Scale Setting in Lattice QCD

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NIC @ DESY, Zeuthen & HU zu Berlin

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Thanks

- M. Lüscher
  ... for very useful corresponence & SU(3) YM data
- Gregorio Herdoiza
  ...for graphs and discussion
- BMW, Nathan Brown, Albert Deuzeman, Georg von Hippel, Roger Horsley, Björn Leder, Harvey Meyer, Albert Ramos, Carsten Urbach
  ... for material, data, graphs

For illustration I use mainly plots from ALPHA Collaboration: M. Bruno & S. Lottini !

What is scale setting

Simplification: ignore QED and Isospin splitting

- N_f parameters, g_0^2, , $m_\mathrm{u} = m_\mathrm{d}$, m_s ...
- Need to fix $\Phi_i,\ i = 1 \ldots N_\mathrm{f}$ dimensionless quantities to take a continuum limit and in the end to have real world QCD (e.g. up to $\mathrm{O}(\Lambda/m_\mathrm{c})$)
- hadronic input

$$m_1^\mathrm{h} = m_\pi\,,\ m_2^\mathrm{h} = m_\mathrm{K}\,,\ (m_3^\mathrm{h} = m_\mathrm{D}\,,\ m_4^\mathrm{h} = m_\mathrm{B})$$

+ a scale

- Length scale

$$Q\,,\ [Q] = -1$$

$$\Phi_i = Q\, m_i^\mathrm{h} \text{ dimensionless}$$

- Natural from the physics point of view:

$$Q = m_\mathrm{proton}^{-1}$$

or

$$Q = f^{-1}$$

(low energy constant of SU(2) chiral Lagrangian)

Why is it so important?

- check for cutoff effects
- decouple scale setting from the rest of the calculation
  e.g. compute $B \to \pi$ form factor ...
- fix quark masses from the beginning
- There have been changes in predictions due to better determinations of the scale

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  - Example 1: $f_{\mathrm{D_s}}$, $N_{\mathrm{f}} = 2+1$

HPQCD, 2007	$f_{\mathrm{D_s}} = 241(3)\,\mathrm{MeV}$	scale from $r_1 = 0.321(5)$
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  - Example 2: $\Lambda_{\overline{\mathrm{MS}}}$, $N_\mathrm{f} = 2$

ALPHA, 2004	$\Lambda_{\overline{\mathrm{MS}}} = 245(16)(16)\,\mathrm{MeV}$	scale from QCDSF ($r_0 \approx 0.5\,\mathrm{fm}$)
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It is important to have a precise and correct scale determination.

Criteria for a good Q

- Precision
  - statistical
  - systematic

  related!
- Quark mass dependence
  - weak dependence is better: it is easier to fix the scale at $(m_{\mathrm{PS}}, m_{\mathrm{K}}) = (m_{\pi}^{\mathrm{phys}}, m_{\mathrm{K}}^{\mathrm{phys}})$
- N_{f} dependence
  - first we need to define this...

The candidates, an overview

Hadronic, experimentally measurable

- m_Ω
  - stable, best S/N ratio of non - pseudo scalars
- $f_\pi, \ f_\mathrm{K}$
  - pseudo scalar: very good S/N
  - but knowledge of V_ud, (or V_us) assumed

Constructed (only we can say what their values are precisely)

- r_0, r_1 (static force)
  - still linked to phenomenology (a bit dirty)
  - solvable S/N problem (in practise)
- t_0, w_0 (gradient flow)
  - artificial
  - very precise

Once r_0, r_1, w_0, w_1 are determined from hadronic quantities, they are just as good (or better).

Being established

- v_0 (my name)
  - from Isospin 1 hadronic vacuum polarisation
  - Harvey Meyer

Vector correlator and scale determination in lattice QCD
Mon, 14:40, Seminar Room C (RW4)

Talks at this conference

Harvey Meyer

Vector correlator and scale determination in lattice QCD
Mon, 14:40, Seminar Room C (RW4)

Mattia Bruno

On the Nf-dependence of gluonic observables
Mon, 15:00, Seminar Room C – Parallels 1C

Nathan Brown

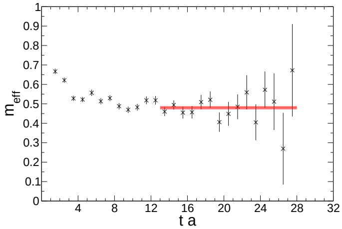
Symanzik flow on HISQ ensembles
Mon, 18:10, Seminar Room G (HS III)

Roger Horsley

SU(3) flavour symmetry breaking and charmed states
Thu, 15:20, Seminar Room G – Parallels 7G

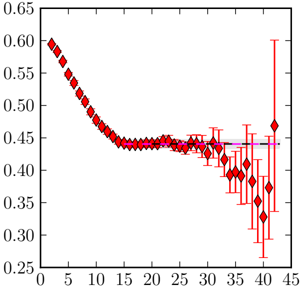
Reasonable signal-to-noise ratio

BMW: $2 + 1$ “HEX”
$a \approx 0.054$ fm, $m_\pi \approx 260$MeV, $L \approx 1.7$fm



aparently very uncorrelated data

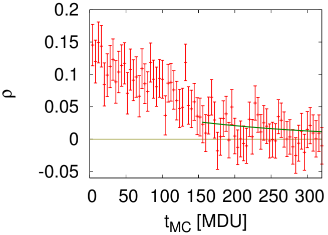
Mainz group, CLS configurations, $N_\mathrm{f} = 2$
$a \approx 0.045$ fm, $m_\pi \approx 340$MeV, $L \approx 2.2$fm



- good signal-to-noise
- small τ_int
  quite weak coupling
  to slow modes

  autocorrelation function of f_π
  $a = 0.045$ fm, large statistics $\rightarrow$

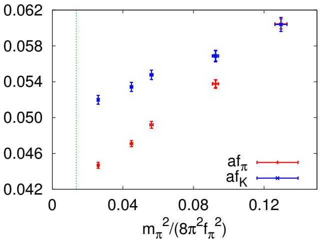
  tail contributes $\approx$ 20 MDU to τ_int



- significant dependence on either
  light quark masses: f_π
  or strange quark mass: f_K

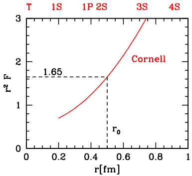
  We do have ChPT for the
  asymptotic behavior

  light quark mass dependence for $N_\mathrm{f} = 2$:



[Figures: S. Lottini, ALPHA]

$$\begin{aligned} r_0^2 F(r_0) &= 1.65 \\ r_1^2 F(r_1) &= 1 \end{aligned}$$



[R.S., 1993]

Issues

- $V(r)$ from large T behavior of Wilson loop (variationally improved) $F(r_\mathrm{I})$ by an interpolation
  - instead: force often through fits to $V(r)$ over a larger range of r

  $\rightarrow$ what excatly one determines depends on the assumed shape (fitfunction)
- Signal-to-noise problem $\frac{\text{signal}}{\text{noise}}(\text{Wloop}) \sim \exp(-\frac{e_1}{a} + \text{finite}T\ )$
  - grows towards the continuum limit
  - e_1 can be reduced by a change of static quark action

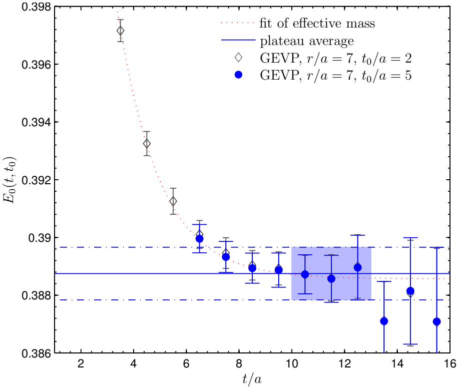
    [A. Hasenfratz & F. Knechtli, 2003; M. Della Morte, A. Shindler & R.S. 2005]
  - r_1 was motivated by an improvement of the signal [MILC, 2000]
    but: shorter distance, larger discretisation effects
    amazing: MILC lattices: r_1/a down to $r_1/a = 2$

A modern computation of r_0

On CLS ensembles, $N_\mathrm{f} = 2$ [M. Donnellan, F. Knechtli, B. Leder and R. S., ALPHA Collaboration, 2010]

Basis of (smeared) parallel transporters, GEVP [M. Lüscher & U. Wolff, 1991]

$$\text{fixed } r \approx r_0 : \qquad C(t)\,\psi_\alpha = \lambda_\alpha(t,t_0)\,C(t_0)\,\psi_\alpha\,, \quad \alpha = 0\ldots M-1$$

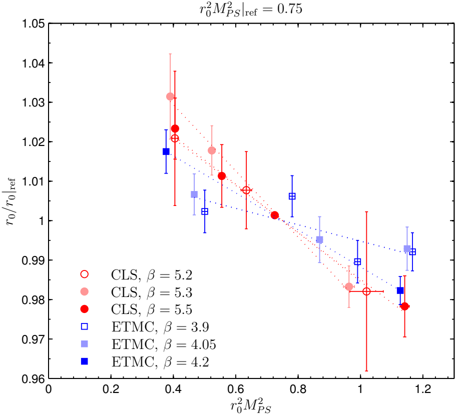


$$E_\alpha(t,t_0) \equiv \ln\left(\lambda_\alpha(t,t_0)/\lambda_\alpha(t+a,t_0)\right) = E_\alpha + \beta_\alpha \mathrm{e}^{-(E_M - E_\alpha)\,t}$$

- Not entirely trivial
- But here solved
- Important: understanding of the corrections!

[ALPHA Collaboration, Blossier et al., 2009]

Quark mass dependence of r_0



[F. Knechtli & B. Leder, 2011, update BL 2013]

$N_{\rm f} = 2$

Data of
ETMC and ALPHA

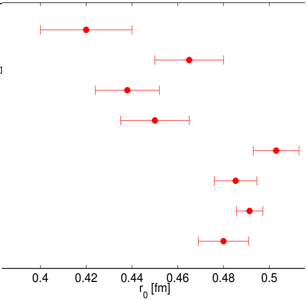
2013 update

$$\frac{r_0(500\ \mathrm{MeV})}{r_0(0)} - 1 \approx -9\%$$

- Weak dependence on quark mass.
- ETMC quark mass dependence seems even weaker at larger lattice spacings.

r_0: some recent results

$N_{\rm f}$	r_0[fm]	Reference
2	$0.420(20)$	ETMC 08 from f_π
2	$0.465(15)$	ETMC 09 from $m_{\rm nucleon}$
2	$0.438(14)$	ETMC 09 from $f_{\rm K}$
2	$0.450(15)$	ETMC from f_π Lat13
2	$0.503(10)$	ALPHA from $f_{\rm K}$
2	$0.485(9)$	ALPHA from f_π Lat13
2	$0.491(6)$	ALPHA from $f_{\rm K}$ Lat13
3	$0.480(11)$	RBC 12



Gradient flow

[M. Lüscher, 2010]

- New renormalised observables depending on $t \neq x_0$, $t > 0$
  - Flow equation $B_\mu(x,t)$ with $B_\mu(x,0) = A_\mu$ = quantum field

$$\frac{\mathrm{d}}{\mathrm{d}t} B_\mu(x,t) = \dot{B}_\mu(x,t) = D_\nu G_{\nu\mu}(x,t) \sim -\frac{\delta S_{YM}[B]}{\delta B_\mu}$$

  - smoothing over a radius of $\sqrt{8t}$, lowest order of PT:

$$B_\mu(x,t) \quad = \quad \int \mathrm{d}^4 y \; (4\pi t)^{-2} \mathrm{e}^{-(x-y)^2/(4t)} \; A_\mu(y) + \mathrm{O}(g_0^2)$$

- Discretisation of the flow equation
  - "Wilson flow"

$$\dot{V}(t) = -\frac{\partial}{\partial V} S_{\mathrm{plaq}}(V)\, V(t)\,, \qquad V(0) = U$$

  - "Symanzik flow"

$$\dot{V}(t) = -\frac{\partial}{\partial V} S_{\mathrm{TLSymanzik}}(V)\, V(t)\,, \qquad V(0) = U$$

5-d formulation

[M. Lüscher and P. Weisz, 2010, M. Lüscher 2012]

- $t \geq 0$ as additional coordinate $[t]$=length^2
- 4-d field theory at $t = 0$ boundary
- For precise understanding of renormalisation and improvement

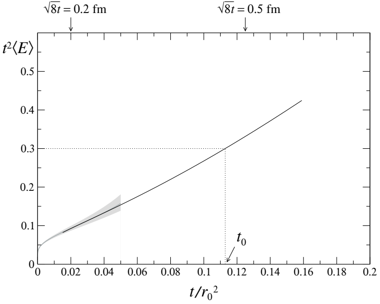
Definition

[M. Lüscher, 1006.4518]

$$E(x,t) = -\frac{1}{4}\,\mathrm{tr}\, G_{\mu\nu}(x,t) G_{\mu\nu}(x,t)$$

Observation in numerical data:

$$t^2\langle E(t)\rangle \approx k\, t \quad \text{for } t = \mathrm{O}(r_0^2)$$
$$t_0^2\langle E(t_0)\rangle = 0.3 \quad \text{defines } t_0$$



Discretisations in use

- $E =$ plaquette
- $G_{\mu\nu}$ = clover (“ symmetric ”)

Definition

[BMW, 1203.4469]

Observation in numerical data:

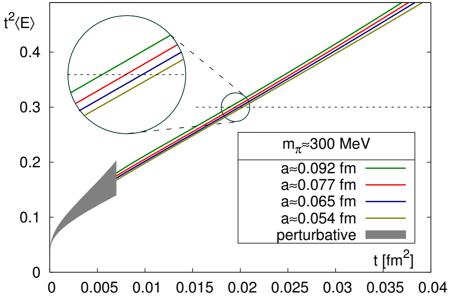
different discretisations:

$\approx$ parallel lines for $\langle t^2 E(t)\rangle$

Different a at fixed m_Ω

$\approx$ parallel lines for $\langle t^2 E(t)\rangle$

$$\left[t\frac{\mathrm{d}}{\mathrm{d}t}\langle t^2 E(t)\rangle\right]_{t=w_0^2} = 0.3$$



Scale through m_Ω

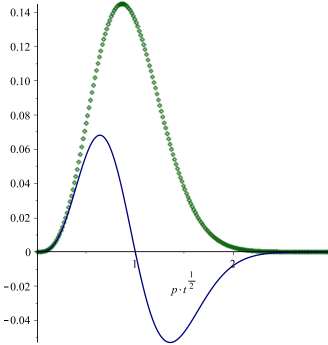
1203.4469: $t^2\langle E(t)\rangle$: "scales between a and $\sqrt{t}$"
w_0: "scales of order w_0 only'

however: leading order of PT:

$$\langle E(t)\rangle \sim g^2 \int \mathrm{d}^4p\mathrm{e}^{-2tp^2}(p^2\delta_{\mu\nu} - p_\mu p_\nu)D(p)_{\mu\nu}$$

$$t^2\langle E(t)\rangle \sim g^2 \int_0^\infty p^3\mathrm{e}^{-2tp^2}\mathrm{d}p$$

$$t\partial_t[t^2\langle E(t)\rangle] \sim g^2 \int_0^\infty p^3\,(1 - t\,p^2)\mathrm{e}^{-2tp^2}\mathrm{d}p$$



Integrands in LO PT vs. $p\sqrt{t}$

Flow quantities: Improvement

The possible interest is in a^2 improvement

For that one needs

- improvement of 4-d action
- improvement of the flow equation (no g_0^2 dependence) ($t > 0$ bulk of 5d theory)
- improvement of flow observables, E (no g_0^2 dependence) ($t > 0$ bulk of 5d theory)
- boundary improvement terms such as

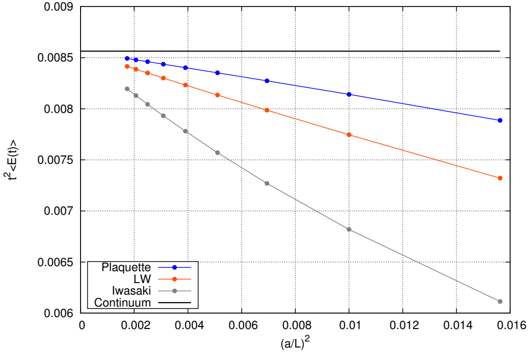
$$a^6 \sum_x \partial_t \, \mathrm{tr}\, G_{\mu\nu} G_{\mu\nu}|_{t=0}$$

at the $t = 0$ boundary

Incomplete improvement may be worse than no improvement

Incomplete improvement can be worse than no improvement

- a^2 improvement of 4-d action, i.e. tree-level Symanzik action
- but no improvement
  of flow
  of flow observables, E



- at LO of PT

$$\langle t^2 E(t) \rangle_{t=0.3L^2}$$

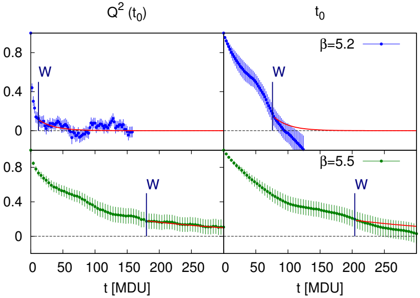
[A. Ramos, 2013]

- SF boundary conditions, $T = L$
- not a SF feature (independent of T)

Here: Plaquette action is better than tree level improved (LW).

t_0, w_0 statistical precision

- Very small and scaling variance.
- Strong coupling to slow modes of HMC. ($\rightarrow$ perfect detector of slow modes)



$N_\mathrm{f} = 2$ [M. Bruno, ALPHA, 2013]

$a = 0.075\,\mathrm{fm}$

$m_\pi = 280\,\mathrm{MeV}$

$a = 0.049\,\mathrm{fm}$

$m_\pi = 340\,\mathrm{MeV}$

- All in all excellent precision.
- No relevant differences between t_0 and w_0.

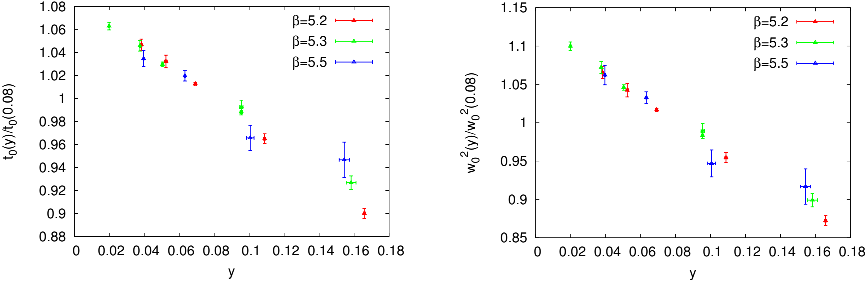
- ALPHA (2): $\tau_\mathrm{int} \approx 50 \ldots 130$ MDU
  $a = 0.049 \ldots 0.075$ fm
- BMW (2+1, 2HEX): $\tau_\mathrm{int} \approx 70$ MDU
  $a \geq 0.054$ fm
- MILC (2+1+1, HISQ): $\tau_\mathrm{int} \approx 20 \ldots 80$ MDU
  $a = 0.06 \ldots 0.15$ fm

$t_0,\ w_0$: quark mass dependence

$N_\mathrm{f} = 2$ [ALPHA, 2013]

$$y = t_0(m)\, m_\pi^2 \sim m_\mathrm{quark}$$

$$\frac{t_0(y)}{t_0(0.8)} \qquad\qquad \frac{w_0^2(y)}{w_0^2(0.8)}$$



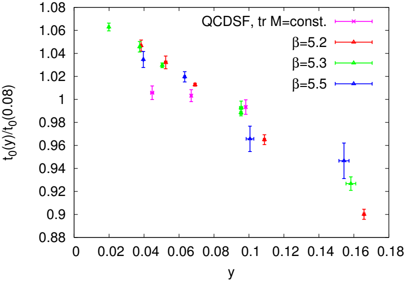
- very linear
- but we have no theory for asymptotic behavior (ChPT)
- comment: no (mass-dependent) discretisation effects visible (e.g. due to missing b_g-terms) with precise data

$t_0,\ w_0$: quark mass dependence

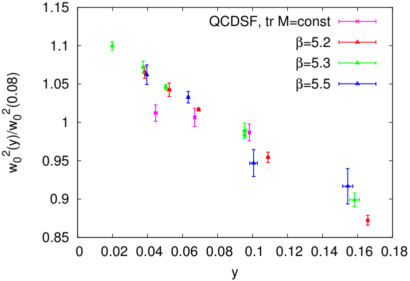
$N_\mathrm{f} = 2$ [ALPHA, 2013] & $N_\mathrm{f} = 2+1$, QCDSF, tr m_quark=const.

$$y = t_0(m)\, m_\pi^2 \sim m_\mathrm{quark}$$

$t_0(y)/t_0(0.8)$



$w_0^2(y)/w_0^2(0.8)$



- tr m_quark=const.: rather flat behavior
- in agreement with $t_0(y) = t_0(y_\mathrm{sym}) + \mathrm{O}((y - y_\mathrm{sym})^2$,
  where "sym" means $m_1 = m_2 = m_3$
- see: Roger Horsley
  SU(3) flavour symmetry breaking and charmed states
  Thu, 15:20, Seminar Room G – Parallels 7G

N_{f}-dependence

- What do we mean by an N_{f}-dependence?
- Different N_{f}: different theory, different coupling ...
- Related by decoupling, up to a change of the coupling $\leftrightarrow$ a change of the scale (effective theory)
  Need to consider dimensionless low energy quantities

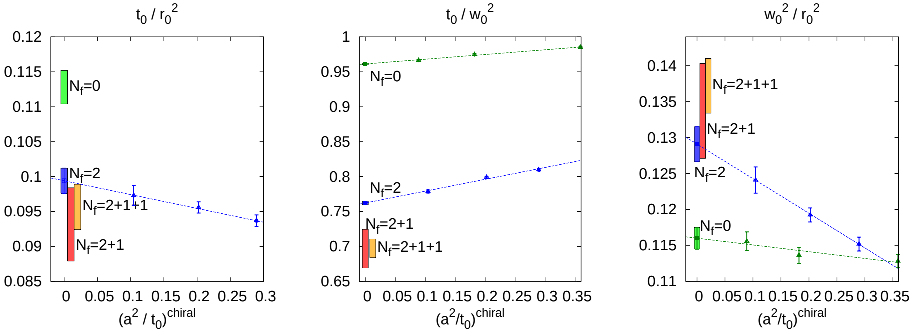
  $R_{x,y}$, here $R_{t_0,r_0^2} = t_0/r_0^2$ etc.

  $m_{N_{\mathrm{f}}} \gg m_{N_{\mathrm{f}}-1} > \ldots$ $\qquad k = 2$ in PT

  $$R_{x,y}^{(N_{\mathrm{f}})}(m_1, \ldots m_{N_{\mathrm{f}}}) = R_{x,y}^{(N_{\mathrm{f}}-1)}(m_1, \ldots, m_{N_{\mathrm{f}}} - 1) + \mathrm{O}((m_{N_{\mathrm{f}}})^{-k}), \quad k \geq 1$$

- N_{f}-dependence and
  quark mass dependence are related

N_f dependence



- Significant differences in gluonic scales between $N_\mathrm{f} = 0$ and 2
- which reduce when you increase the quark mass in $N_\mathrm{f} > 2$
- Small differences for $N_\mathrm{f} > 2$

$N_\mathrm{f} = 0$ $\langle E(t) \rangle$ data by M. Lüscher

- $N_f = 2 + 1$

Quantity [fm]	ref.
$r_0 = 0.480(10)(4)$	RBC, '12
$\sqrt{t_0} = 0.1465(21)(13)$	BMW, '12
$w_0 = 0.1755(18)(4)$	BMW, '12

- $N_f = 2 + 1 + 1$

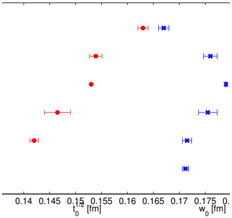
Quantity	ref.
$r_0/r_1 = 1.508$	HotQCD, '11
$\sqrt{t_0}/w_0 = 0.835(8)$	HPQCD, '13
$r_1/w_0 = 1.790(25)$	HPQCD, '13

$\sqrt{t_0}$, w_0 in physical units

Uncertainties dominated by

- the physical observables used to set the scale
- the extrapolations to the physical point

N_f	$\sqrt{t_0}$ [fm]	w_0 [fm]	Reference
0	$0.1630(10)$	$0.1670(10)$	$r_0 = 0.49\,\mathrm{fm}$ Lat13*
2	$0.1539(12)$	$0.1760(13)$	ALPHA from f_K Lat13
3	0.1530	0.1790	QCDSF from Octet Lat13
3	$0.1465(25)$	$0.1755(18)$	BMW 12
4	$0.1420(8)$	$0.1715(9)$	HPQCD 13
4		$0.1712(6)$	MILC from f_π Lat13



* $N_\mathrm{f} = 0$: $E(t)$ data of M. Lüscher [1006.4518]

- some VERY small uncertainties are being cited
  f_π to 0.3 % including FSE, effects of strange tuning, charm tuning
- Some numbers are preliminary; should be discussed at Lat14

t_0, w_0^2: a comparison

Quark mass dependence (roughly extracted): consider $t_0(m_\pi), w_0(m_\pi)$

N_f	$\frac{t_0(500\mathrm{MeV})}{t_0(0)} - 1$	$\frac{w_0^2(500\mathrm{MeV})}{w_0^2(0)} - 1$	Reference
2	-12%	-20%	ALPHA @ Lat13
2+1		-18%	BMW 2012
2+1+1		-13%	MILC Lat13

Lattice spacing dependence (roughly extracted) $t_0(a)$, $w_0(a)$

clover discretisation of $G_{\mu\nu}$

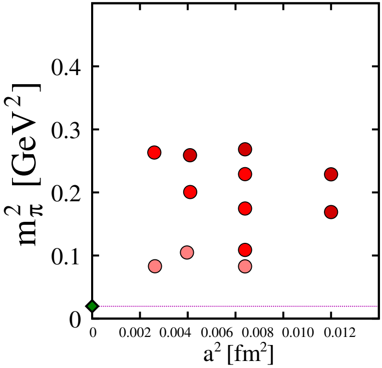
N_f	$\frac{t_0(0.1\,\mathrm{fm})}{t_0(0)} - 1$	$\frac{w_0^2(0.1\,\mathrm{fm})}{w_0^2(0)} - 1$	ref.scale	Reference
0	-1%	-3%	r_0	Lat13 [data: 1006.4518]
2	-8%	-19%	r_0	ALPHA @ Lat13
2+1	-19%	≈ 0	m_Ω	BMW 2012
2+1+1		≈ 0	f_π	HPQCD 13
2+1+1		≈ 0	f_π	MILC Lat13

This depends on many parameters:
$4-d$ action, boundary terms, flow equation, discretisation of $G_{\mu\nu}$, ref.scale
Not surprisingly: see various numbers.

A Problem: r_0 for $N_\mathrm{f} = 2$

There were differences: r_0, $N_\mathrm{f} = 2$ ETMC

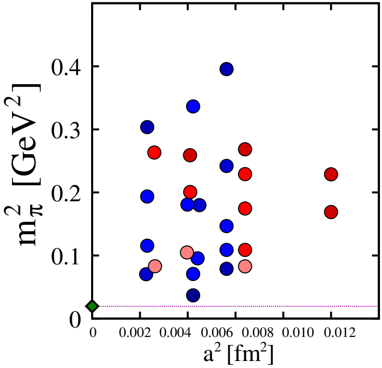
The ensembles:



light color: $3.5 \lesssim m_\pi L < 4$

There were differences: r_0, $N_\mathrm{f} = 2$ ETMC vs. ALPHA

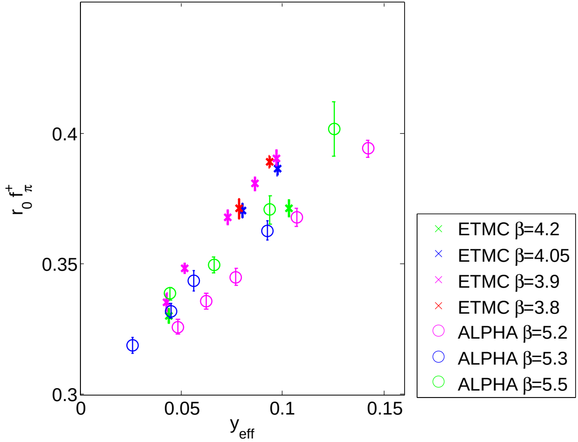
The ensembles:



light color: $3.5 \lesssim m_\pi L < 4$

raw data for ETMC[0911.5061] and ALPHA [Lat13]

plot against $y = (m_\pi^+ / f_\pi^+)^2 / (8\pi^2)$

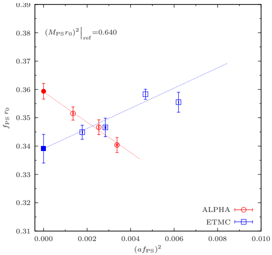


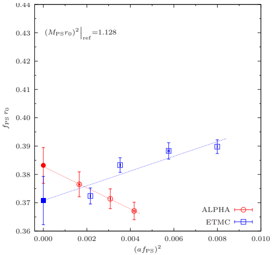
ETMC:
$\frac{\mathrm{d}}{\mathrm{d}a}[f_\pi\, r_0] > 0$

ALPHA:
$\frac{\mathrm{d}}{\mathrm{d}a}[f_\pi\, r_0] < 0$

...but they cross [G. Herdoiza, S. Lottini, ALPHA + ETMC, Lat13]

interpolated to fixed reference quark masses, e.g. by $m_\pi^2\, r_0^2$ =fixed





- Is this data ready for continuum extrapolations?

...but they cross

[G. Herdoiza, S. Lottini, ALPHA + ETMC, Lat13]

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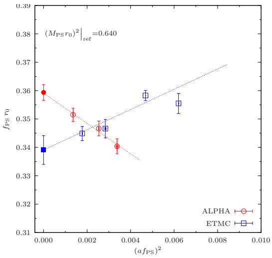
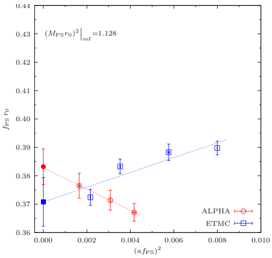
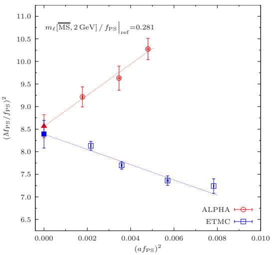
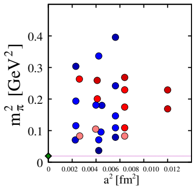






- Is this data ready for continuum extrapolations?
- There are better quantities;
  on the right r_0 does not enter
  and it is a 500MeV m_π
- ETMC: small differences when r_0 does not enter.
- But: maybe differences are there for small masses.



The discretisations

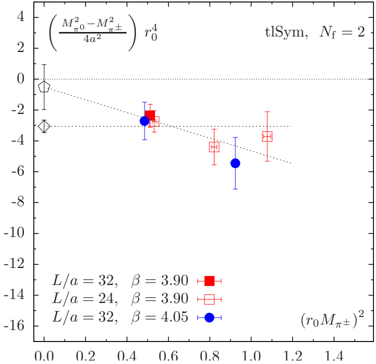
To twist or not to twist or how to twist

$\mathrm{O}(a)$ improved fermions	twisted mass fermions
NP determination of c_{sw}	tune to maximal twist
NP determination of $c_{\mathrm{A}}, Z_{\mathrm{A}}$ ...	isospin breaking

The splitting

$$\Delta m_\pi^2 \quad = \quad (m_\pi^0)^2 - (m_\pi^+)^2 \,,$$

was summarised in 1303.3516 [ETMC, G. Herdoiza, K. Jansen, C. Michael, K. Ottnad & C. Urbach].



$$a^2 \Delta m_\pi^2 \quad \approx \quad -(A + B\,(m_\pi^+ r_0)^2)\,(a/r_0)^4$$
$$A = 2\,, \quad B = 16$$

For $a \approx 0.1$ fm

$$\Delta m_\pi^2 \approx -(m_\pi^{\rm phys})^2$$

but there are considerable uncertainties

This motivates to use power counting $m_{\rm quark} \sim a^2\Lambda_{\rm QCD}^4$ in WChPT: [O. Bär, 2010]

$$f_\pi^+ = f\left(1-(y_+\log(y_+)+y_0\log(y_0)) + \underbrace{\alpha}_{\rm LEC}(y_+ + y_0) + {\rm O}(y^2)\right)$$

$$y_c = \frac{(m_\pi^c)^2}{16\pi^2 f^2} = \frac{(m_\pi^c)^2}{16\pi^2 f_\pi^{+2}} + {\rm O}(m_{\rm quark}^4)$$

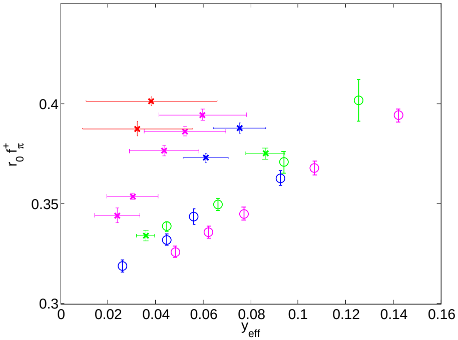
This suggests to use as a variable (e.g. in plots)

$$y_{\rm eff}: \qquad y_{\rm eff}\log(y_{\rm eff}) \equiv \frac{y_+\log(y_+)+y_0\log(y_0)}{2}$$

with a ChPT prediction

$$f_\pi^+ = f\,(1-2\,y_{\rm eff}\log(y_{\rm eff}) + \alpha\, y_{\rm eff} + {\rm tiny} + {\rm O}(y^2))$$

Plot $r_0\, f_\pi^+$ against $y_{\rm eff}$.



"error bars":
half of difference $y - y_{\rm eff}$

applied also NLO FS correction

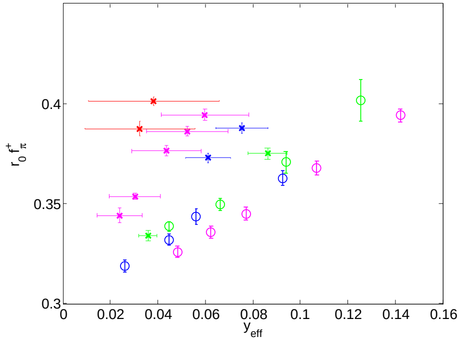
[G. Colangelo & U. Wenger, 2010; O. Bär, 2010]

(not so important)

- NLO WChPT with $m_{\rm quark} \sim a^2\Lambda_{\rm QCD}^4$ does not seem to capture the effect !
- Large other (additional) a^2 effects would be needed.

Decay constant

Plot $r_0 f_\pi^+$ against $y_{\rm eff}$.



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- NLO WChPT with $m_{\rm quark} \sim a^2 \Lambda_{\rm QCD}^4$ does not seem to capture the effect !
- Large other (additional) a^2 effects would be needed.
- or the isospin splitting is not as large as usually thought
  the determination of m_π^0 is a difficult computation

Will we solve the problem? how?

- The isospin-splitting seems not to be as big as often said (measurement of m_π^0 is very difficult)
- Motivation
  for $N_\mathrm{f} = 2$ is fading away ... will there be work on the problem?
- Replace
  r_0 by t_0, w_0 to gain statistical precision!!
- Ultimately:
  reduce the lattice spacing
- Wishfull
  thinking: a computation differing ONLY by the twist angle.

Conclusions & predictions

- Intermediate and relative scale setting
  - Wilson flow observables are very precise
  - Gluonic observables, no inversions, no valence quark mass dependences
  - We have t_0, w_0

    – there is little obvious difference

    – there are probably other useful quantities

    – cutoff effects are non-universal

    a distinction between t_0 and w_0 is not clear

    – The N_f dependence of w_0 is weaker than the one of t_0
  - The use of r_0, r_1 will slowly fade away in favor of t_0, w_0 (and maybe others)

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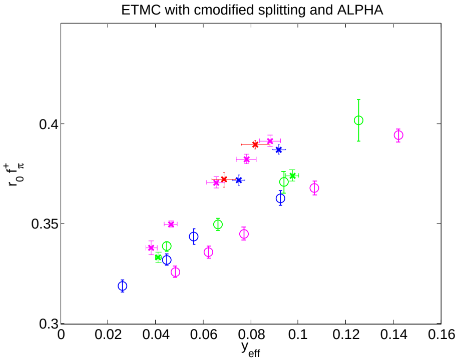
- Hadronic scales
  - f_π seems the best but is dependent on V_ud
  - m_Ω appears fine
  - the isovector vector correlator is being developed
  - m_P would be nice $\leftarrow$ solve signal/noise problem

$r_0\, f_\pi^+$ against $y_{\rm eff}$

Assume reduced splitting by a factor 4: $A, B \to A/4, B/4$



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This looks more reasonable.