young scientists
- Significant challenges need to be overcome most important: basis of interpolating fields, contractions, dealing with coupled inelastic channels $NN \to NN\pi$
- 3+ particle formalism will soon be applied to numerical results
- Burden of proof on HALQCD to demonstrate consistency with other lattice results - more statistics and interpolating fields which look more like bound states

Acknowledgements

- In particular, I would like to thank Raúl Briceño and Max Hansen for many detailed conversations

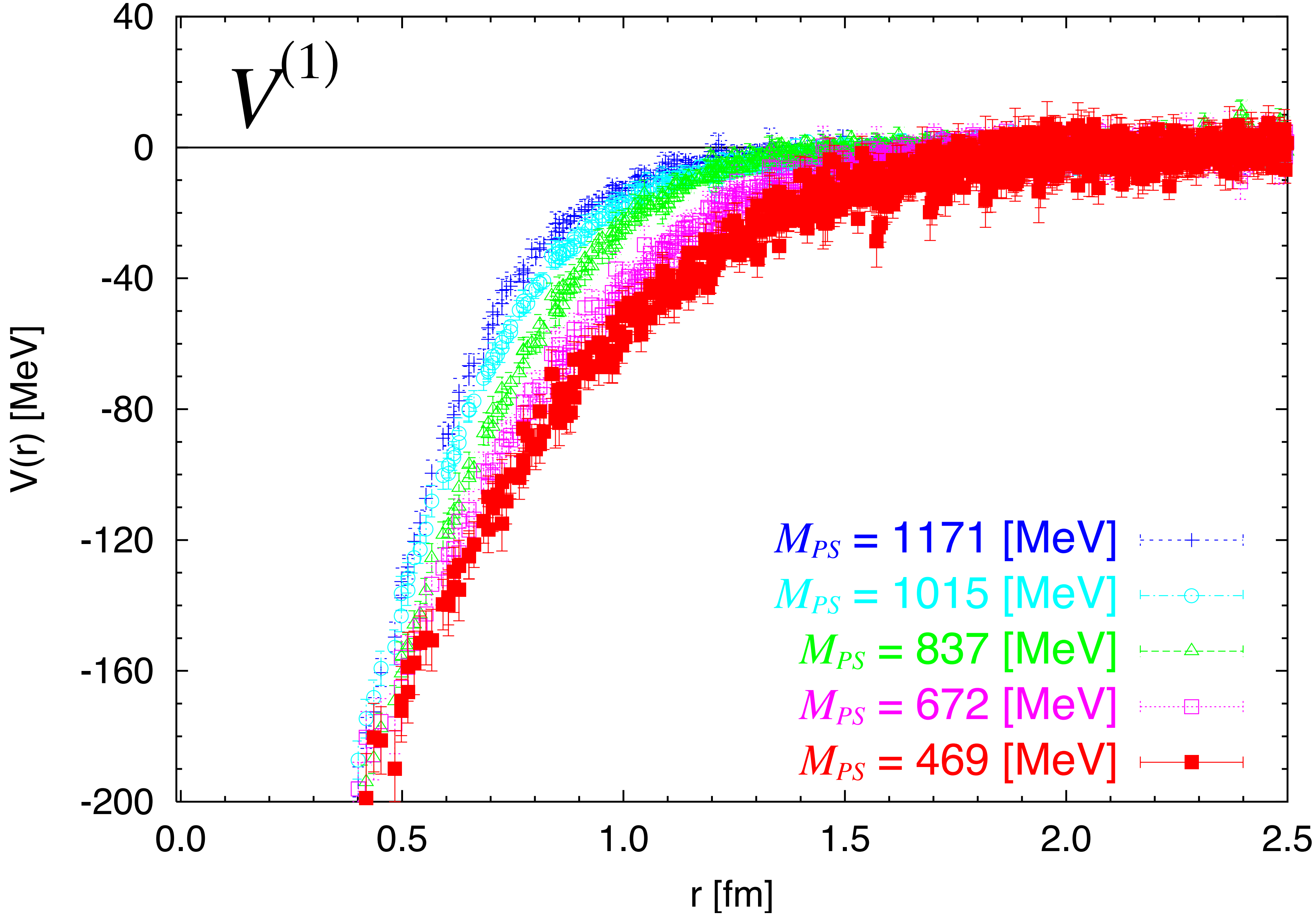
- I would also like to thank Bruno Charron, Takumi Doi and Tetsuo Hatsuda for fruitful discussions

- I would also like to thank the members of HALQCD for generously sharing with me many details of the numerical work, as well as intermediate numerical data for my preparation

- I would like to thank everyone else who provided me with material for this review

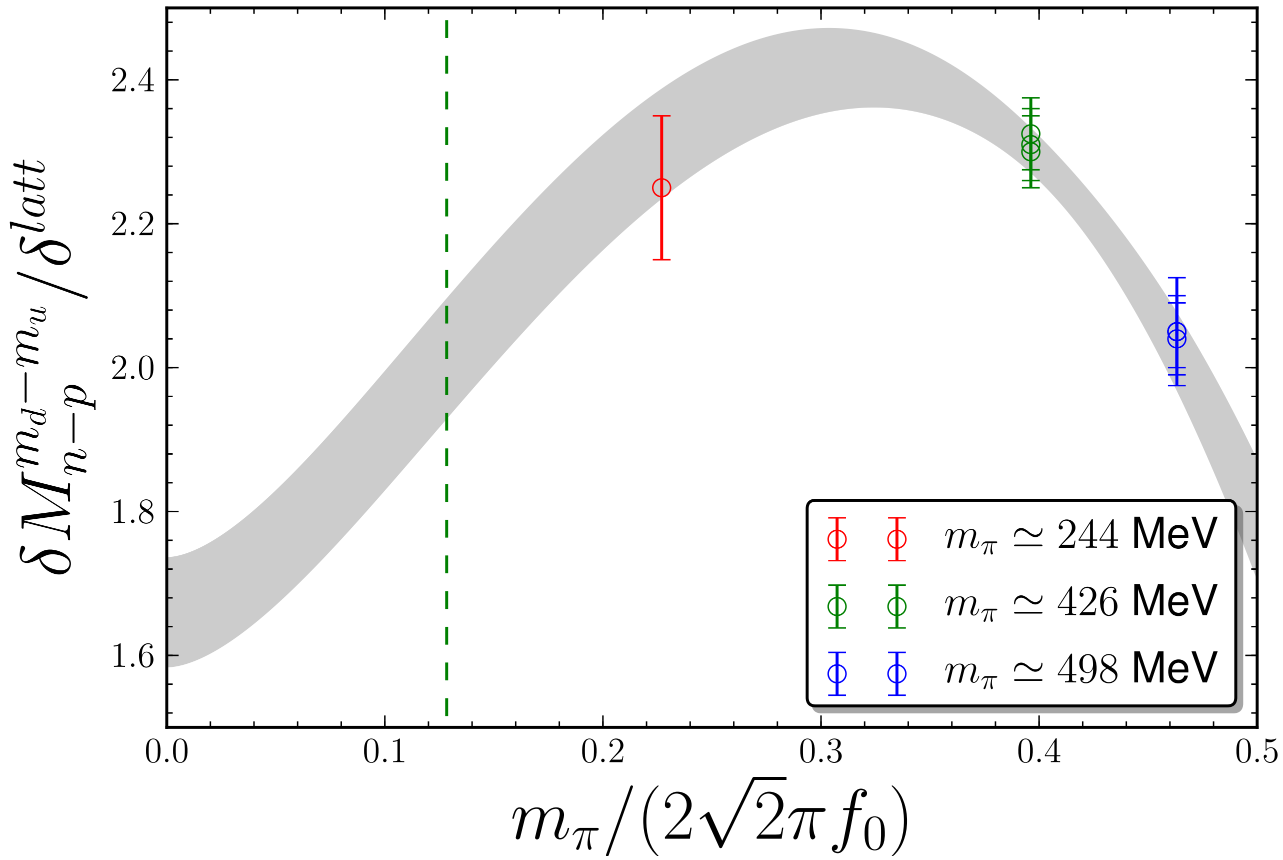
Thank You
(again)

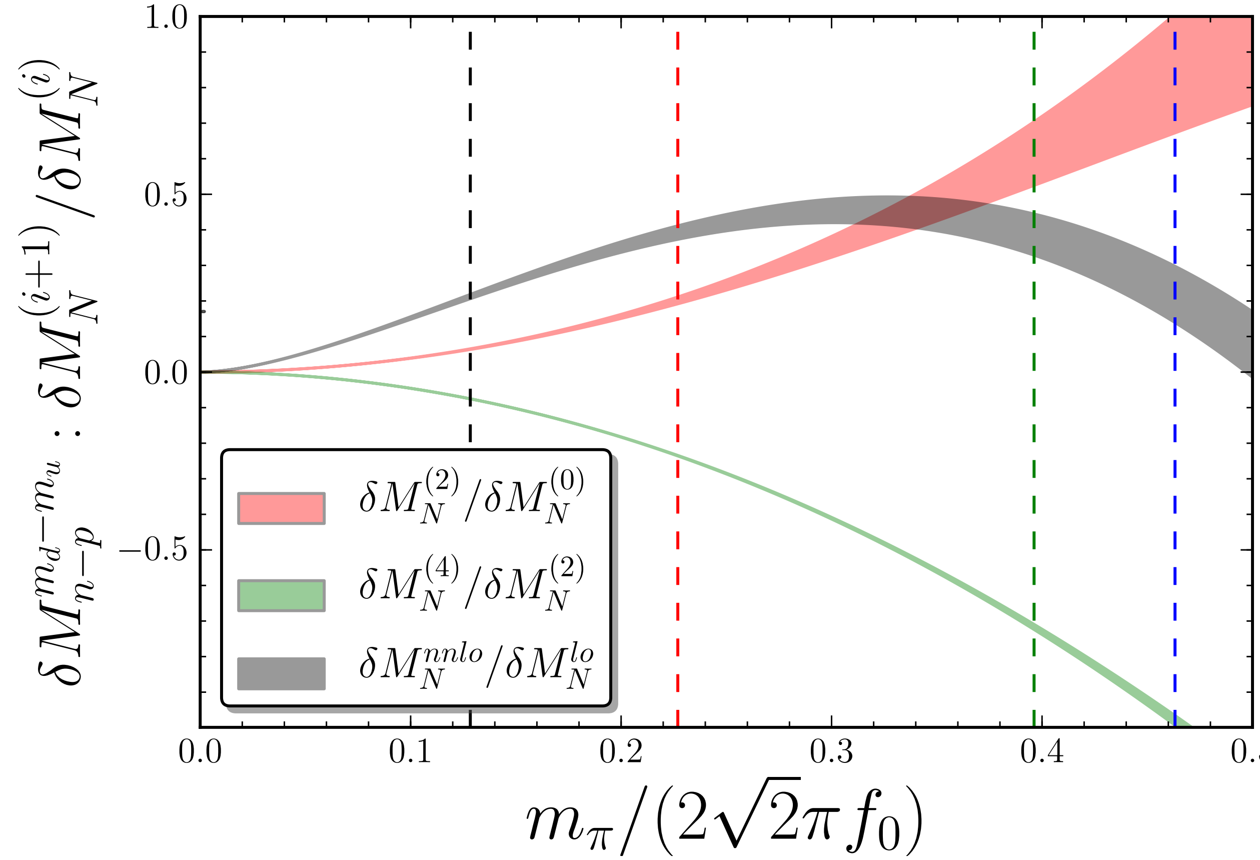
Methods and Results



HALQCD NPA 881 (2012)
arXiv:1112.5926

Strong Isospin Breaking: m_d - m_u PRELIMINARY





$$\delta M^{\delta}_{n-p} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$$\delta M^{\delta}_{n-p} = \delta\left\{\alpha + \beta\frac{m_\pi^2}{8\pi^2 f_\pi^2} + \gamma\frac{m_\pi^4}{(8\pi^2 f_\pi^2)^2}\right\}$$

adding $\gamma\frac{m_\pi^4}{(8\pi^2 f_\pi^2)^2}$ counterterm does not improve fit: γ consistent with zero

higher order polynomial gives good fit but poorer convergence