$\pi^0 \to \gamma\gamma$ and chiral anomaly on the lattice

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$$\pi^0 \to \gamma\gamma$$

$\pi^0 \to \gamma\gamma$

- π^0 decay into $\gamma\gamma$ with a branching rate of 98.8%

- $\pi^0 \to \gamma\gamma$ process is described by transition amplitude

$$\langle \gamma(p_1, \lambda_1)\gamma(p_2, \lambda_2)|\pi^0(q)\rangle$$

- integrating out the γ-field

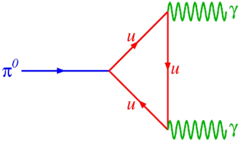
$$M_{\mu\nu}(p_1, p_2) = \langle 0|J_\mu(p_1)J_\nu(p_2)|\pi^0(q)\rangle \equiv \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$$

- decay width is given by

$$\Gamma_{\pi^0\gamma\gamma} = \frac{\pi\alpha_e^2 m_\pi^3}{4}\mathcal{F}^2_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)$$

History of $\pi^0 \to \gamma\gamma$

- early theoretical work [Sutherland & Veltman, 1967]
  - using PCAC relation $\Rightarrow \mathcal{F}_{\pi^0\gamma\gamma}(0,0,0) = 0$
  - pion should *not* decay, but experimentally it decays!
- paradox is solved by existence of chiral anomaly (ABJ anomaly)
  - considering quantum fluctuation, PCAC relation has to be modified



$$\partial_\lambda J_\lambda^5 = m_\pi^2 F_\pi \phi_\pi + \frac{e^2}{16\pi^2}\epsilon_{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

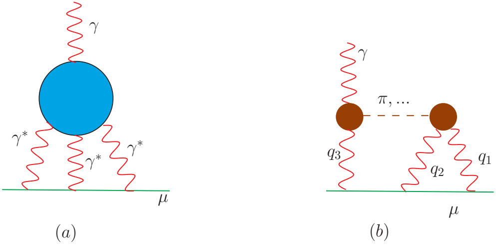
  - Anomaly leads to

$$\mathcal{F}_{\pi^0\gamma\gamma}(0,0,0) = \frac{N_c}{12\pi^2 F_\pi} \quad \Rightarrow \quad \Gamma_{\pi^0\gamma\gamma} = 7.76\ \mathrm{eV}$$

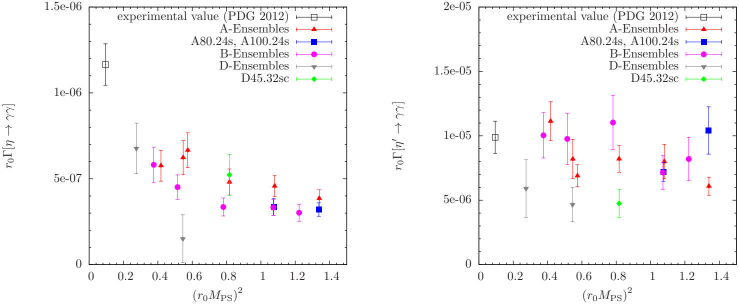
- PrimEx@Cornell (1974) measured $\Gamma_{\pi^0\gamma\gamma} = 7.92(42)$ eV
  - stringent test for existence of anomaly
  - one of the evidences for $N_c = 3$

Current motivation

- PrimEx@JLab: $\Gamma_{\pi^0\gamma\gamma} = 7.82(22)$ eV [PrimEx, PRL106, 2011]
  - Precision: 2.8% $\rightarrow$ 1.4% (projected goal)
  - Benchmark test of chiral anomaly in QCD
- may help to determine muon g-2, as it dominates the contribution to hadronic light-by-light scattering



- past lattice work on $\pi^0 \to \gamma\gamma$
  - Cohen, Lin, Dudek, Edwards [LAT 08]
  - Shintani et.al., JLQCD collaboration [LAT 09, 10]
  - Lin, Cohen [Confinement X, 2012]
  - Feng et.al. JLQCD collaboration [PRL, 2011] (*)
- $\eta, \eta' \to \gamma\gamma$ is also related to anomaly
  - work by Ottnad, Michael and Urbach, ETM collaboration



  - results are obtained by putting η-η' mixing angles and decay constants in to ChPT, more details, see Ottnad Thu, 17:10, Seminar Room G

Lattice QCD and chiral anomaly

Anomaly on the lattice

- Conventional fermion formulation, e.g. Wilson, break chiral symmetry
  - No conserved current: $\partial_\mu A_\mu - 2mP \neq 0$
  - Non-zero term yields anomaly in continuum limit [Karsten, Smit, 1981]
- Ginsparg-Wilson fermions ⇒ modified chiral symmetry on the lattice

$$\delta\bar{\psi} = i\alpha\bar{\psi}(1 - aD/2)\gamma_5, \quad \delta\psi = i\alpha\gamma_5(1 - aD/2)\psi$$

  - under chiral transformation, measurement of fermion fields produce a Jacobian [Adams 98, Hasenfratz et.al. 98, Lüscher 98, Fujikawa 98]

$$\mathcal{D}\bar{\psi}'\mathcal{D}\psi' = \mathcal{J}\mathcal{D}\bar{\psi}\mathcal{D}\psi, \quad \mathcal{J} = \exp[-i\mathrm{Tr}\ \alpha\gamma_5 D] \neq 1$$

  - if gauge configuration is sufficiently smooth ⇒ chiral anomaly

$$\frac{1}{2}\mathrm{Tr}\ \gamma_5 D = \frac{1}{32\pi^2}\epsilon_{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Lattice setup

- we are using overlap fermion to test the chiral anomaly
  - at $a \sim 0.1$ fm, gauge field is far from smooth $\Rightarrow$ chiral anomaly may not be guaranteed
- $m_u = m_d$, neglect the small isospin breaking effect at this moment
- all-to-all propagator to construct correlator + disconnected diag.
- calculation of $\pi^0 \to \gamma\gamma$ is nontrivial
  - γ is not an asympototic state of QCD
  - conventional method to extract the eigenstate fails
  - 1^{--} interpolating operator yields vector meson rather than γ
- new method is needed

Analytic continuation method

From $\pi^0 \to \gamma\gamma$ to photon vacuum polarization

- $\pi^0 \to \gamma\gamma$ has non-QCD final state

$$\Rightarrow \quad M_{\mu\nu}(p_1, p_2) = \int d^4x\, e^{ip_1x} \langle 0 | T\{J_\mu(x) J_\nu(0)\} | \pi^0(q) \rangle$$

- a simpler case: photon hadronic vacuum polarization (HVP)

$$\Rightarrow \quad \Pi_{\mu\nu}(p) = \int d^4x\, e^{ipx} \langle 0 | T\{J_\mu(x) J_\nu(0)\} | 0 \rangle$$

Analytic continuation in the HVP

- HVP function in Euclidean space-time

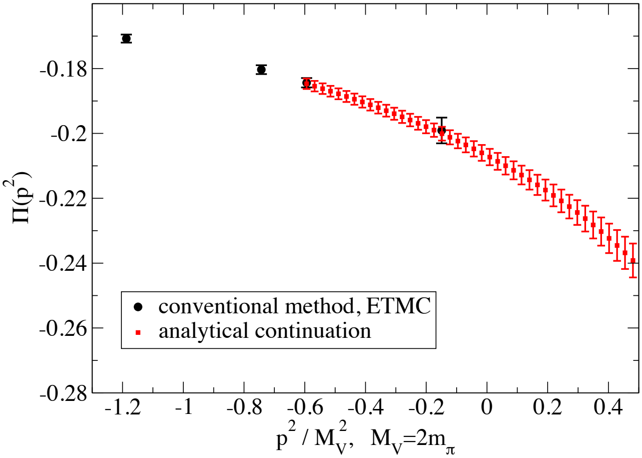
$$\Pi_{\mu\nu}(p) = \int d^4x\, e^{ipx} \langle 0|T\{J_\mu(x)J_\nu(0)\}|0\rangle$$

- what we use (proposed by Ji & Jung, 2001)

$$\int dt\, e^{\omega t} \int d^3\vec{x}\, e^{i\vec{p}\vec{x}} \langle 0|T\{J_\mu(x)J_\nu(0)\}|0\rangle$$

  - $p_0 \to -i\omega$: analytic continuation
  - ω means photon energy, input by hand, thus can be tuned continuously
  - $p^2 = \omega^2 - \vec{p}^2$, both space-like and time-like
  - worry about $e^{\omega t}$ divergent for large t?
    - ★ $\langle 0|T\{J_\mu(t)J_\nu(0)\}|0\rangle$ exponentially decreases as $e^{-E_V t}$
    - ★ an important constraint: $\omega < E_V$ or $p^2 = \omega^2 - \vec{p}^2 < M_V^2$
  - demonstration of the method: see also [XF, Hashimoto, Hotzel, Jansen, Petschlies, Renner, arXiv:1305.5878]

Results for HVP function



- analytic continuation: $p^2 = \omega^2 - \vec{p}^2$, $\vec{p}$ discrete but ω continous
- more details: poster by Karl Jansen [LAT 13]

Back to $\pi^0 \to \gamma\gamma$

Analytic continuation in $\pi^0 \to \gamma\gamma$

- observable: $M_{\mu\nu}(p_1, p_2) = \int d^4x \; e^{ip_1x} \langle 0|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle$
- analytic continuation

$$M_{\mu\nu}(p_1, p_2) = \lim_{t_{1,2}-t_\pi \to \infty} \frac{1}{\frac{\phi_{\pi,\vec{q}}}{2E_{\pi,\vec{q}}} e^{-E_{\pi,\vec{q}}(t_2-t_\pi)}} \int dt_1 \; e^{\omega(t_1-t_2)} C_{\mu\nu}(t_1, t_2, t_\pi)$$

$$C_{\mu\nu}(t_1, t_2, t_\pi) \equiv \int d^3\vec{x} \; e^{-i\vec{p}_1\cdot\vec{x}} \int d^3\vec{z} \; e^{i\vec{q}\cdot\vec{z}} \langle 0|T\{J_\mu(\vec{x}, t_1)J_\nu(\vec{y}, t_2)\pi^0(\vec{z}, t_\pi)\}|0\rangle$$

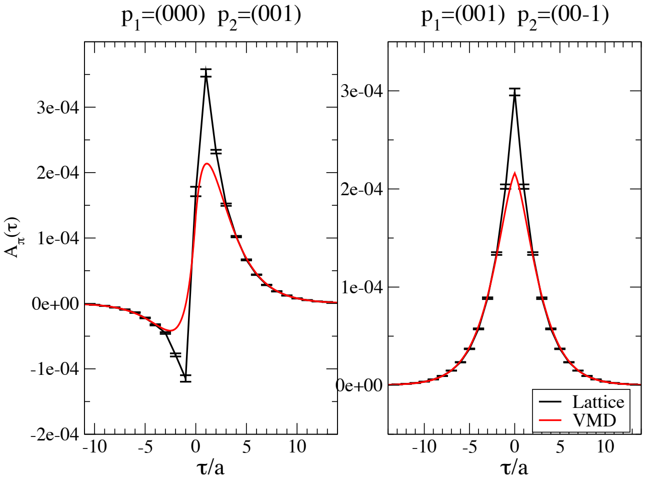
  - large $t_{1,2} - t_\pi$ limit to pick up pion
  - $e^{\omega(t_1-t_2)}$ divergent for $t_1 > t_2$? No, suppression by $C_{\mu\nu}(t_1, t_2, t_\pi)$

- we want to study the $t_1 - t_2$ dependence of $C_{\mu\nu}(t_1, t_2, t_\pi)$
- define amplitude $A_\pi(\tau)$

$$A_\pi(\tau) \equiv \lim_{t-t_\pi \to \infty} \frac{C_{\mu\nu}(t_1, t_2, t_\pi)}{e^{-E_\pi(t-t_\pi)}}, \quad \tau = t_1 - t_2, \quad t = \min\{t_1, t_2\}$$

Time dependence of $A_\pi(\tau)$

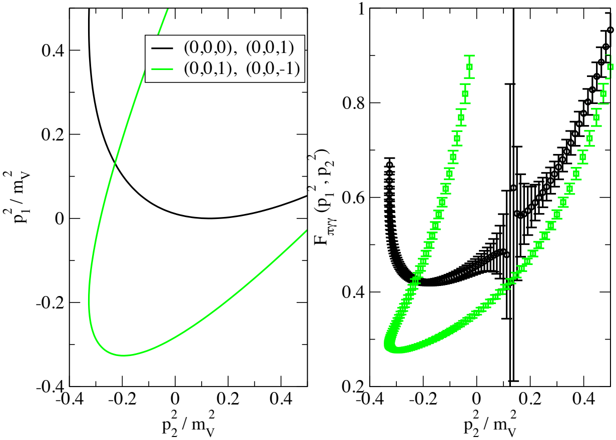
- use VMD as a guideline



- assume at large $|\tau|$, lowest states saturate

Form factor

- $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = \mathcal{M}_{\mu\nu}(p_1, p_2)/\epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta$



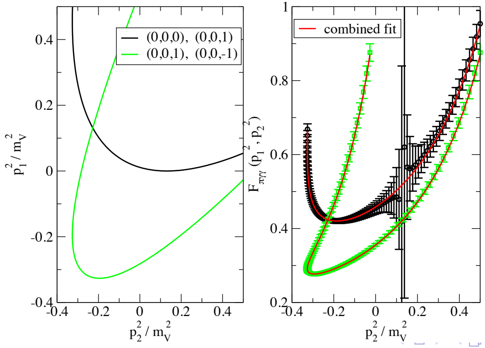
- first photon momentum $p_1 = (\omega, \vec{p}_1)$, second one $p_2 = (E_{\pi,\vec{q}} - \omega, \vec{p}_2)$
- form factor is calculated on a curve of (p_1^2, p_2^2)

Combined fit

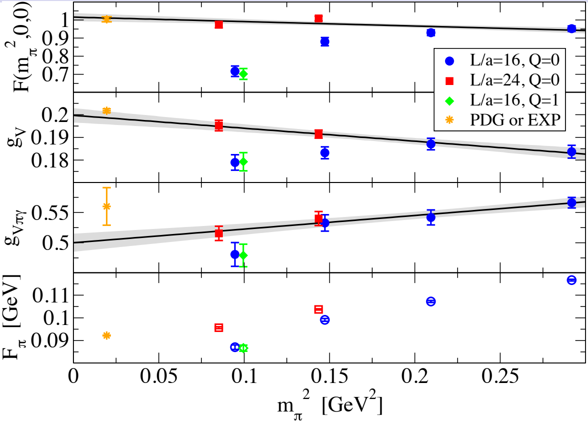
- fit ansatz $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2) +$

$$\sum_m c_m \left((p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2)\right) + \sum_{m,n} c_{m,n} (p_1^2)^m (p_2^2)^n$$

  - first term from VMD, $G_V(p^2) = \frac{M_V^2}{M_V^2 - p^2}$ is vector meson propagator
  - residual contributions are accounted for by including polynomials of $p_{1,2}^2$
  - combined fit of lattice data with parameters c_V, c_0, $c_{0,0}$, $c_{0,1} = c_{1,0}$



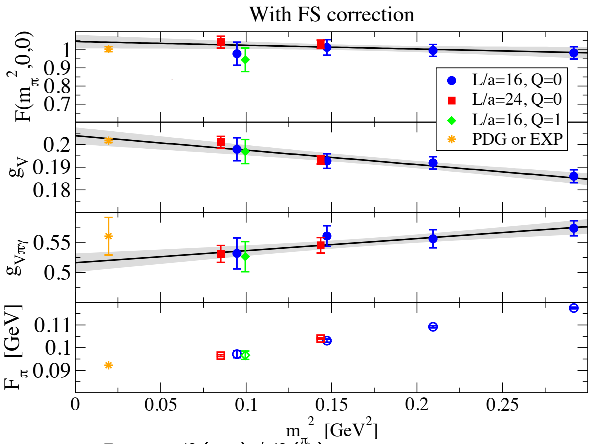
On-shell photon limit



- $F(m_\pi^2, 0, 0) \equiv \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)/\mathcal{F}^{\mathrm{ABJ}}_{\pi^0\gamma\gamma}$
- data with $m_\pi L \geq 4$: consistent with ABJ and PrimEx
- $L/a = 16$: smallest two quark mass, big FS effects
- expand the correlator into three hadronic matrix elements:

$$\langle J_\mu J_\nu \pi^0 \rangle \to \langle 0|J_\mu|V\rangle\langle V|J_\nu|\pi^0\rangle\langle\pi^0|\pi^0|0\rangle \to g_V \times g_{V\pi\gamma} \times F_\pi$$

Finite-size corrections



- FS corrections $R_{\mathcal{O}} \equiv \mathcal{O}(\infty)/\mathcal{O}(L)$
- R_{g_V}, $R_{g_{V\pi\gamma}}$ treated by adding a correction term, $e^{-m_\pi L}$, to the fit
- R_{F_π} treated using NNLO SU(3) ChPT
- FS correction to $F(m_\pi^2, 0, 0)$: $R_{F(m_\pi^2,0,0)} = R_{g_V} R_{g_{V\pi\gamma}} R_{F_\pi}$
- chiral extrapolation: only $m_\pi L > 4$ data or all data set (FS corrected)

Results

- we check possible systematic effects
  - conventional finite-size effect
  - fixing-topology effect
  - disconnected diagram contribution (few percent)
- final results yield

$$\begin{aligned} F(0,0,0) &= 1.009(22)(29) \\ F(m^2_{\pi,\mathrm{phy}},0,0) &= 1.005(20)(30) \\ \Gamma_{\pi^0\gamma\gamma} &= 7.83(31)(49)\ \mathrm{eV} \end{aligned}$$

- ABJ anomaly and PrimEx measurement

$$\begin{aligned} F(0,0,0) &= 1 \\ F(m^2_{\pi,\mathrm{phy}},0,0) &= 1.004(14) \\ \Gamma_{\pi^0\gamma\gamma} &= 7.82(22)\ \mathrm{eV} \end{aligned}$$

Conclusions

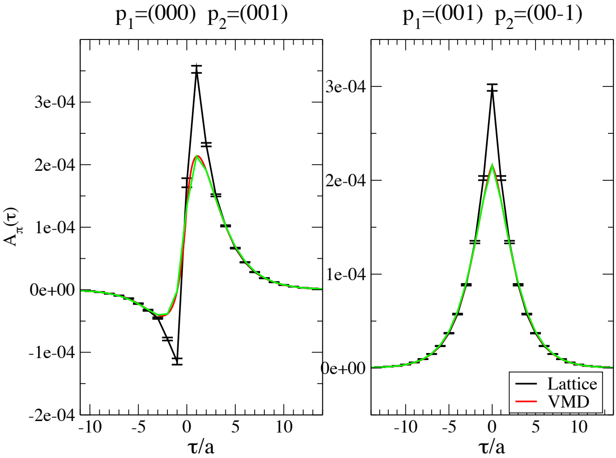
- $\pi^0 \to \gamma\gamma$ calculation is successfully carried through
  - by analytic continuation
  - using all-to-all propagators
- chiral lattice fermion works well here
  - ABJ anomaly confirmed in the chiral limit
  - although expensive, theoretically clean formulation is helpful
- worthwhile to try other fermion formulations, large lattice volumes
- isospin breaking effect need to be understood
- extend the study to new projects, where non-QCD states are involved
  - $\eta, \eta' \to \gamma\gamma$

Backup slides

Analysis of systematic effects

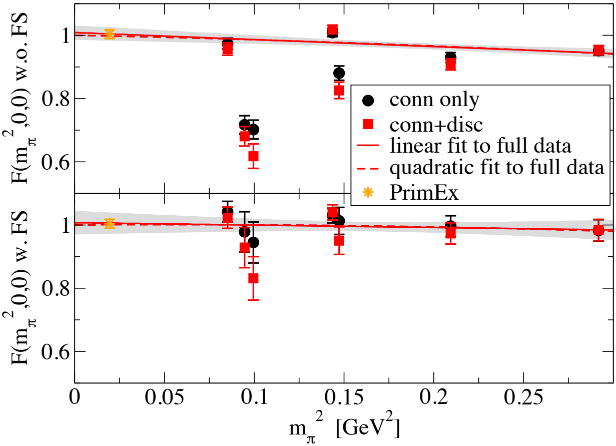
Lattice artifacts

- discrete data v.s. continuum case?



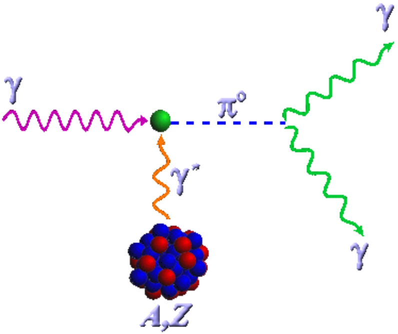
- disc. effects in VMD model: less than 5×10^{-4}, neglegiable

Disconnected-diagram effects



- all-to-all propagator: control error of disc. contribution
- although not significant, conn+disc systematically shift down
- precision level (3% for form factor): disc. diagram should be included

Primakoff effect



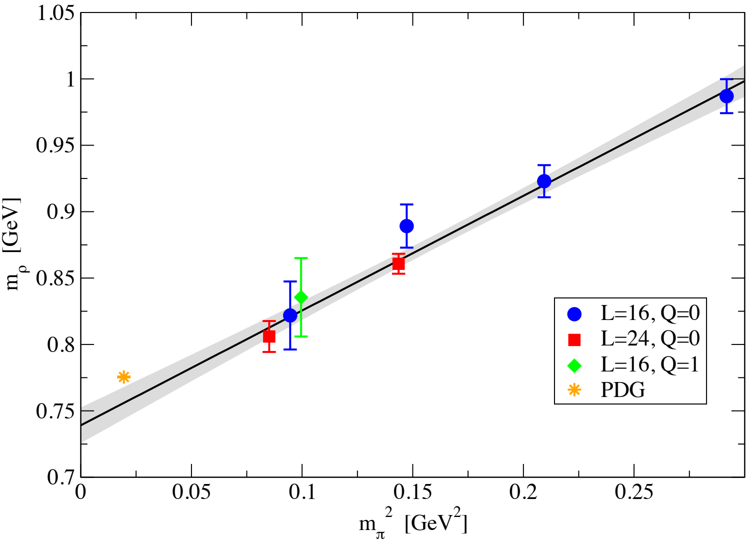
- high-energy photon interact with an atomic nucleus
- at small angles this reaction is dominated by $\gamma + \gamma^* \to \pi^0$
- γ^* is due to the Coulomb field of the nucleus

$\pi^0, \eta, \eta' \to \gamma^*\gamma^*$ contribution to LbyL scattering

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops + other subleading in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

- summary table [Jegerlehner, Nyffeler, Phys.Rept.477:1-110,2009]
  - $\pi^0, \eta, \eta' \to \gamma^*\gamma^*$ contributions are consistent with total ones
  - among three PS mesons, π^0 takes about $\sim$70% contribution
  - calulation on the $\pi^0 \to \gamma^*\gamma^*$ is a first step towards the η, η' sector

Rho mass



1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0
0.05
0.1
0.15
0.2
0.25
m_ρ [GeV]
m_π^2 [GeV^2]
L=16, Q=0
L=24, Q=0
L=16, Q=1
PDG